On Graviton non-Gaussianities in the Effective Field Theory of Inflation

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Outline

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1. Primer for Bootstrapping Graviton Bispectra
2. Effective Field Theory of Inflation (EFToI)
3. Bootstrapping Graviton Bispectra in the EFToI
4. Conclusions and Future Directions

Wavefunction of the universe

- Late-time observables in the CMB and LSS can be traced back to correlators at the boundary from the end of inflation η_0.
- In cosmology, we compute the wavefunction of the universe

$$\Psi[\bar{\phi};\eta_0] = \int_{\phi(-\infty)=\Omega_{BD}}^{\bar{\phi}=\phi(\eta_0)} \mathcal{D}\phi\mathcal{D}\pi\, e^{i\int d^4x\, [\phi'\pi - \mathcal{H}(\phi,\pi)]} \,. \qquad (1)$$

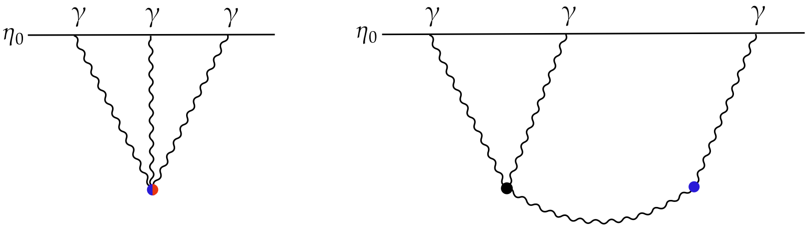
- For the spin-2 graviton γ, we can parameterise Ψ at η_0 as

$$\Psi[\gamma;\eta_0] = \exp\left[-\sum_{n=2}^{\infty}\frac{1}{n!}\int\left(\prod_a^n\sum_{h_i=\pm}\gamma_{\mathbf{k}_a}^{h_i}\, d^3\mathbf{k}_a\, \psi_n^{h_1,\ldots,h_n}\right)\right]. \qquad (2)$$

Wavefunction of the universe (cont.)

- We do a saddle point approximation and compute ψ_n at tree-level, i.e. contact and exchange Feynman diagrams.
- Wavefunction coefficients ψ_n are related to correlators B_n, e.g. from the CMB and LSS, we aim to measure B_3 given by

$$B_3 = -\frac{2\,\mathrm{Re}\,\psi_3}{\prod_{a=1}^{3} 2\,\mathrm{Re}\,\psi_2(k_a)} \quad . \tag{3}$$



Graviton Bispectra

- We write a three-point wavefunction coefficient ψ_3 as

$$\psi_3^{h_1,h_2,h_3}(\{k\},\{\mathbf{k}\}) = \sum_{\text{contractions}} [e^{h_1}(\mathbf{k}_1)e^{h_2}(\mathbf{k}_2)e^{h_3}(\mathbf{k}_3)\mathbf{k}_1^{\alpha_1}\mathbf{k}_2^{\alpha_2}\mathbf{k}_3^{\alpha_3}] \times \psi_3^{\text{trimmed}}(\{k\}) \quad , \tag{4}$$

where the total number of spatial momenta $\alpha = \alpha_1 + \alpha_2 + \alpha_3$.

- We contract momenta with one of these two tensor structures

$$e^{h_1}_{i_1 i_2} e^{h_2}_{i_3 i_4} e^{h_3}_{i_5 i_6} \text{ (even)} \quad \text{or} \quad \epsilon_{i_1 i_2 i_3} e^{h_1}_{i_4 i_5} e^{h_2}_{i_6 i_7} e^{h_3}_{i_8 i_9} \text{ (odd)} \quad . \tag{5}$$

- For parity-even $\alpha \in 2\mathbb{Z} \leq 6$ and for parity-odd $\alpha \in 2\mathbb{Z}+1 \leq 7$.

Graviton Bispectra from Inflation

Symmetries observed in nature:

- Translations
- Rotations
- Scale invariance

dS w/ boosts

Full SO(4,1) isometries

- 2 parity-even shapes
- 0 parity-odd shapes

(Maldacena & Pimentel 2011)

dS w/o boosts (incl. EFToI, Solid Inflation)

ISO(3) with dilations

- ∞ parity-even shapes
- 3 parity-odd shapes

(Cabass, Pajer, Stefanyszyn, Supeł 2021)

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Building blocks for the EFToI

- For the case of graviton interactions we set $\zeta = 0$ in the usual ADM formalism with the following line element

$$ds^2 = -dt^2 + a^2(t)(e^\gamma)_{ij}dx^i dx^j \quad . \qquad (6)$$

- In this gauge the most general action we can write is

$$S = \int d^4x\sqrt{-g}\, F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t) \quad . \qquad (7)$$

- Geometric identities and field redefinitions simplify (7) to

$$S = S_{EH} + M_{\rm pl}^2 \int d^4x\sqrt{-g}\left[\boldsymbol{\delta K^{ij}\mathcal{O}_{(0)}\delta K_{ij} + \mathcal{O}(\delta K^3)}\right] \quad . \qquad (8)$$

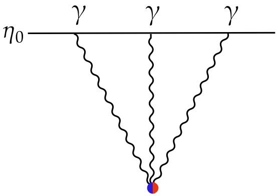
First time it has been shown that the action up to cubic order can be constructed out of the extrinsic curvature only. Potential repercussions for scalars?

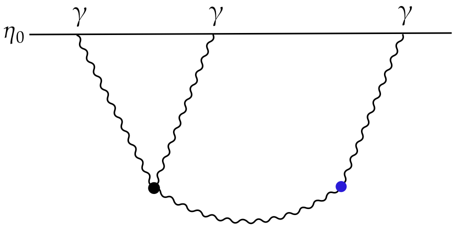
Type-I and Type-II

- δK^i_i vanishes to all orders in γ, hence the most general action that can contribute to graviton bispectra at tree-level is

$$S = S_{EH} + M_{\rm pl}^2 \int d^4x\sqrt{-g}\,\Big[\underbrace{\delta K^{ij}\mathcal{O}_{(0)}\delta K_{ij}}_{\text{Type-I}} + \underbrace{\mathcal{O}(\delta K^3)}_{\text{Type-II}}\Big] \quad . \qquad (9)$$

- For Type-I operators, contact and single-exchange diagrams' contributions are comparable; required for consistency relations.
- For Type-II operators only contact diagrams contribute.





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Type-II Graviton Bispectra

- Type-II$\sim \mathcal{O}(\delta K^3)$, so we only need δK_{ij} to linear order in γ_{ij}

$$\delta K_{ij} \propto \gamma'_{ij} + \mathcal{O}(\gamma^2) \propto e^h_{ij}(\mathbf{k}) k^2 e^{ik\eta} \quad . \tag{10}$$

$\therefore$ Type-II bispectra contain an overall factor of $(k_1 k_2 k_3)^2$.

- We use spinor-helicity variables to encapsulate tensor structures; the wavefunction coefficient's scalar part is dealt with separately.

$$\underbrace{\psi_3^{h_1,h_2,h_3}}_{=[k^3](\text{scale invariance})} = \underbrace{\mathrm{SH}^{h_1,h_2,h_3}}_{=[k^0]} \times \underbrace{h_\alpha}_{=[k^\alpha]} \times \underbrace{\psi_3^{\text{trimmed}}}_{=[k^{3-\alpha}]} \quad , \tag{11}$$

- We write an initial ansatz for ψ_3^{trimmed} to be of the form

$$\psi_3^{\text{trimmed}} = (k_1 k_2 k_3)^2 f_{-3-\alpha}(k_1, k_2, k_3) \quad . \tag{12}$$

Manifestly Local Test (MLT)

- For massless gravitons, we have the Manifestly Local Test (MLT)

$$\frac{\partial}{\partial k_a} \psi_3^{\text{trimmed}} \Big|_{k_a=0} = 0 \,, \qquad \forall\, a = 1, 2, 3 \,. \tag{13}$$

- Assumption of a Bunch-Davies vacuum and enough derivatives in our EFToI operators restricts our ψ_3^{trimmed} ansatz to

$$\psi_3^{\text{trimmed}} = \underbrace{\frac{\overbrace{(k_1 k_2 k_3)^2}^{\delta K_{ij} \propto \gamma'_{ij}}}{(k_1 + k_2 + k_3)^p}}_{\text{BD vacuum}} \underbrace{\text{Poly}_{p-3-\alpha}(k_1, k_2, k_3)}_{\text{polynomial symmetric in } k_1, k_2, k_3} \,. \tag{14}$$

Pajer [2020]

- ψ_3^{trimmed} trivially satisfies the MLT.

Bootstrapping Type-II Graviton Bispectra

- Next we construct the full $\psi_3^{h_1,h_2,h_3}$ by multiplying the tensor structure encapsulated in $\mathrm{SH}^{h_1,h_2,h_3}$ with a solution to the MLT, and then use standard techniques to convert these to bispectra

$$B_{3,\,\alpha=0}^{+++} = \frac{\mathrm{SH}^{+++}}{(k_1 k_2 k_3) k_T^p} \mathrm{Poly}_{p-3}(k_1, k_2, k_3) \quad , \tag{15}$$

$$B_{3,\,\alpha=6}^{+++} = \frac{\mathrm{SH}^{+++}}{(k_1 k_2 k_3) k_T^p} I_1^2 I_2^2 I_3^2 \mathrm{Poly}_{p-9}(k_1, k_2, k_3) \quad , \tag{16}$$

$$B_{3,\,\alpha=6}^{++-} = \frac{\mathrm{SH}^{++-}}{(k_1 k_2 k_3) k_T^p} I_1^2 I_2^2 k_T^2 \mathrm{Poly}_{p-9}(k_1, k_2, k_3) \quad , \tag{17}$$

where $k_T = k_1 + k_2 + k_3$ and $I_a \equiv k_T - 2k_a$.

Outline

Conclusions and Future Directions

- Derived **all** tree-level graviton bispectra in the EFToI.
- Showed for the **first time** that they can be constructed out of the extrinsic curvature only.
- Classified all graviton bispectra for a large class of single-field inflationary models.
- Used a combination of bulk and bootstrap tools.

Future work involves:

- Reproducing these results in a fully “bootstrap” way using consistency relations, i.e. soft theorems to sub-leading order.
- Construct mixed **and especially scalar** correlators, by developing bootstrap tools to construct these correlators.
- Constrain bispectra by demanding consistency of higher-point functions such as the trispectrum.