Voronoi Volume Function

A new probe of cosmology & galaxy evolution

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Cosmology From Home
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(in collaboration with Shadab Alam)

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Outline

- ✦ Introduction
  - Development of the Cosmic Web (non-Gaussian spatial distribution of matter)

- ✦ Information beyond 2-pt statistics
  - Voronoi tessellation
  - Voronoi volume function (VVF): definition and analytical expectations
  - Simulations: VVF dependence on halo mass, large-scale clustering, redshift, nature of dark matter, RSD, substructure

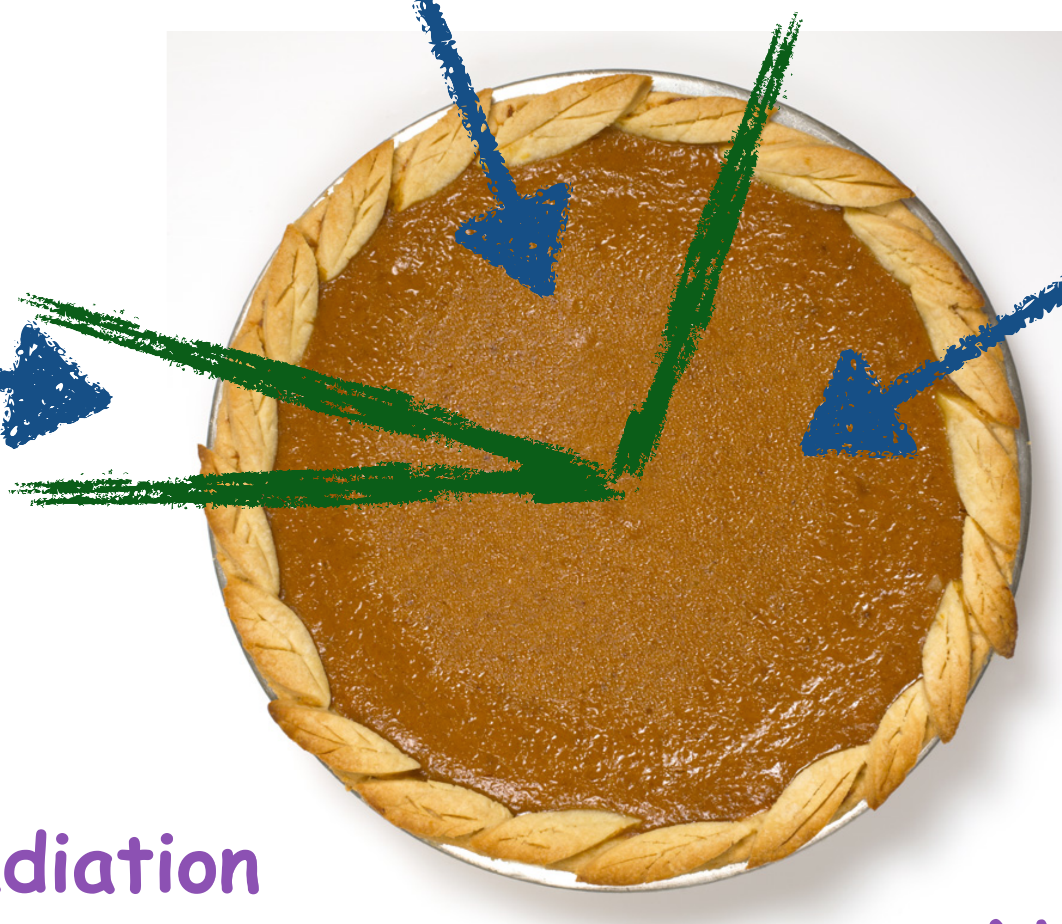
- ✦ Preliminary comparisons with observations

Standard Model of Cosmology

Matter
(cold, dark)
Ω_c

Energy
(dark, constant)
Ω_Λ

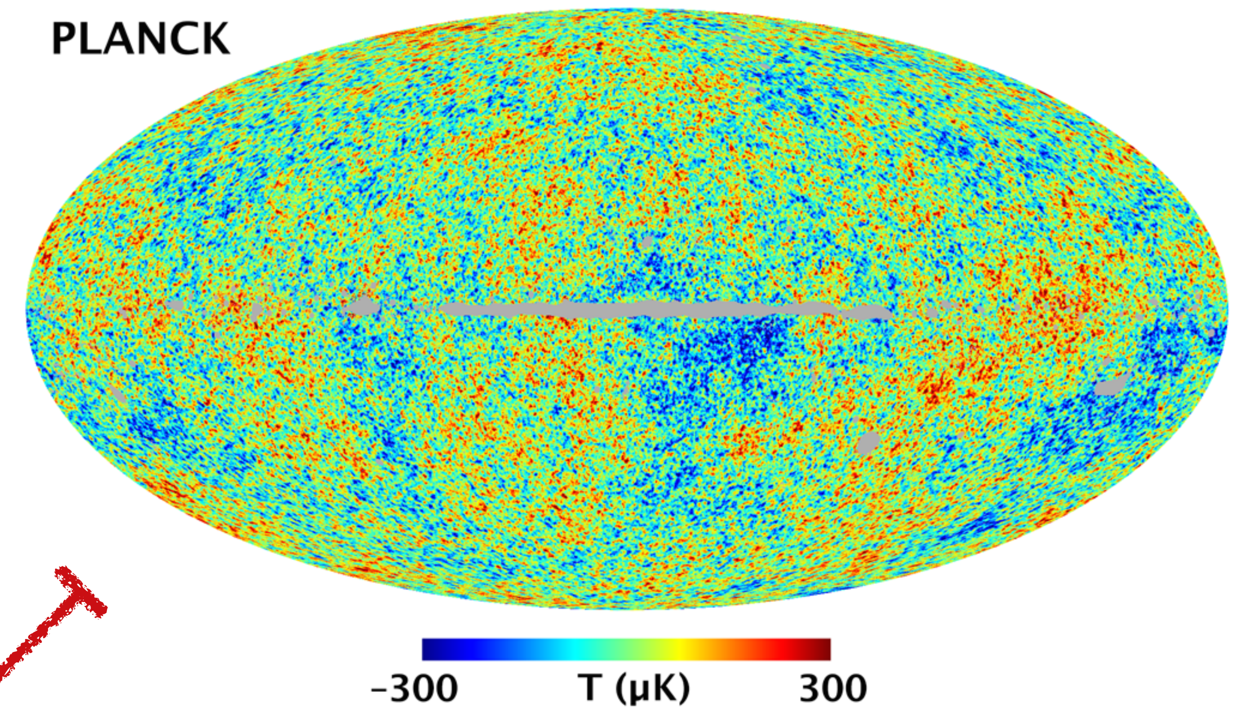
Baryons
Ω_b

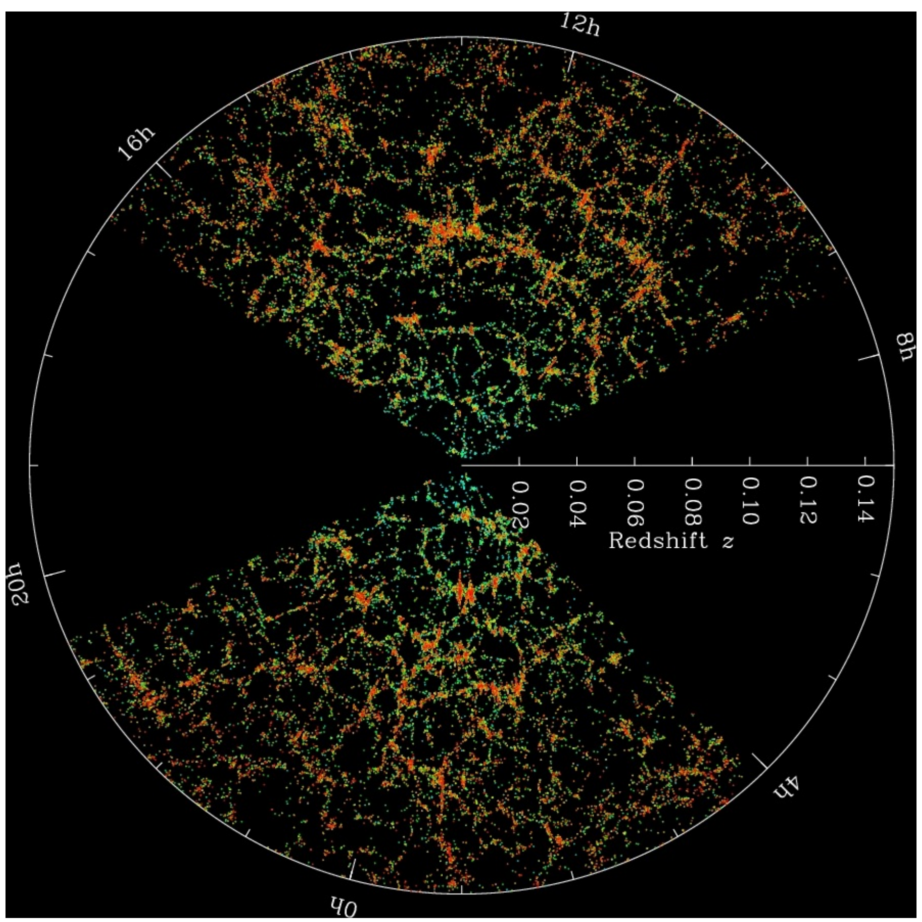


Garnish with radiation
Ω_r

Neutrinos to taste
Ω_ν

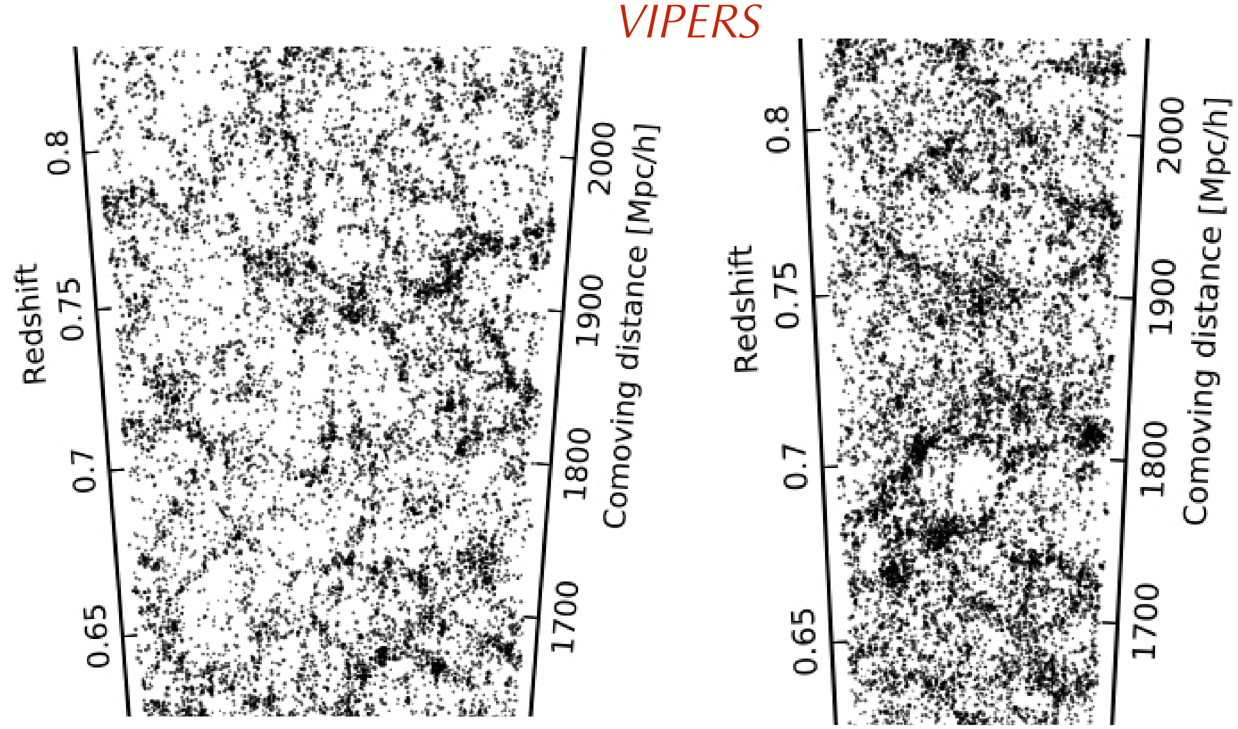
Growth of Structure





Sloan Digital Sky Survey

VIPERS

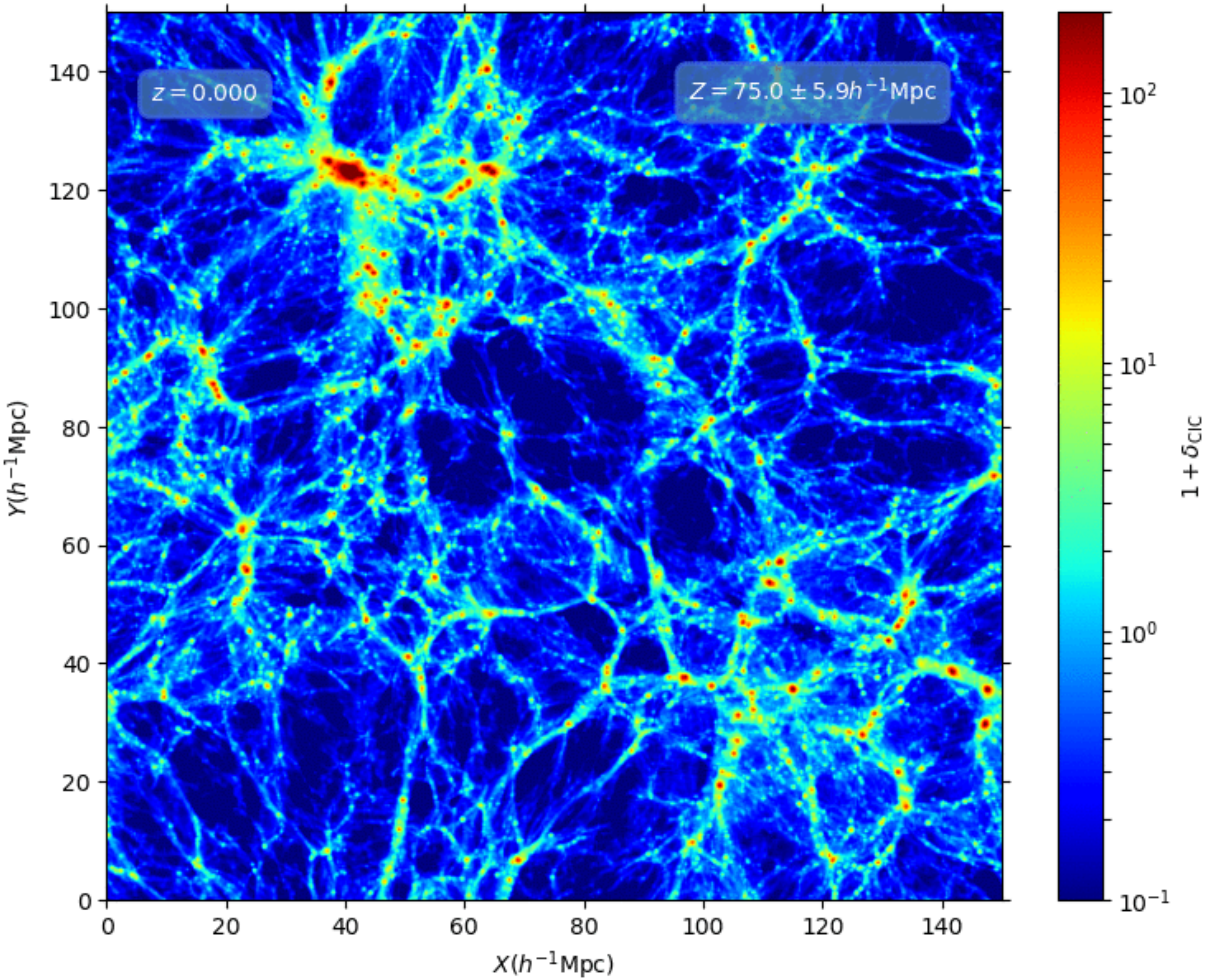


Growth of Structure

Standard ΛCDM cosmology.

Collisionless cold dark matter.

Newtonian self-gravity.



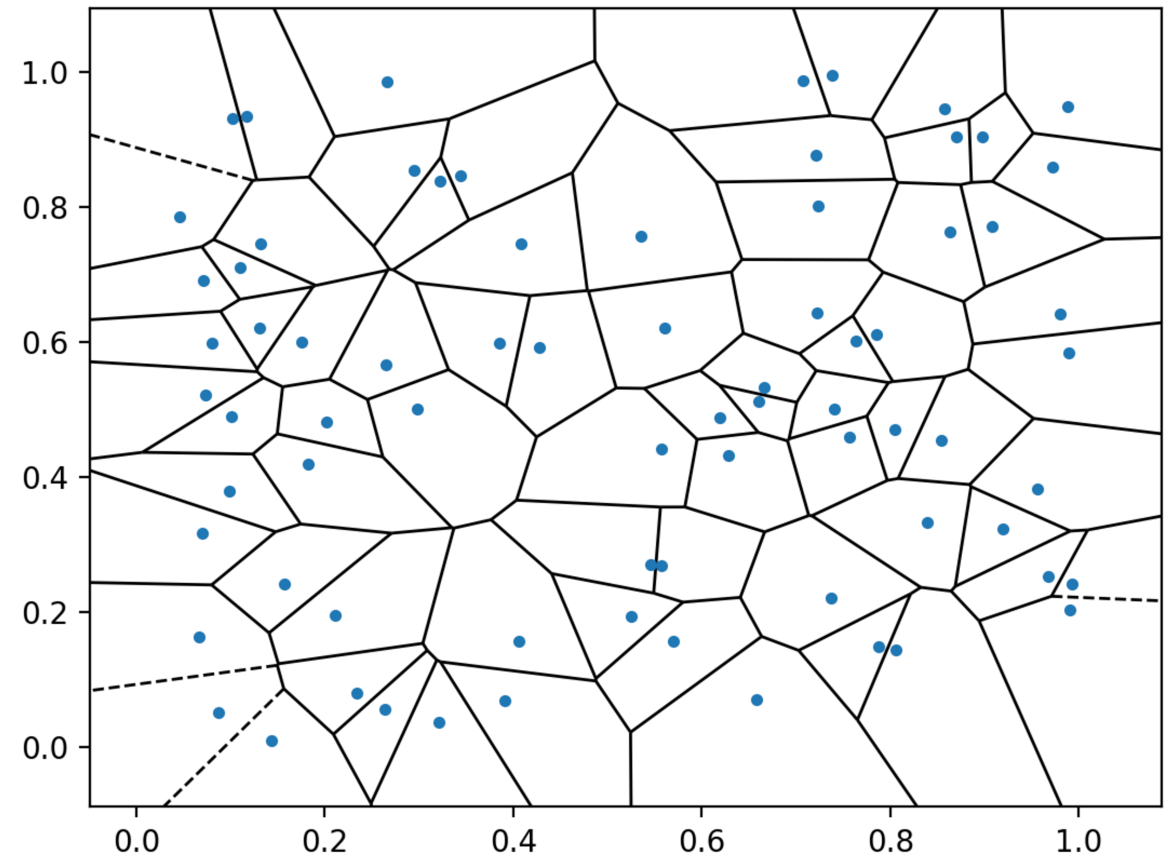
Voronoi tessellation

Given $N_{\rm trc}$ tracers at positions $\{\mathbf{x}_t\}$ with $1 \leq t \leq N_{\rm trc}$, the Voronoi tessellation is a partition of space into $N_{\rm trc}$ cells $\{\mathscr{C}_t\}$ such that, for a given tracer t, $\mathscr{C}_t$ is the set of points closer to t than to any other t'.

— Used in various fields such as meteorology, epidemiology, geophysics, computational fluid dynamics (e.g. AREPO) etc.

— Several applications in cosmology too. E.g., cosmic web classification, void identification, etc.

This talk:
Volume function of Voronoi cells of 3d clustered tracers



SciPy implementation of Voronoi tessellation of uniformly distributed 2d tracers.

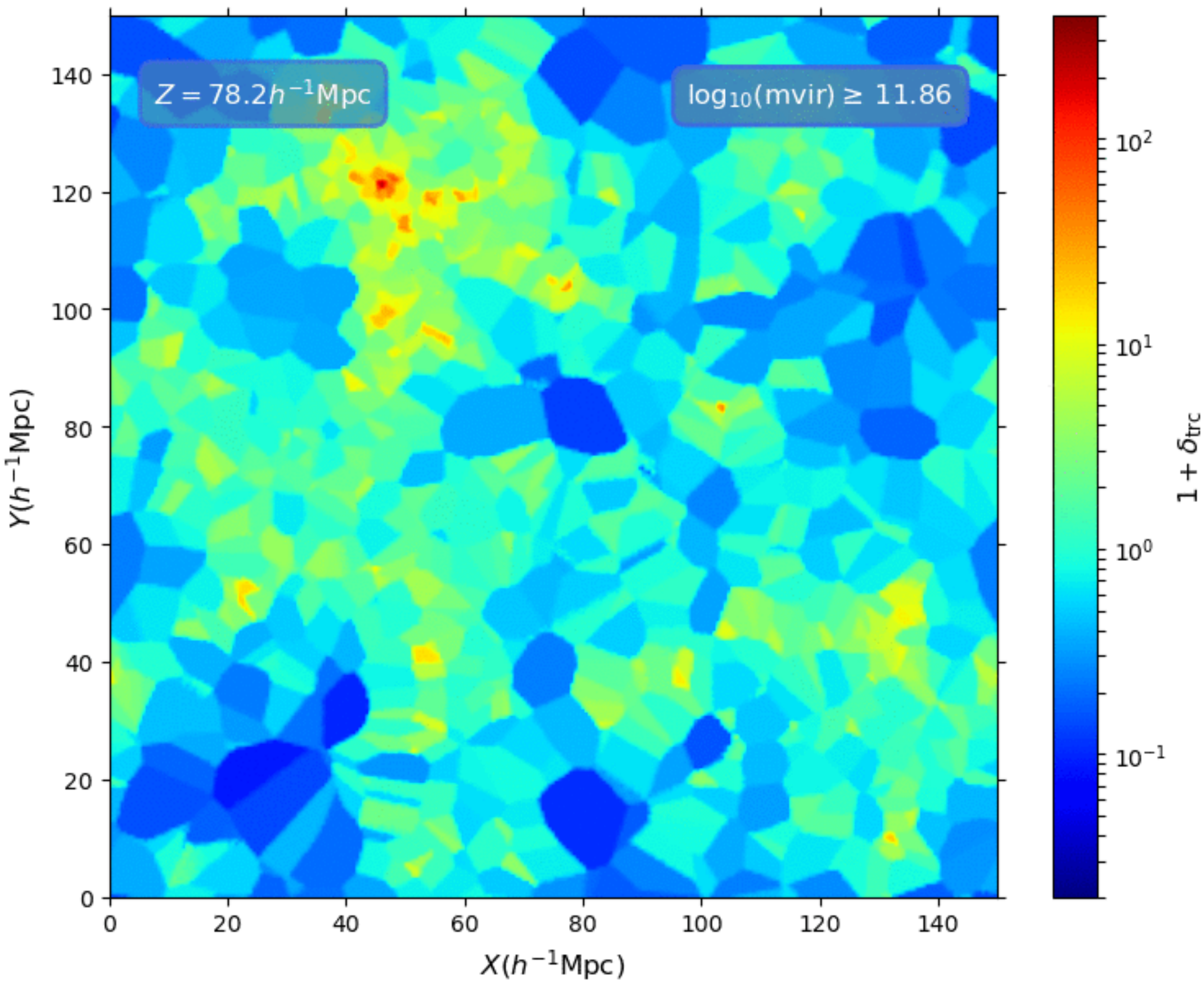
Voronoi tessellation

simulation:
WMAP7 ΛCDM

tracers:
mass-thresholded haloes at $z = 0$

tessellation:
Monte Carlo algorithm

colour:
$1 + \delta_{\rm trc} = (n_{\rm trc} V)^{-1}$



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Voronoi volume function

Definitions

If $V(t)$ is volume of cell $\mathscr{C}_t$ containing tracer t then

$$\langle V \rangle = \frac{1}{N_{\rm trc}} \sum_t V(t) = \frac{V_{\rm tot}}{N_{\rm trc}} = n_{\rm trc}^{-1}$$

Define

$$y \equiv V/\langle V \rangle = n_{\rm trc} V$$

We will denote the probability distribution $p(y)$ as the Voronoi volume function (VVF).

Clearly $\langle y \rangle = \int {\rm d}y \, p(y) \, y = 1.$

Voronoi volume function

Uniformly distributed (Poisson) tracers

For **uniformly distributed (Poisson)** tracers, $\langle y^2 \rangle$ is known analytically *[Gilbert 1962]*

$$\langle y^2 \rangle_{\rm Poisson} = \frac{8\pi^2}{3} \int_0^\infty \mathrm{d}z\, z^2 \int_{-1}^{1} \mathrm{d}\mu \frac{1}{v(z,\mu)^2} \simeq 1.179$$

where $v(z,\mu) = \frac{\pi}{3}\left[2z^3 + 3\mu z(z^2+1) - (3\mu^2 z^2 + 1)z + 3(1-\mu z)T + 2T^{3/2}\right]$
with $T = \left| z^2 + 1 - 2\mu z \right|$.

Although $p(y)$ is not known analytically, accurate fitting functions exist, e.g.:

$$p_{\rm Poisson}(y) = \frac{c\, b^{a/c}}{\Gamma(b/c)}\, y^{a-1} \exp(-b y^c)$$

with $a = 4.8065$, $b = 4.06342$, $c = 1.16391$ *[Tanemura 2003]*

Voronoi volume function

Clustered tracers

For **clustered** tracers, generalising *[Gilbert 1962]* we can write

$$\langle y^2 \rangle = \frac{8\pi^2}{3} \int_0^\infty \mathrm{d}z \, z^2 \int_{-1}^{1} \mathrm{d}\mu \frac{1}{v(z,\mu)^2} \, n_{\rm trc}^2 \int_0^\infty \mathrm{d}V_U \, V_U \exp\left(W_0(n_{\rm trc}, V_U) \right)$$

where $\exp\left(W_0(n_{\rm trc}, V)\right)$ is the *void probability function* for the volume V *[White 1979]*:

$$W_0(n_{\rm trc}, V) = \sum_{k=1}^{\infty} \frac{\left(-n_{\rm trc} V\right)^k}{k!} \, \bar{\xi}_k(V) \equiv \left(-n_{\rm trc} V\right) \chi(n_{\rm trc}, V)$$

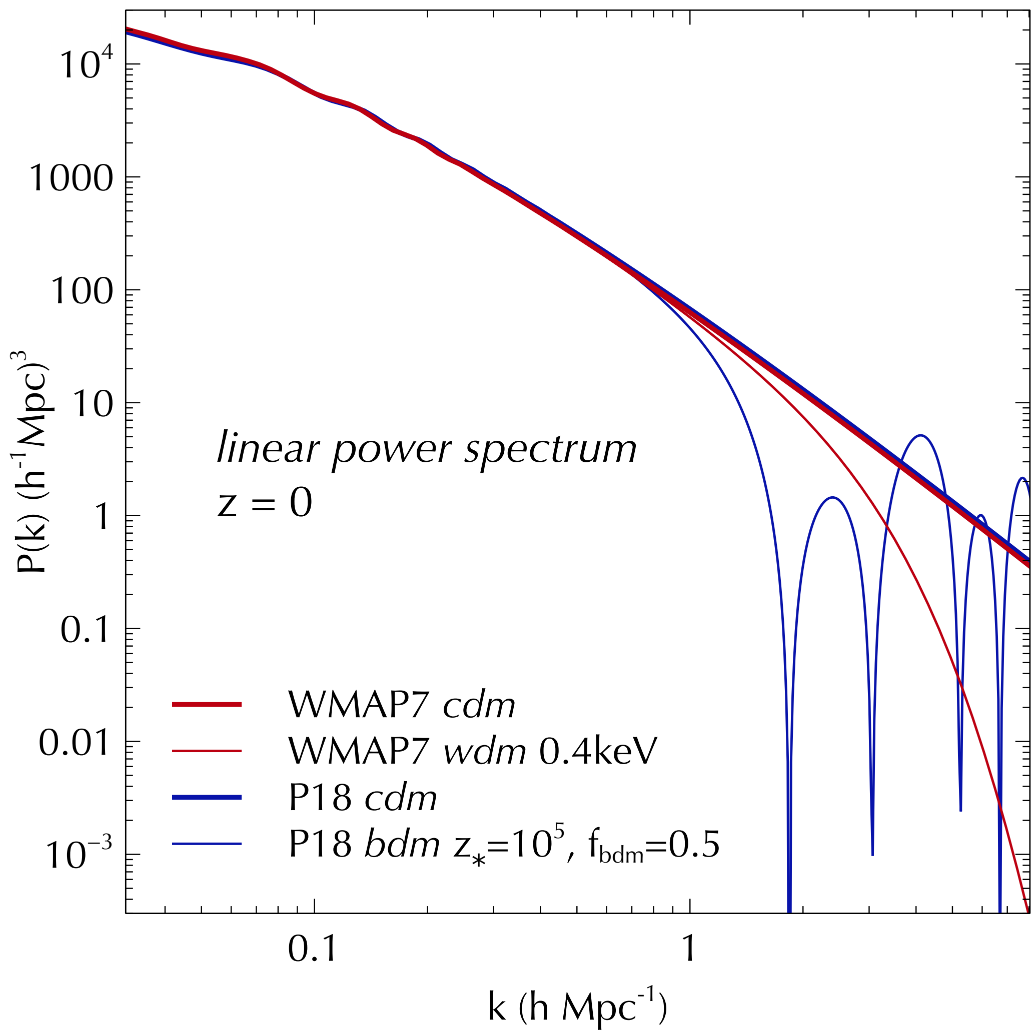
where $\bar{\xi}_k(V)$ is the connected k-point correlation function averaged over V.

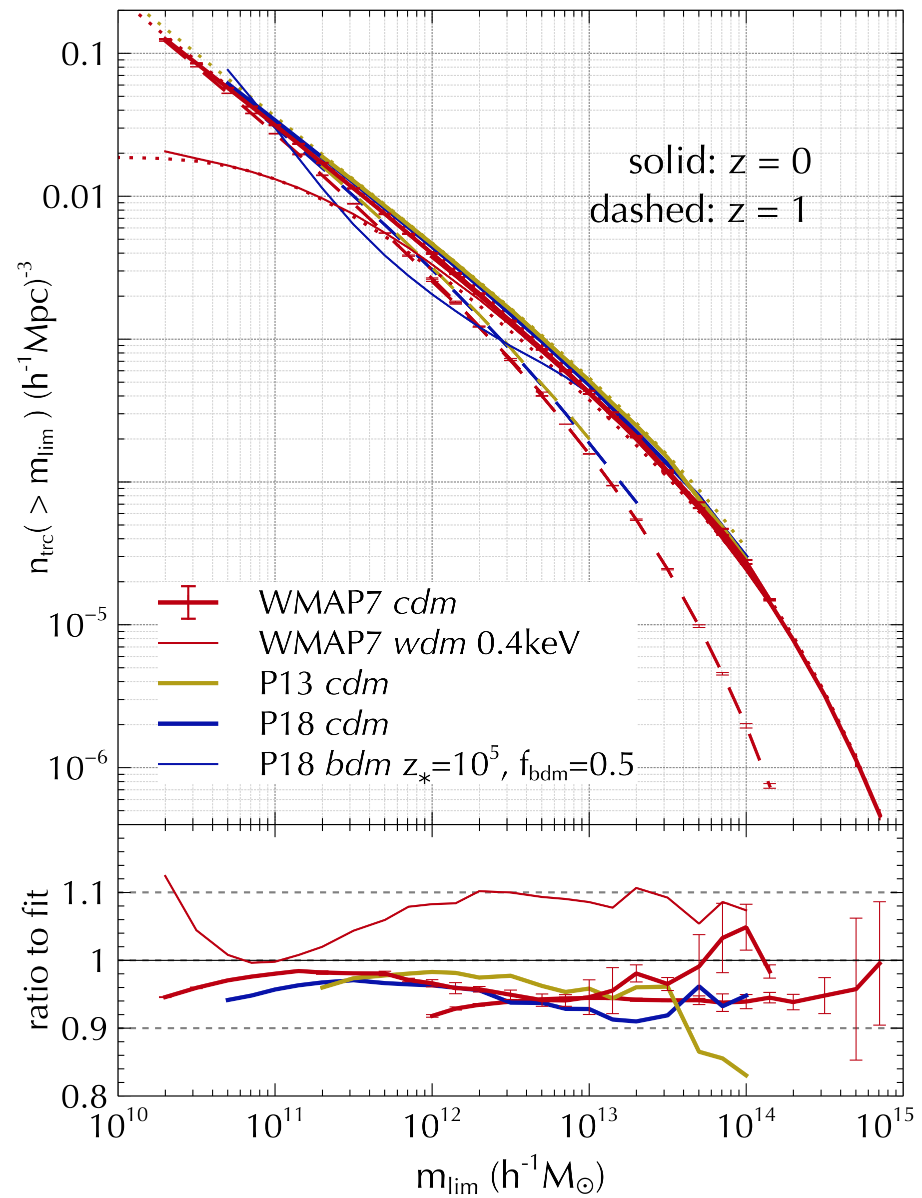
Thus $\langle y^2 \rangle$ depends on the infinite hierarchy of tracer correlation functions.

[For Poisson distributed tracers, $\chi(n_{\rm trc}, V) = 1$ and we recover $\langle y^2 \rangle_{\rm Poisson} \simeq 1.179$.]

Simulation suite

Gravity-only GADGET-2 *N-body runs*





Cold Dark Matter

P13 $(\Omega_m = 0.315, h = 0.673, \sigma_8 = 0.829)$
L150_N512

WMAP7 $(\Omega_m = 0.276, h = 0.7, \sigma_8 = 0.811)$
L150_N1024, L300_N1024, L600_N1024

P18 $(\Omega_m = 0.306, h = 0.678, \sigma_8 = 0.815)$
L200_N1024

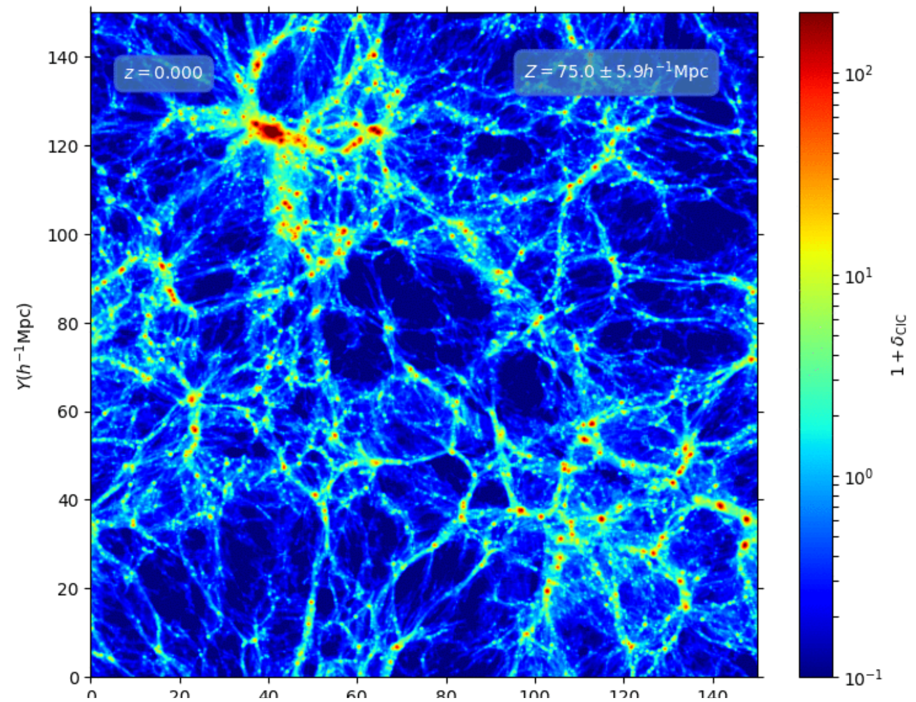
Alternate Dark Matter *(paired to CDM)*

WMAP7 warm DM $(m_{DM} = 0.4\,\mathrm{keV})$
L150_N1024

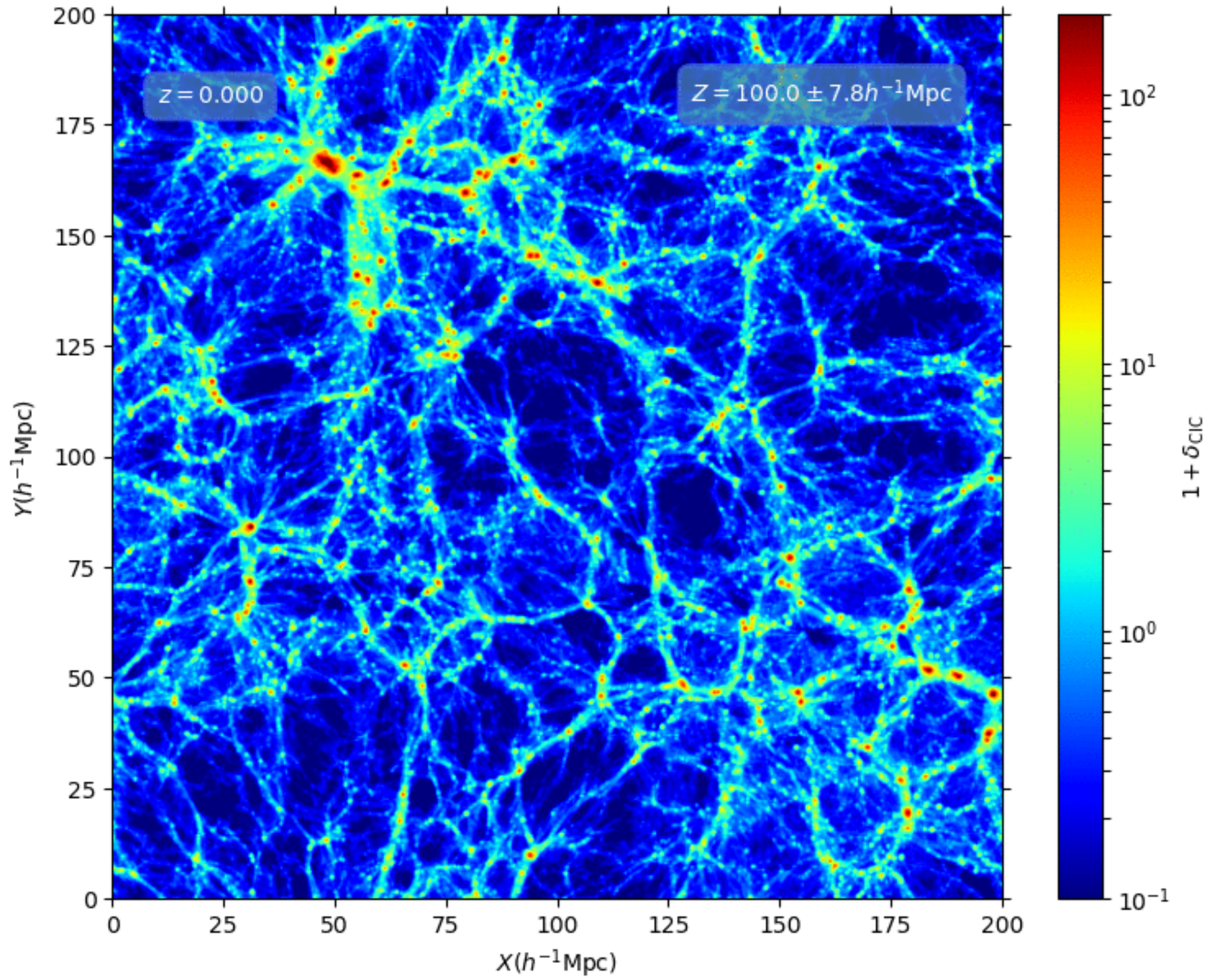
P18 ballistic DM $(z_* = 10^5, f_{bdm} = 0.5)$
L200_N1024

Simulation suite

WMAP7 *cdm*

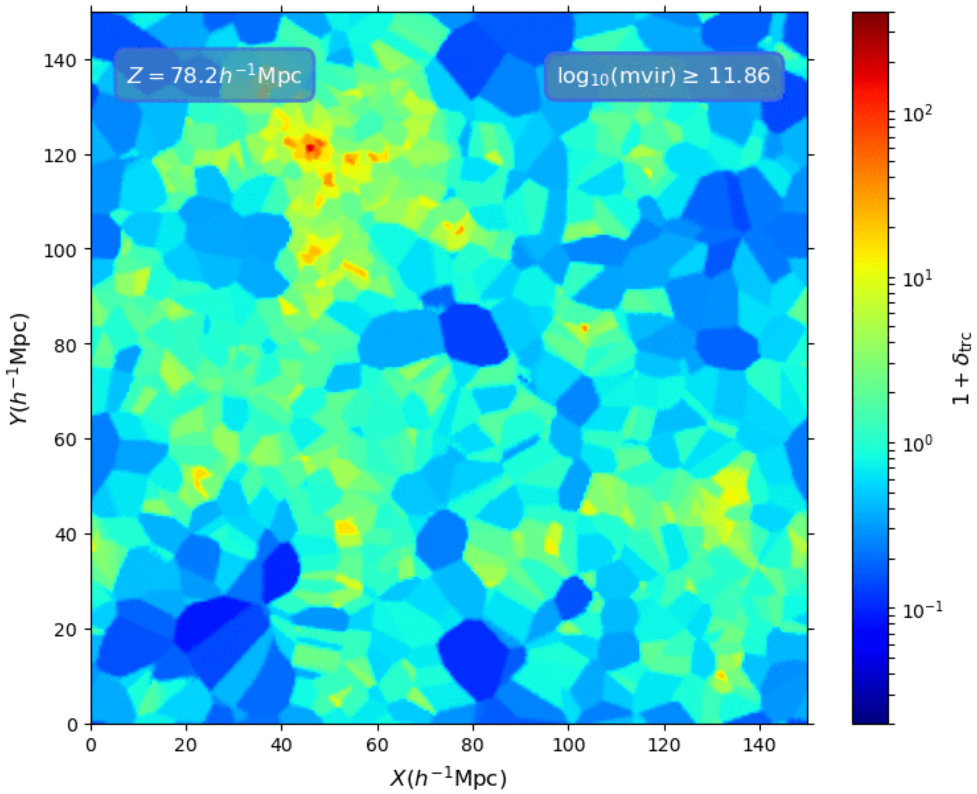


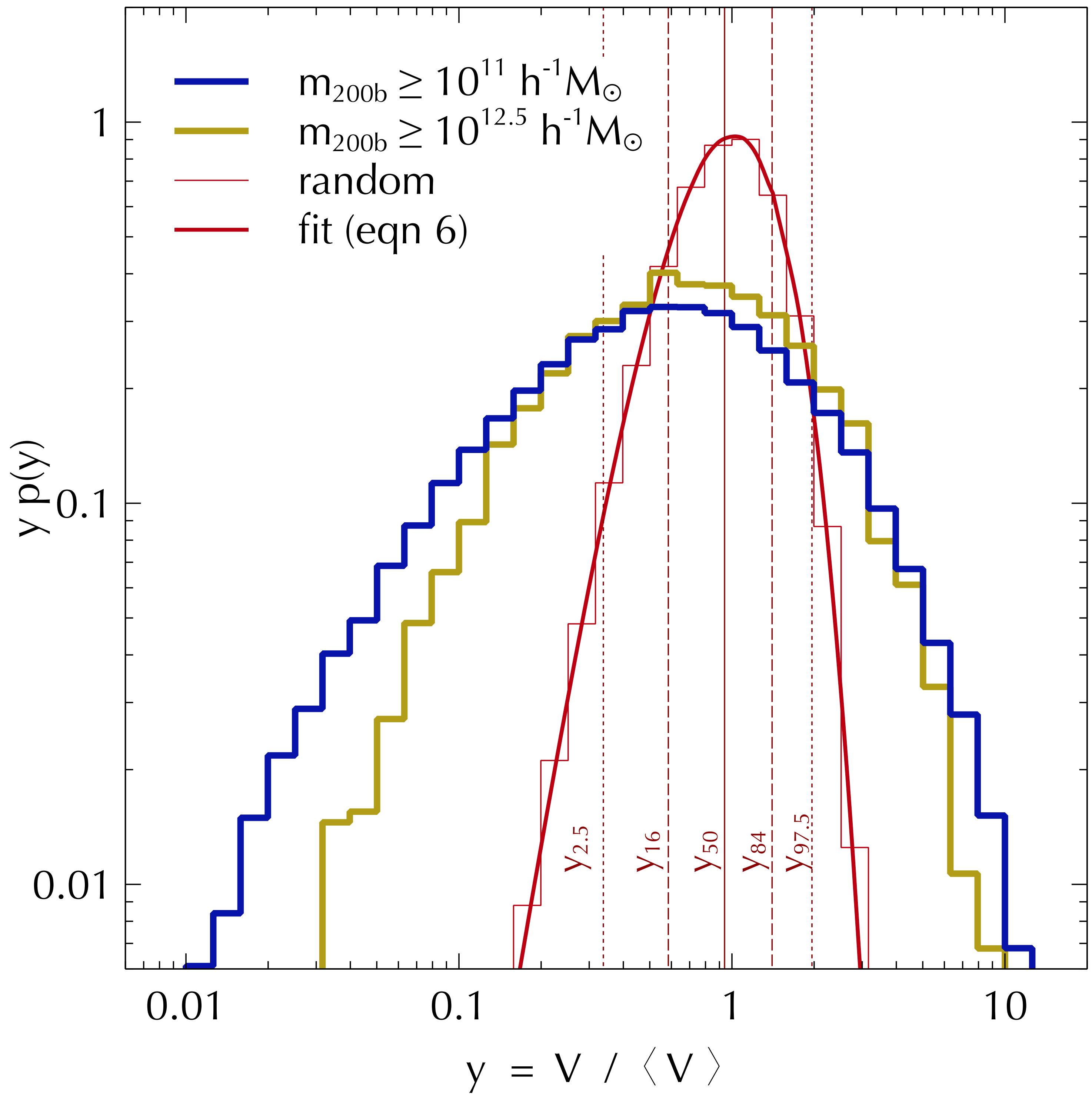
P18 *cdm*



VVF shape

Dependence on halo mass

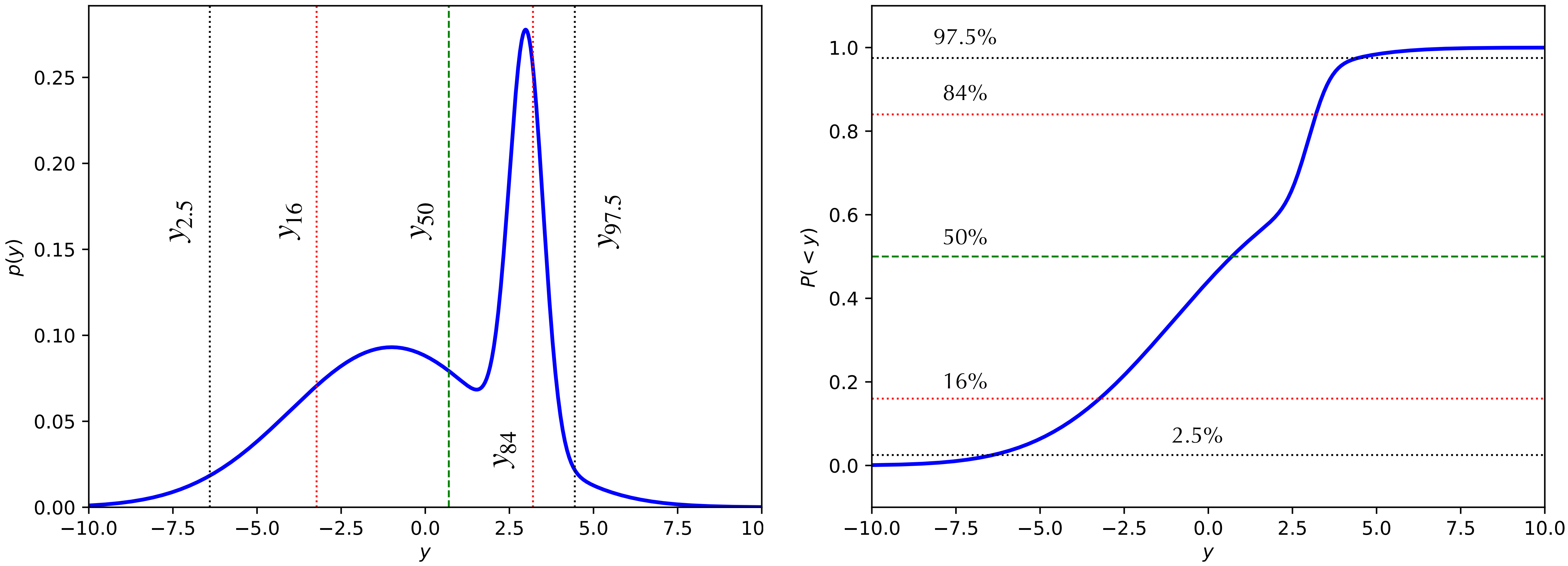




tracers:
mass-thresholded
haloes in real space

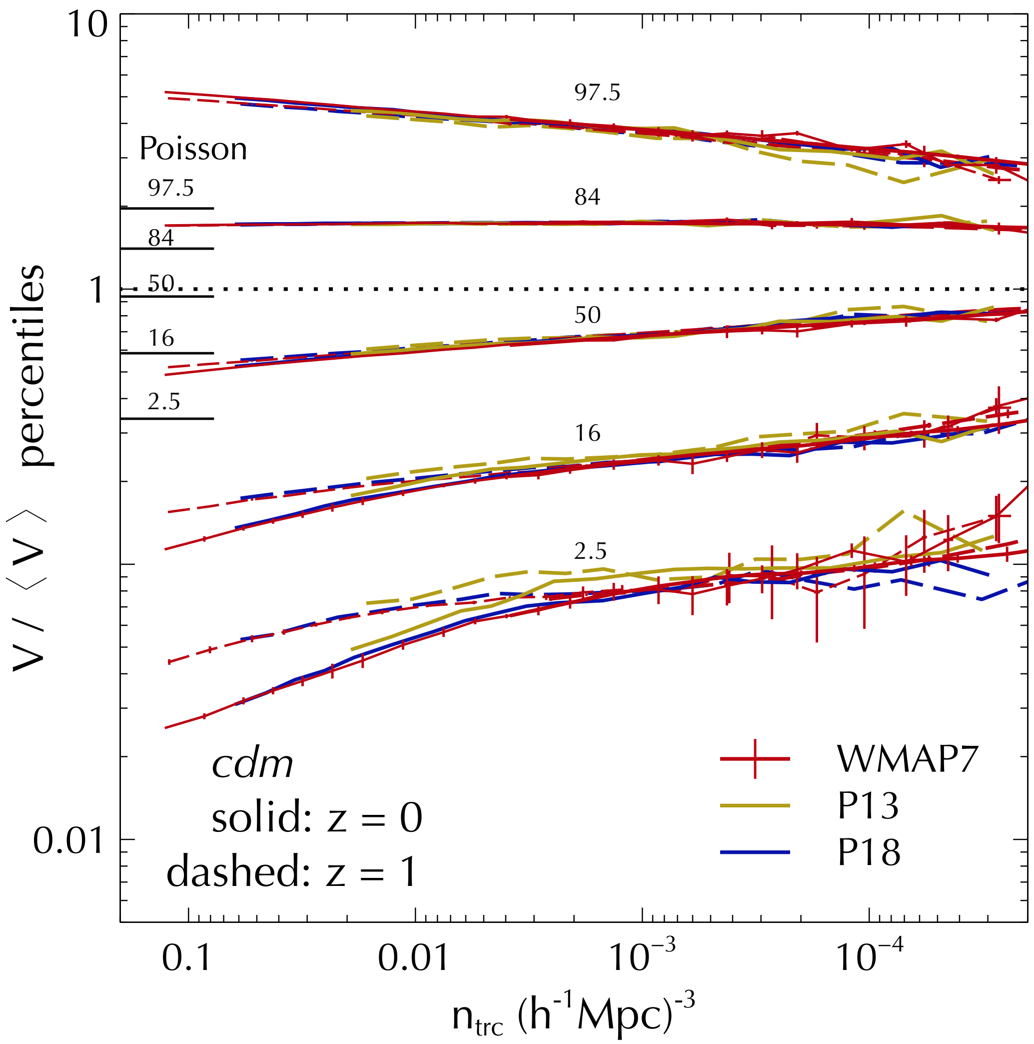
Aside

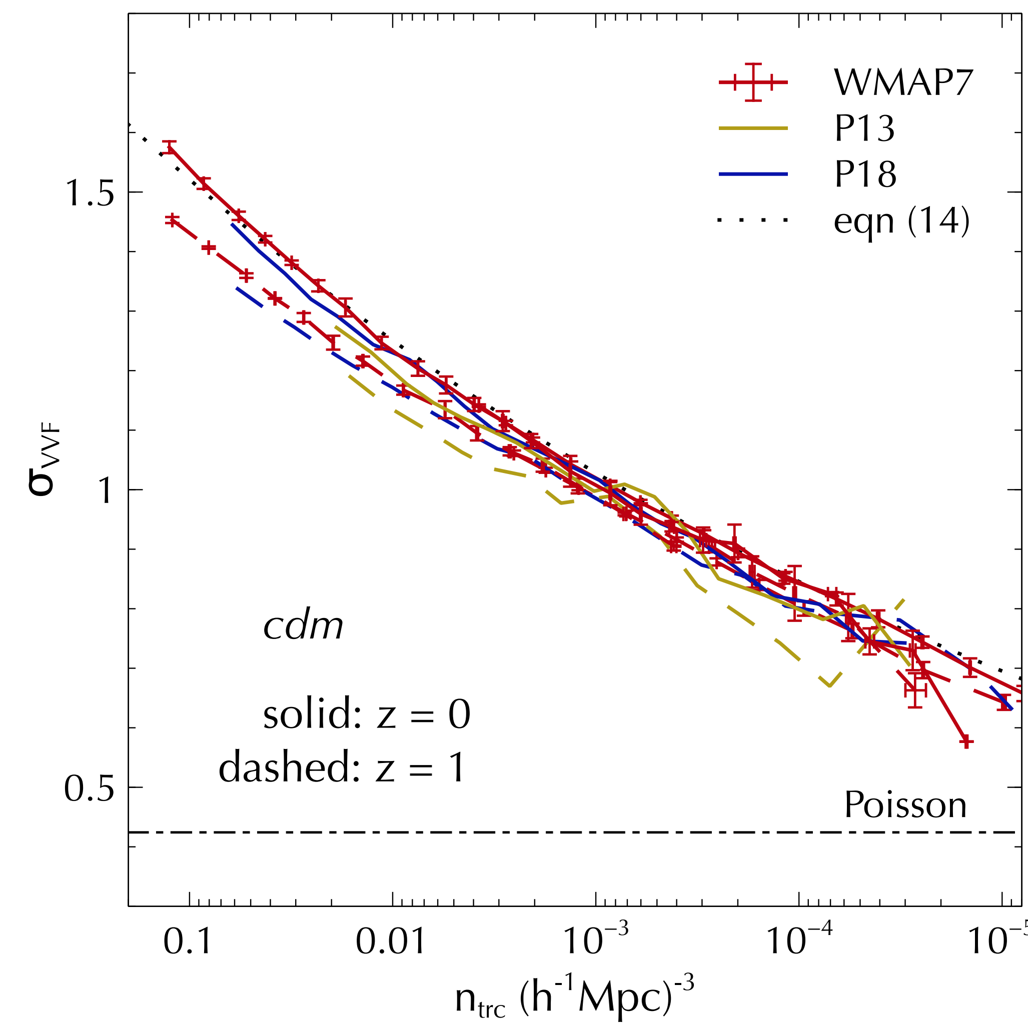
Percentiles of a distribution

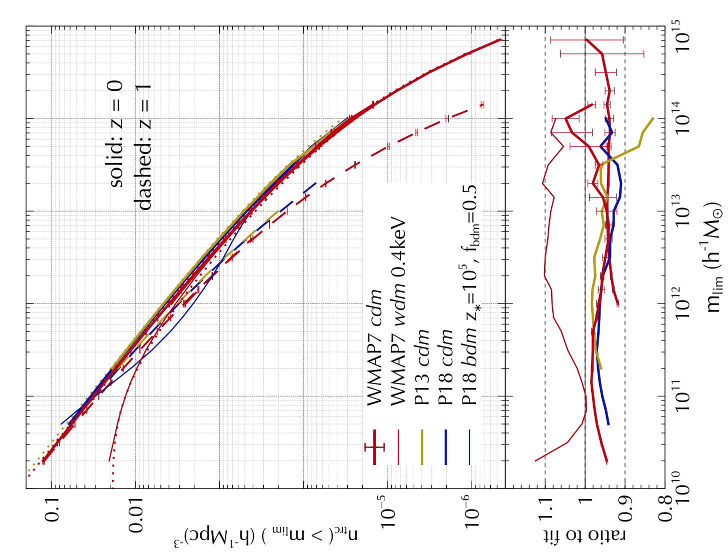


VVF percentiles + std dev

Dependence on halo mass and redshift



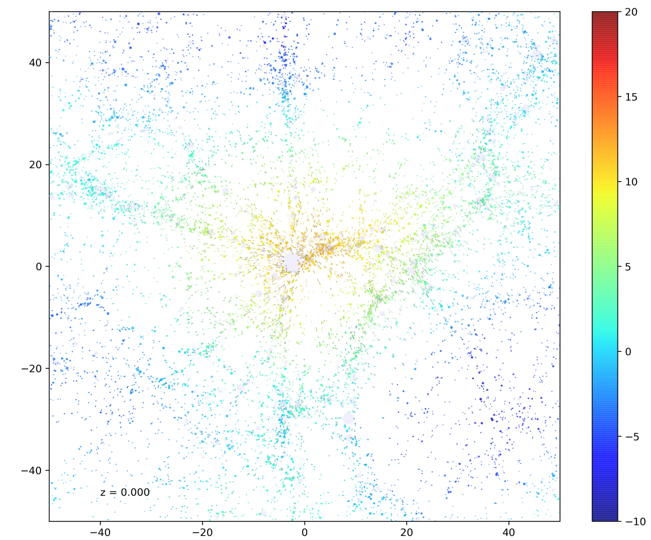




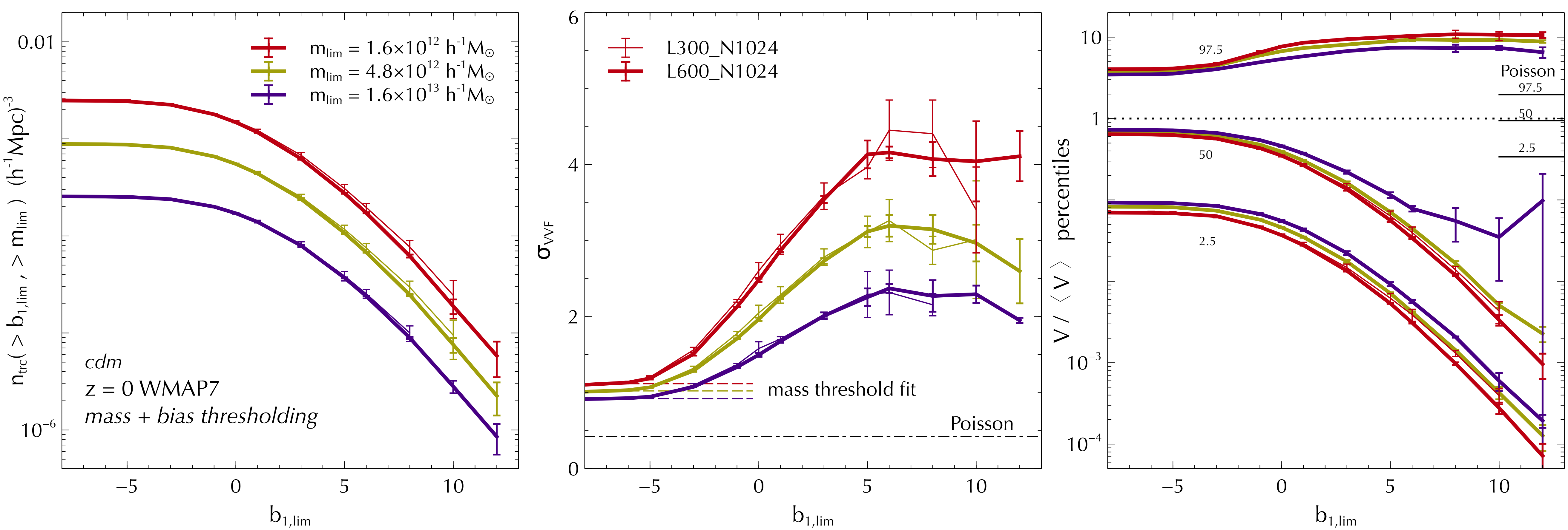
tracers:
mass-thresholded haloes in real space

VVF percentiles + std dev

Dependence on halo clustering



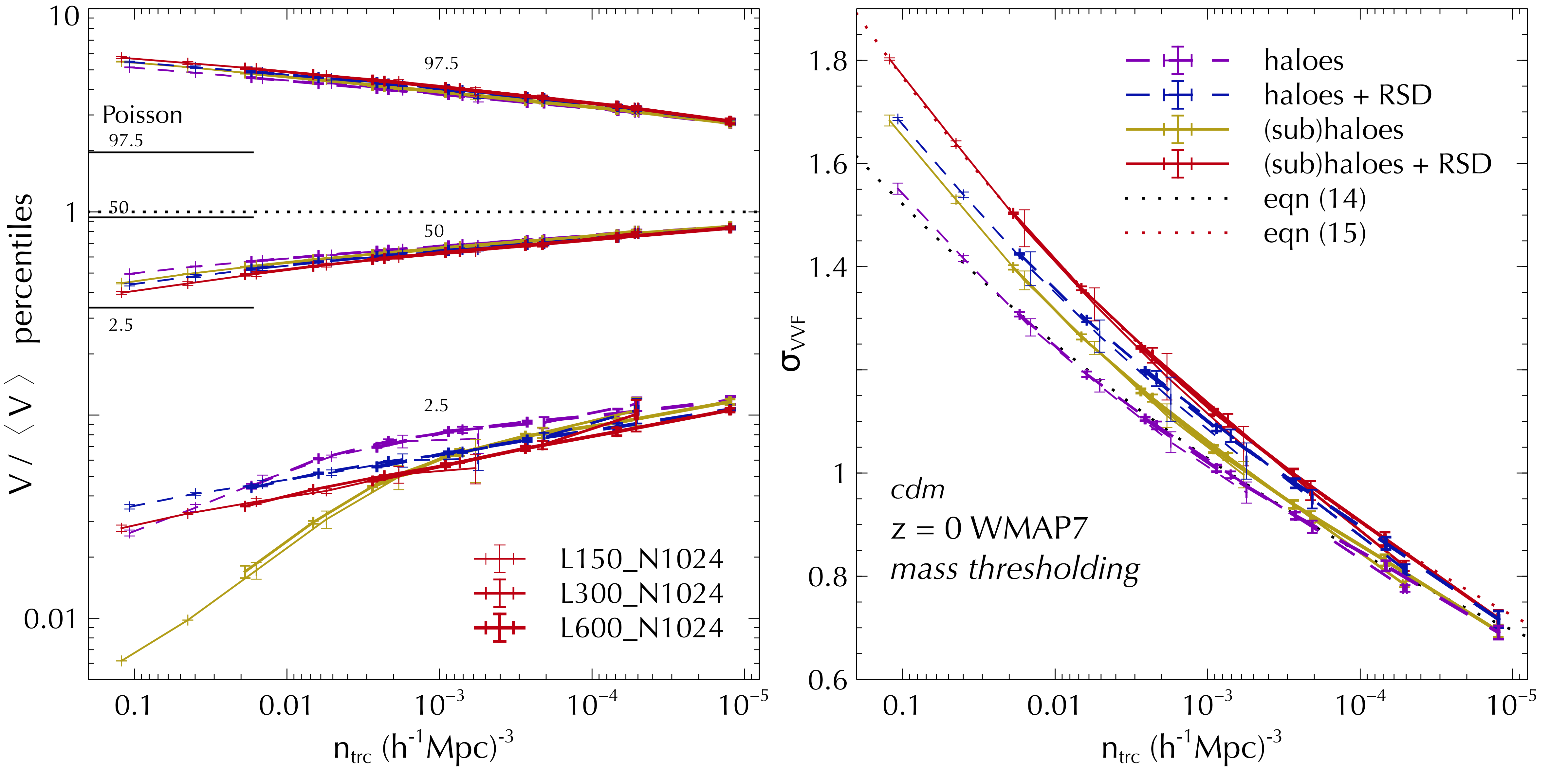
[AP, Hahn & Sheth 2018]



tracers: (mass + bias)-thresholded haloes in real space at $z = 0$

VVF percentiles + std dev

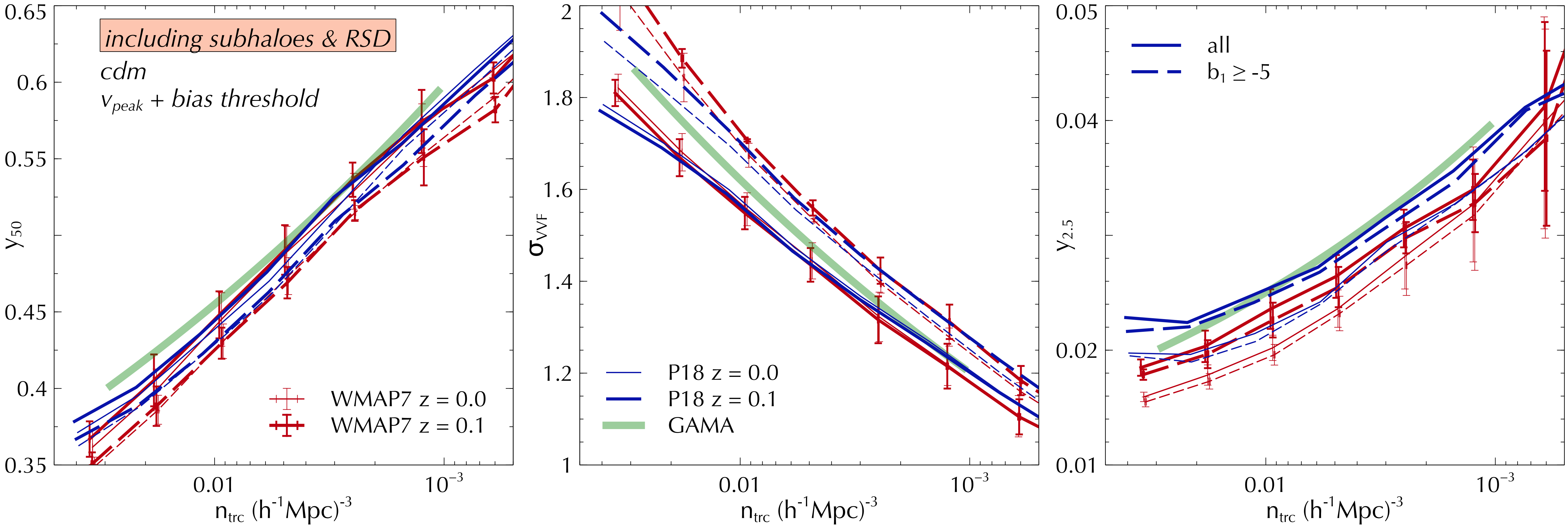
Effects of substructure and redshift-space distortions



tracers: mass-thresholded (sub)haloes in real and redshift space at $z = 0$

Comparison with data

Subhalo abundance matching: CDM simulations



tracers: ($v_{\rm peak}$+bias)-thresholded (sub)haloes in redshift space at $z = 0$ and $z = 0.1$

Simulation suite

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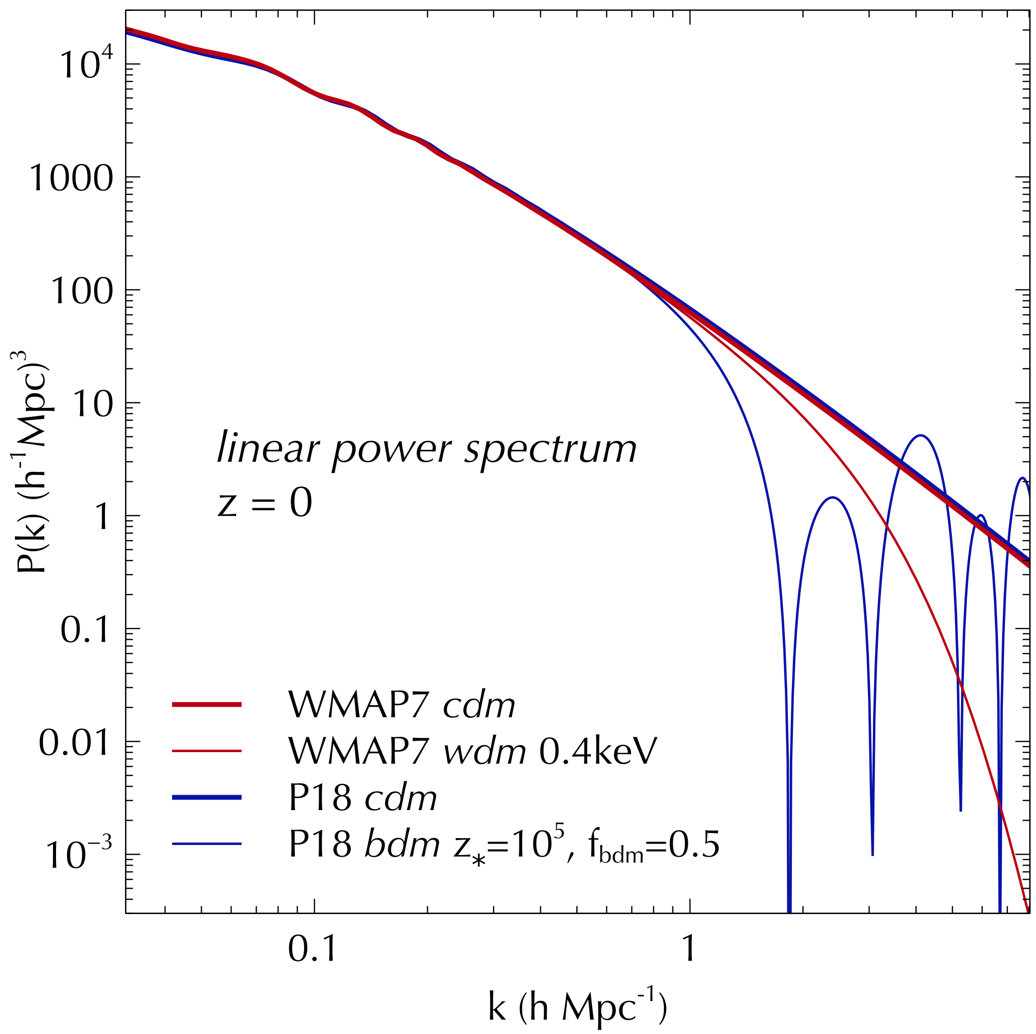

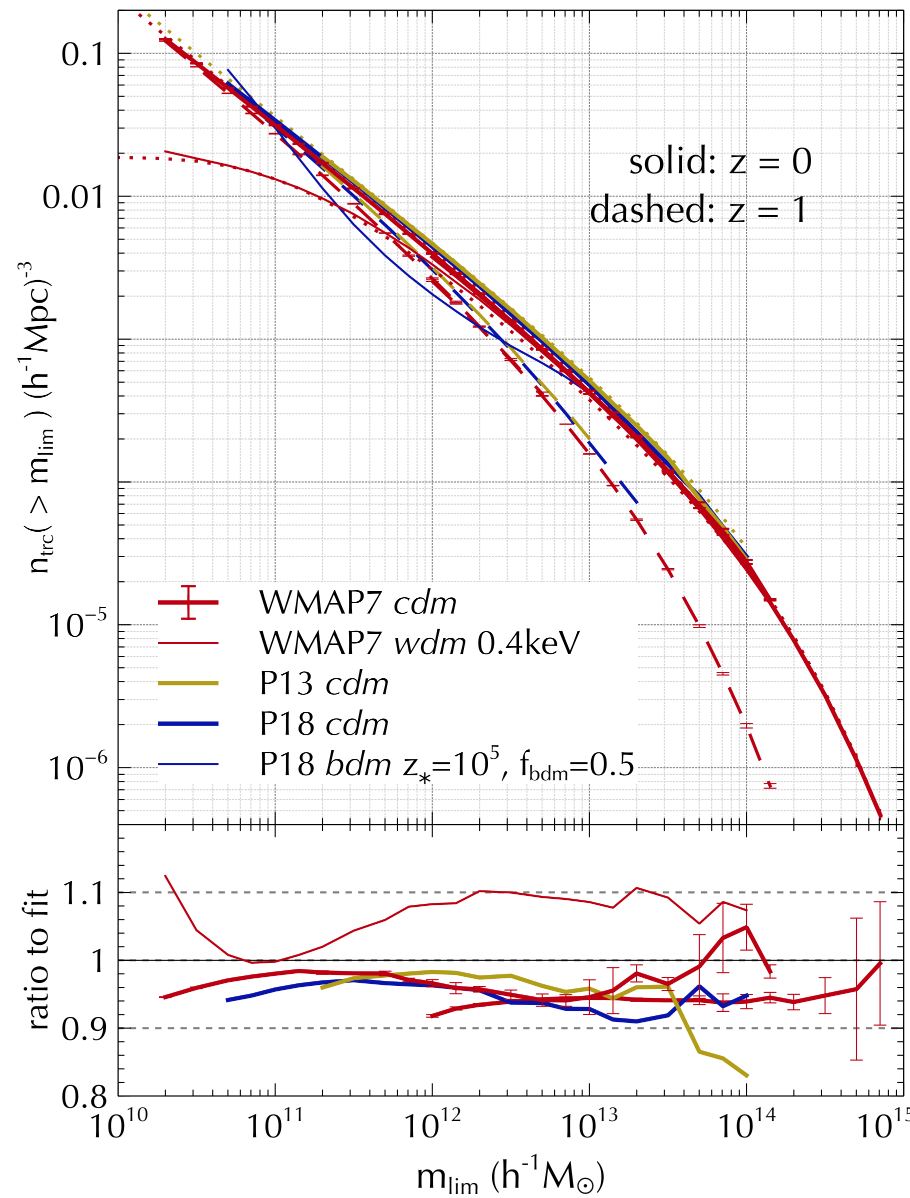

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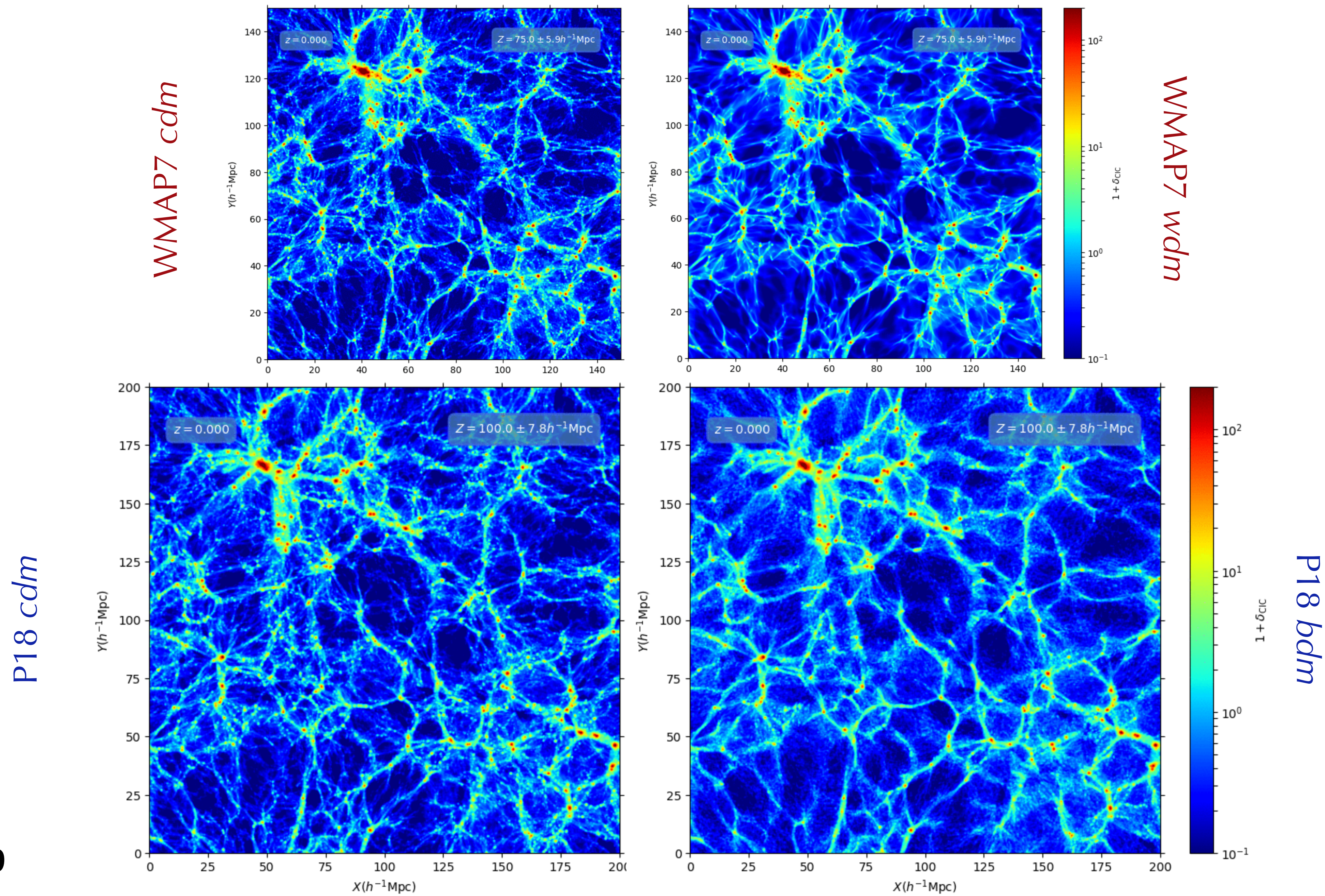
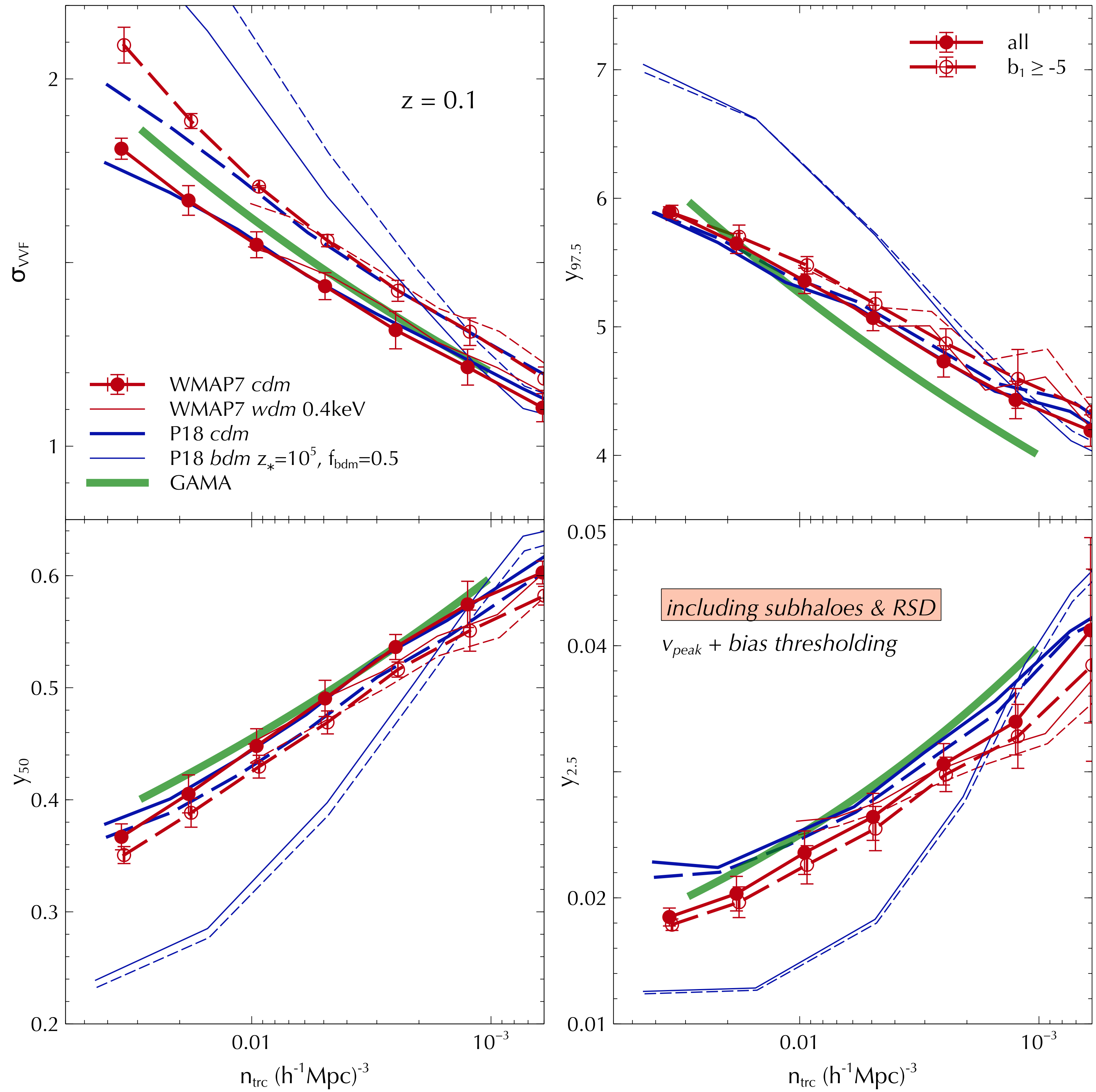


Comparison with data

Subhalo abundance matching: cosmology dependence



Conclusions

★ Voronoi volume function is a novel probe of nonlinear structure, sensitive to variety of tracer properties

- Halo mass
- Substructure content
- Kinematics (RSD)
- Large-scale clustering
- Redshift evolution

★ Cosmology dependence (*cdm* / *wdm*):

- VVF of mass-selected samples essentially universal
- Weak cosmology dependence in realistic (SHAM) samples

★ Cosmology dependence (*bdm*):

- Strong sensitivity to oscillatory features in $P(k)$