Effective Field Theory Approach to Thermal Nucleation

Phys. Rev. D 104, 096015 with Oliver Gould

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12.7.2022

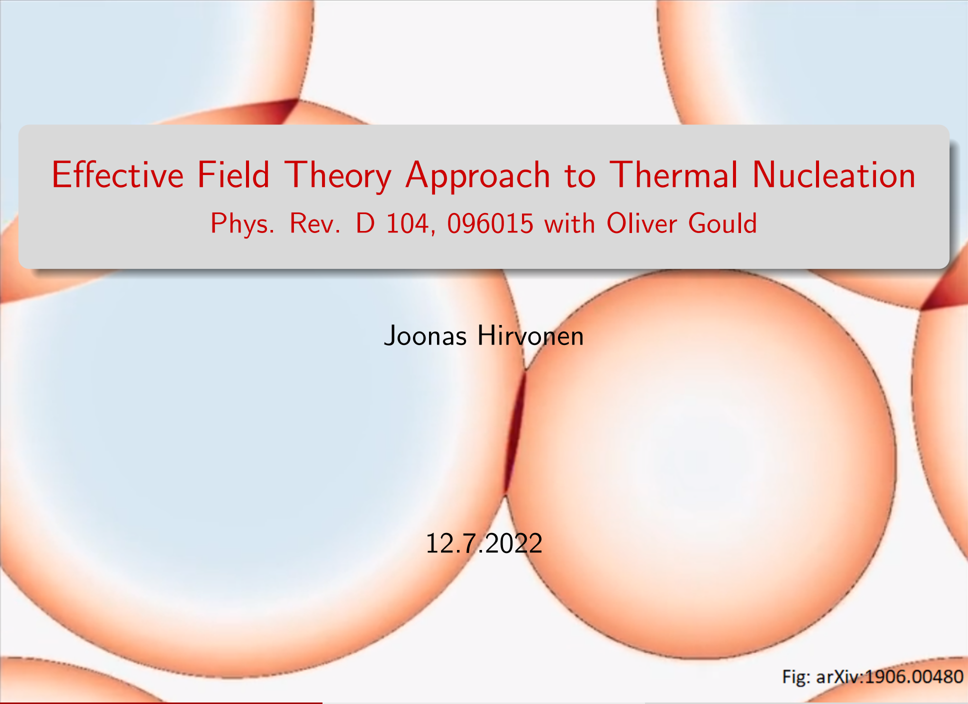


Fig: arXiv:1906.00480

Contents

- Introduction
- EFTs: natural framework
- Comparison to previous literature
- Eases calculations [Hirvonen '22]

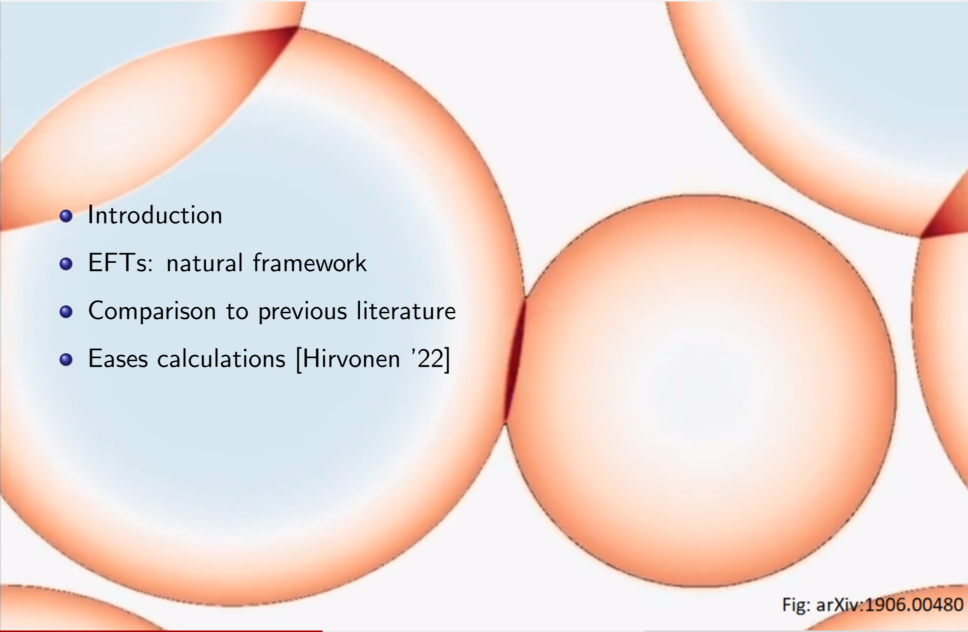


Fig: arXiv:1906.00480

Motivation

- Cosmological first order phase transition
  - Matter production
  - Gravitational wave signature
- Nucleation rate: large uncertainties

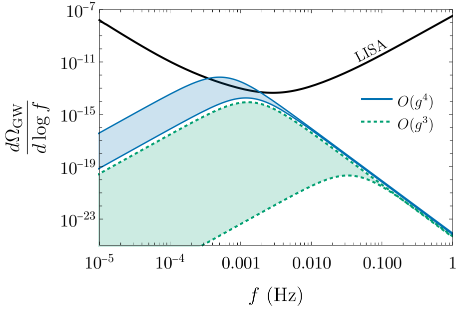


Figure: Gravitational wave spectrum [Gould, Tenkanen '21]

Bubbles

- First-order phase transitions
- Field configurations interpolating between the phases

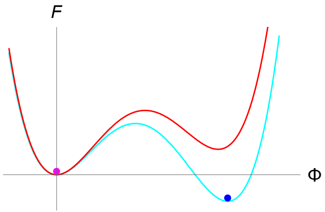


Figure: Free energy with a first-order transition

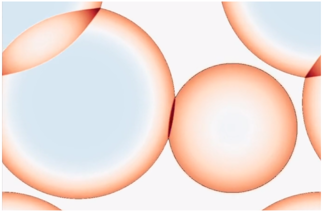


Figure: Bubbles [Cutting et al. '20]

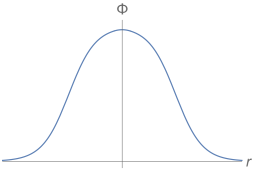


Figure: Bubble configuration

Nucleation scale

- Length scale of the bubbles, R_{bubble}
- Mass of the nucleating field, m_{nucl}

$$R_{\text{bubble}} \gtrsim m_{\text{nucl}}^{-1}$$

- Describe this scale – describe nucleation

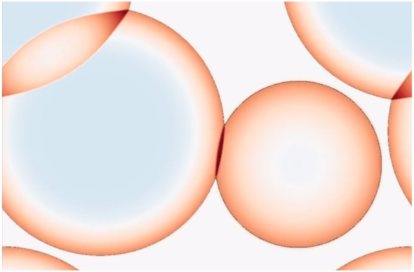


Figure: Nucleation scale

Thermal scale

- Scale of temperature
- "Initiates" the transition
  - Change in temperature

  $\longrightarrow$ Change in the free energy
- Needs to be accounted for bubbles

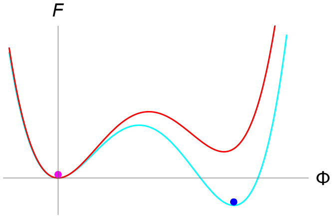


Figure: Changing free energy

Scale hierarchy

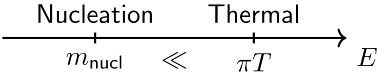


Figure: High-temperature scale hierarchy

- Between nucleation and thermal scale
- Temperature affects free energy at tree level:

$$\lambda T^2 \sim m_{\text{nucl}}^2$$

- Effective field theories!

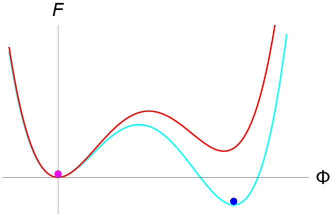
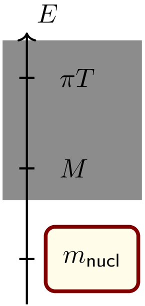


Figure: Changing free energy

Nucleation scale EFT

$$Z = \int \mathcal{D}\phi \, e^{-S_{\mathsf{E}}[\phi]}$$

- Statistical description for nucleation scale
- EFT techniques
  - Matching
  - Integrating out a scale [Hirvonen '22]
- Nucleation on tree level of S_{nucl}



$$\longrightarrow \quad Z = \int \mathcal{D}\phi_{\mathsf{nucl}} \, e^{-S_{\mathsf{nucl}}[\phi_{\mathsf{nucl}}]}$$

Nucleation rate

- Thermal classical nucleation [Langer '67, Ekstedt '22]
  - Integrate over nucleating configurations
- Suppression from the critical bubble
- Growth rate, κ, unknown

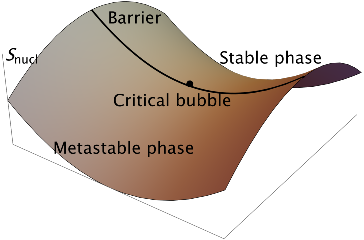


Figure: Field configuration space

$$\Gamma \propto \int \mathcal{D}\phi_{\text{nucl}}\, \delta_{\text{barrier}}\, e^{-S_{\text{nucl}}[\phi_{\text{nucl}}]}$$

$$\Rightarrow \frac{\Gamma}{V} \sim \kappa\, m_{\text{nucl}}^3\, e^{-S_{\text{nucl}}[\phi_{\text{CB}}]}$$

Conventional rate

- Imaginary part by analytic continuation [Linde, 1981]
- 1PI action, $\Gamma[\phi]$
- Obscurities
  - Double counting
  - Diverging derivative series
  - $\Gamma[\phi_{\mathsf{CB}}] \notin \mathbb{R}$

$$\Gamma = -2\,\mathrm{Im}\,F$$

$$\frac{\Gamma}{V} \overset{?}{\sim} T^4\, e^{-\Gamma[\phi_{\mathsf{CB}}]}$$

Computational improvements

- Obscurities eradicated
  - Related to the nucleation scale
  - EFT respects the nucleation scale
- Improved perturbation theory
  - Gauge invariance [Hirvonen et al. '21, Löfgren et al. '21]
  - Easier to compute higher orders

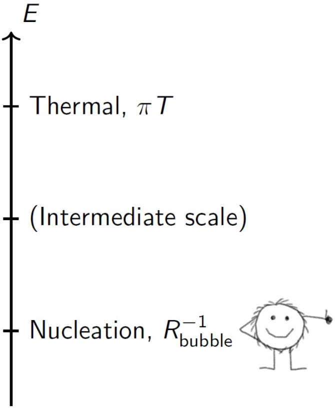


Figure: Happy bubble on the nucleation scale

Integrating out a scale [Hirvonen '22]

- Nearly the 1PI action for ϕ, $\Gamma[\phi]$
- Expand the propagators in IR quantities
  - m^2 and external momentum
  - IR scale removed from the diagrams

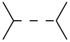
$$\frac{1}{p^2+m^2}\Bigg|_{\substack{\text{IRq}\\\text{exp.}}} = \frac{1}{p^2} - \frac{m^2}{p^4} + \dots$$

- Automatizes resummations

$$S_{\mathsf{E}}[\phi,\chi] = \int_x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{1}{2}(\partial_\mu \chi)^2 + \frac{M^2}{2}\chi^2 + \frac{g}{2}\chi\phi^2 \right)$$

$\downarrow$ Integrating out the scale of M^2 ($\gg m^2$)

$$S_{\mathsf{eff}}[\phi_{\mathsf{IR}}] = \int_x \left(\frac{1}{2}(\partial_\mu \phi_{\mathsf{IR}})^2 + \frac{m^2}{2}\phi_{\mathsf{IR}}^2 \right)$$

$+$  $\Big|_{\substack{\text{IRq}\\\text{exp.}}}$ $+$  $\Big|_{\substack{\text{IRq}\\\text{exp.}}}$

$+ \dots$

Thank you for keeping the bubbles happy!