Related works:

- [astro-ph: 2205.03392]
- [astro-ph: 2206.05578]

Cosmology from home 2022

# Structure of dark matter halos with differential elastic scattering cross sections

**Daneng Yang (UCR)**

**in collaboration with:**

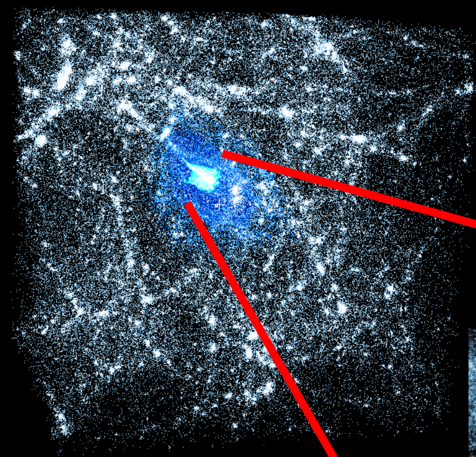
**Hai-Bo Yu (UCR)**

**July 4-15, 2022**

# Dark matter halos from cosmic structure formation

Large scales

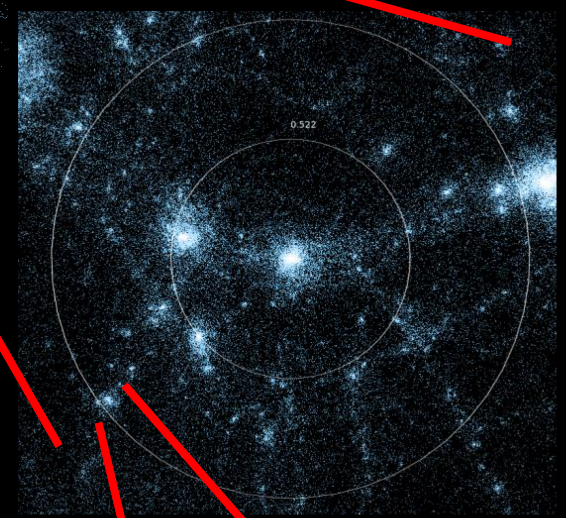
Cold Dark Matter



Halo growth

Merger + Tidal

interactions




Small scales

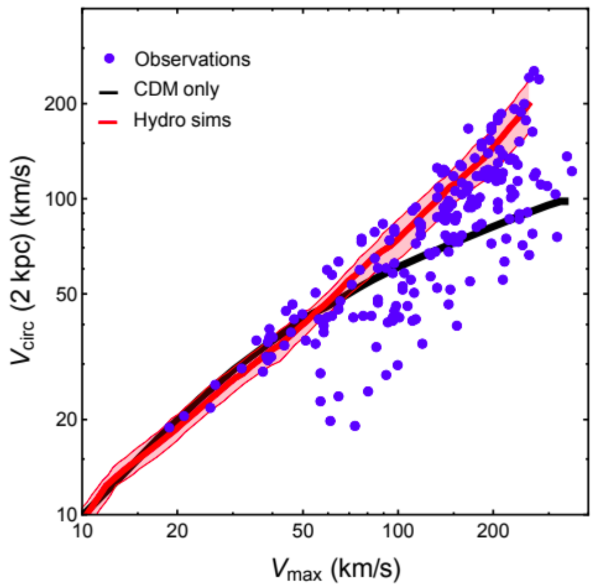
Re-establish pressure equilibrium

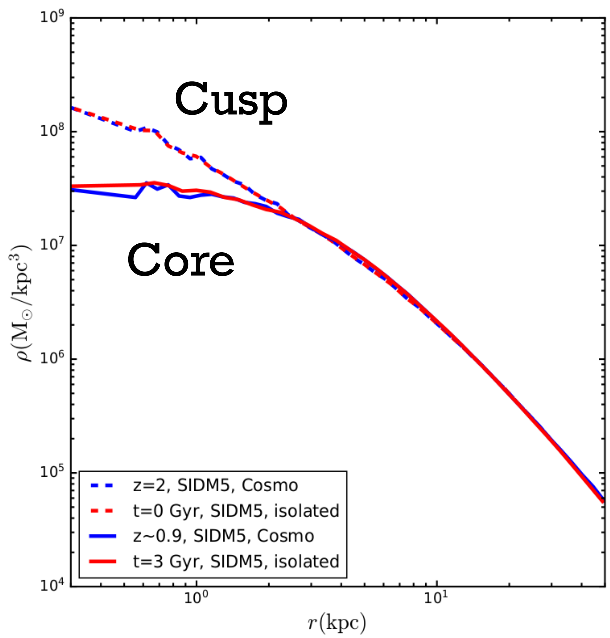
+ Could be self-interacting

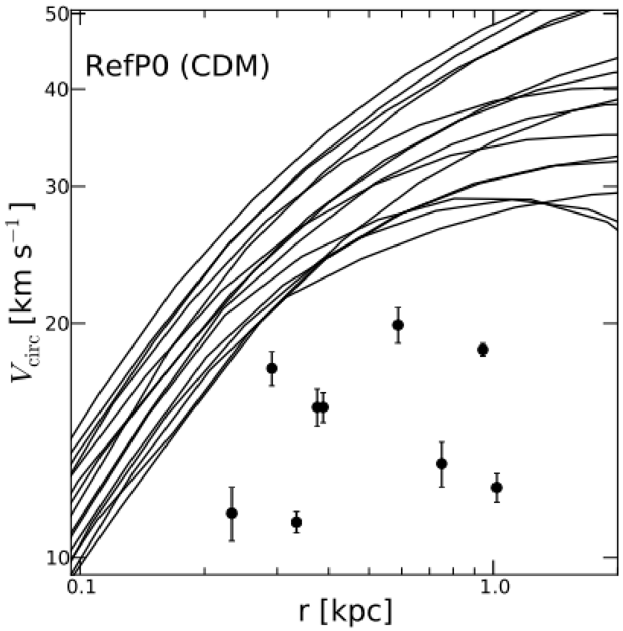
Tulin and Yu 2017, with data compiled in Oman+ 2015

***Previously,*** self-interacting dark matter (SIDM) has been used to address the

- Diversity problem
- Core cusp problem
- Too big to fail problem



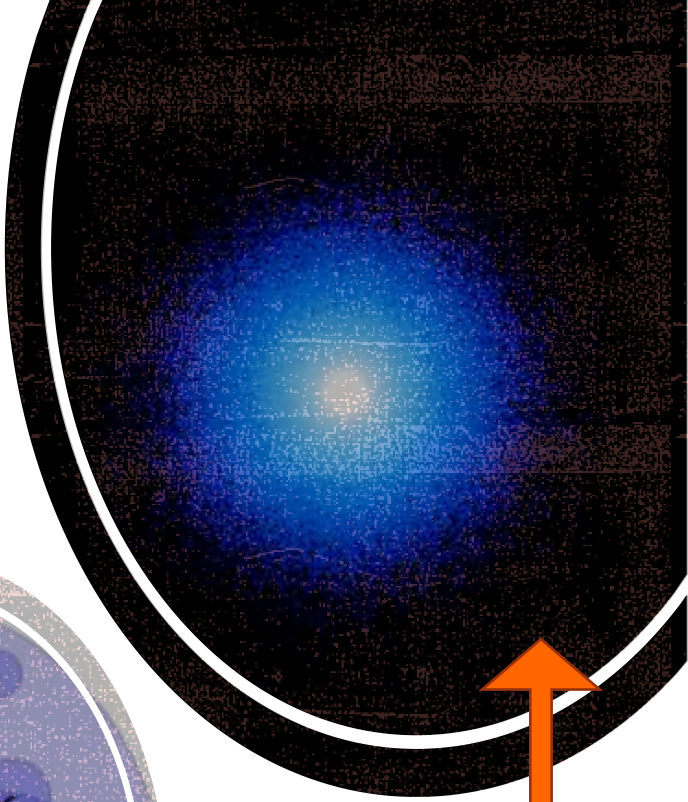



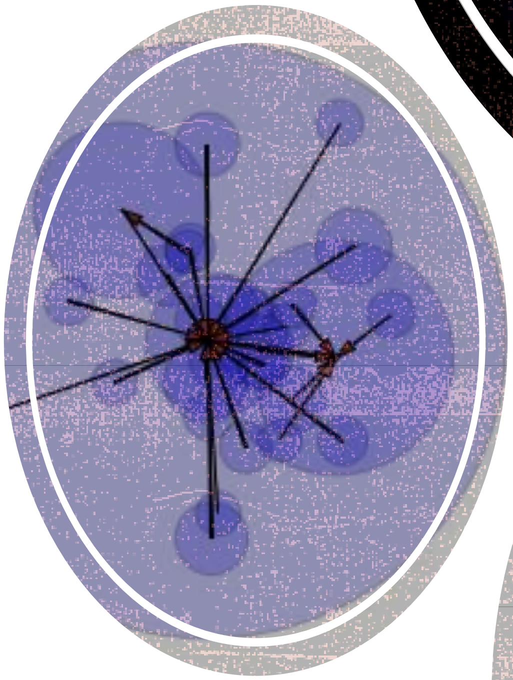



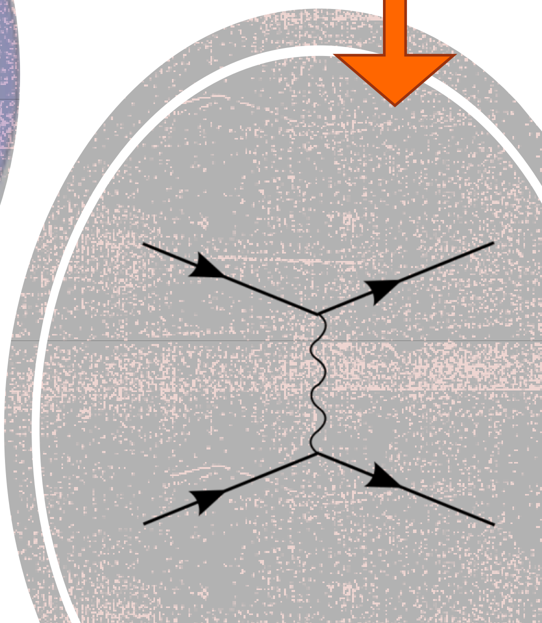

Vogelsberger+ 2012

# THIS TALK

- Thermodynamic properties of dark matter halos
- Quantifying the correlation between SIDM and halo structures
- Halo level characteristics
- A network of dark matter halos







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# Thermodynamic properties of dark matter halos

- [astro-ph: 2205.03392]

A thermalized system has a temperature & a Maxwellian velocity distribution

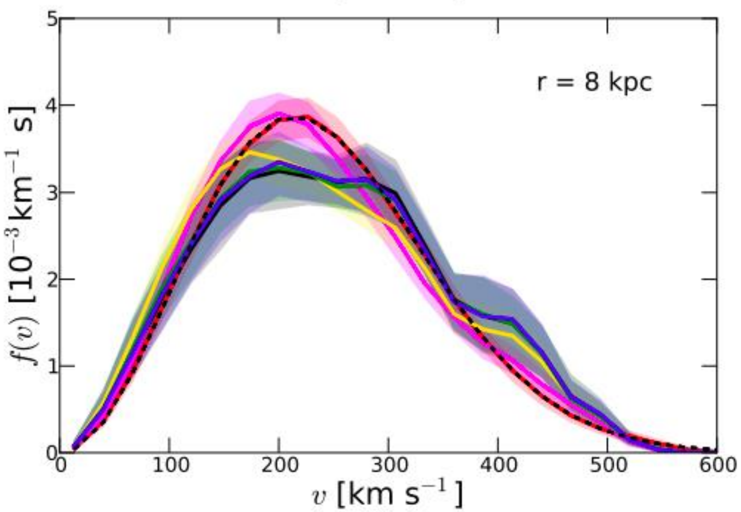
- A halo of finite mass cannot be a thermalized system

➢ Scatterings drive the velocity distribution to Maxwellian

$$f(\mathbf{v}, \mathbf{x}, t) = \rho(\mathbf{x}, t)\left(\frac{m_D}{2\pi T}\right)^{\frac{3}{2}} e^{-\frac{m_D(\mathbf{v}-\mathbf{u})^2}{2T}}$$

✓ An SIDM halo demonstrates thermodynamic features

✓ The "temperature" of a halo is a function of the radius




https://arxiv.org/pdf/1211.1377.pdf

*SIDM10 in red*

***Maxwellian fit in black-dashed***

# *Formulation of a thermodynamic description*

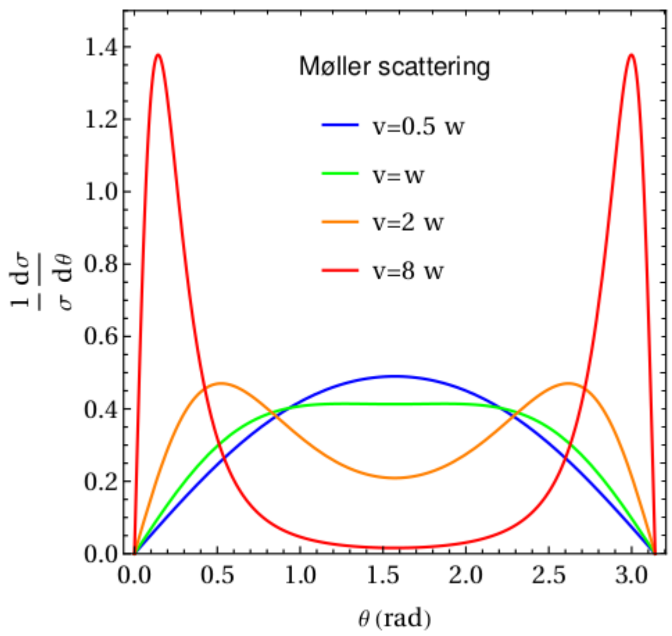
SIDM effective here

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \mathbf{v}} = C[f]$$

$$\nabla(\rho \sigma_{1D}^2) = -\rho \nabla \Phi$$ Pressure equilibrium

$$\frac{1}{4\pi} \frac{\partial L}{\partial r} = -\rho \sigma_{1D}^2 \frac{Ds}{Dt}$$ Heat/Energy transport

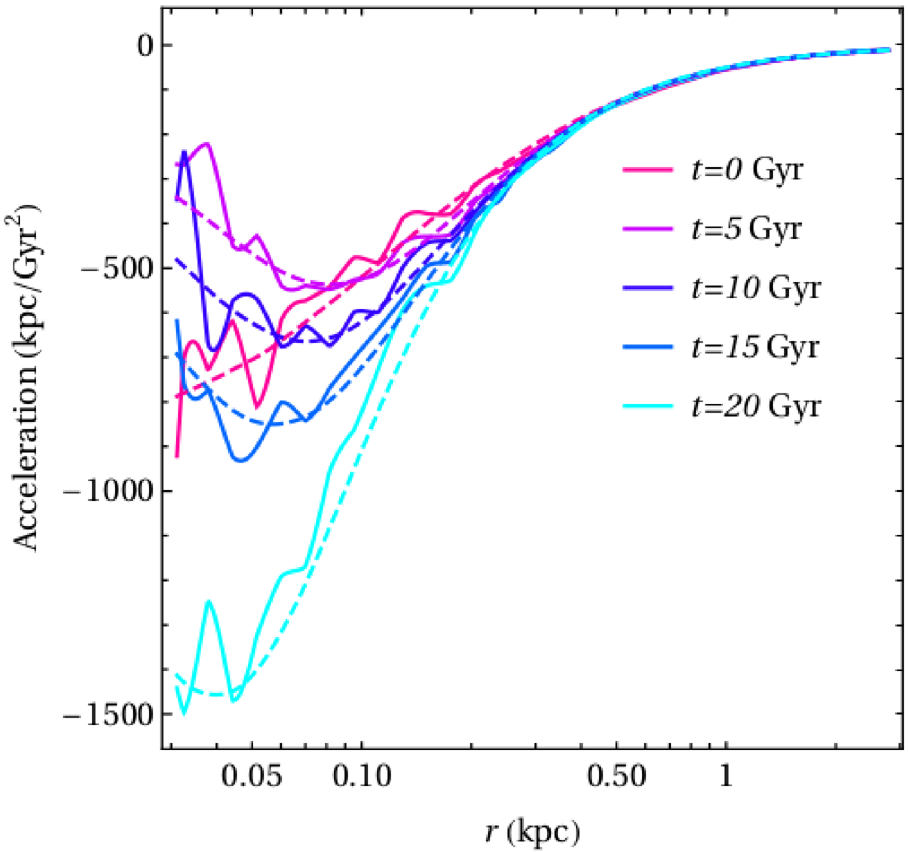
Thermodynamic equations



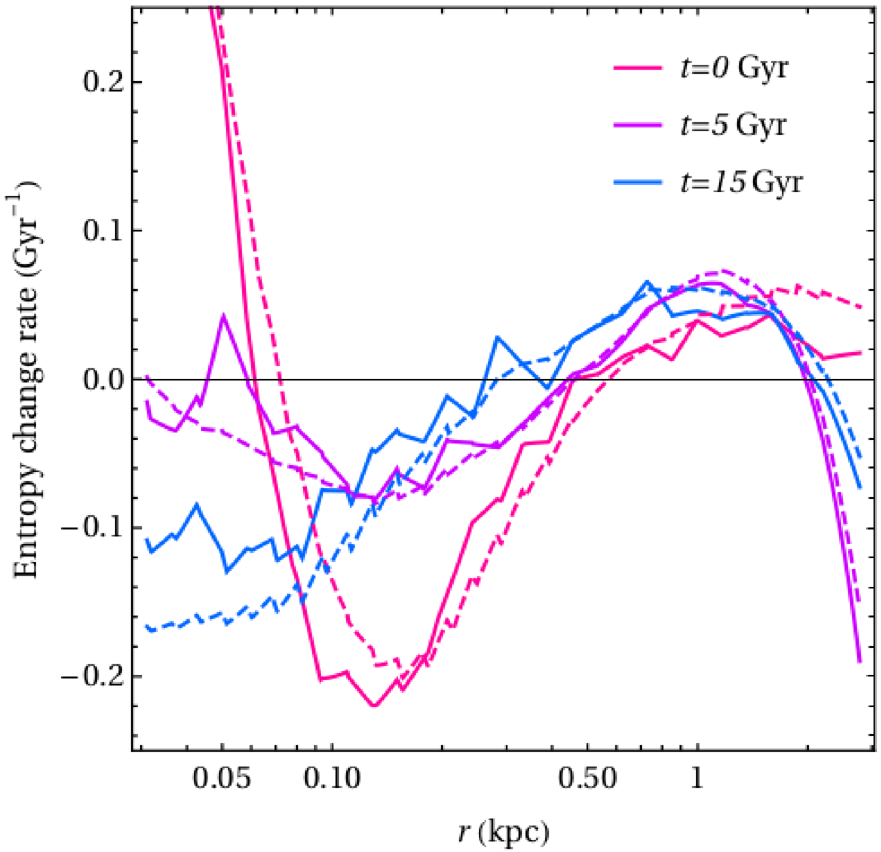

*Moller & Rutherford like scatterings are angular and velocity dependent*

*Reconstruct two sides of thermodynamic equations from N-body simulations*

## Pressure equilibrium




## Heat/Energy transport




$\frac{\partial s}{\partial t}$ (solid) $\quad \frac{1}{\sigma^2}\frac{\partial \mathcal{E}}{\partial t}$ (dashed)

# *Halo "structures" in terms of thermodynamic quantities*

- Temperature $= \frac{\Delta\mathcal{E}}{\Delta s}$

  Our N-body results supports the use of T=m$\sigma^2$

$$\frac{1}{\sigma^2}\frac{D\mathcal{E}}{Dt} = \frac{Ds}{Dt}$$

$$s = \ln(\sigma^3/\rho)$$

$$\sim -\frac{1}{\rho}\int d^3v f \ln f$$

$$\mathcal{E} = \frac{v^2}{2} + \Phi \approx \frac{3\sigma^2}{2} + \Phi$$

- Heat Capacity

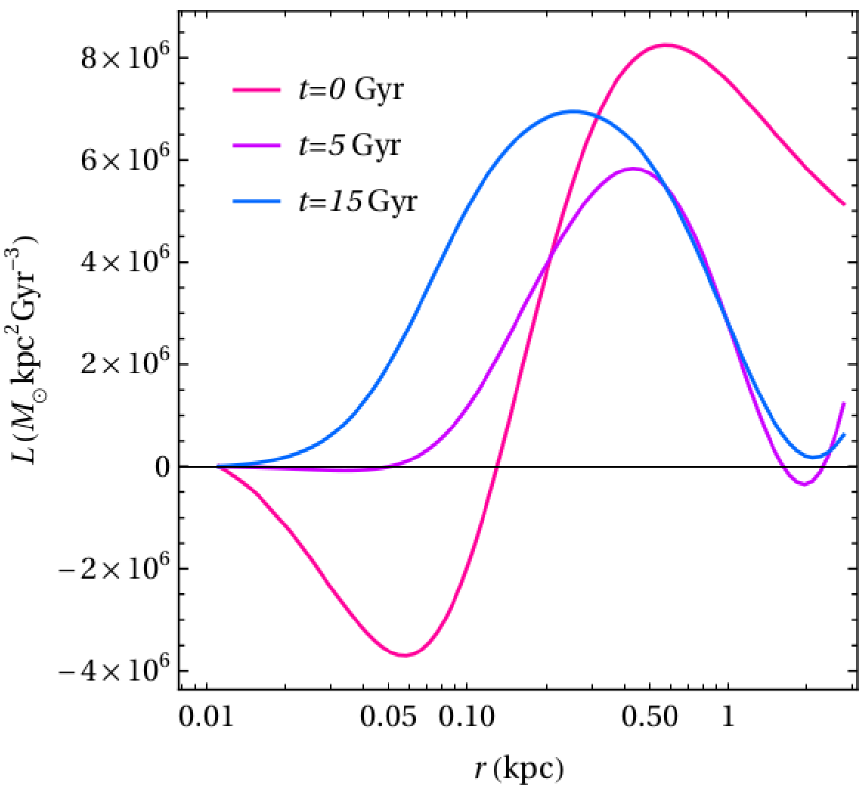
$$C(r) = dE/d\sigma^2_{\rm 1D}$$

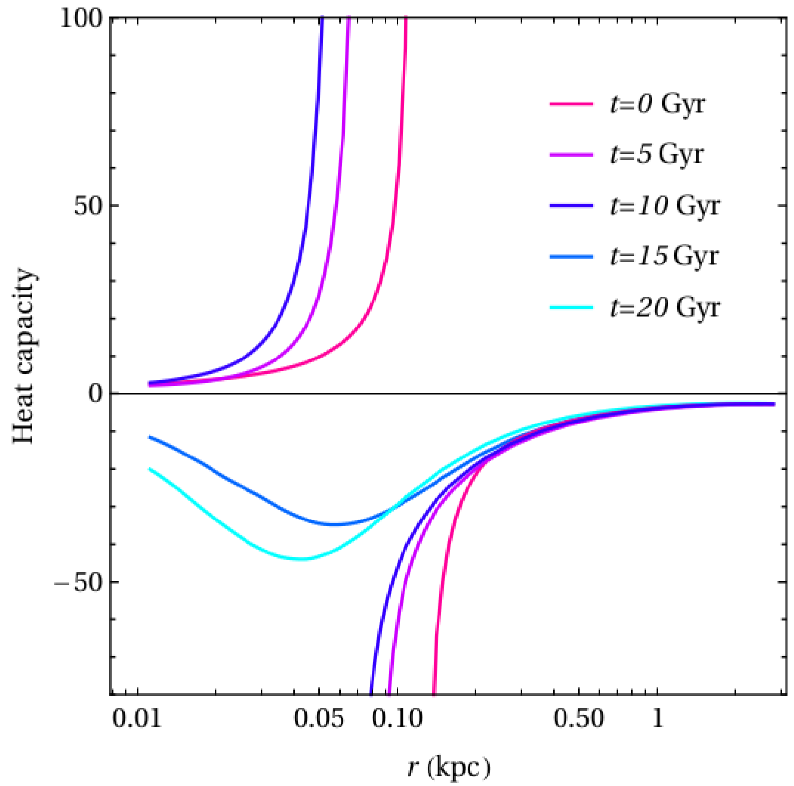
- Luminosity

$$L = -4\pi \int_0^r dr' r'^2 \rho(r') \frac{DE(r)}{Dt}$$

## Novel radius dependence of SIDM effect in time

Reconstructed from N-body

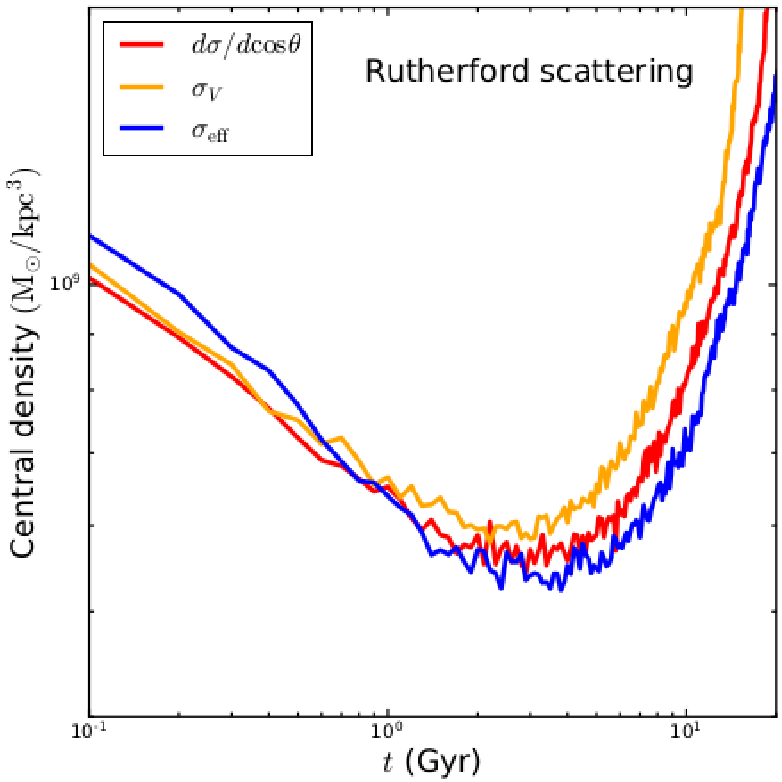


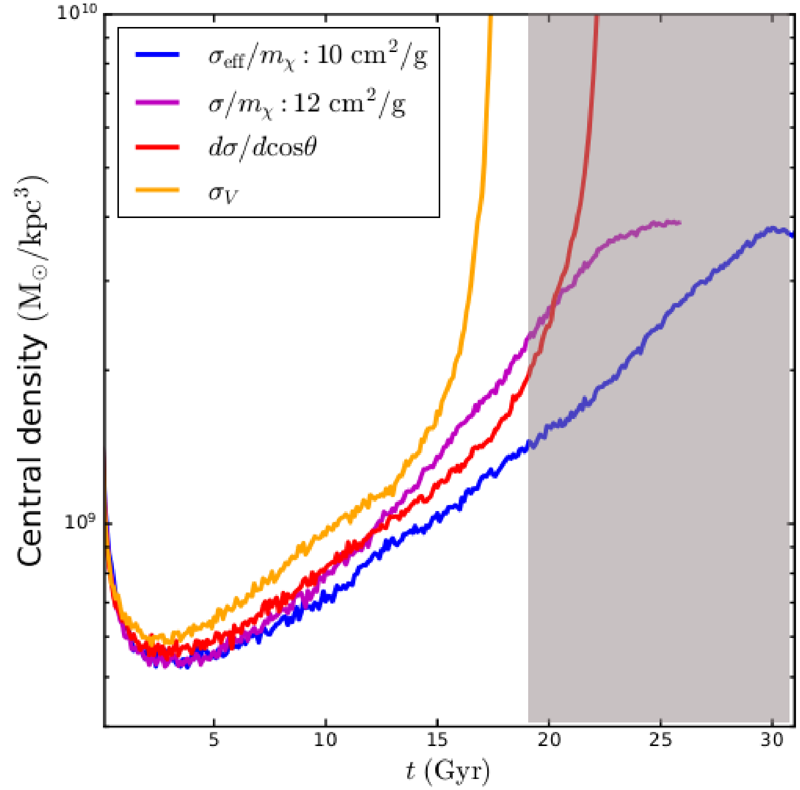

$$L \sim -4\pi r^2 \kappa \nabla T$$

**Thermodynamical explanation**

| | |
|---|---|
| Early times: | -heat flux + capacity => core formation |
| Intermediate times: | +heat flux + capacity => stable core |
| Late times: | +heat flux - capacity => core collapse |




Log scale




Linear scale

Early times: -heat flux + capacity => core formation
Intermediate times: +heat flux + capacity => stable core
Late times: +heat flux - capacity => core collapse
*Very late times:* +/- heat flux - capacity => catastrophe or stall of the collapse

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# Quantifying the correlation between SIDM and halo structures

- [astro-ph: 2205.03392]

Heat/Energy transport drives the thermodynamics of a halo

- It is complicated by the inclusion of potential energy
- and by the long-mean-free-path of particle collisions

Thermal conductivity can be computed using kinetic theory, but:

$$\frac{m\nabla(\kappa\nabla\sigma^2)}{\rho\sigma^2} = \frac{D}{Dt}\ln\frac{\sigma^3}{\rho}$$

$$L = -4\pi\int_0^r dr' r'^2 \rho(r') \frac{DE(r)}{Dt}$$

$$\begin{aligned}\kappa_{\rm est} m &\approx \frac{L}{4\pi r^2 2\sigma_{1D}(\partial\sigma_{1D}/\partial r)} \\ &\approx \frac{5\times10^6\ \mathrm{M_\odot kpc^2 Gyr^{-3}}}{4\pi(0.5\ \mathrm{kpc})^2(2\times5\ \mathrm{kpc/Gyr})(3\ \mathrm{kpc/Gyr}/(1\ \mathrm{kpc}))} \\ &\approx 0.5\times10^5\ \mathrm{M_\odot kpc^{-1}Gyr^{-1}},\end{aligned}$$

$$\kappa_{smfp} m\Big|_{r=0.5\ \mathrm{kpc}} \approx 5\times10^9\ \mathrm{M_\odot kpc^{-1}Gyr^{-1}}$$

Effectively, the heat conductivity can be orders of magnitude smaller than the ones computed using kinetic theory!

Reason:

- Mean free path >> scale radius of the halo
- Heat conduction regulated by size of the halo

**In N-body simulation, there is no need for a "long-mean-free-path $\kappa_{lmfp}$"**

◆ Post scattering evolution is governed by gravity

➢ Can use the weighting kernel of kinetic theory thermal conductivity to average out a differential cross section

A radius dependent conductivity cross section can be introduced:
Kernel:

$$\sigma_\kappa(r) = \frac{2\int v^2 dv d\cos\theta \frac{d\sigma}{d\cos\theta}\sin^2\theta v^5 \exp\left[-\frac{v^2}{4\sigma_{\rm 1D}^2(r)}\right]}{\int v^2 dv d\cos\theta \sin^2\theta v^5 \exp\left[-\frac{v^2}{4\sigma_{\rm 1D}^2(r)}\right]}$$

## Our equation demonstrates how the halo structures (velocity and its dispersion) correlate to a differential cross section

<σv>

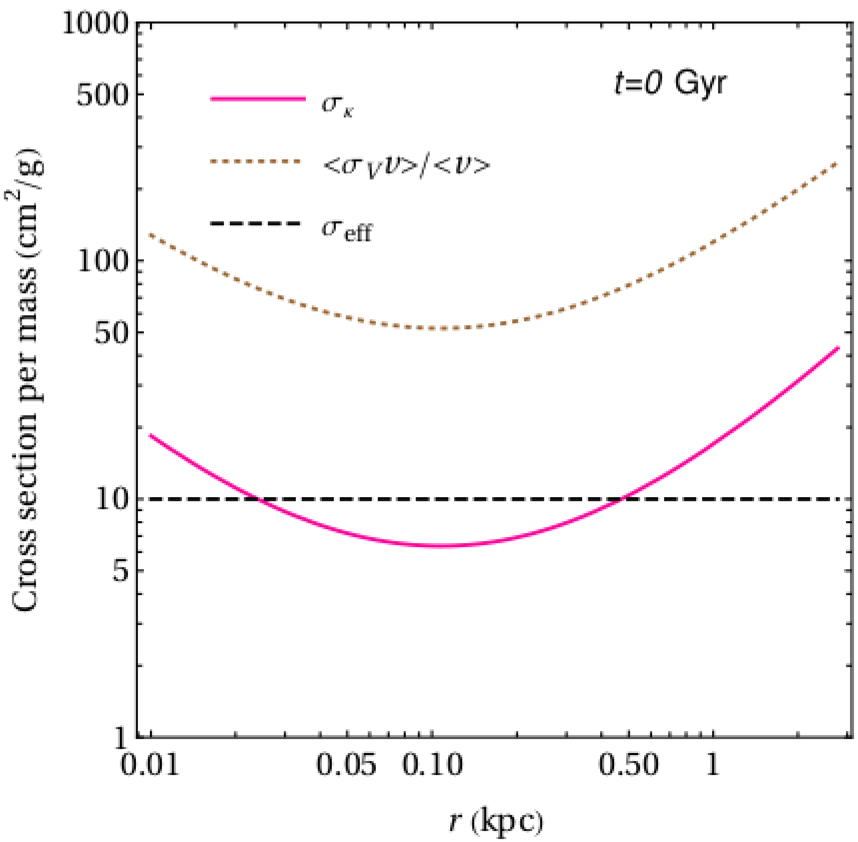
$$\sigma_n(r) = \frac{\int dv \sigma(v) v^3 e^{-v^2/(4\sigma_{1D}^2(r))}}{\int dv v^3 e^{-v^2/(4\sigma_{1D}^2(r))}}$$

$$\sigma_\kappa(r) = \frac{2\int v^2 dv d\cos\theta \frac{d\sigma}{d\cos\theta} \sin^2\theta v^5 \exp\left[-\frac{v^2}{4\sigma_{1D}^2(r)}\right]}{\int v^2 dv d\cos\theta \sin^2\theta v^5 \exp\left[-\frac{v^2}{4\sigma_{1D}^2(r)}\right]}$$

Collision rate

vs

Heat conduction ✓




- The integrand reveals how a differential cross section couples to the halo velocity and its dispersion

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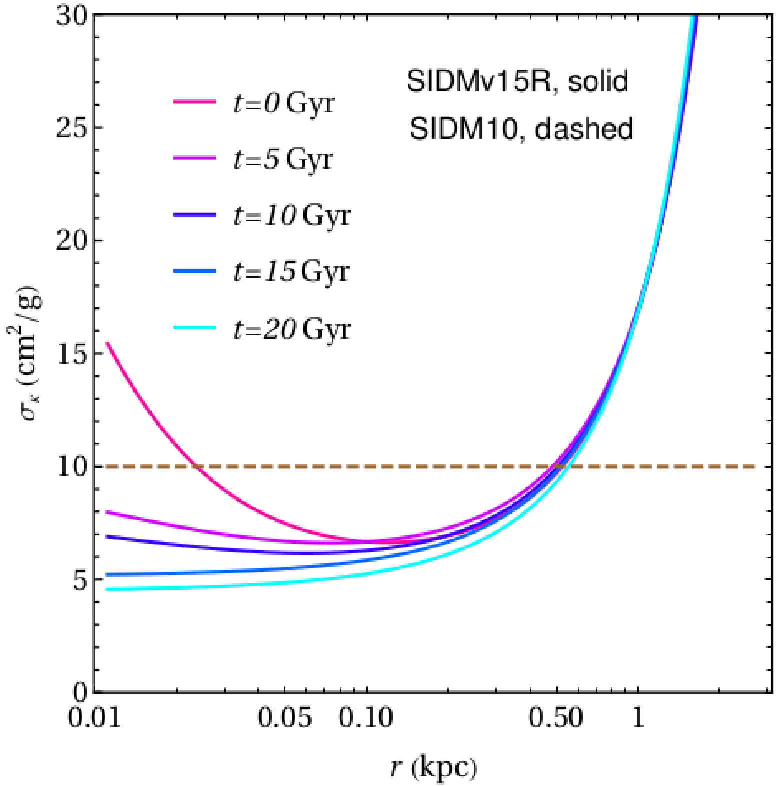
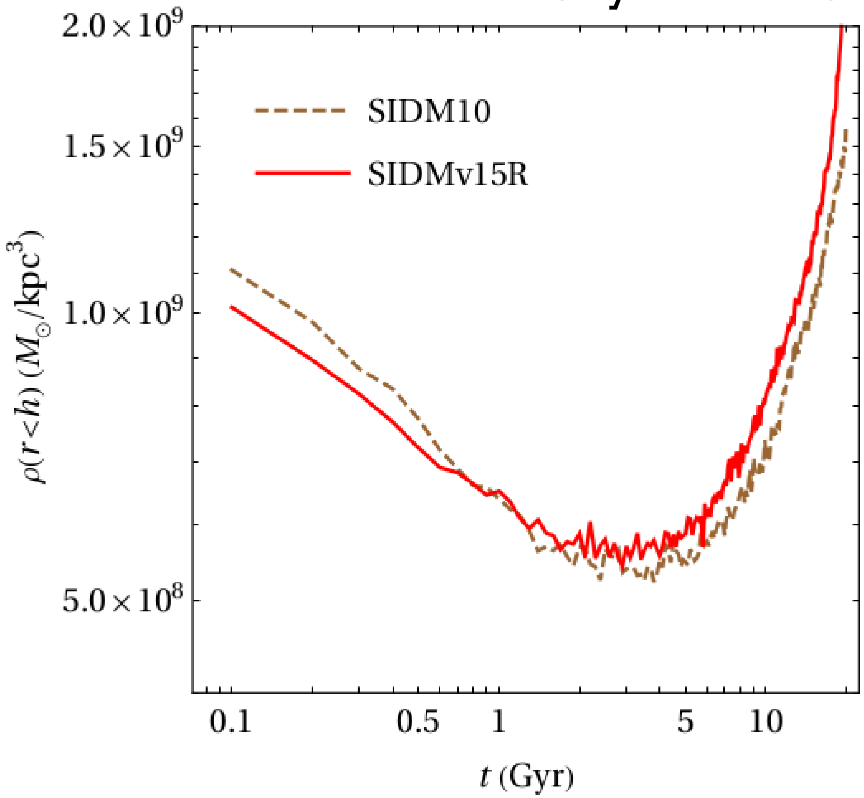
# Halo level characteristics

- [astro-ph: 2205.03392]
- [astro-ph: 2206.05578]

# We can further eliminate the radius dependence




N-body simulation




SIDM10 approximate the differential cross section pretty well

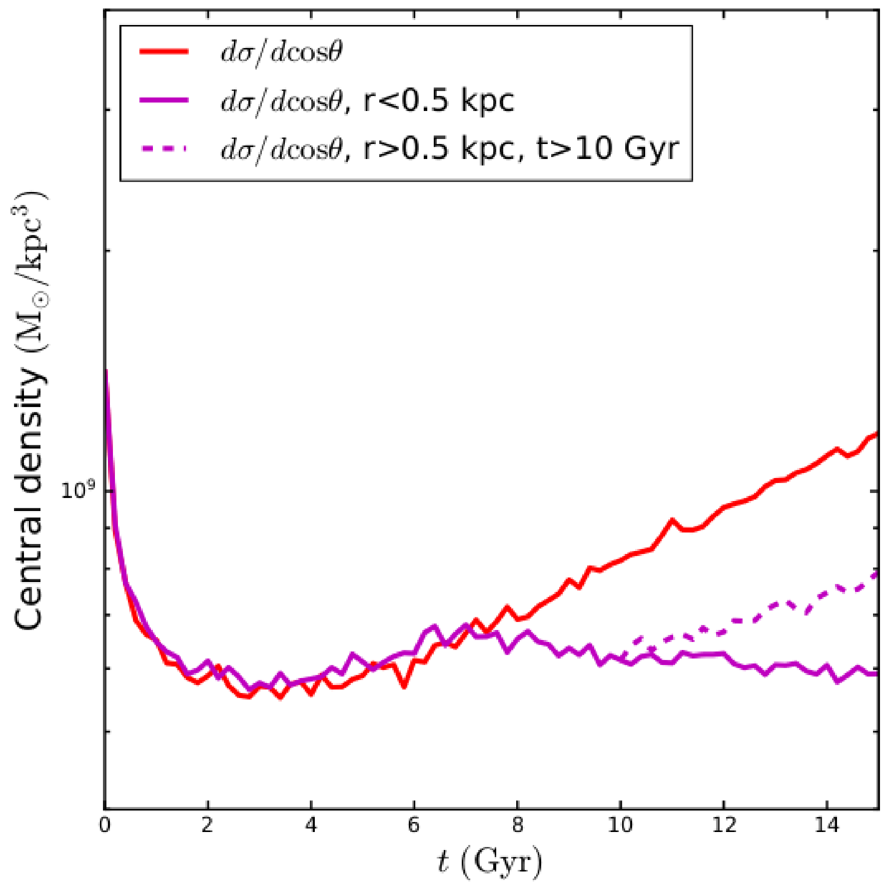
There is a characteristic velocity dispersion: $\nu_{\rm eff} = V_{\rm max}/\sqrt{3}$

Could SIDM effect at large radii be more important at late times?!

N-body simulation

**Yes!**

- No core-collapse without SIDM at r>0.5 kpc (rs~0.14 kpc)
- *Core-collapse* reappears if turning it back on at t=10 Gyr




SIDM at large radii plays a crucial role!

# A constant effective cross section

$$\sigma_{\rm eff} = \frac{2\int dv d\cos\theta \frac{d\sigma(v)}{d\cos\theta}\sin^2\theta v^7 e^{-\frac{v^2}{4\nu_{\rm eff}^2}}}{\int dv d\cos\theta \sin^2\theta v^7 e^{-\frac{v^2}{4\nu_{\rm eff}^2}}}$$

$$= \frac{1}{512\nu_{\rm eff}^8}\int dv d\cos\theta \frac{d\sigma(v)}{d\cos\theta} v^7 \sin^2\theta e^{-\frac{v^2}{4\nu_{\rm eff}^2}}$$

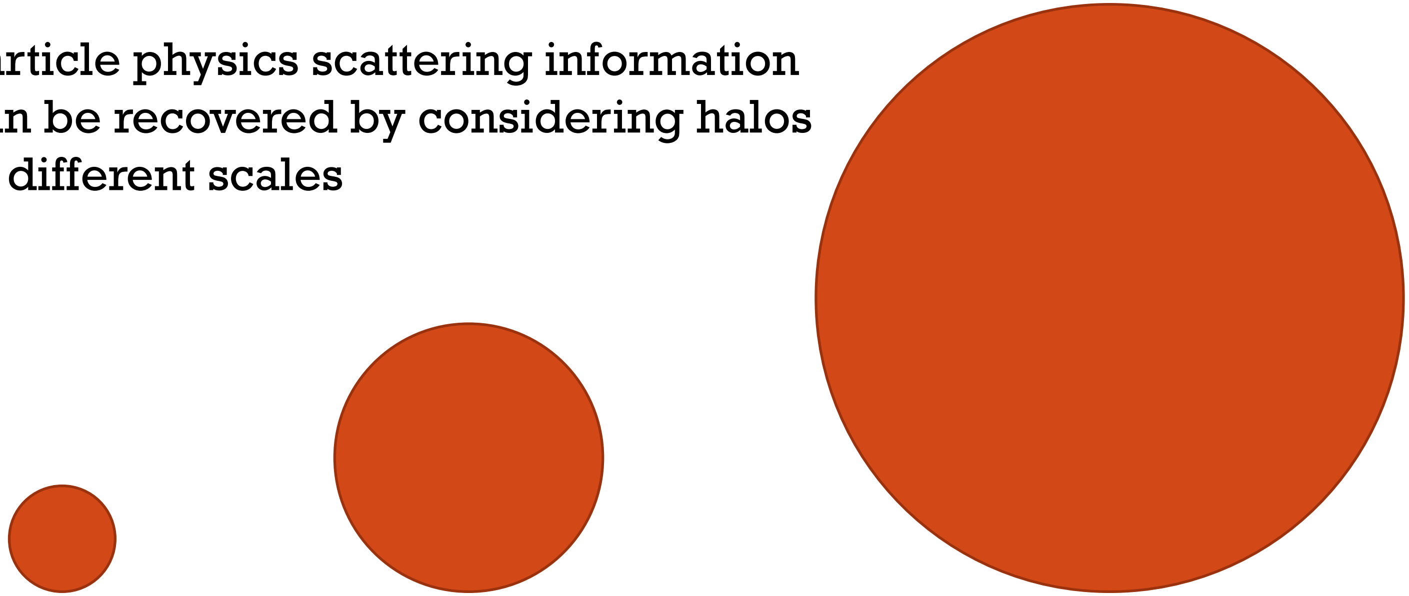
$$\nu_{\rm eff} = V_{\rm max}/\sqrt{3}$$

- *Details of the particle dynamics are hidden in a single halo*
- Only the velocity dependence couples to the halo velocity dispersion
- A characteristic velocity scale can be used to capture the majority of the correlation
- Only *at late times* of the core-collapse, there will be a deviation

*See also, astro-ph:2204.06568 astro-ph: 2205.02957 for a discussion on the universal gravothermal evolution in the conducting fluid model*

➢ Need to study a population of halos to probe details of the scattering

Particle physics scattering information can be recovered by considering halos of different scales



| | **Halo 1** | **Halo 2** | **Halo 3** |
|---|---|---|---|
| Model 0 | SIDM 1 | SIDM 1 | SIDM 1 |
| Model 1 | SIDM 10 | SIDM 1 | SIDM 0.1 |
| Model 2 | SIDM 100 | SIDM 10 | SIDM 0.01 |

- Halo level characteristics can be assigned to nodes in a **graph**, enabling graph neural network studies

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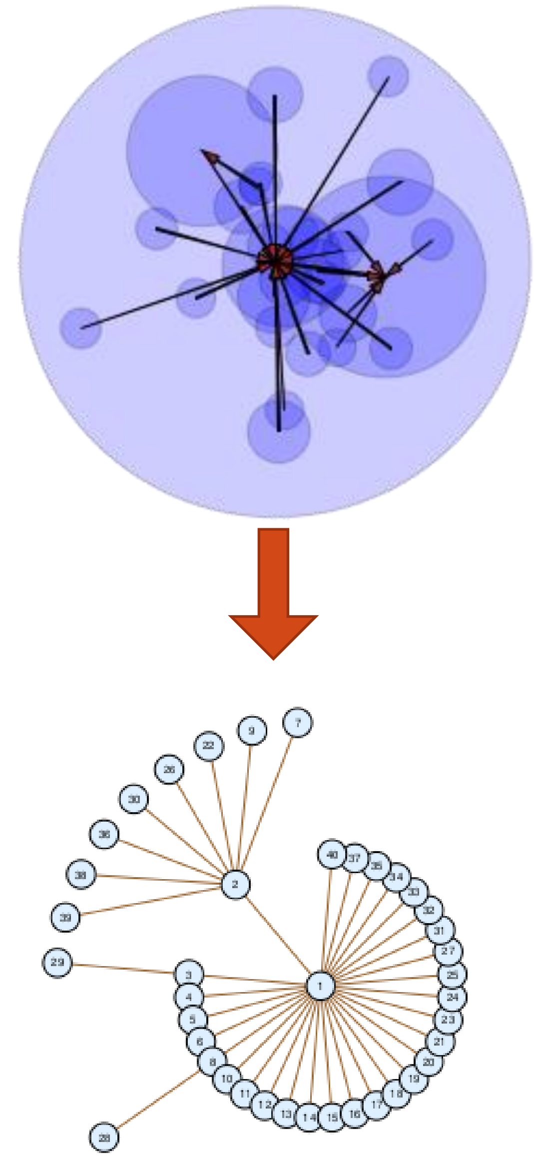
# A network of dark matter halos

- [astro-ph: 2205.03392]
- [astro-ph: 2206.05578]

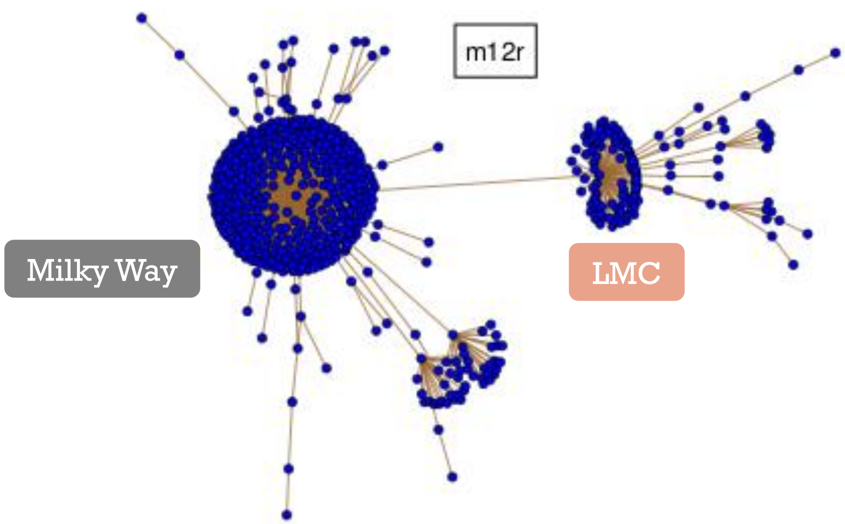
# Graph/networks from dark matter halos

Example: N = 40 most massive halos FIRE2 m12r

$$\{N_{\mathrm{sub},1}, N_{\mathrm{sub},2}, N_{\mathrm{sub},3}, N_{\mathrm{sub},8}\} = \{29, 8, 1, 1\}$$

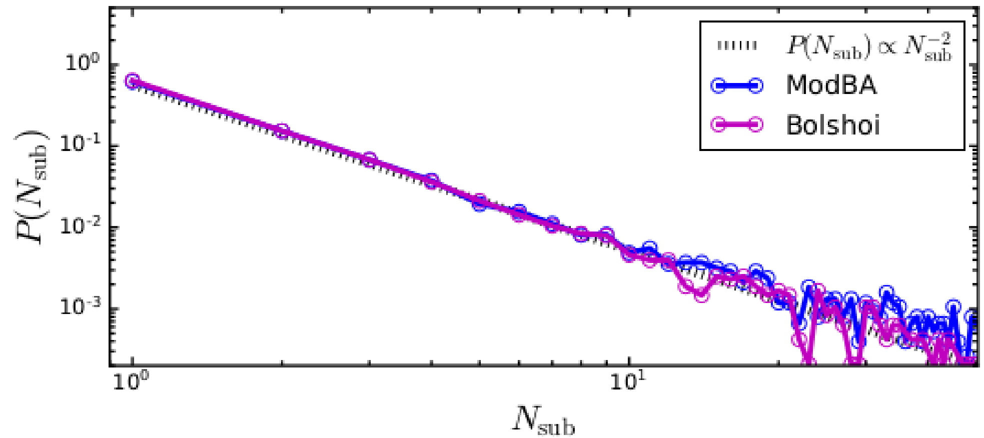


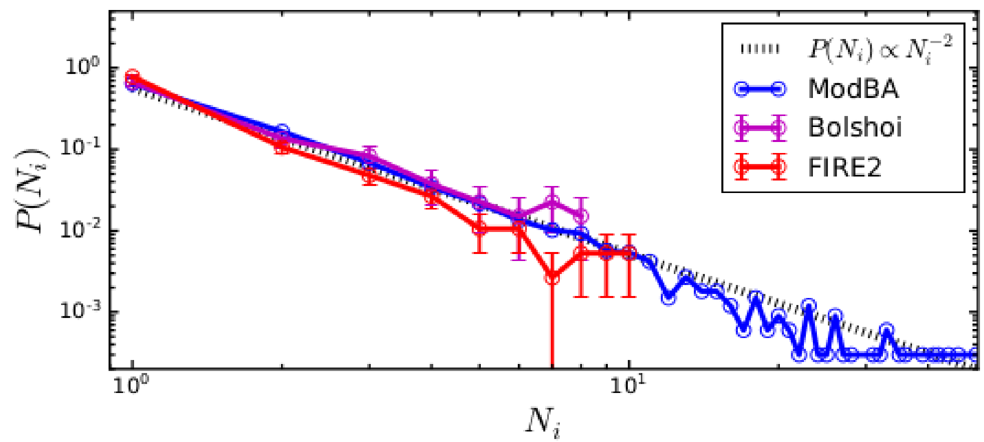
If consider N=600



Our recent work

- A random graph model for the clustering of halos
- Effectively incorporates the major merger, minor merger, and tidal effects
- Provides a new example of scale-free network




[2206.05578] A graph model for the clustering of dark matter halos (arxiv.org)

# Possible applications

Topological data analysis

- Semi-analytic model building
- Metrics based on the adjacency matrix could have a sensitivity to scale-violating physics

Graph neural network

- Halo level characteristics can be encoded into *weights* of nodes/links
- Graph Fourier Transformation + Low pass filter

$$\mathbf{h}_u^{(k+1)} = \text{UPDATE}^{(k)}\left(\mathbf{h}_u^{(k)}, \text{AGGREGATE}^{(k)}(\{\mathbf{h}_v^{(k)}, \forall v \in \mathcal{N}(u)\})\right)$$

- Has been applied to study halo shape, orientation, etc *(P. Villanueva-Domingo et al. 2021,2022, Jagvaral Y. et al, 2022)*

# SUMMARY

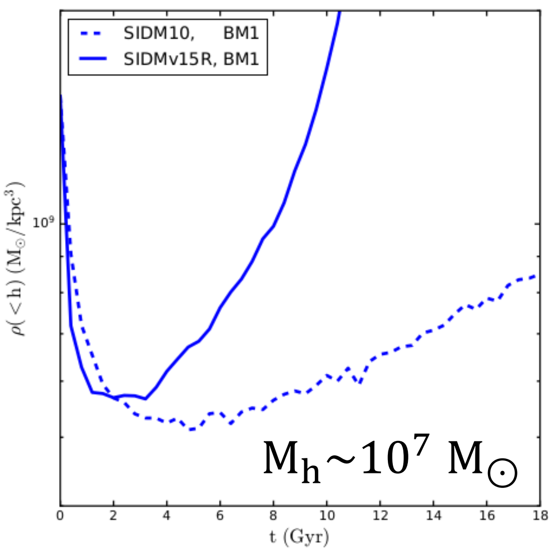
- SIDM leads to novel gravothermal evolution of a halo.
- The effect of a differential cross section is largely degenerates to a constant cross section in a single halo.
- A population of halos can be used to probe the scattering structures.
- It is promising to apply graph-based techniques to explore small-scale physics.

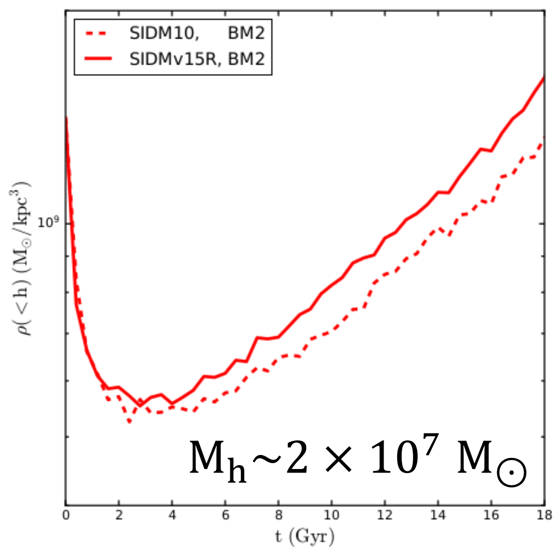
## OUTREACH

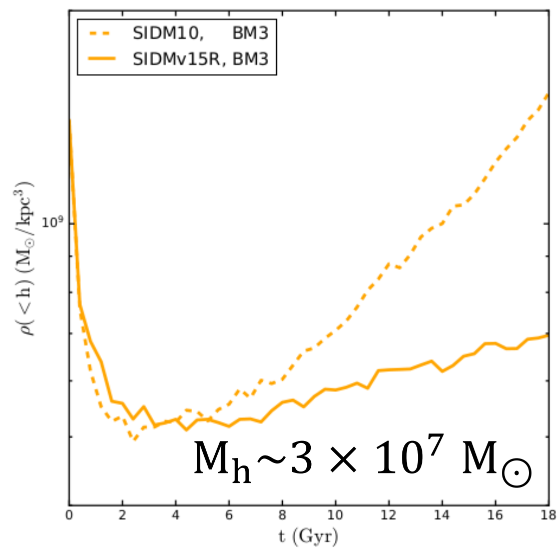
- Graph neural network
- Persistent homology
- Deep learning and preferential attachment

# 26 BACKUP

## The gravothermal evolution is sensitive to a change in the halo parameters



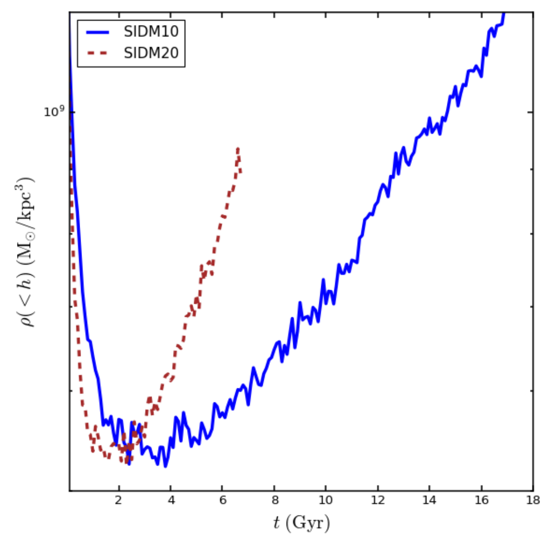




… and in a change of the effective cross section

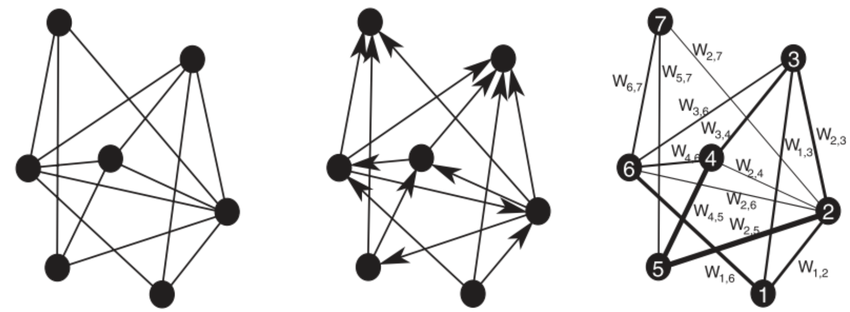
SIDM10

SIDM20

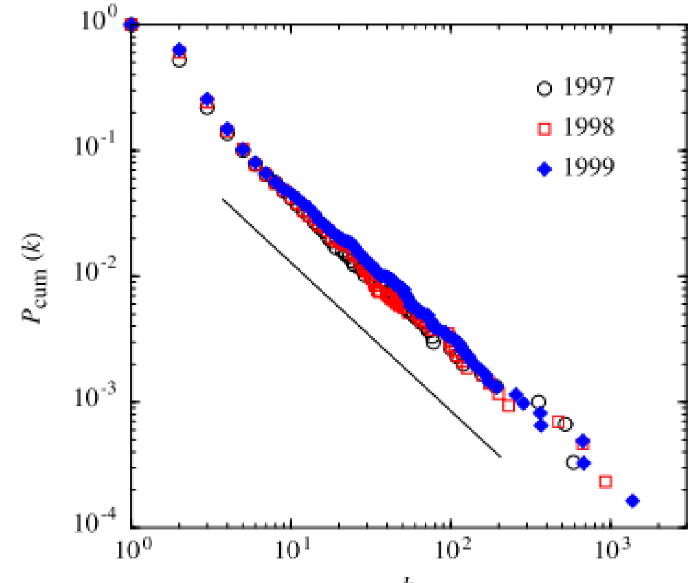
$M_h \sim 2 \times 10^7\ M_\odot$



# Power-law networks from complex systems


(a) (b) (c)

| Network | $N$ | $\gamma$ |
|---|---|---|
| AS2001 | 11,174 | 2.38 |
| Routers | 228, 263 | 2.18 |
| Gnutella | 709 | 2.19 |
| WWW | $\sim 2 \times 10^8$ | 2.1/2.7 |
| Protein | 2,115 | 2.4 |
| Metabolic | 778 | 2.2/2.1 |
| Math1999 | 57, 516 | 2.47 |
| Actors | 225,226 | 2.3 |
| e-mail | 59,812 | 1.5/2.0 |



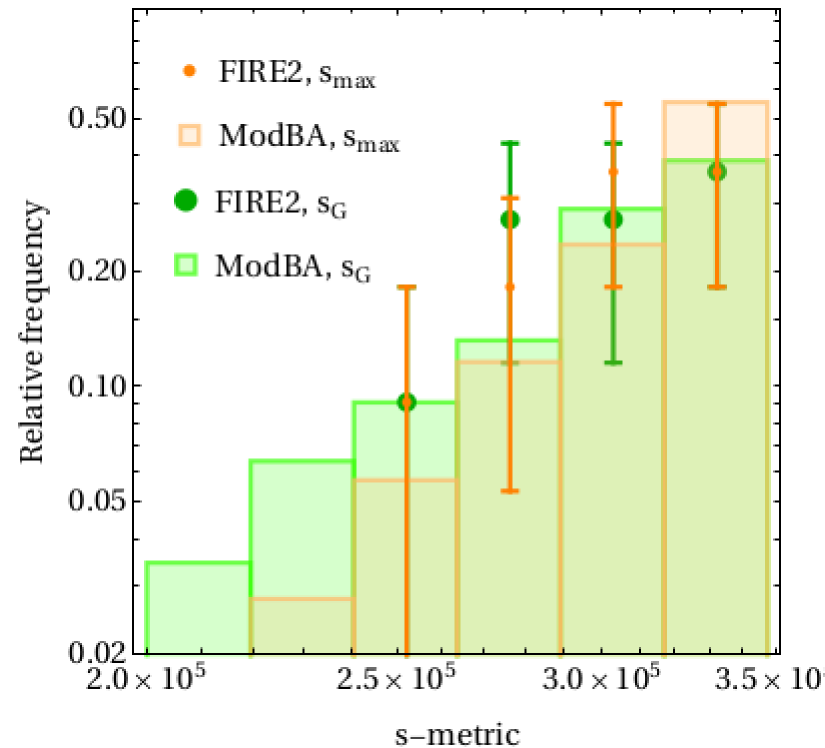
Cumulative degree distributions of the Internet graphs for three different years.

*Sources: S. Boccaletti et al. / Physics Reports 424 (2006) 175–308*

# Scale-free networks

Properties (Li Lun, et al 2006)
- Power-law degree distribution
- Hub-like core
- Preserved by random degree-preserving rewiring
- Self-similar

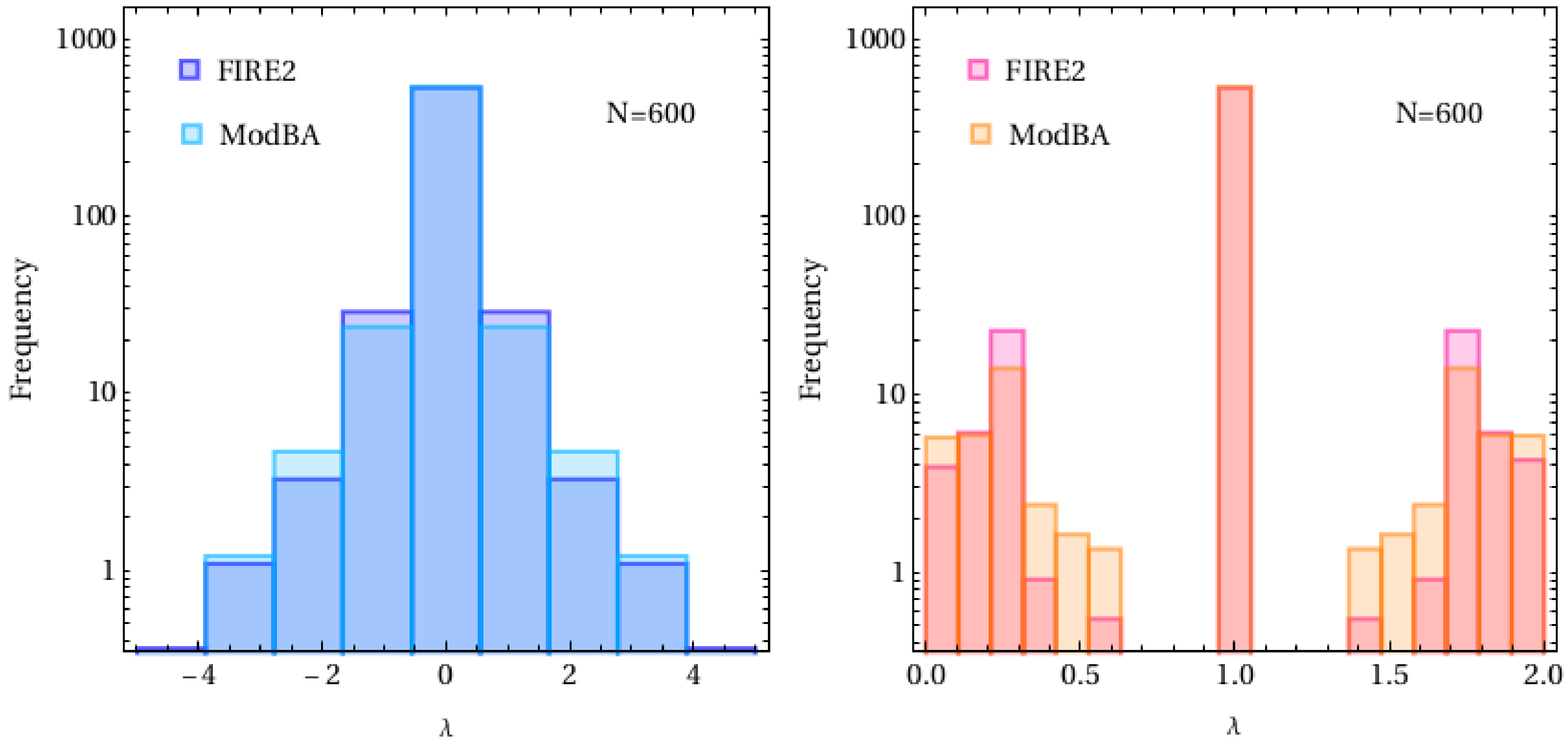
$$s_G = \sum_{(i,j)\in E} k_i k_j.$$




Given a degree sequence D={$k_1, k_2, \ldots, k_N$}, one can construct a graph maximizing $s_G$ (Li Lun, et al 2006)
- $s_G/\ s_{max}$=0.98 (**FIRE2 simulations**)
- $s_G/\ s_{max}$=0.93 (**Model constructions**)

**Preferential attachment naturally leads to scale-free networks: Early attachment advantage**

# Model vs. Simulation




Eigenvalue spectra of the adjacency matrix and the Laplacian matrix

- Adjacency matrix: $X_{ij} = X_{ji}$=1 if i,j nodes are connected
- Normalized Laplacian matrix: $L$=$I$-$D^{-1/2}XD^{-1/2}$