



UNIVERSITY OF
CAMBRIDGE



The Stephen Hawking
Centre for Theoretical Cosmology

Boostless Cosmological Collider Bootstrap

with Guilherme Pimentel, arXiv:2205.00013

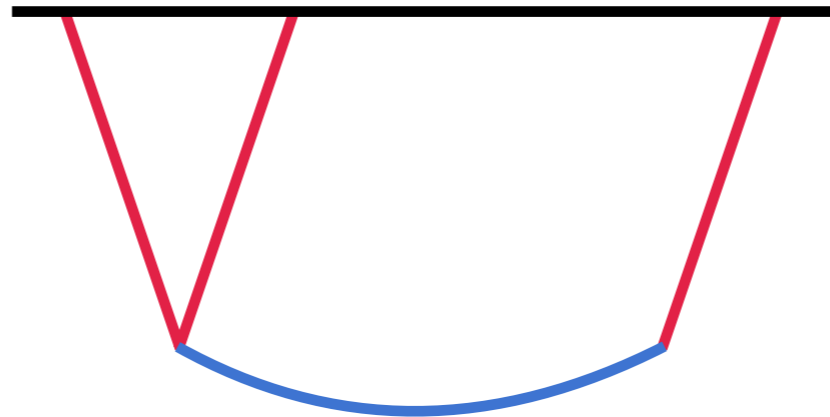
Dong-Gang Wang (王东刚)

DAMTP Cambridge

Cosmology from Home 2022

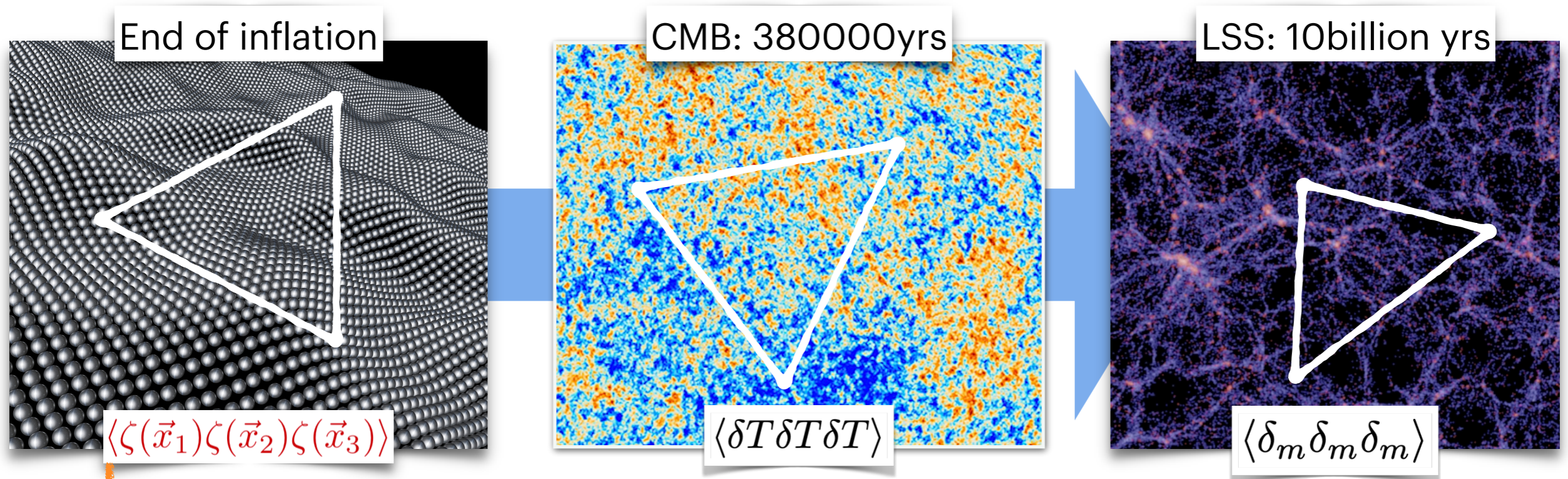
Goal:

Find all Cosmological Colliders using the **Bootstrap** method, and identify the large signals and new shapes for upcoming observations.



Triangles in the Sky

3-point correlation function in the primordial perturbations



→ **primordial bispectrum**: the Fourier transf. of the 3pt correlation function

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \sim f_{\text{NL}} S(k_1, k_2, k_3) P_\zeta^2$$

How easy/hard
to be detected

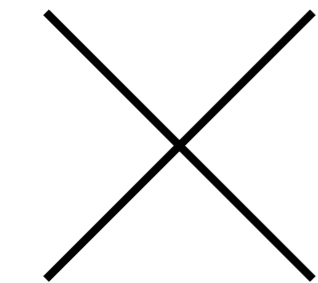
size

shape

Lots of information;
Focus of this talk
Major target of CMB, LSS, 21cm..

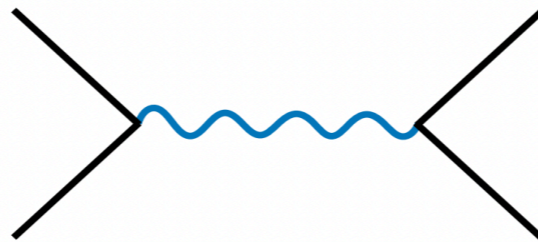
Fantastic New Physics and Where to Find Them

Scattering Amplitudes

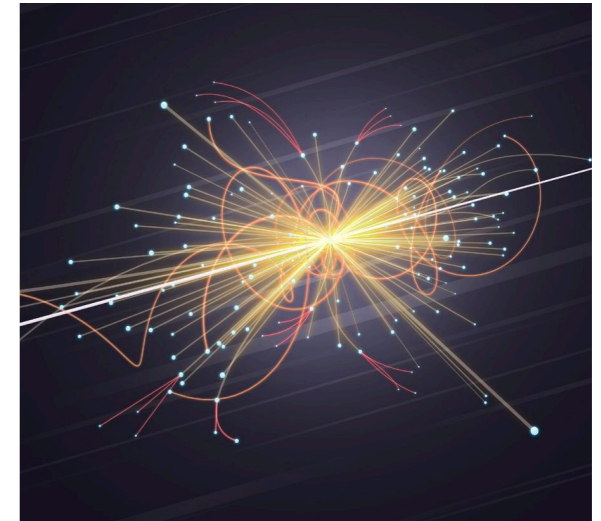
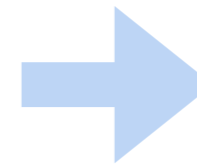


Contact diagram

+

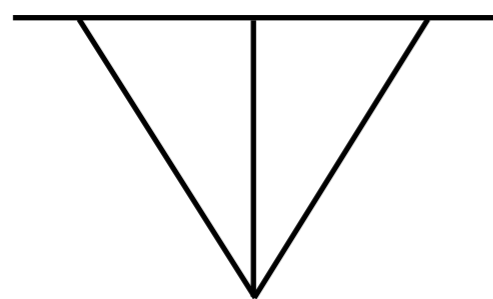


Exchange diagram



$\sim 1\text{TeV}$

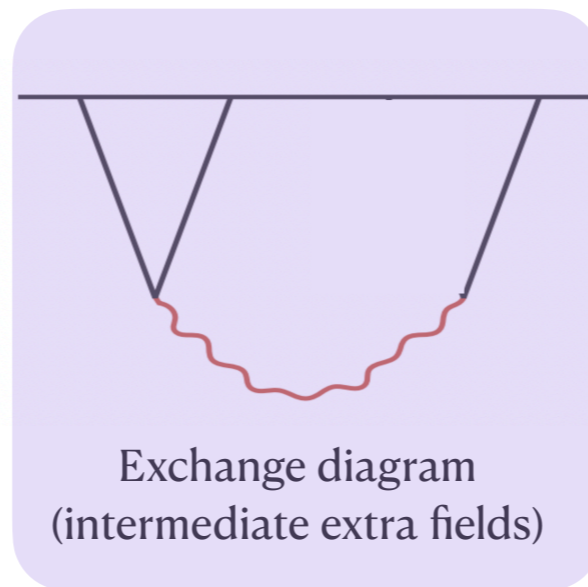
Cosmological Correlators $\langle \zeta(x_1)\zeta(x_2)\zeta(x_3) \rangle$



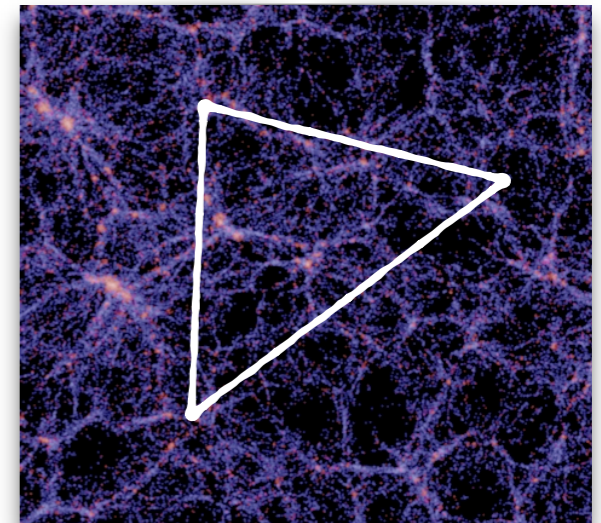
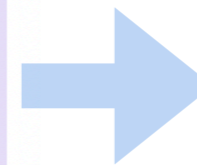
Contact diagram
(Self-interaction of the inflaton)

$\eta = 0$

+



Exchange diagram
(intermediate extra fields)



$< 10^{14}\text{GeV}$

Cosmological Collider Physics

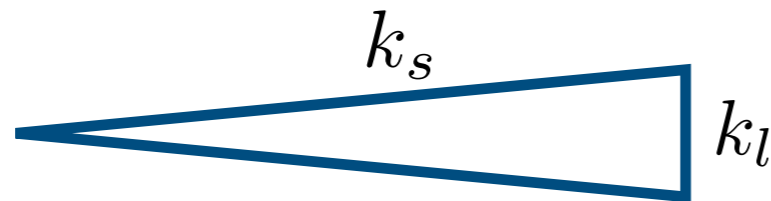
Squeezed limit of the inflationary bispectrum

Arkani-Hamed, Maldacena 2015

Chen, Wang 2009

Baumann, Green 2010

Noumi et al 2012



contains information of heavy particles in the high energy environment of inflation

$$\lim_{k_l \rightarrow 0} \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_s} \zeta_{\mathbf{k}_s} \rangle \propto \left(\frac{k_l}{k_s} \right)^{3/2} \cos \left[\frac{m}{H} \ln \left(\frac{k_l}{k_s} \right) + \delta \right] P_s(\mathbf{k}_1 \cdot \mathbf{k}_s)$$

oscillation measures mass, **angular** dependence measures spin

A Large Menu of Possibilities

EFT of cosmo collider

Lee, Baumann, Pimentel 2016

Standard Model mass spectrum

Chen, Wang, Xianyu 2016

Non-perturbative regimes

An, McAneny, Ridgway, Wise 2017

Gauge theories + GUT

Kumar, Sumdrum 2017; 2018

Chemical Potential

Wang, Xianyu 2019

SM fermions and Higgs

Hook, Huang, Racco 2019

Curved field manifold

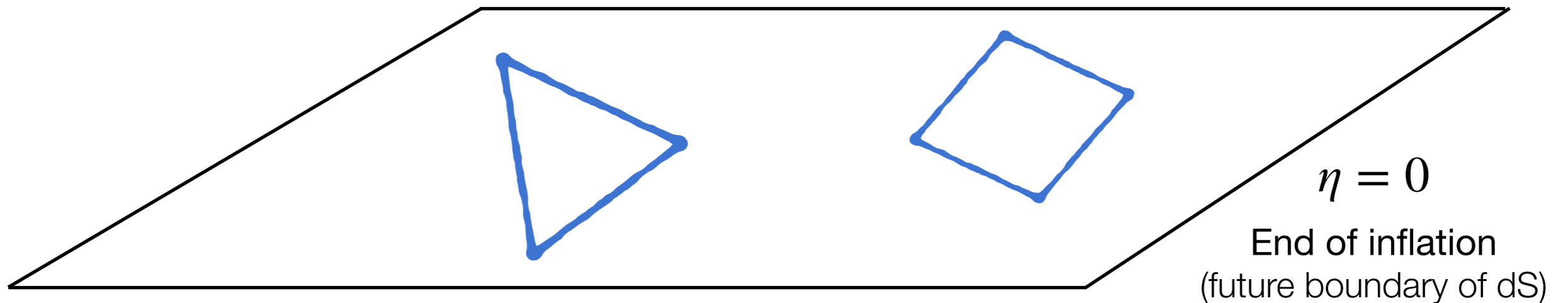
DGW 2019

.....

This Talk:

Find all Cosmo Collider bispectra via

Cosmological Bootstrap



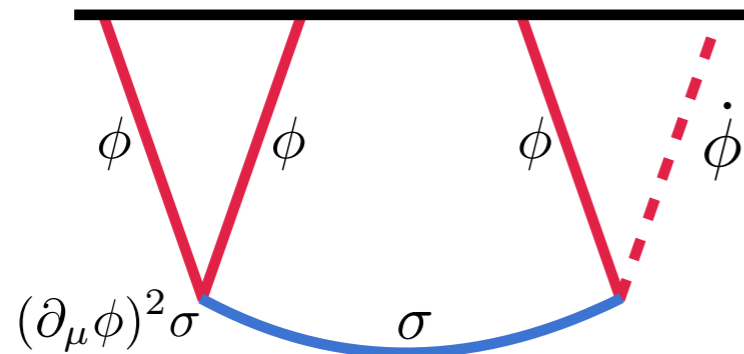
Cosmological Correlators from Symmetries, Locality & Unitarity

- Fully model-independent;
- Powerful computational tools.

See [H. Goodhew's talk](#)
for general introduction

De Sitter Bootstrap

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Baumann, Duaso Pueyo, Joyce, Lee,
Pimentel 2019, 2020



Pros

dS symmetries are nicely manifested;
Fully analytical control for correlators;

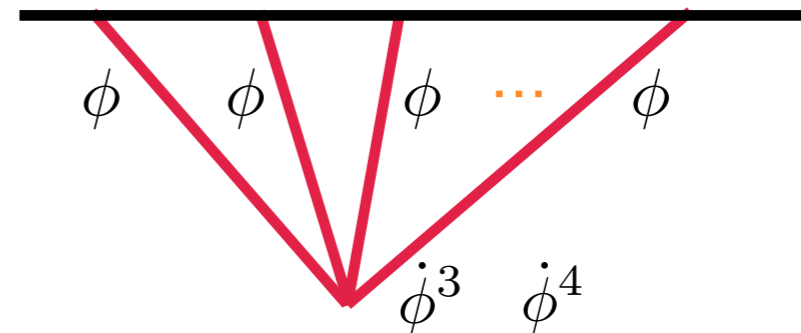
Cons

Non-Gaussianity signals are *very small*;
Need 4-pt first, before computing 3-pt.

v.s.

Boostless Bootstrap

Pajer 2020
Jazayeri, Pajer, Stefanyszyn 2021
Bonifacio, Pajer, DGW 2021;



Pros

Large signals are possible;
A complete set of single field correlators.

Cons

Only for correlators from massless
field interactions

See M.H. G. Lee and A. Thavanesan's talks

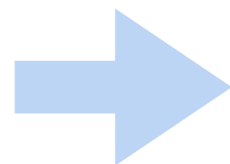
De Sitter Bootstrap + Boostless Bootstrap



Boostless Cosmological Collider Bootstrap

Pimentel, DGW 2022

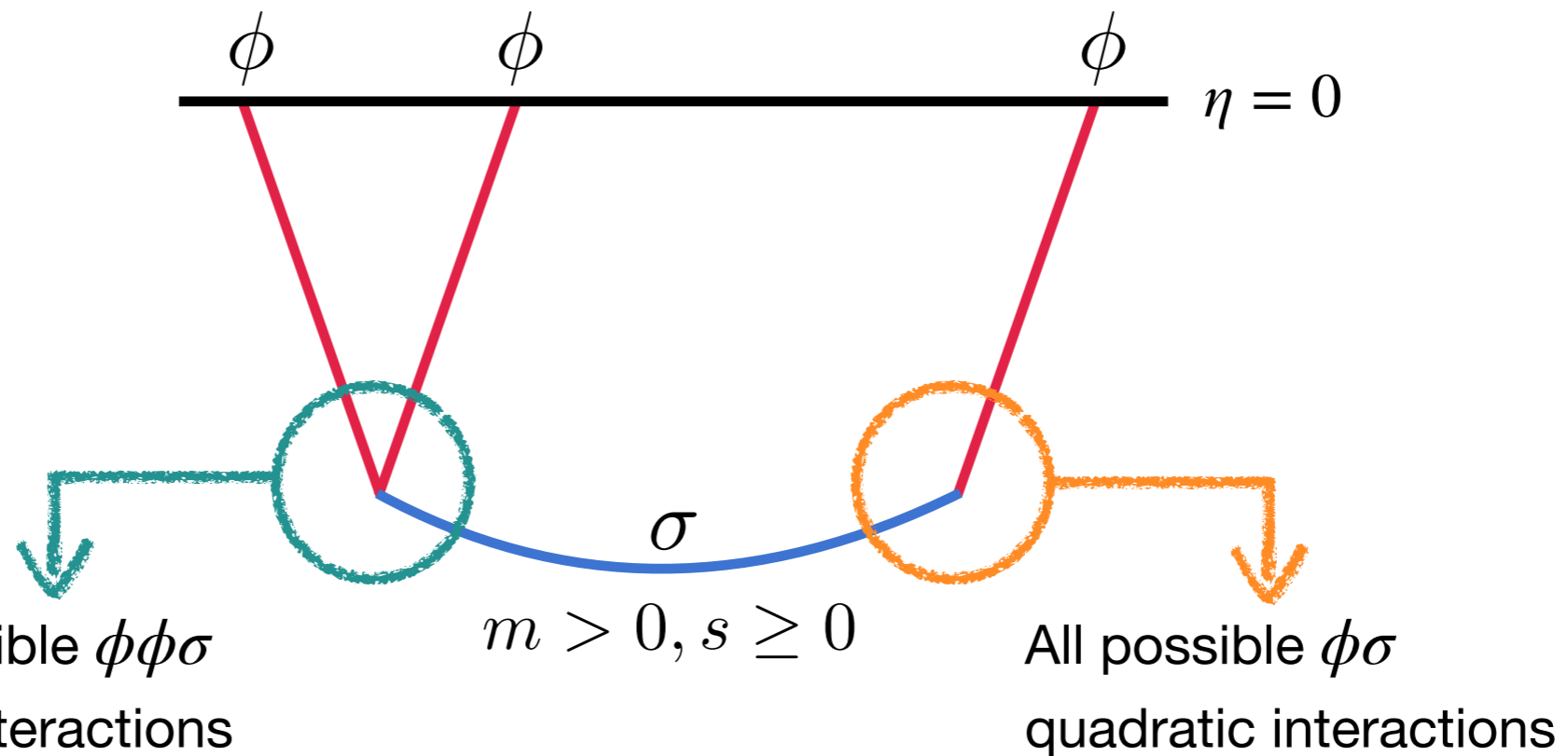
- Large **boost-breaking interactions** with massive particles;
- Bootstrap inflationary **three-point functions** directly;



A **complete menu** of possibilities for massive-exchange bispectra.

To be more specific...

Inflaton Bispectrum $\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle$



$$\dot{\phi}^2 \sigma, (\partial_i \phi)^2 \sigma, \ddot{\phi}^2 \sigma, \dots$$

scalar exchange

$$\dot{\phi} \sigma, \partial^2 \phi \sigma, \ddot{\phi} \sigma, \dots$$

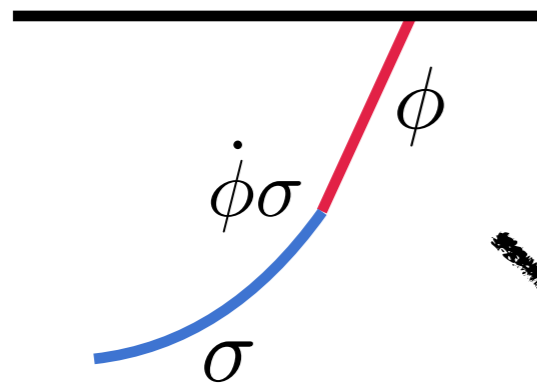
$$\dot{\phi} \partial_{ij} \dots \dot{\phi} \sigma_{ij} \dots, \dots$$

spinning exchange

$$\partial_{ij} \dots \dot{\phi} \sigma_{ij} \dots, \dots$$

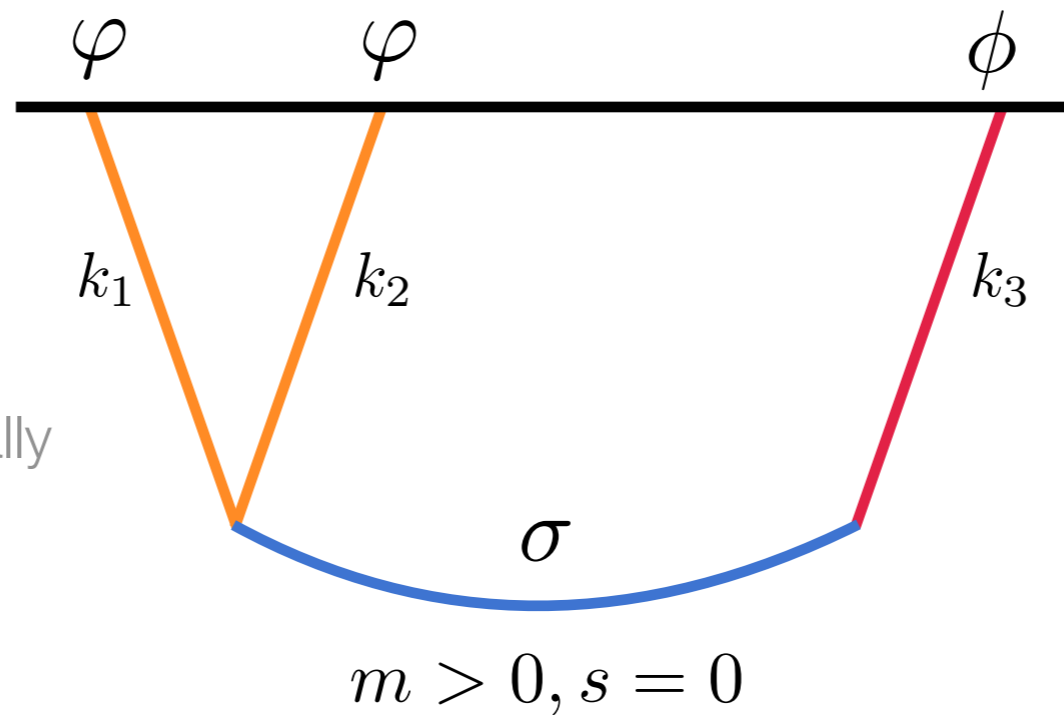
The Building Blocks of BCCB

0. A Mixed Propagator



I. Three-Point Scalar Seed $\langle \varphi_{k_1} \varphi_{k_2} \phi_{k_3} \rangle \sim \hat{I}(u)$

the conversion from massive scalar σ to the inflaton ϕ



$$u \equiv \frac{k_3}{k_1 + k_2}$$

φ — conformally coupled scalar
 $m^2 = 2H^2$

$$m > 0, s = 0$$

I. Solving the Scalar Seed

A differential equation for the scalar seed

$$\left(\Delta_u + \frac{m^2}{H^2} \right) \hat{\mathcal{I}} = \frac{u}{1+u}$$

with

$$\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$$

$$u \equiv \frac{k_3}{k_1 + k_2}$$

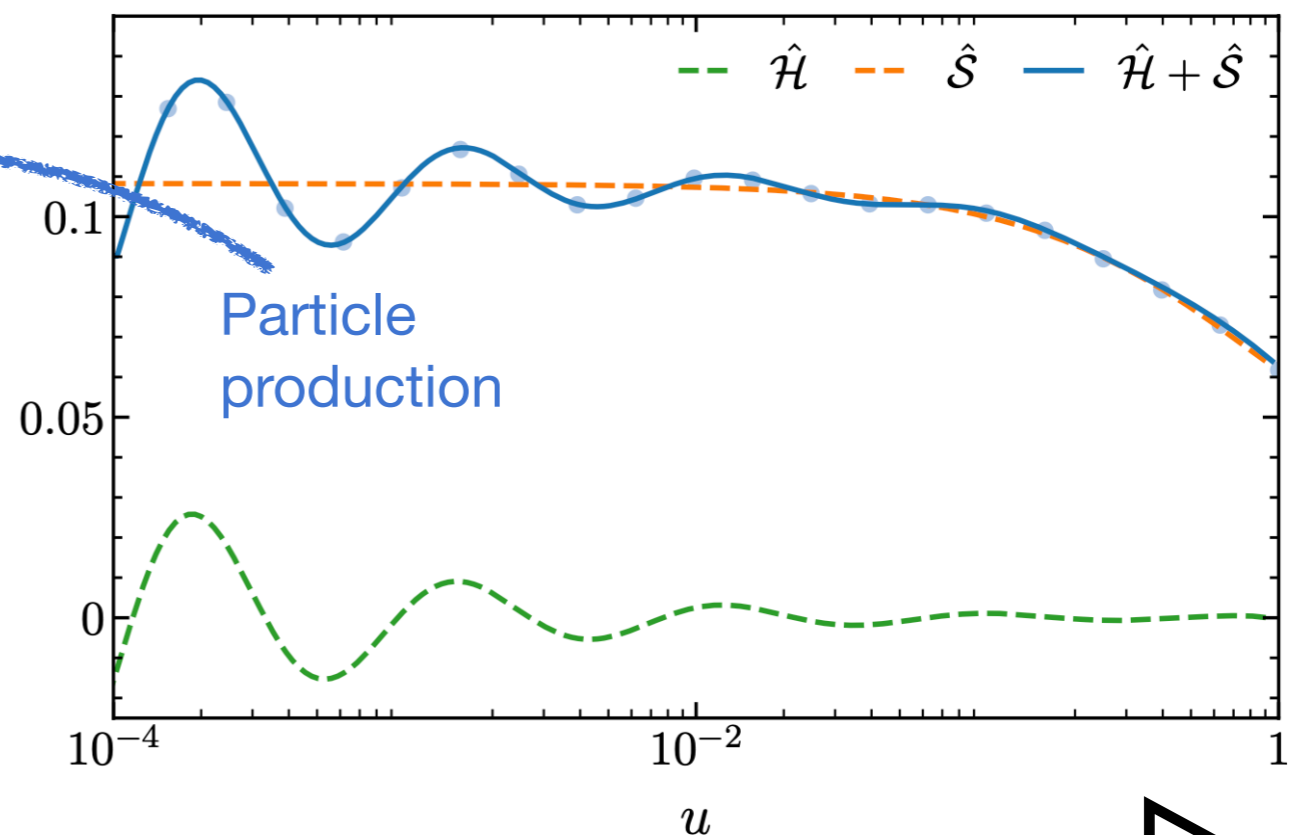
Analytical solution:

$$\hat{\mathcal{I}}(u)/u$$

The squeezed-limit oscillations caused by massive particles

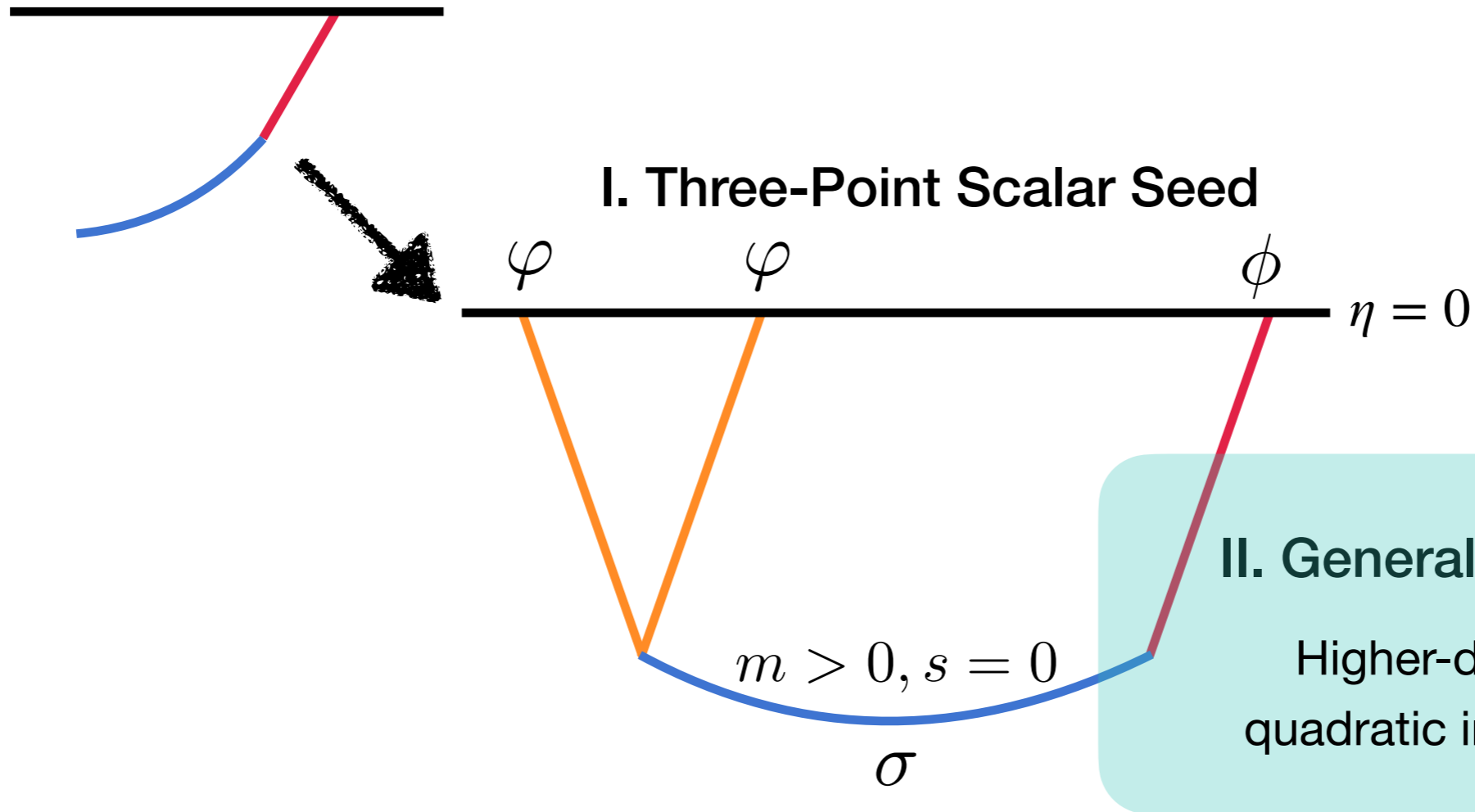
$$\hat{\mathcal{I}}(u \rightarrow 0) = -\frac{i}{2} \sum_{\pm} B_{\pm} \left(\frac{u}{2} \right)^{\frac{1}{2} \pm i\mu} \propto \sin \left(\frac{M}{H} \log(k_L/k_S) \right)$$

Cosmological Collider Signal



Strategy of BCCB

0. A Mixed Propagator



$$\left(\Delta_u + \frac{m^2}{H^2} \right) \hat{\mathcal{I}}^{(n)} = \left(\frac{u}{1+u} \right)^{n+1}$$

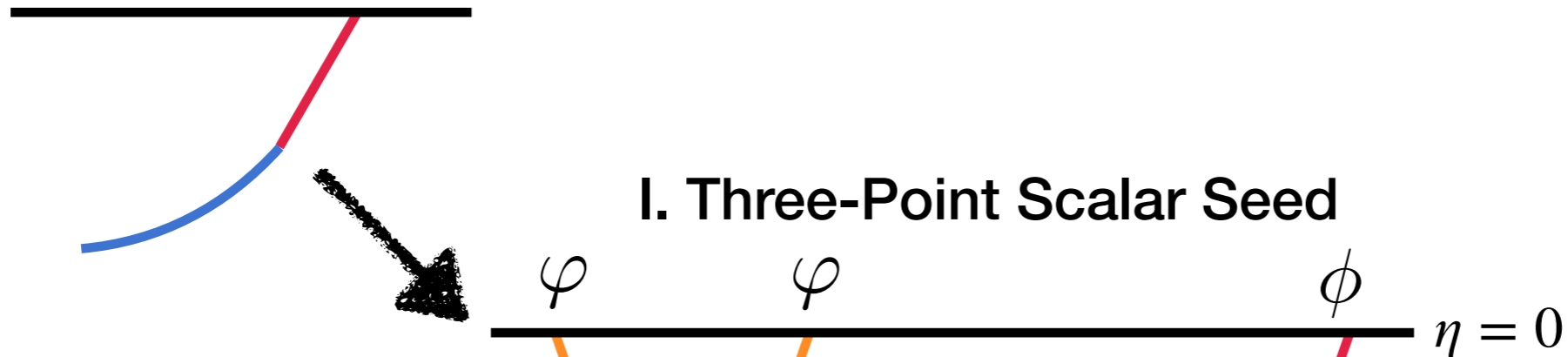
$$\dot{\phi}\sigma \longrightarrow a^{-n\partial_i} \partial_i^{n\partial_i} \left(\partial_t^{n\partial_t} \phi \right) \left(\partial_t^{\tilde{n}\partial_t} \sigma \right)$$

$$\hat{\mathcal{I}}(u) \longrightarrow \hat{\mathcal{I}}^{(n)}$$

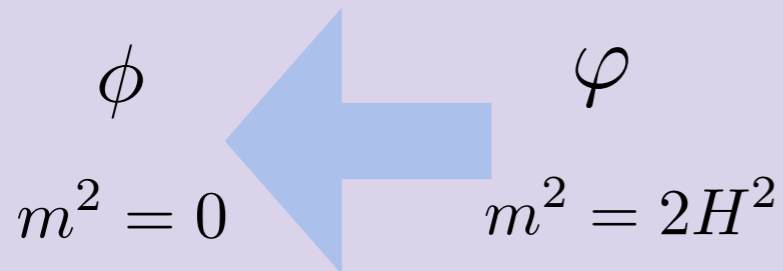
(for any number of time and spatial derivatives)

Strategy of BCCB

0. A Mixed Propagator



III. Weight-Shifting



II. Generalized Seeds

Higher-derivative quadratic interactions

$m > 0, s = 0$

σ

$$\partial_i^{n_s} (\partial_t^{n_1} \phi \partial_t^{n_2} \phi \partial_t^{n_3} \sigma)$$

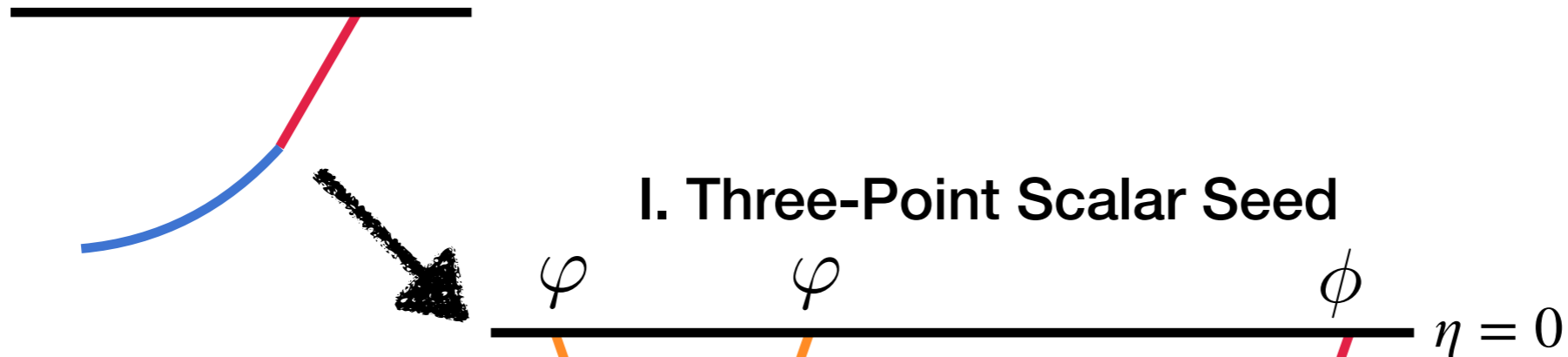
Any boost-breaking cubic interactions

Weight-shifting operators $\langle \phi \phi \phi \rangle = \mathcal{W} \langle \varphi \varphi \phi \rangle \sim \mathcal{W} \hat{\mathcal{I}}$

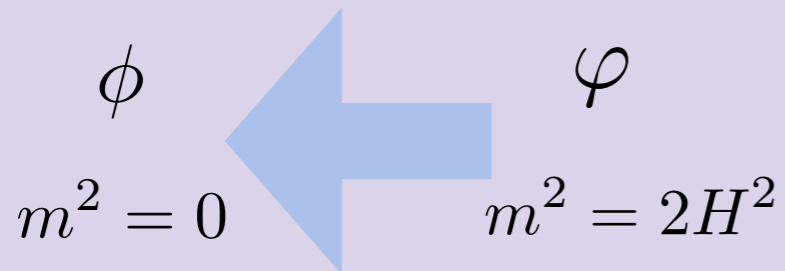
$$\mathcal{W}_{12} \equiv -c_s^{2-n_s} (\mathbf{k}_a \cdot \mathbf{k}_b)^{n_s/2} k_1^{n_1-1} k_2^{n_2-1} (1 - n_1 - k_1 \partial_{k_1}) (1 - n_2 - k_2 \partial_{k_2}) \partial_{k_{12}}^{\tilde{n}_T-2}$$

Strategy of BCCB

0. A Mixed Propagator



III. Weight-Shifting



$$m > 0, s = 0$$

$$\sigma$$

IV. Spin-Raising

$$\sigma_{\mu_1 \dots \mu_s}$$

$$m > 0, s \geq 0$$

II. Generalized Seeds

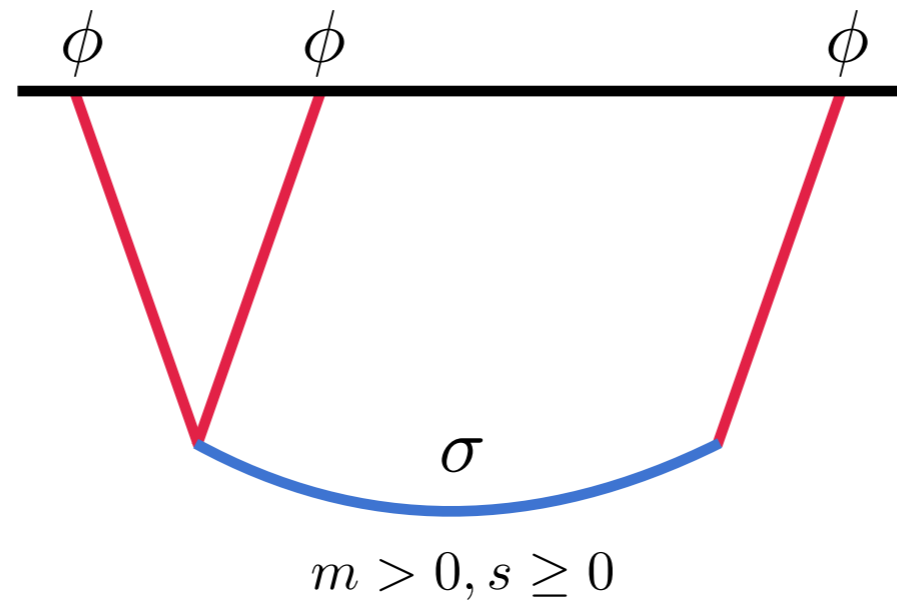
Higher-derivative
quadratic interactions

Spin-raising operators:

$$\mathcal{D}_{23}^{(s)} \equiv \sum_{m=0}^s (i c_s)^{m-s} k_3^{2s-m-1} a_m^{(s)} \mathcal{U}_{k_3}^{(m)} \partial_{k_2}^{2s-m-1}$$

One Formula to Find Them All

A complete set of inflationary bispectra from:



$$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle \sim P_s(\cos \theta) \cdot \mathcal{W} \cdot \mathcal{U} \cdot \hat{\mathcal{I}}^{(n)}$$

Weight-
shifting

Spin-
raising

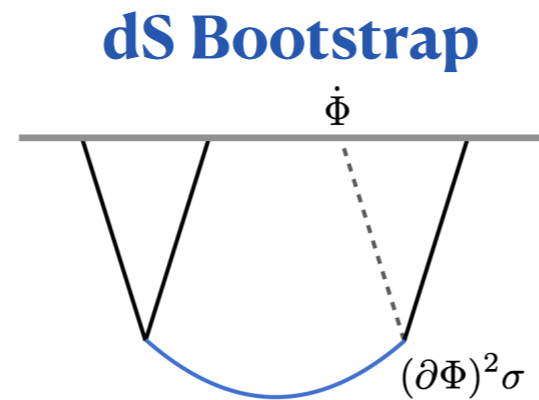
Scalar
Seeds

● Full shapes

● Large signals

● Richer pheno

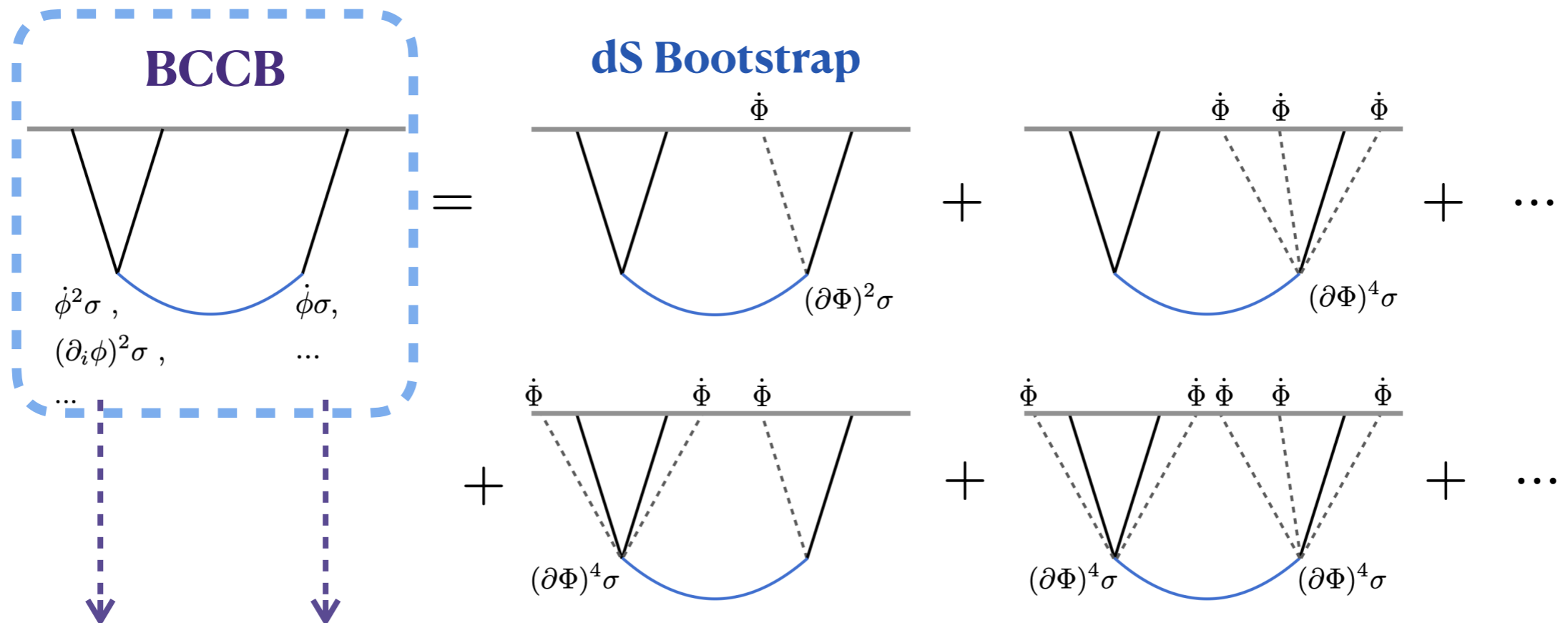
Why results from dS bootstrap are small?



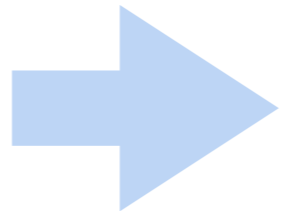
mild breaking of dS boost symmetries

$\rightarrow \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$ Slow-roll suppressed $\sim O(\epsilon)$

Signals are boosted in strong boost-breaking scenarios



Cubic & quadratic interactions from the EFT of inflation



The EFT of Cosmo Collider

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \sim f_{\text{NL}} S(k_1, k_2, k_3) P_\zeta^2$$

size \leftarrow shape

$$f_{\text{NL}} \lesssim \mathcal{O}(10)$$

Baumann, Lee, Pimentel 2016
Bordin, Creminelli, et al 2018
Pimentel, DGW 2022

Phase of Cosmological Colliders

Oscillatory signals in the squeezed limit of the shape function

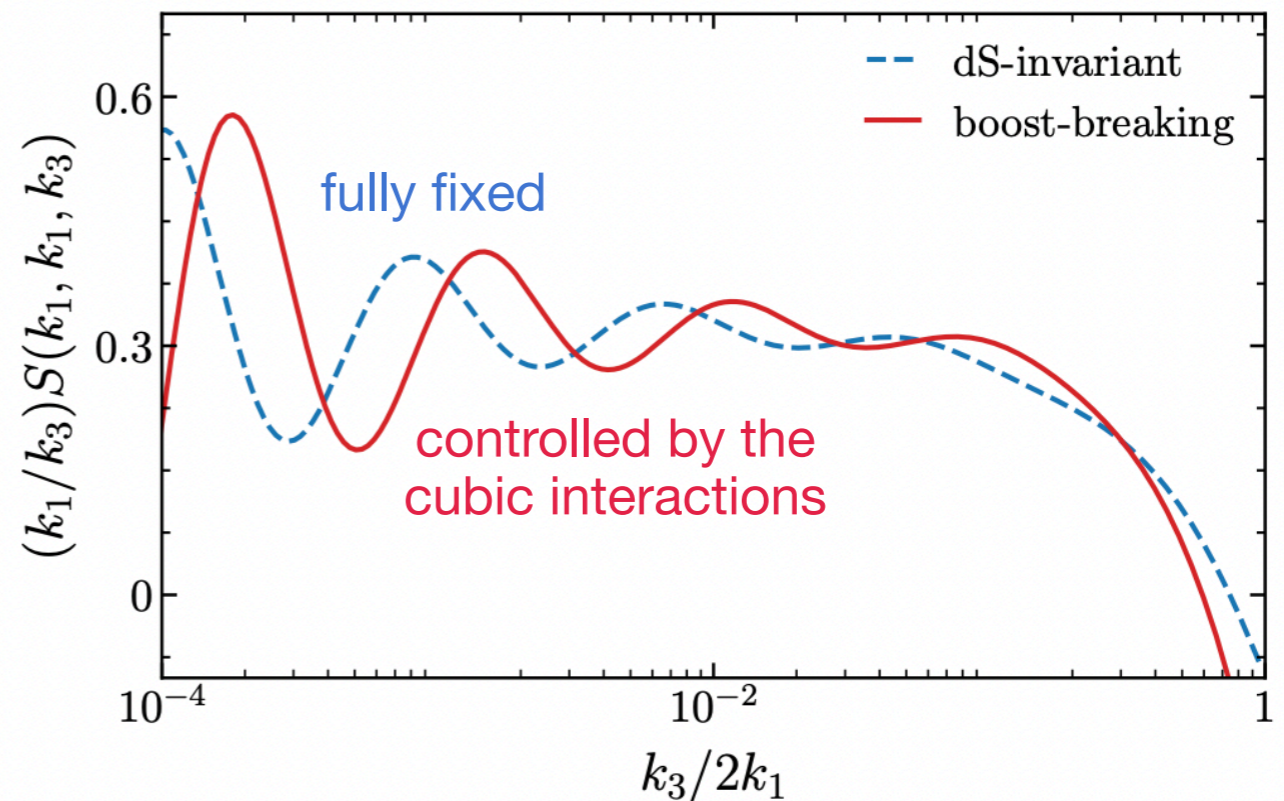
$$\lim_{k_3 \rightarrow 0} S^{(0)}(k_1, k_2, k_3) \sim \left(\frac{k_3}{k_1}\right)^{1/2} \cos \left[\underbrace{\mu \log \left(\frac{c_\sigma k_3}{4c_s k_1} \right)}_{\text{two sound speeds}} + \delta(\mu) \right]$$

phase

For dS-invariant theories:

$$c_s = c_\sigma = 1,$$

$$\delta^{\text{dS}}(\mu) = \arg \left[i \frac{\Gamma(\frac{7}{2} + i\mu)}{\Gamma(1 + i\mu)} \frac{(1 + i \sinh \pi\mu)}{\frac{1}{2} + i\mu} \right]$$



Collider Signals around the Equilateral Limit

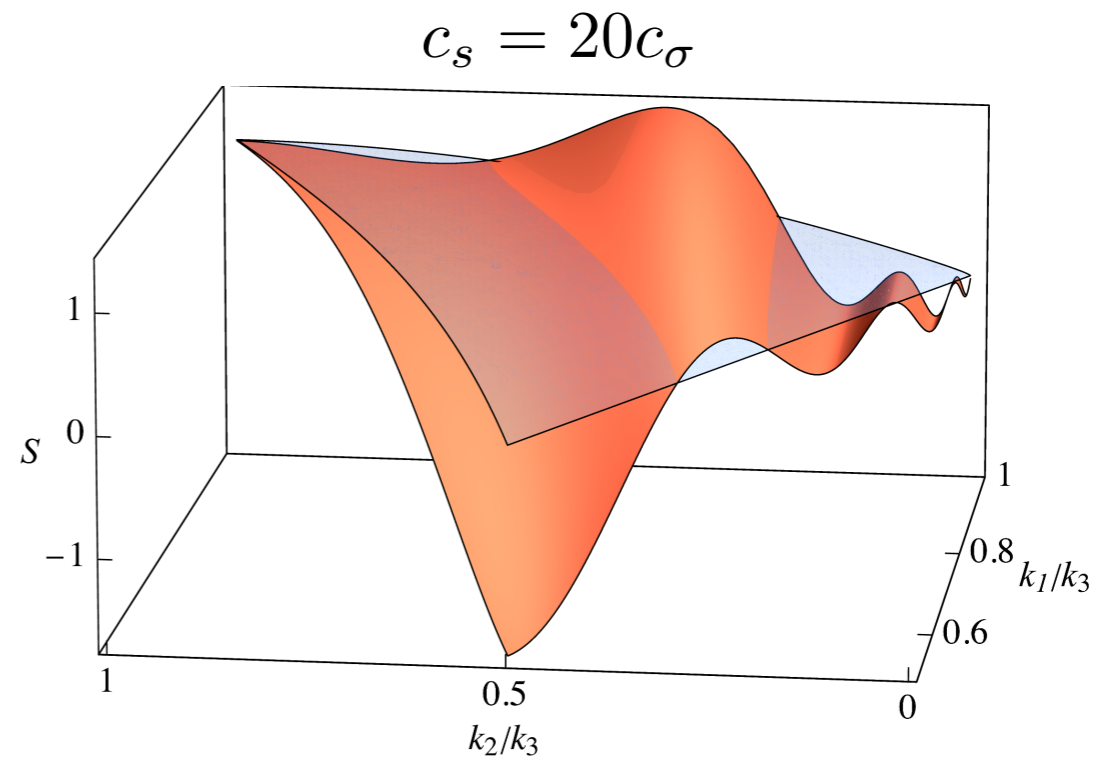
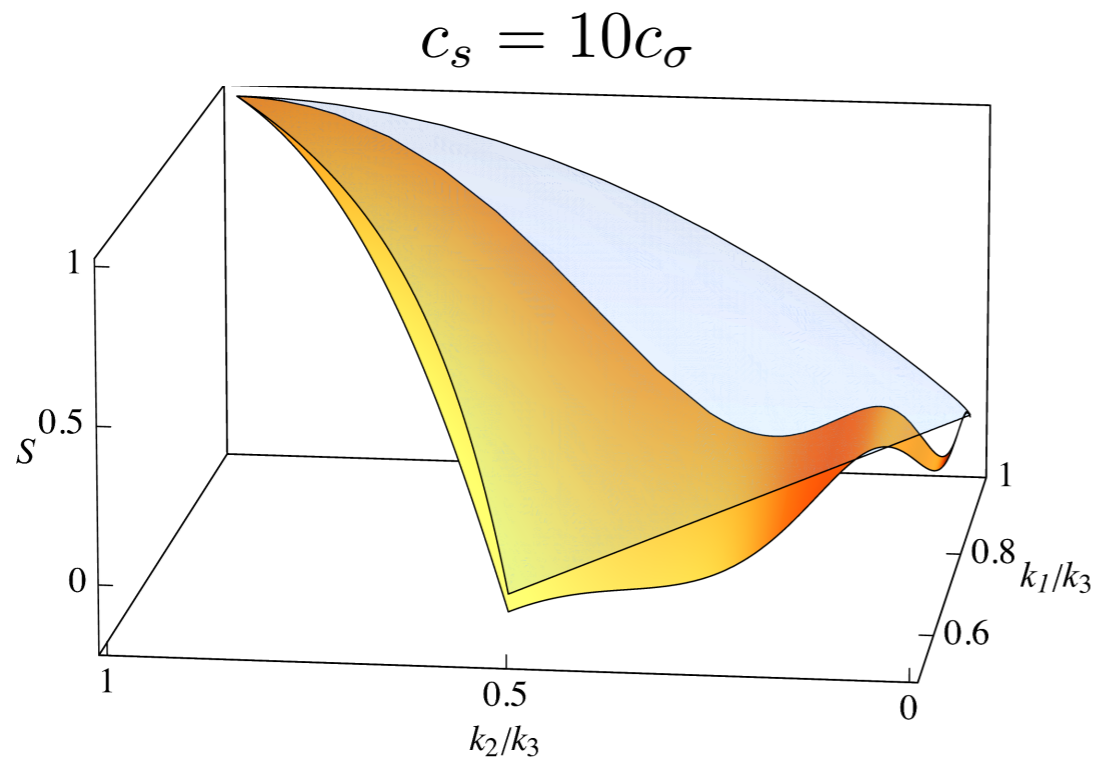
One special case:

$$c_\sigma \ll c_s$$

Collider signals are shifted outside of the squeezed limit.

The Equilateral Collider Shape

$$S^{\text{eq.col.}}(k_1, k_2, k_3) = \frac{k_1 k_2}{(k_1 + k_2)^2} \left(\frac{k_3}{k_1 + k_2} \right)^{1/2} \cos \left[\mu \log \left(\frac{c_\sigma k_3}{2c_s(k_1 + k_2)} \right) + \delta \right] + \text{perms.}$$



Take-Away Messages

- ☑ A complete set of cosmo collider bispectra, Bootstrapped!!
- ☑ The size of the bispectra can be large, as strong breaking of the dS boosts is allowed in BCCB.
- ☑ New pheno with the phase of cosmo colliders and the equilateral collider shape
- ☑ These bispectra shapes provide theoretically well-motivated (and consistent) **targets for upcoming LSS/CMB/21cm surveys.**

New formalism. A lot to be done!

Looking forward to questions in the discussion session.