Going Beyond the Galaxy Power Spectrum: an Analysis of BOSS data with Wavelet Scattering Transforms

Georgios Valogiannis
Harvard University

Cosmology from Home
Parallel Talk
July 2022

Background from
Millennium Simulation, 2005

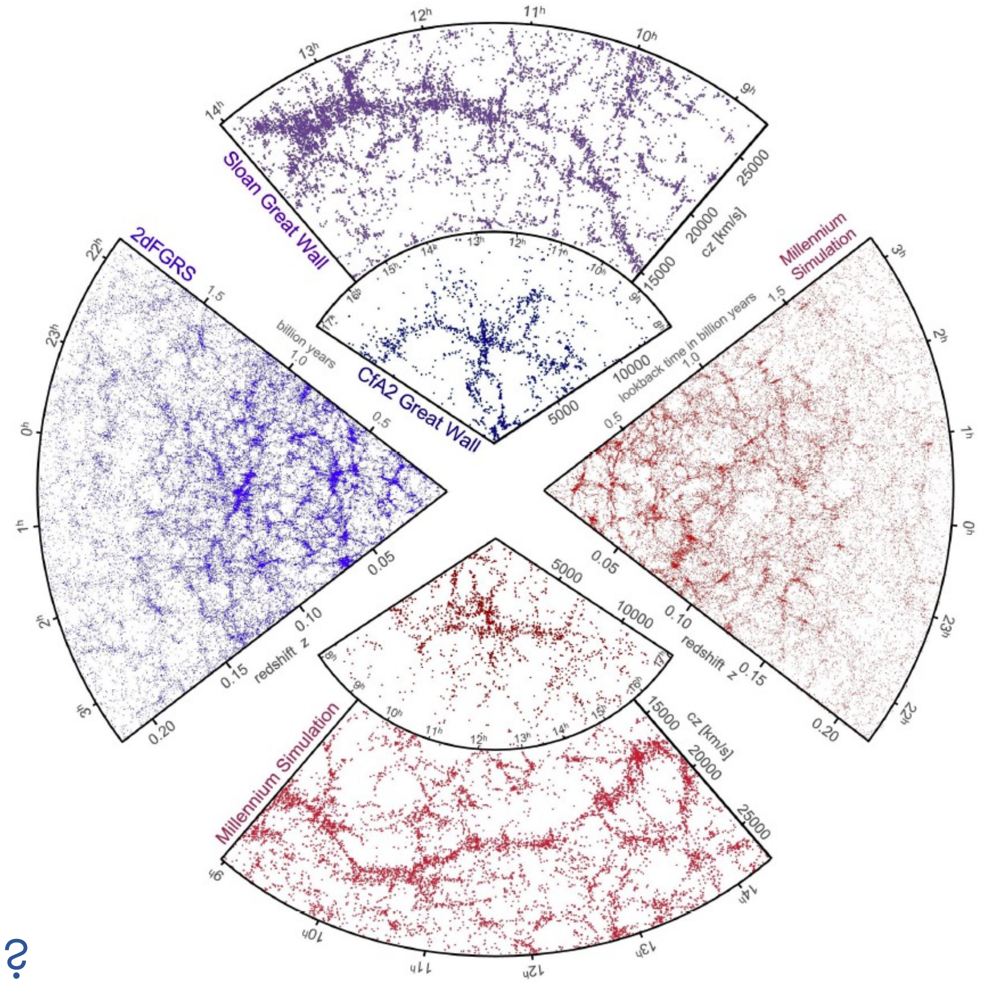
Based on
arXiv: 2204.13717 & 2108.07821
in collaboration with Cora Dvorkin

Challenges in the era of precision cosmology

- Large-Scale Structure (LSS) of the universe a powerful probe of *fundamental physics*
  - Dark energy
  - Dark matter
  - Massive neutrinos
  - Gravity

- Will soon be mapped precisely by:
  - Dark Energy Scientific Instrument (DESI)
  - V. Rubin Observatory LSST
  - Euclid
  - Nancy Grace Roman Space Telescope
  - SPHEREx
  - + Synergies with CMB

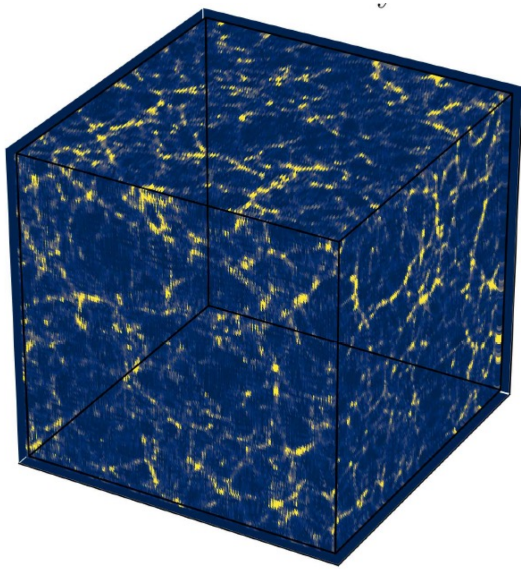
- How do we *optimally* extract information from the LSS??



V. Springel et al. (2006)

The quest for an ideal estimator

- Attempts to describe the information encoded in the 3D cosmic density field

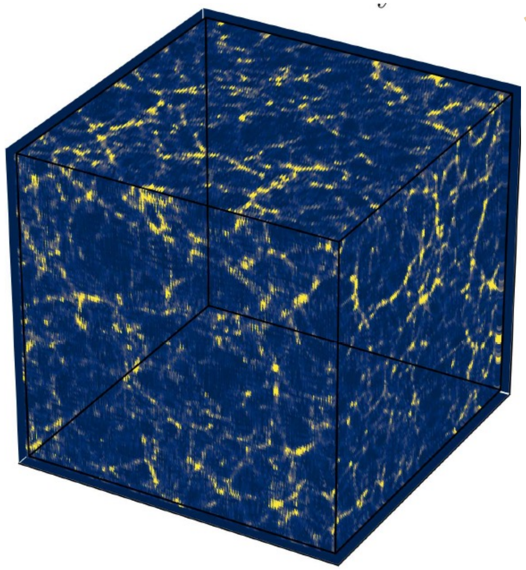


F. Villaescusa-Navaro et al. (2019)

The quest for an ideal estimator

- Attempts to describe the information encoded in the 3D cosmic density field

Power Spectrum



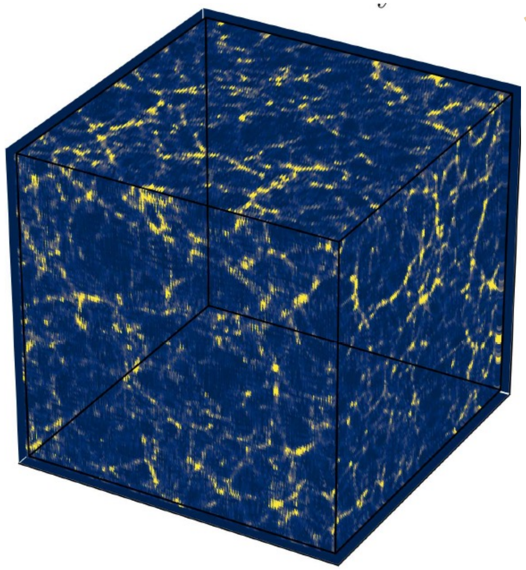
F. Villaescusa-Navaro et al. (2019)

Physical Information

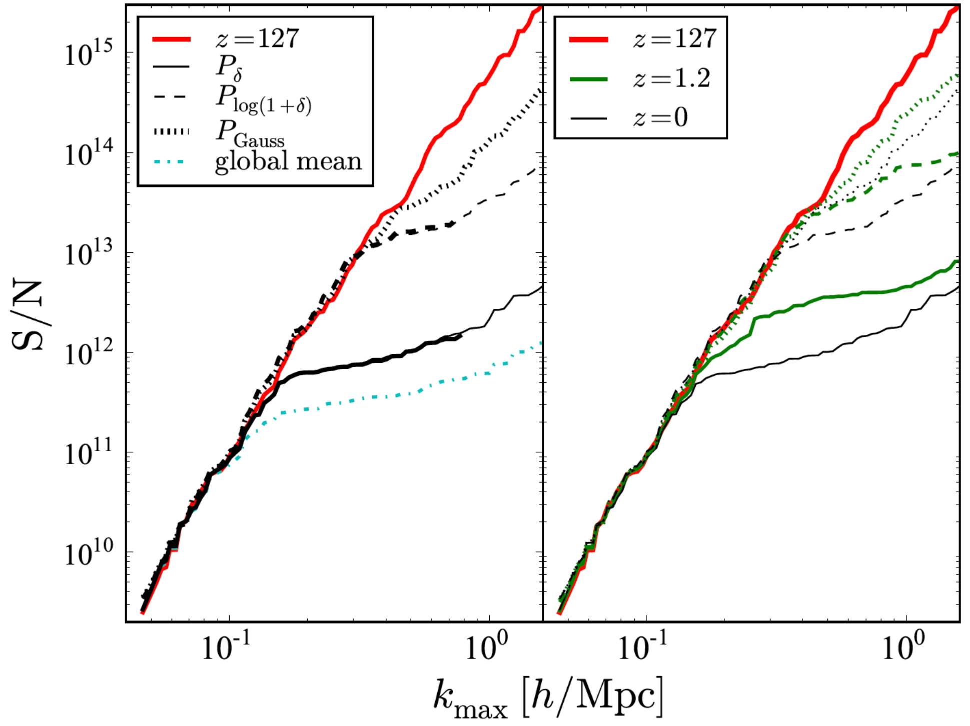
The quest for an ideal estimator

- Attempts to describe the information encoded in the 3D cosmic density field

Power Spectrum (Incomplete)



F. Villaescusa-Navaro et al. (2019)



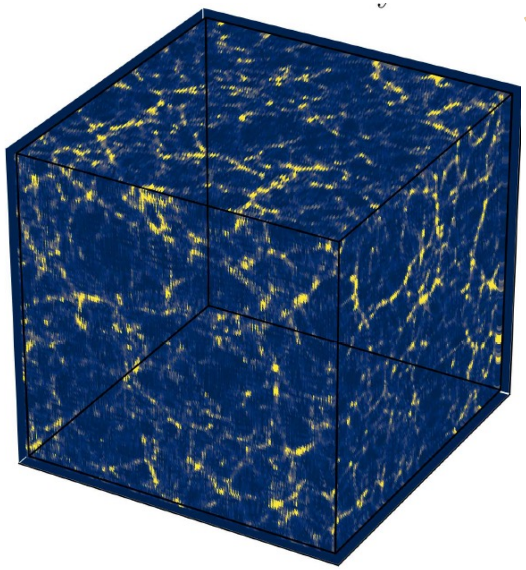
M. Neyrinck et al. (2009)

Power Spectrum information saturates in nonlinear regime. Inadequate!
(Carron 2011,2012)

The quest for an ideal estimator

- Attempts to describe the information encoded in the 3D cosmic density field

Power spectrum + Higher order statistics (expensive)



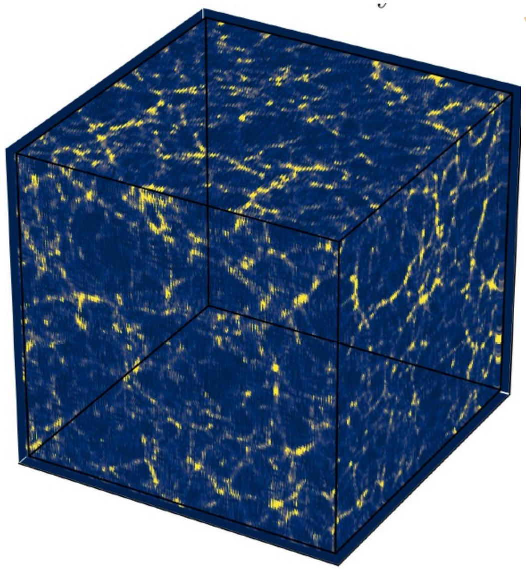
F. Villaescusa-Navaro et al. (2019)

Physical Information

The quest for an ideal estimator

- Attempts to describe the information encoded in the 3D cosmic density field

Power spectrum + Higher order statistics



F. Villaescusa-Navaro et al. (2019)

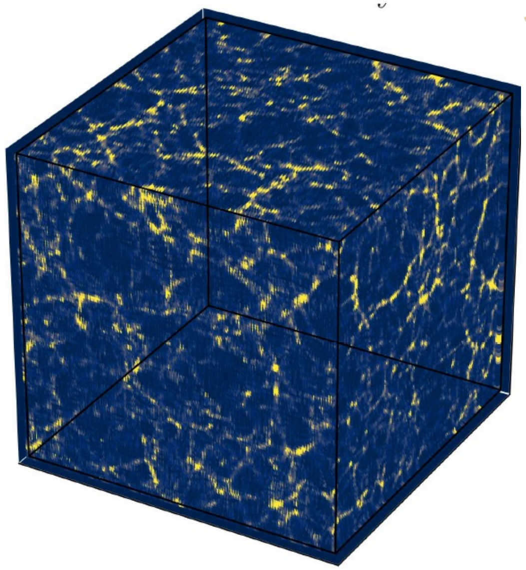
Physical Information

Convolutional Neural Networks (CNNs)
(Training, interpretability)

The quest for an ideal estimator

- Attempts to describe the information encoded in the 3D cosmic density field

Power spectrum + Higher order statistics



F. Villaescusa-Navaro et al. (2019)

Wavelet Scattering Transform (WST)
S. Mallat (2012)

Physical Information

Convolutional Neural Networks (CNNs)

The Wavelet Scattering Transform (WST)

"Scattering Network" image by G. Exarchakis (2018)

Convolution

Modulus

WST: $\langle \left| I_0 \star \psi^{j_1, l_1} \right| \rangle$ → Averaging

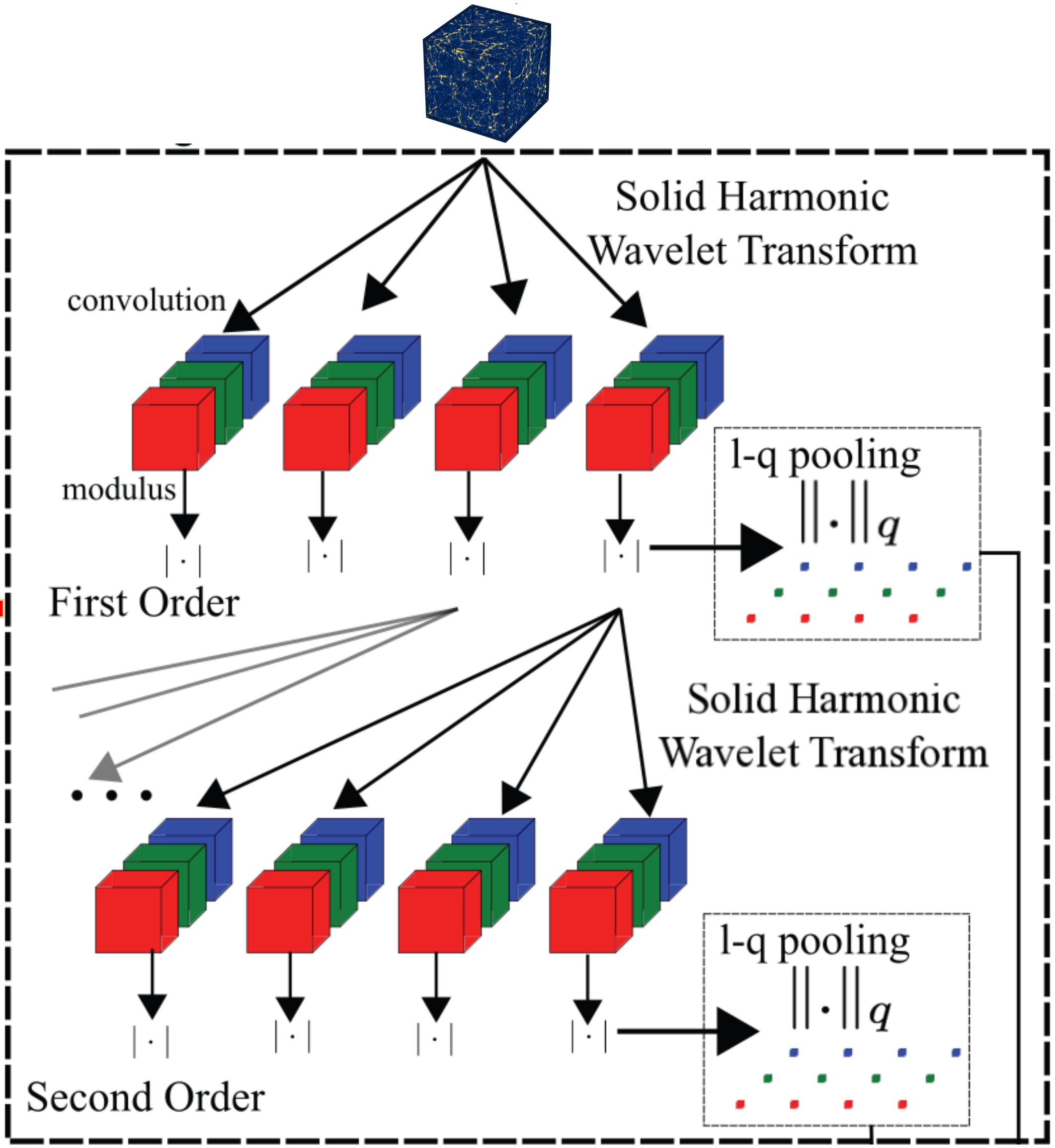
Input field

Family of Wavelets
- Dilated by 2^{j_1} - J scales
- Rotated by l_1 - L orientations

$$S_0 \equiv \langle I_0 \rangle$$

$$S_1^{j_1,l_1} \equiv \langle I_1^{j_1,l_1} \rangle = \langle \left| I_0 \star \psi^{j_1,l_1} \right| \rangle$$

$$S_2^{j_1,l_1,j_2,l_2} \equiv \langle I_2^{j_1,l_1,j_2,l_2} \rangle = \langle \left| \left| I_0 \star \psi^{j_1,l_1} \right| \star \psi^{j_2,l_2} \right| \rangle$$



The Wavelet Scattering Transform (WST)

Physical interpretation of WST coefficients

- $S_0 \equiv \langle I_0 \rangle$: Mean field
- $S_1^{j_1,l_1} = \langle \left| I_0 \star \psi^{j_1,l_1} \right| \rangle$: ~ P(k). In fact, P(k) ⟶ $\langle |I \star e^{-ikx}|^2 \rangle$
- $S_2^{j_1,l_1,j_2,l_2} = \langle \left| |I_0 \star \psi^{j_1,l_1}| \star \psi^{j_2,l_2} \right| \rangle$: *Non-Gaussian* information (up to 2^2 = 4pcf , for n=2)
- Basis $S_0 + S_1 + S_2$ reflects clustering properties of target field $I_0(x)$
- Retaining all *desirable* properties of regular P(k) ✅ Mallat (2012)

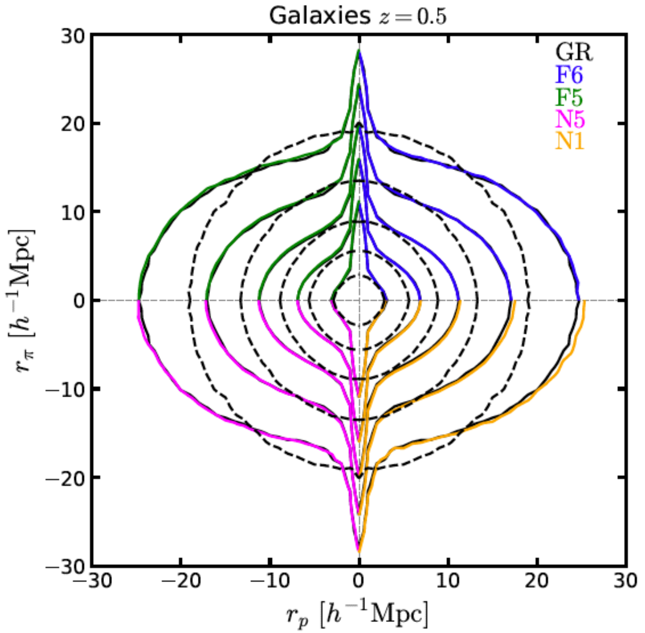
  +

- Compactness ✅ (Anden & Mallat, 2011,2014, Bruna & Mallat, 2013)
- Robustness/Stability ✅ (Carron 2011,2012, Cheng & Menard 2021b)
- A CNN with fixed weights, but interpretable! (Bruna & Mallat 2013)
  - Performance on par with a CNN in WL applications! (Cheng et al. 2020b, Cheng & Menard 2021a)
- WST exceeds performance of regular & marked P(k) in 3D LSS studies **(Valogiannis & Dvorkin 2021)**

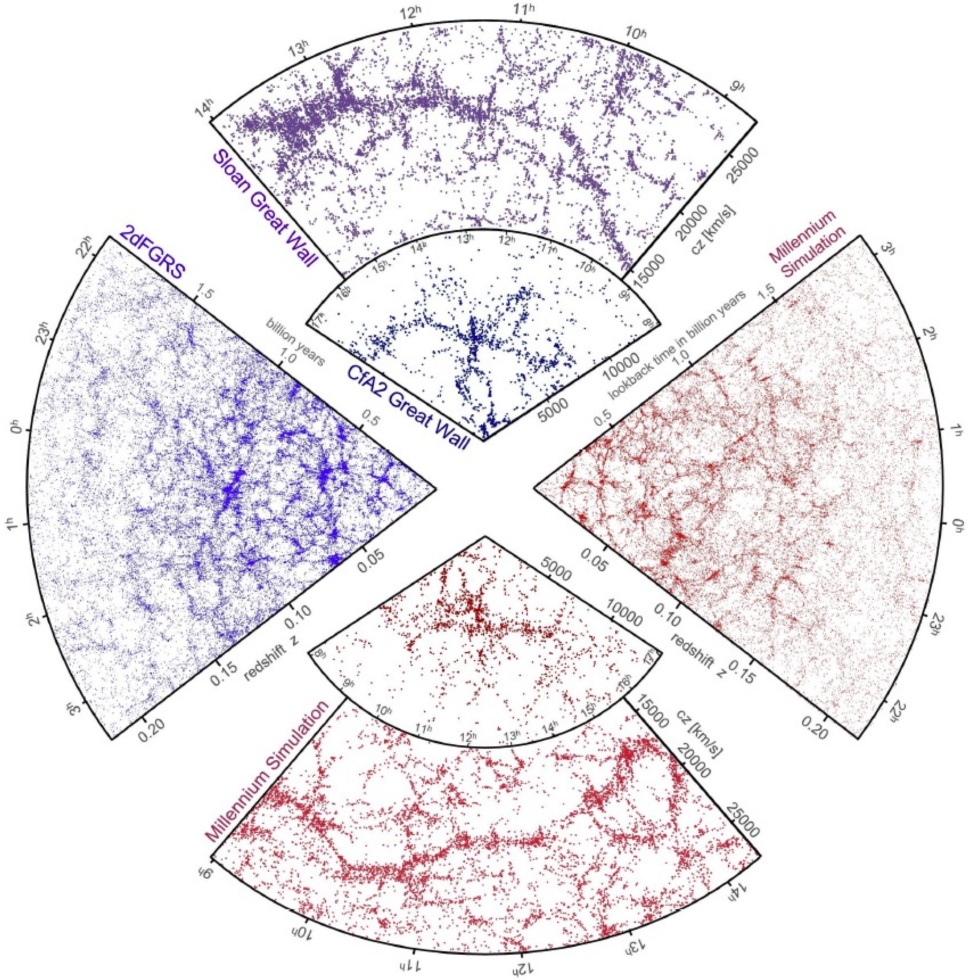
Realistic galaxy survey data

However

- LSS surveys observe *galaxies*:
  - Biased tracers of dark matter field
  - Redshift-Space Distortions (RSD)
  - Systematics (Geometry, fiber collisions, etc..)
  - Lightcone
  - etc



C. Hernández-Aguayo et al., 2018



V. Springel et al. (2006)

First WST application on BOSS

- **First** WST application on 3D *redshift-space galaxy* density field! (**Valogiannis & Dvorkin 2022**)
  - Working with *BOSS CMASS DR12* sample at 0.46<z<0.60
  - Northern + Southern Galactic Cap
- For survey data, fundamental quantity of interest is the *FKP field* (Feldman, Kaiser, Peacock et al., 1994) :

$$F(\mathbf{r}) = \frac{w_{\mathrm{FKP}}(\mathbf{r})}{I_2^{1/2}} \left[w_c(\mathbf{r}) n_g(\mathbf{r}) - \alpha_r n_s(\mathbf{r}) \right]$$

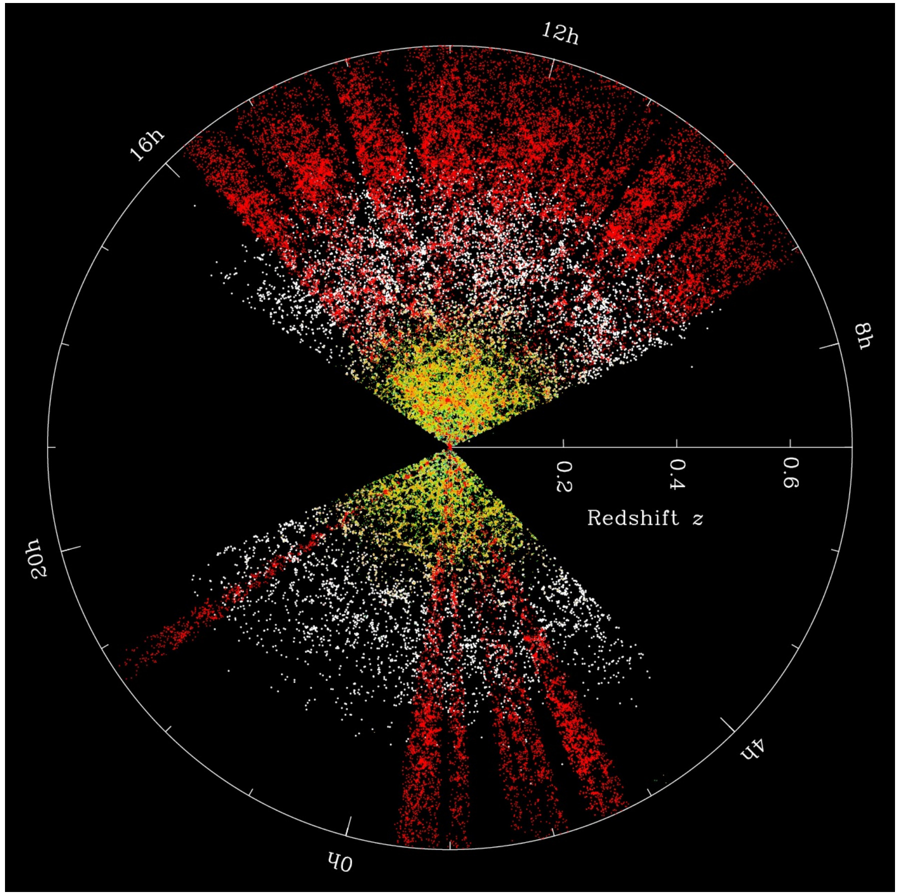
Galaxies Randoms

- Systematic + FKP weights

$$w_c(\mathbf{r}) = \left(w_{\mathrm{rf}}(\mathbf{r}) + w_{\mathrm{fc}}(\mathbf{r}) - 1.0 \right) w_{\mathrm{sys}}(\mathbf{r})$$

$$w_{\mathrm{FKP}}(\mathbf{r}) = \left[1 + \bar{n}_g(\mathbf{r}) P_0 \right]^{-1}$$

- Serves as input into WST network
  - With $N_{grid} = 282^3$ and $L_{Box} = 2820\ Mpc/h$



SDSS https://blog.sdss.org/

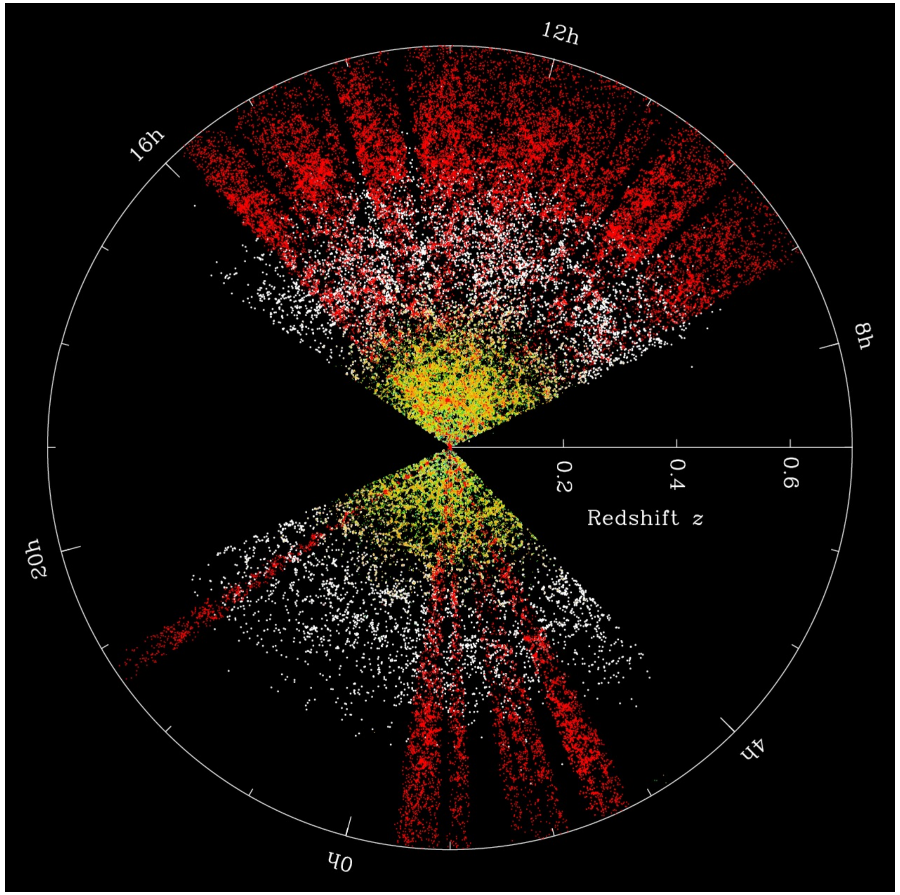
Likelihood analysis

- Data

$$\log \mathcal{L}(\theta|\mathbf{d}) \propto -\frac{1}{2}\left[\mathbf{X_d} - \mathbf{X}_t(\theta)\right]^{\mathrm{T}} C^{-1}\left[\mathbf{X_d} - \mathbf{X}_t(\theta)\right]$$

- Use vector of WST coefficients as observable
- Extracted from BOSS CMASS FKP field, using J=4 scales and L=4 orientations
- $\boldsymbol{S_0 + S_1 + S_2 = 76}$ WST coefficients

- Also, use galaxy power spectrum multipoles $\boldsymbol{P_{l=0,2}(k)}$ **(**$\boldsymbol{k_{max} = 0.25}$ **Mpc/h)** as benchmark



SDSS https://blog.sdss.org/

Likelihood analysis

- Theory model

$$\log \mathcal{L}(\theta|\mathbf{d}) \propto -\frac{1}{2}\left[\mathbf{X_d} - \mathbf{X}_t(\theta)\right]^{\mathrm{T}} C^{-1} \left[\mathbf{X_d} - \mathbf{X}_t(\theta)\right]$$

- Capture cosmological dependence using

Abacus Summit simulations (Maksimova et al. 2021, Garrison et al. 2019&2021)

HOD tuned to BOSS CMASS at 0.46<z<0.60 with AbacusHOD (**Yuan et al. 2021**)

Box L=2000 Mpc/h, $N_{grid} = 200^3$

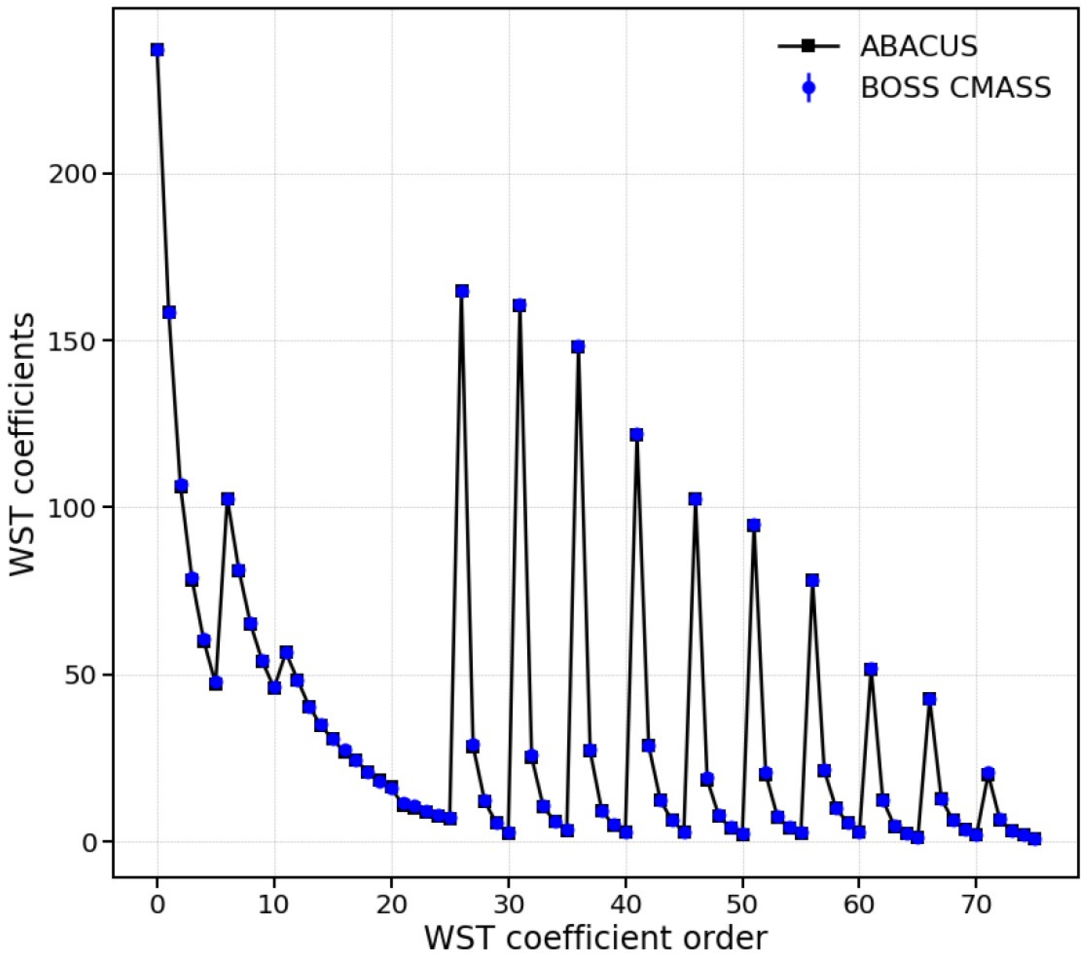
- Fiducial cosmology from Planck 2018 $\{\omega_b, \omega_c, n_s, \sigma_8\} = \{0.02237, 0.120, 0.9649, 0.8114\}$
- + Fixed angular size of sound horizon at last scattering. $100\theta_\star = 1.041533$
- + 7 HOD model paramerers (vanilla HOD + velocity bias)

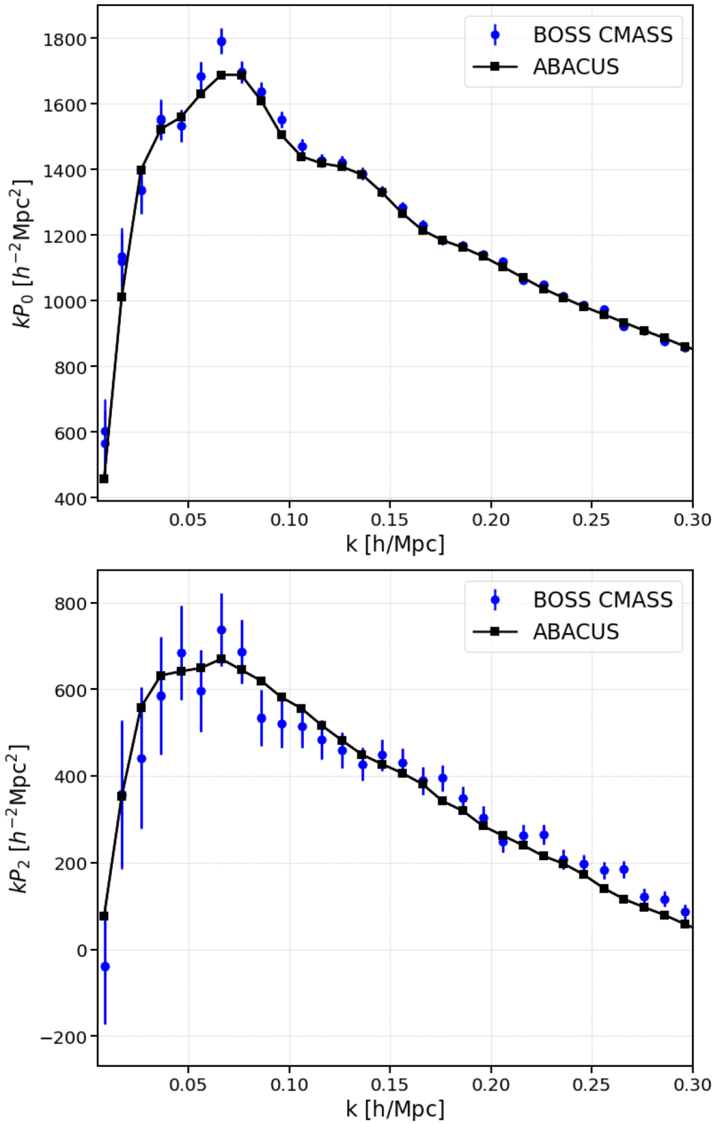
$$\{\alpha, \alpha_{\mathrm{c}}, \alpha_{\mathrm{s}}, \kappa, \log M_1, \log M_{\mathrm{cut}}, \sigma\} = \{0.9022, 0.2499, 1.1807, 0.3288, 14.313, 12.8881, 0.02084\}$$

- We cut Abacus cubic boxes into actual CMASS geometry
    - Using 'make survey' (White et al., 2013)

Likelihood analysis

- Fiducial cosmology predictions

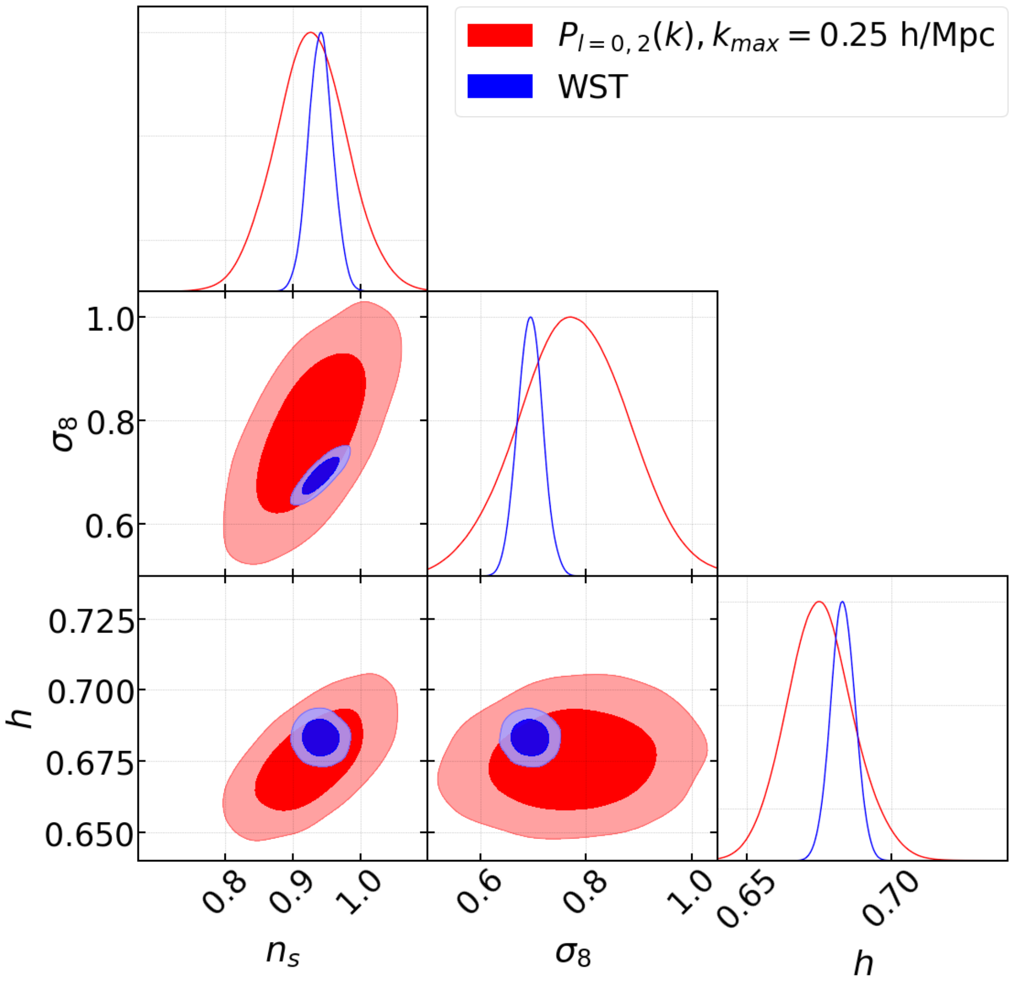




Valogiannis & Dvorkin 2022

Likelihood analysis

- Likelihood analysis using a BBN prior on ω_b $\omega_b = 0.02268 \pm 0.00038$



	BBN prior on ω_b		unrestricted priors	
	P(k)	WST	P(k)	WST
ω_b	$0.02267^{+0.00045}_{-0.00045}$	$0.02268^{+0.00036}_{-0.00036}$	$0.0217^{+0.0043}_{-0.0043}$	$0.01946^{+0.0008}_{-0.0008}$
ω_c	$0.1223^{+0.0031}_{-0.0028}$	$0.1202^{+0.00013}_{-0.00013}$	$0.1217^{+0.0058}_{-0.0058}$	$0.11672^{+0.001}_{-0.001}$
n_s	$0.928^{+0.075}_{-0.075}$	$0.942^{+0.018}_{-0.018}$	$0.921^{+0.057}_{-0.049}$	$0.959^{+0.019}_{-0.019}$
σ_8	$0.77^{+0.14}_{-0.14}$	$0.695^{+0.024}_{-0.024}$	$0.762^{+0.11}_{-0.094}$	$0.716^{+0.025}_{-0.025}$
h	$0.676^{+0.010}_{-0.012}$	$0.6831^{+0.0042}_{-0.0042}$	$0.668^{+0.024}_{-0.024}$	$0.66^{+0.0055}_{-0.0055}$

- Parameter mean values from WST & P(k) always consistent with each other within 1σ (of the P(k))
- Much **tighter** errors from WST compared to P(k)!
- H_0 determined from WST with 0.6% accuracy!

Valogiannis & Dvorkin 2022

Conclusions

- Wavelet Scattering Transform: a novel statistic that efficiently extracts non-Gaussian information from physical fields. *Ideal* middle ground between CNN and traditional estimators
- *First* WST application on actual spectroscopic data **(Valogiannis & Dvorkin , arXiv: 2204.13717)**
  - Worked with BOSS CMASS galaxy sample at 0.46<z<0.60
  - Great improvement in the 1σ errors over traditional galaxy P(k) multipoles
  - Much **tighter** errors both using BBN prior on ω_b and using flat unrestricted priors!
  - 0.6% determination of the Hubble constant!
- Future improvements (in progress)
  - *Construct full emulator for WST coefficients (Eg. Yuan et al. 2022)*
  - Include lightcone effects in galaxy mocks
- Future applications
  - Constrain neutrino mass (Eg. as in **Valogiannis & Dvorkin, arXiv: 2108.07821, Phys. Rev. D 105, 103534, 2022)**
  - Analysis explicitly varying H_0 (rather than derived parameter)
  - Comparison with higher-point function analyses (Eg. Philcox et al., 2021)
  - *Application on DESI data*


Thank you!

Likelihood analysis

- Theory model

$$\log \mathcal{L}(\theta|\mathbf{d}) \propto -\frac{1}{2}\left[\mathbf{X_d} - \mathbf{X}_t(\theta)\right]^{\mathrm{T}} C^{-1}\left[\mathbf{X_d} - \mathbf{X}_t(\theta)\right]$$

- To model WST (and P(k)) cosmological dependence, we use the *approximation:*

$$\mathbf{X}_t(\theta) = \mathbf{X}_t(\theta_{\mathrm{fid}}) + (\theta - \theta_{\mathrm{fid}})\nabla_\theta \mathbf{X}$$

Prediction for fiducial cosmology

Constructed from 'Linear derivative grid' of cosmologies

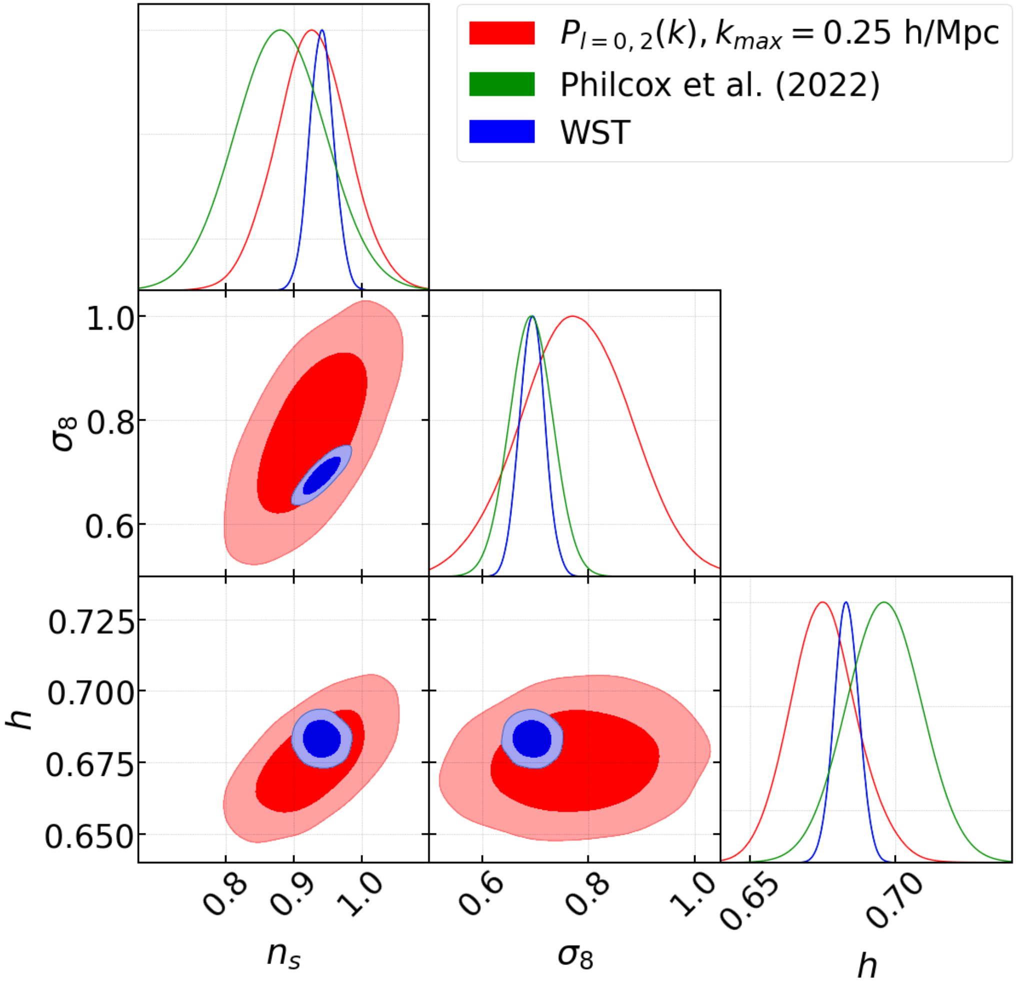
ω_b	ω_c	n_s	σ_8
0.02237	0.1200	0.9649	0.8114
0.02282	0.1200	0.9649	0.8114
0.02193	0.1200	0.9649	0.8114
0.02237	0.1240	0.9649	0.8114
0.02237	0.1161	0.9649	0.8114
0.02237	0.1200	1.0249	0.8114
0.02237	0.1200	0.9049	0.8114
0.02237	0.1200	0.9649	0.8698
0.02237	0.1200	0.9649	0.7532

- + Additional derivative steps in the 7 HOD parameters

Valogiannis & Dvorkin 2022

Likelihood analysis

- Likelihood analysis using a BBN prior on ω_b $\omega_b = 0.02268 \pm 0.00038$



- BOSS with WST:

$H_0 = 68.31^{+0.42}_{-0.42}$ km/s/Mpc $\sigma_8 = 0.695^{+0.024}_{-0.024}$

- Planck results:

$H_0 = 67.4 \pm 0.5$ km/s/Mpc $\sigma_8 = 0.811 \pm 0.006$

- Parameter mean values from WST & P(k) always consistent with each other within 1σ (of the P(k))
- Much **tighter** errors from WST compared to P(k)!
- H_0 determined from WST with 0.6% accuracy!
- $\sigma_8 = 0.695^{+0.024}_{-0.024}$ in tension with Planck result

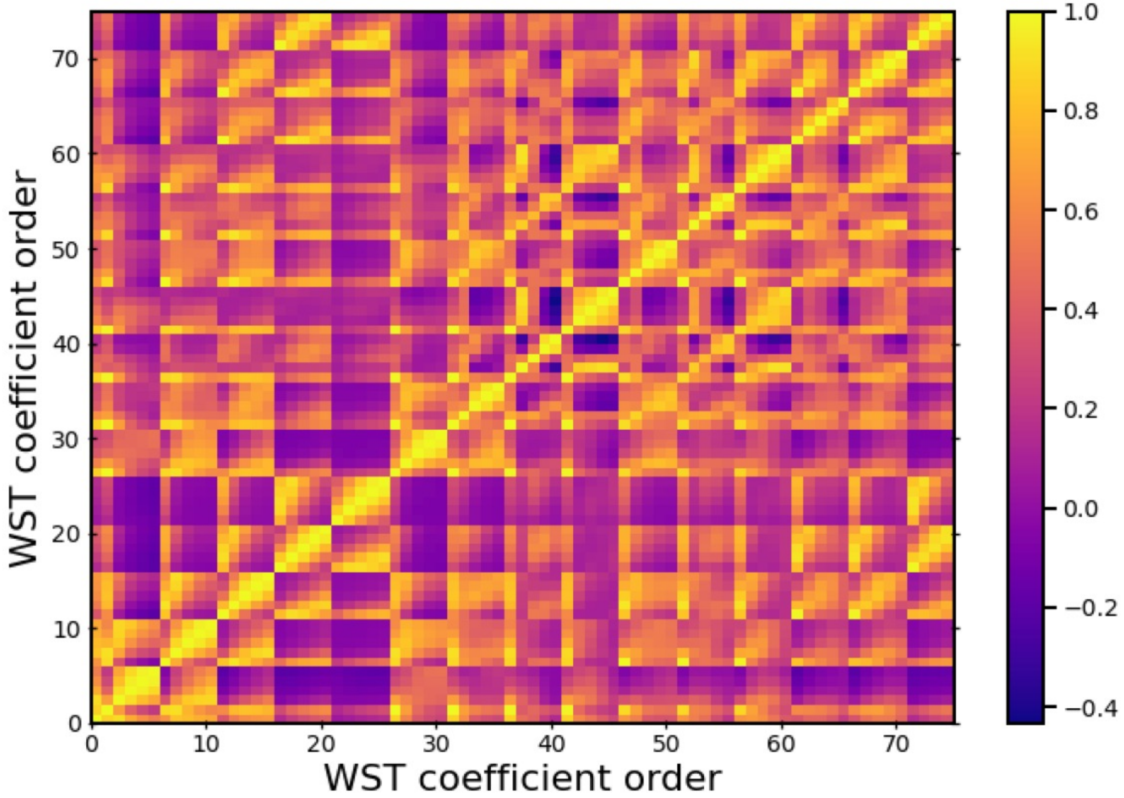
In agreement with recent BOSS analyses
(Philcox & Ivanov, 2022, Chen et al. 2022 a & b)

Valogiannis & Dvorkin 2022

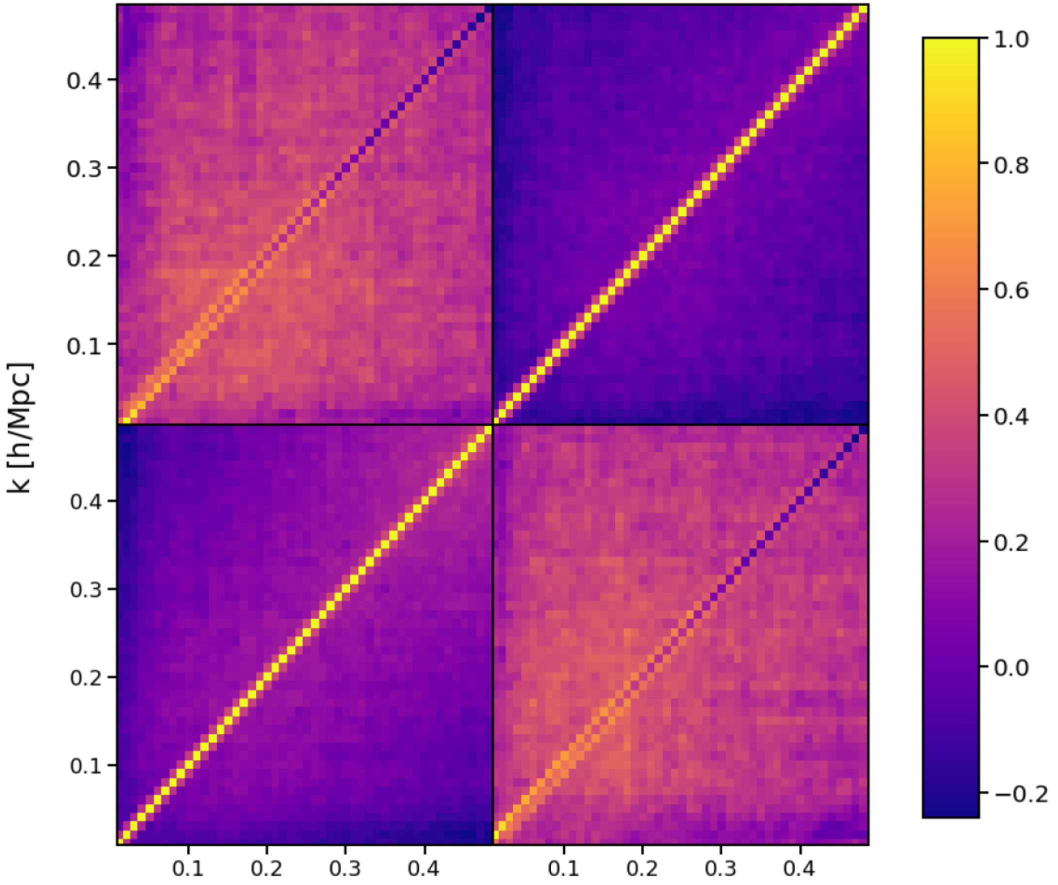
Likelihood analysis

- Covariance matrix obtained from N=2048 *PATCHY* mocks (S. A. Rodriguez-Torres et al., 2016)

$$\log \mathcal{L}(\theta|\mathbf{d}) \propto -\frac{1}{2}\left[\mathbf{X}_\mathbf{d} - \mathbf{X}_t(\theta)\right]^\mathrm{T} C^{-1} \left[\mathbf{X}_\mathbf{d} - \mathbf{X}_t(\theta)\right]$$



Valogiannis & Dvorkin 2022



The Wavelet Scattering Transform (WST)

- 3-dimensional WST implementation with 'solid harmonic' wavelets (Eickenberg et al. (2018))

$$S_0 = \langle |I(\vec{x})|^q \rangle,$$

$$S_1(j_1, l_1) = \left\langle \left(\sum_{m=-l_1}^{m=l_1} |I(\vec{x}) * \psi^m_{j_1,l_1}(\vec{x})|^2 \right)^{\frac{q}{2}} \right\rangle,$$

$$S_2(j_2, j_1, l_1) = \left\langle \left(\sum_{m=1}^{m=l_1} |U_1(j_1, l_1)(\vec{x}) * \psi^m_{j_2,l_1}(\vec{x})|^2 \right)^{\frac{q}{2}} \right\rangle$$

$$U_1(j_1, l_1)(\mathbf{x}) = \left(\sum_{m=-l_1}^{m=l_1} |I(\mathbf{x}) * \psi^m_{j_1,l_1}(\mathbf{x})|^2 \right)^{\frac{1}{2}}$$

$$\psi^m_l(\mathbf{x}) = \underbrace{\frac{1}{(2\pi)^{3/2}} e^{-|\mathbf{x}|^2/2\sigma^2}}_{\text{Gaussian envelope}} \underbrace{|\mathbf{x}|^l Y^m_l \left(\frac{\mathbf{x}}{|\mathbf{x}|} \right)}_{\text{Solid Harmonics}}$$

- Dilated by dyadic scales 2^{j_1}

$$\psi^m_{j_1,l_1}(\mathbf{x}) = 2^{-3j_1} \psi^{m_1}_{l_1}(2^{-j_1}\mathbf{x})$$

- Wavelets in the literature
  - Bump steerable wavelets (Eickenberg et al, 2022, Allys et al, 2020)
  - Morlet wavelets (Cheng et al. 2020b, Cheng & Menard 2021a)

- Implemented in KYMATIO package (Andreux et al. 2019)

The Wavelet Scattering Transform (WST)

- Raising modulus to powers q<1 emphasizes on cosmic voids (while q>1 on density peaks)

$$S_0 = \langle |I(\vec{x})|^q \rangle,$$

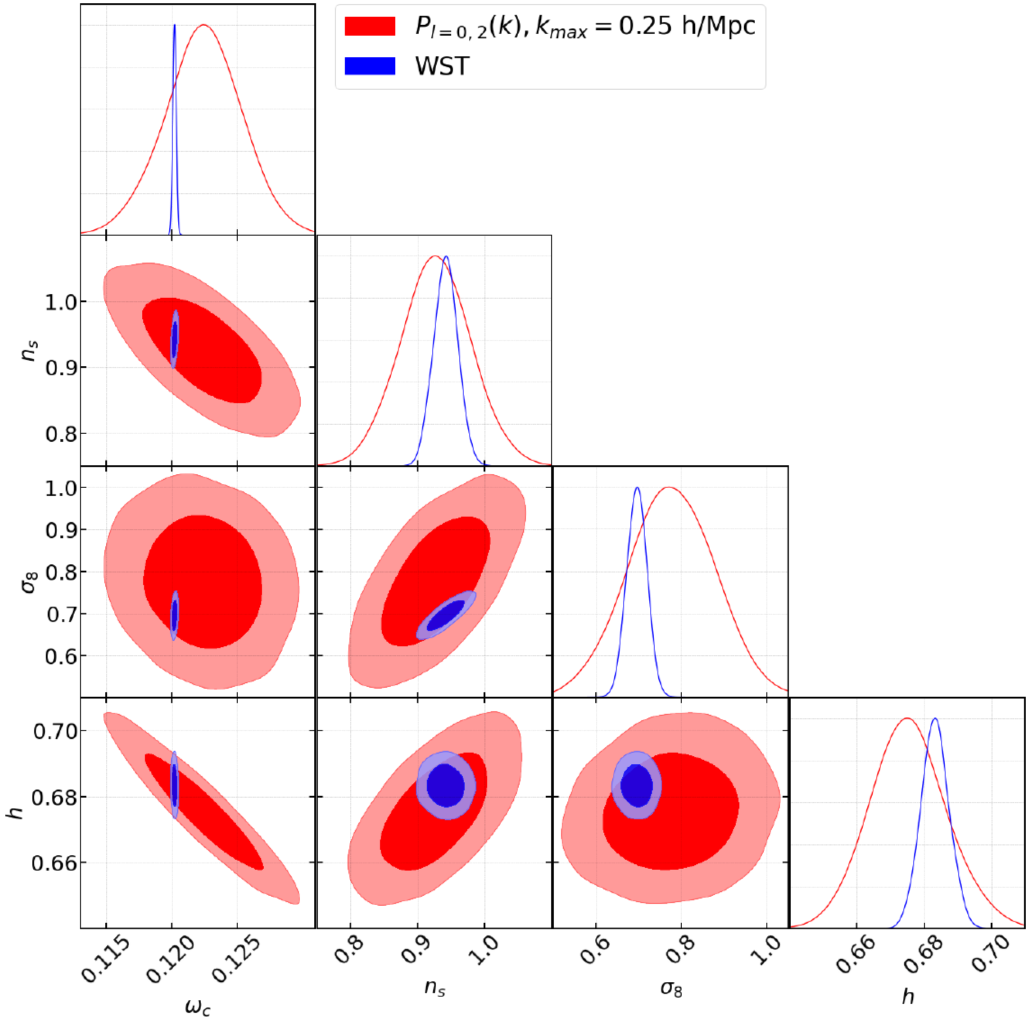
$$S_1(j_1, l_1) = \left\langle \left(\sum_{m=-l_1}^{m=l_1} |I(\vec{x}) * \psi^m_{j_1,l_1}(\vec{x})|^2 \right)^{\frac{q}{2}} \right\rangle,$$

$$S_2(j_2, j_1, l_1) = \left\langle \left(\sum_{m=1}^{m=l_1} |U_1(j_1, l_1)(\vec{x}) * \psi^m_{j_2,l_1}(\vec{x})|^2 \right)^{\frac{q}{2}} \right\rangle$$

- q=0.8 found to be optimal for 3D LSS studies (**Valogiannis & Dvorkin 2021)**
  - Applied WST on 3D matter over-density field from *Quijote* simulations (F. Villaescusa-Navaro et al., 2019)
  - Particularly sensitive to neutrino mass
  - WST matches and exceeds performance of marked P(k) (studied by Massara et al, PRL 2020)

Likelihood analysis

- Likelihood analysis using a BBN prior on ω_b $\omega_b = 0.02268 \pm 0.00038$



	BBN prior on ω_b		unrestricted priors	
	P(k)	WST	P(k)	WST
ω_b	$0.02267^{+0.00045}_{-0.00045}$	$0.02268^{+0.00036}_{-0.00036}$	$0.0217^{+0.0043}_{-0.0043}$	$0.01946^{+0.0008}_{-0.0008}$
ω_c	$0.1223^{+0.0031}_{-0.0028}$	$0.1202^{+0.00013}_{-0.00013}$	$0.1217^{+0.0058}_{-0.0058}$	$0.11672^{+0.001}_{-0.001}$
n_s	$0.928^{+0.075}_{-0.075}$	$0.942^{+0.018}_{-0.018}$	$0.921^{+0.057}_{-0.049}$	$0.959^{+0.019}_{-0.019}$
σ_8	$0.77^{+0.14}_{-0.14}$	$0.695^{+0.024}_{-0.024}$	$0.762^{+0.11}_{-0.094}$	$0.716^{+0.025}_{-0.025}$
h	$0.676^{+0.010}_{-0.012}$	$0.6831^{+0.0042}_{-0.0042}$	$0.668^{+0.024}_{-0.024}$	$0.66^{+0.0055}_{-0.0055}$

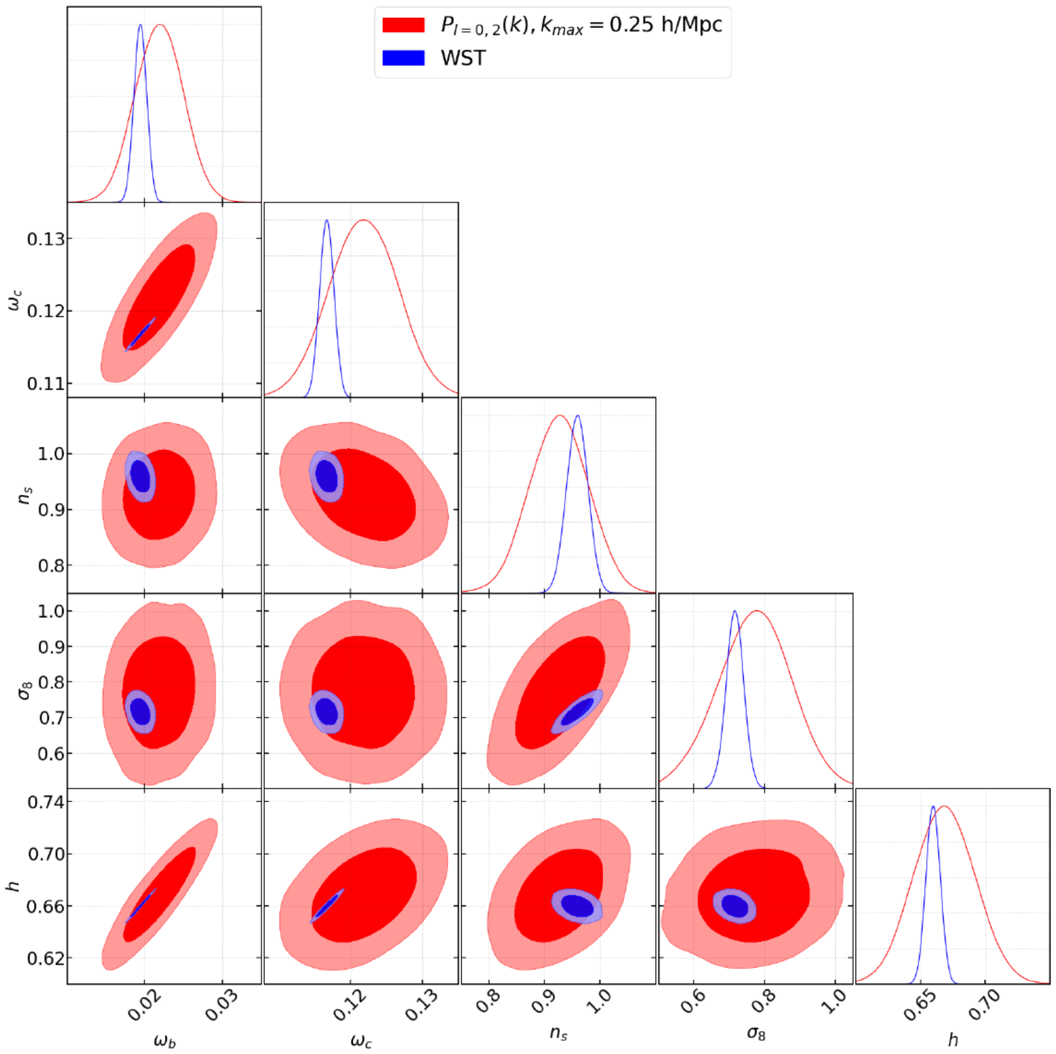
- Parameter mean values from WST & P(k) always consistent with each other within 1σ (of the P(k))
- ~4x28x **tighter** errors from WST compared to P(k)!
- H_0 determined from WST with 0.6% accuracy!
- $\sigma_8 = 0.695^{+0.024}_{-0.024}$ in tension with Planck result

In agreement with recent BOSS analyses (Philcox & Ivanov, 2022, Chen et al. 2022 a & b)

Valogiannis & Dvorkin 2022

Likelihood analysis

- Likelihood analysis using flat unrestricted priors

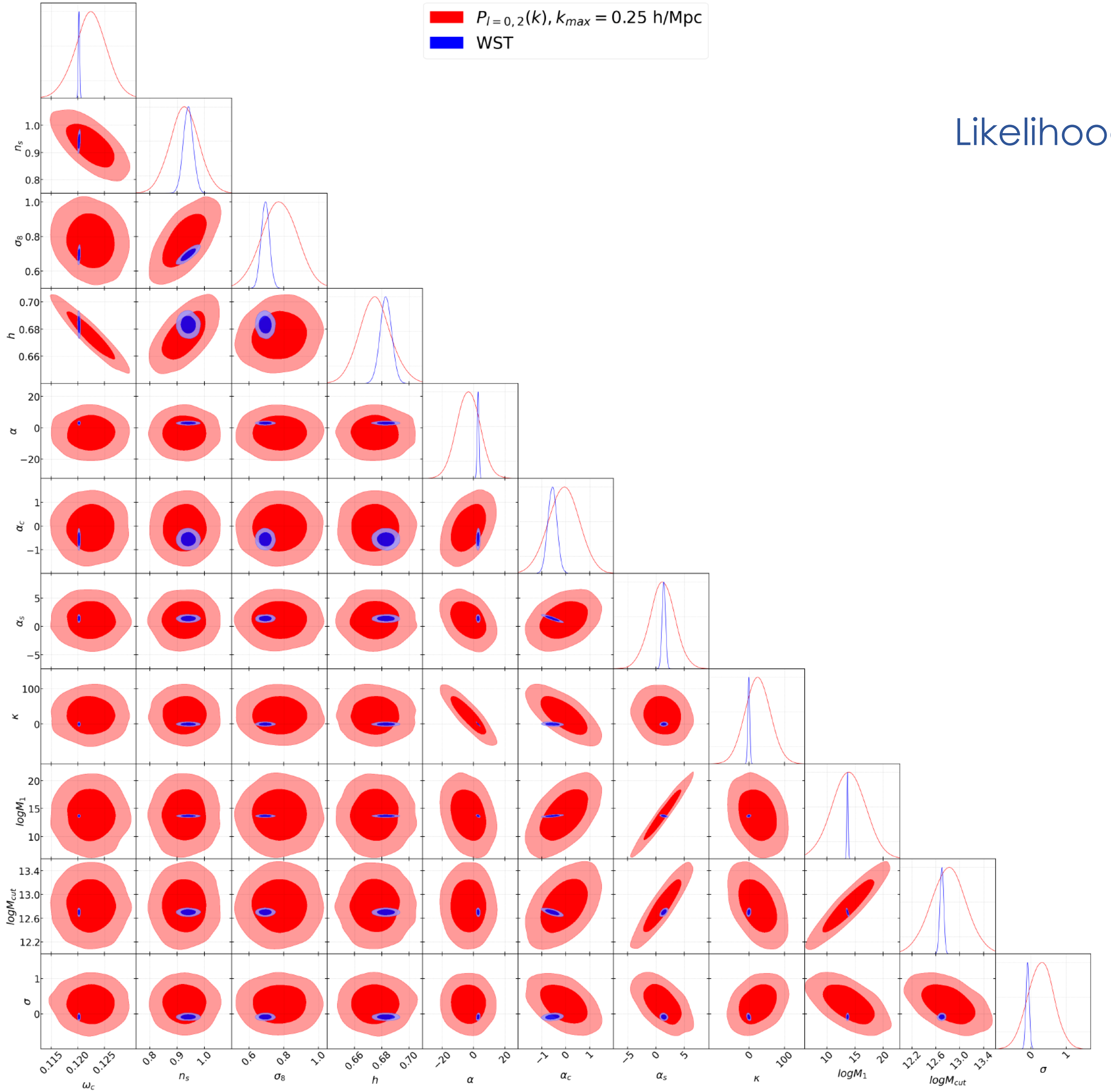


	BBN prior on ω_b		unrestricted priors	
	P(k)	WST	P(k)	WST
ω_b	$0.02267^{+0.00045}_{-0.00045}$	$0.02268^{+0.00036}_{-0.00036}$	$0.0217^{+0.0043}_{-0.0043}$	$0.01946^{+0.0008}_{-0.0008}$
ω_c	$0.1223^{+0.0031}_{-0.0028}$	$0.1202^{+0.00013}_{-0.00013}$	$0.1217^{+0.0058}_{-0.0058}$	$0.11672^{+0.001}_{-0.001}$
n_s	$0.928^{+0.075}_{-0.075}$	$0.942^{+0.018}_{-0.018}$	$0.921^{+0.057}_{-0.049}$	$0.959^{+0.019}_{-0.019}$
σ_8	$0.77^{+0.14}_{-0.14}$	$0.695^{+0.024}_{-0.024}$	$0.762^{+0.11}_{-0.094}$	$0.716^{+0.025}_{-0.025}$
h	$0.676^{+0.010}_{-0.012}$	$0.6831^{+0.0042}_{-0.0042}$	$0.668^{+0.024}_{-0.024}$	$0.66^{+0.0055}_{-0.0055}$

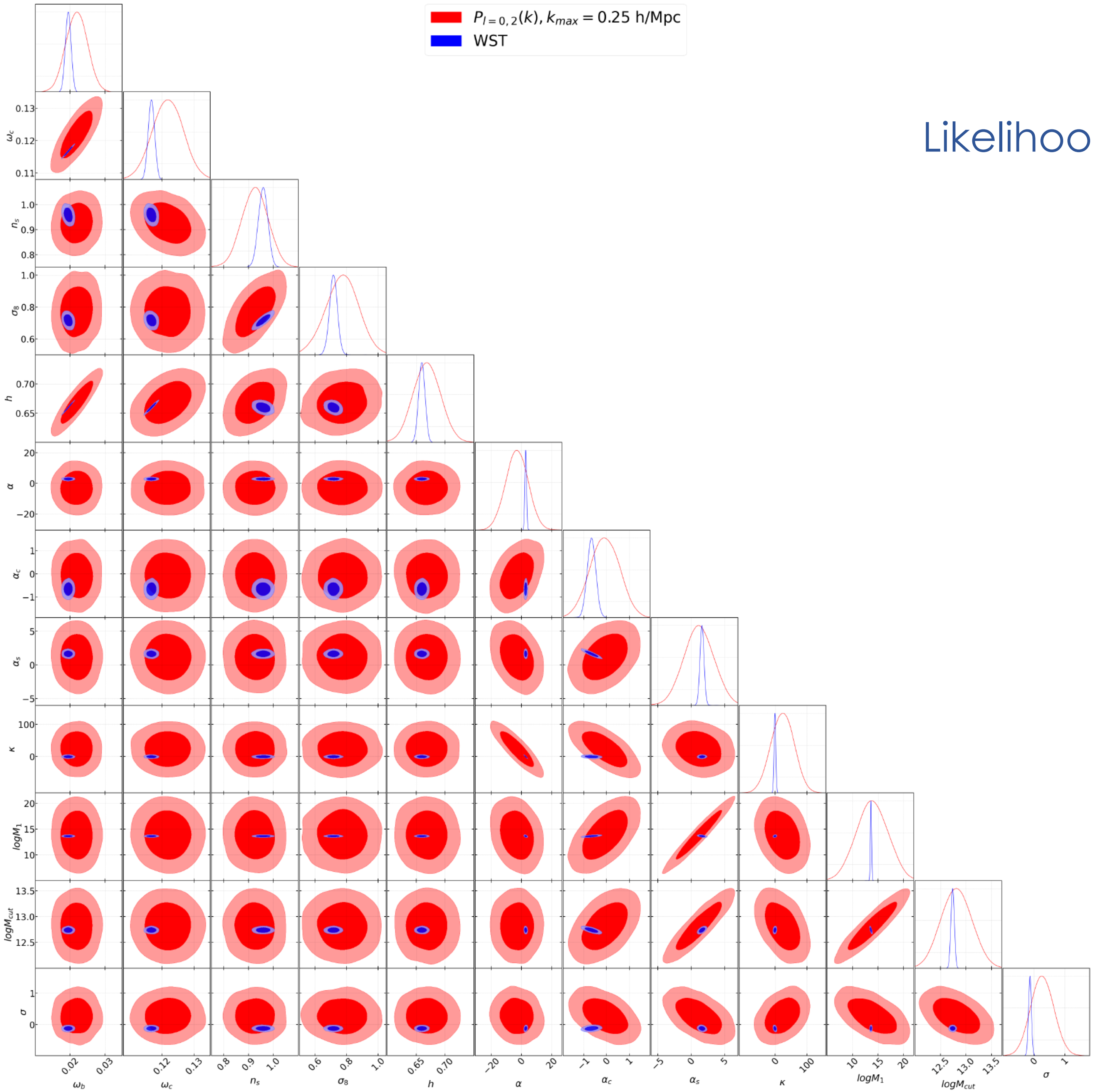
- Parameter mean values from WST & P(k) again always consistent with each other within 1σ (of the P(k))
- ~3x6x **tighter** errors from WST compared to P(k)!

Valogiannis & Dvorkin 2022

Likelihood analysis using a BBN prior on ω_b



Valogiannis & Dvorkin 2022



Likelihood analysis using flat unrestricted priors

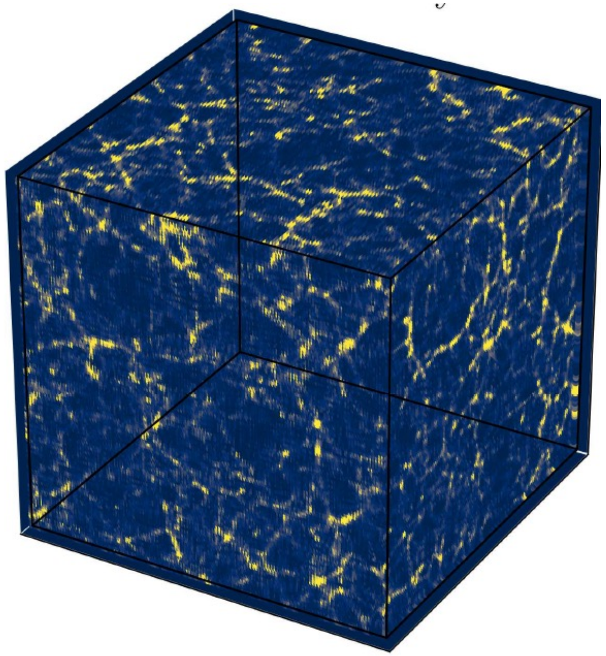
Valogiannis & Dvorkin 2022

First WST application on 3D LSS

- **First** WST application on 3D matter density field! **(Valogiannis & Dvorkin, 2021)**

$$I(\vec{x}) \equiv \delta_m(\vec{x}) = \frac{\rho_m(\vec{x})}{\bar{\rho}_m} - 1.0$$

, resolution $N_{grid} = 256^3$



- Evaluated from the *Quijote* simulations (F. Villaescusa-Navaro et al., 2019)
- Fiducial cosmology

$$\Omega_{\rm m} = 0.3175, \ \Omega_{\rm b} = 0.049, \ h = 0.6711$$

$$n_s = 0.9624, \sigma_8 = 0.834, M_\nu = 0.0 \text{ eV, and } w = -1$$

Box L=1.0 Gpc/h

- In presence of massive neutrinos, trace both:
- $\delta_m = \delta_{CDM} + \delta_b + \delta_\nu$ Total 'm' field
- $\delta_{cb} = \delta_{CDM} + \delta_b$ 'cb' field

Fisher forecast

Fisher forecasting

- 15,000 realizations for fiducial cosmology
- 7,000 for linear derivatives in parameters

$$F_{\alpha\beta} = \frac{\partial O_i}{\partial \theta_\alpha} C_{ij}^{-1} \frac{\partial O_j^T}{\partial \theta_\beta}$$

- Marginalized $\sigma_\alpha = \sqrt{(F^{-1})_{\alpha\alpha}}$

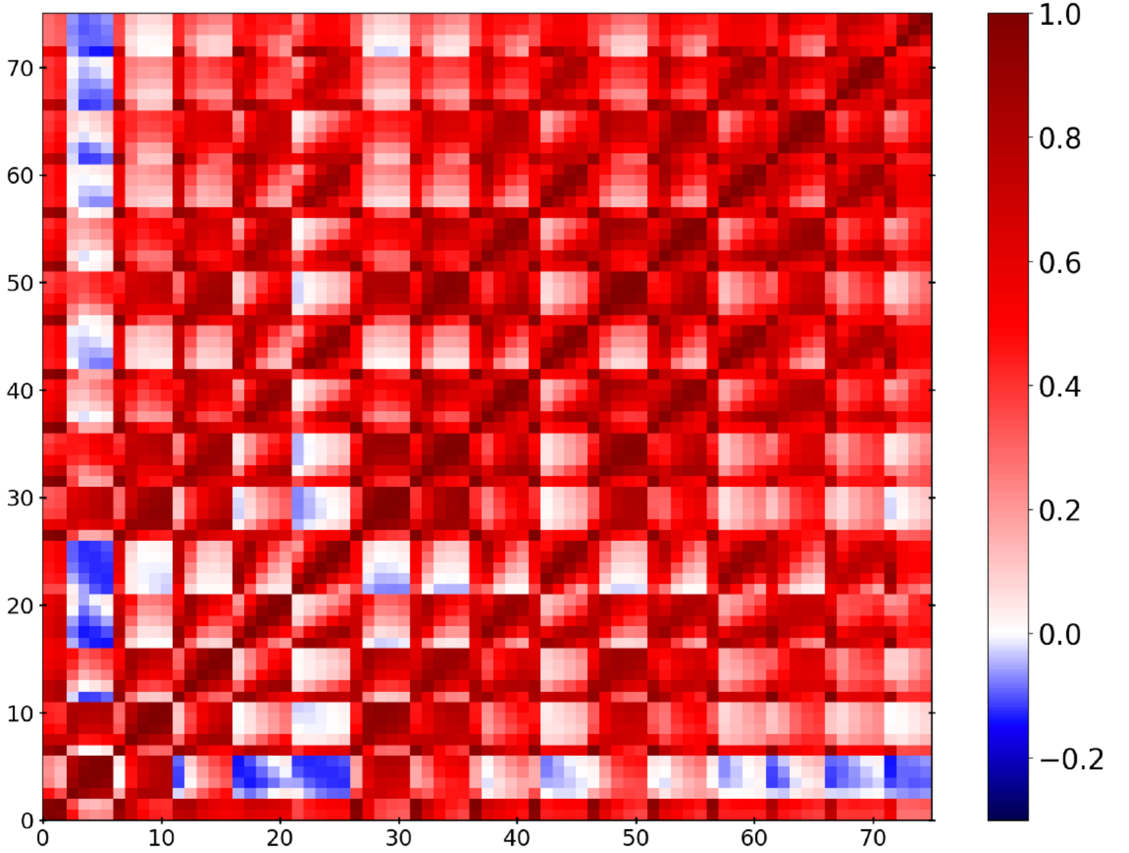
for $\theta_\alpha = \{\Omega_m, \Omega_b, H_0, n_s, \sigma_8, M_\nu\}$, z=0

Comparing 3 observables O_i:

- Power spectrum **P(k)**
- Marked power spectrum **M(k)**
- $S_0 + S_1 + S_2$ **WST** coefficients

Evaluated using *kymatio* package (Andreux et al. 2019)

https://www.kymat.io/



WST coefficients - correlation matrix
Valogiannis & Dvorkin 2021

Marked Power Spectrum

- Marked correlation function generalizes 2-point function

$$\mathcal{M}(r) = \frac{1}{n(r)\bar{m}^2} \sum_{ij} \delta_D(|\mathbf{x}_i - \mathbf{x}_j| - r) m_i m_j = \frac{1 + W(r)}{1 + \xi(r)}$$

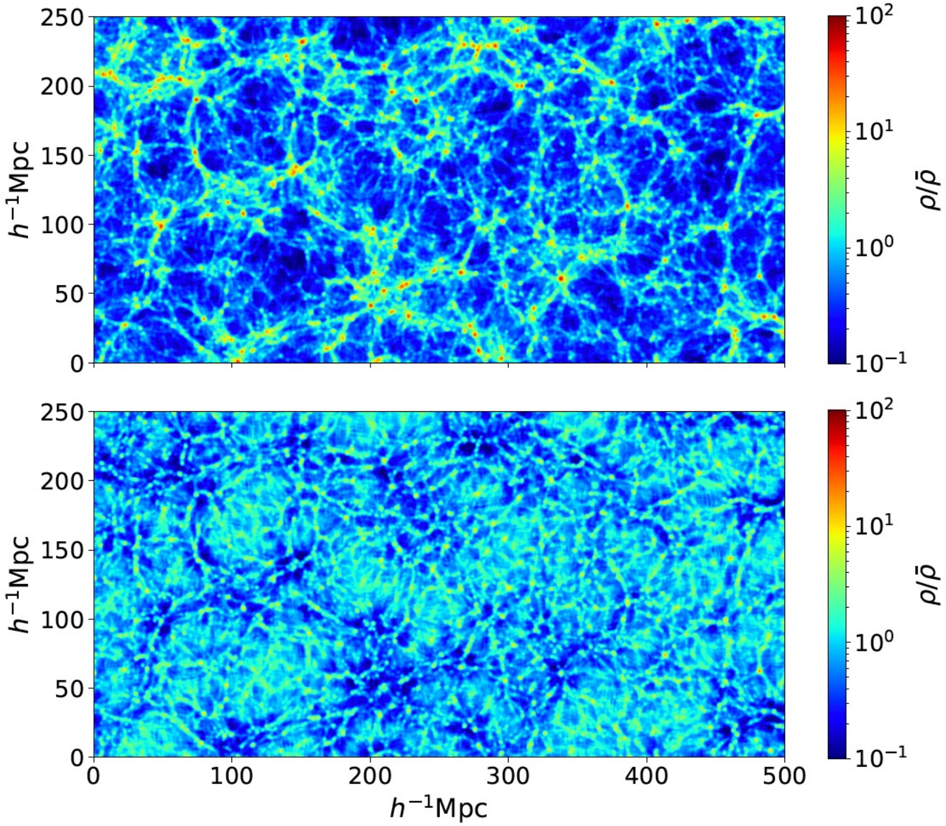
- Each galaxy weighted by mark ‘m’
- Inverse density weighted mark (highlights voids)

$$m\left[\mathbf{x}, R, \delta_s, p\right] = \left(\frac{1 + \delta_s}{1 + \delta_s + \delta_R(\mathbf{x})}\right)^p$$

- Can constrain MG (M. White 2016, **Valogiannis & Bean 2018**, Alam et al., 2021)
- Can constrain neutrino mass (Massara et al 2020)

$\delta(x)$

m(x)



Massara et al 2020

WST sensitivity to neutrino mass

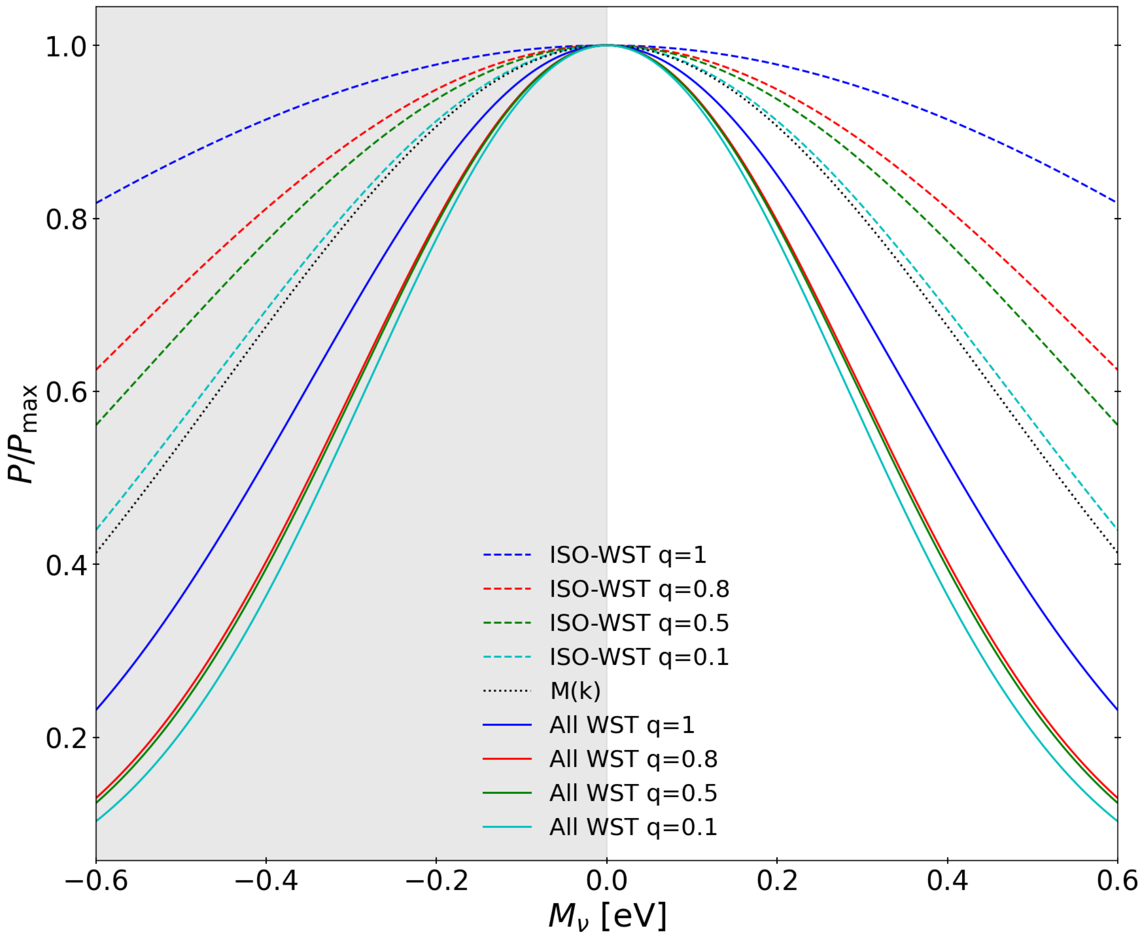
- Raising modulus to powers q<1 emphasizes on cosmic voids
- Very sensitive to neutrino mass!

$$S_0 = \langle |I(\vec{x})|^q \rangle,$$

$$S_1(j_1, l_1) = \left\langle \left(\sum_{m=-l_1}^{m=l_1} |I(\vec{x}) * \psi^m_{j_1,l_1}(\vec{x})|^2 \right)^{\frac{q}{2}} \right\rangle,$$

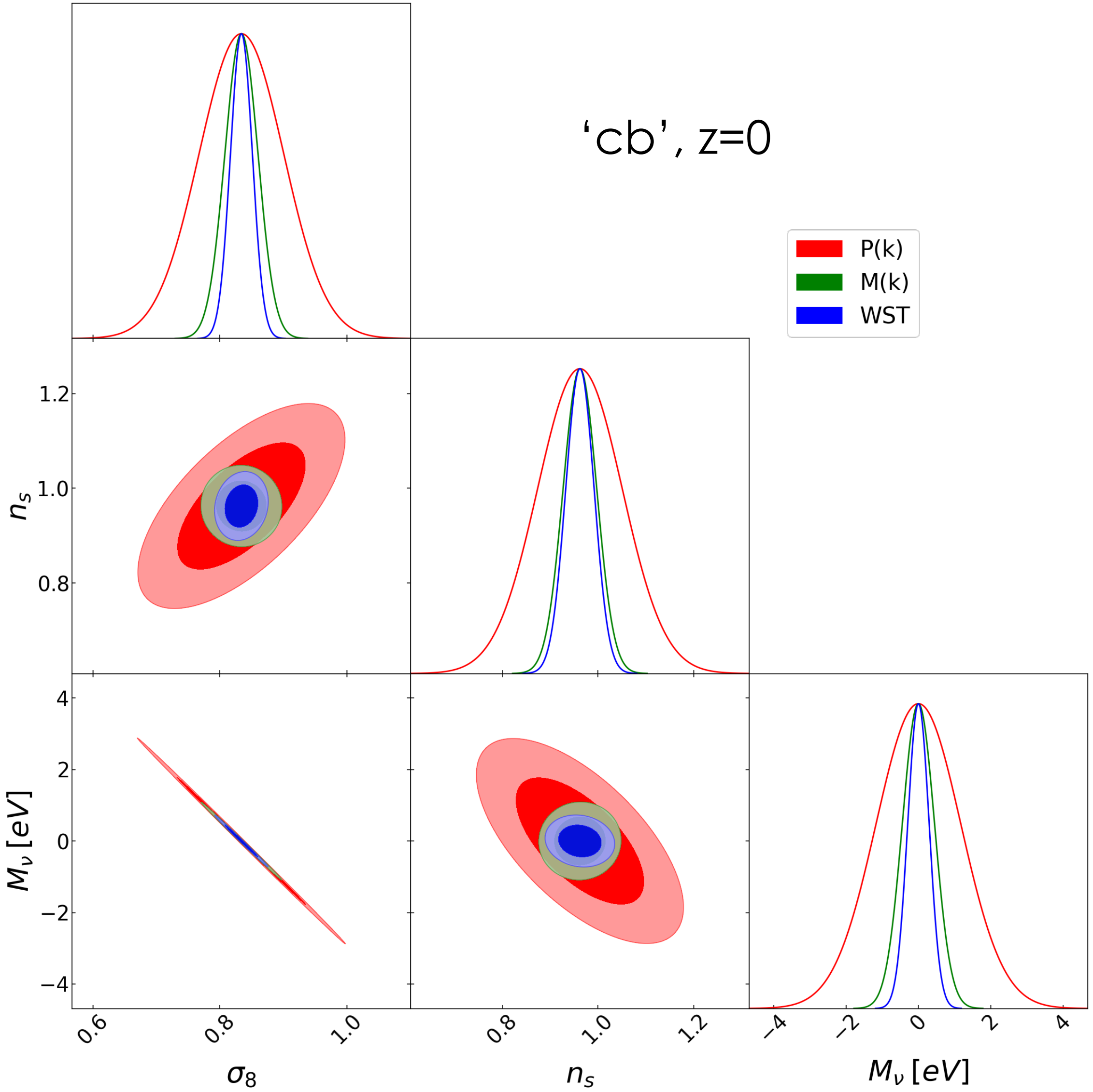
$$S_2(j_2, j_1, l_1) = \left\langle \left(\sum_{m=1}^{m=l_1} |U_1(j_1, l_1)(\vec{x}) * \psi^m_{j_2,l_1}(\vec{x})|^2 \right)^{\frac{q}{2}} \right\rangle$$

- q=0.8 found to be optimal



Valogiannis & Dvorkin 2021

Great improvement over P(k)!



- WST delivers ***very large*** improvement in the 1-σ errors for *all* parameters!
- ~1.2-4x **tighter** errors than from 'cb' P(k)!
- Constrains on neutrino mass:
    - ~4x **tighter** than 'cb' P(k)!
    - ~1.6x **tighter** than 'cb' M(k)!
- ~3x100x **tighter** errors than from 'm' P(k)

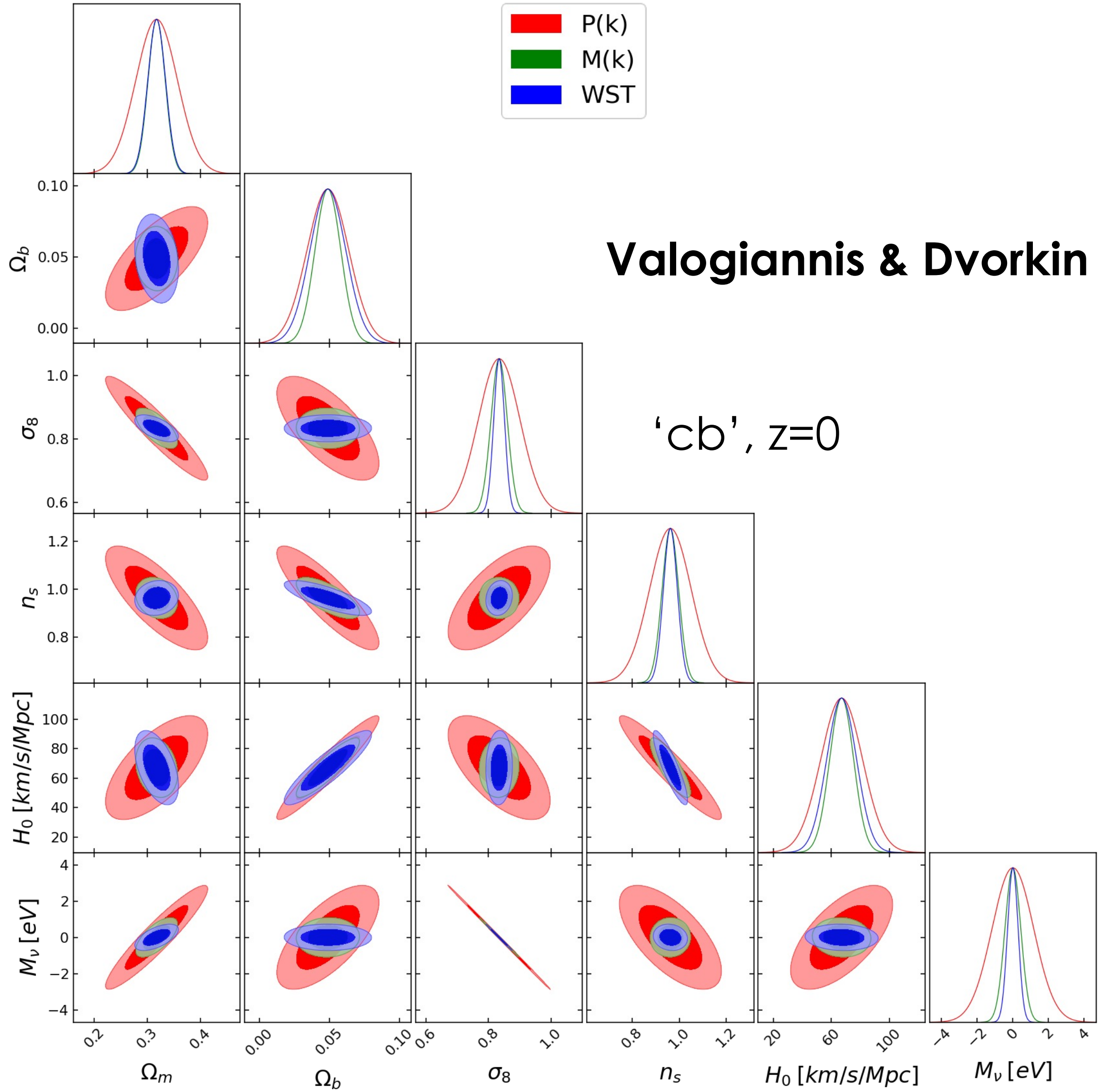
Matter type	'm'			'cb'		
Statistic	$P(k)$	$M(k)$	WST	$P(k)$	$M(k)$	WST
$\sigma(\Omega_m)$	0.076	0.013	0.014	0.040	0.016	0.016
$\sigma(\Omega_b)$	0.033	0.010	0.012	0.015	0.009	0.012
$\sigma(\sigma_8)$	0.01	0.002	0.001	0.067	0.026	0.017
$\sigma(n_s)$	0.39	0.044	0.031	0.088	0.035	0.029
$\sigma(H_0)$ [km/s/Mpc]	40.62	9.50	10.34	14.42	8.28	10.32
$\sigma(M_\nu)$ [eV]	0.72	0.016	**0.008**	1.17	0.45	**0.29**

Valogiannis & Dvorkin 2021

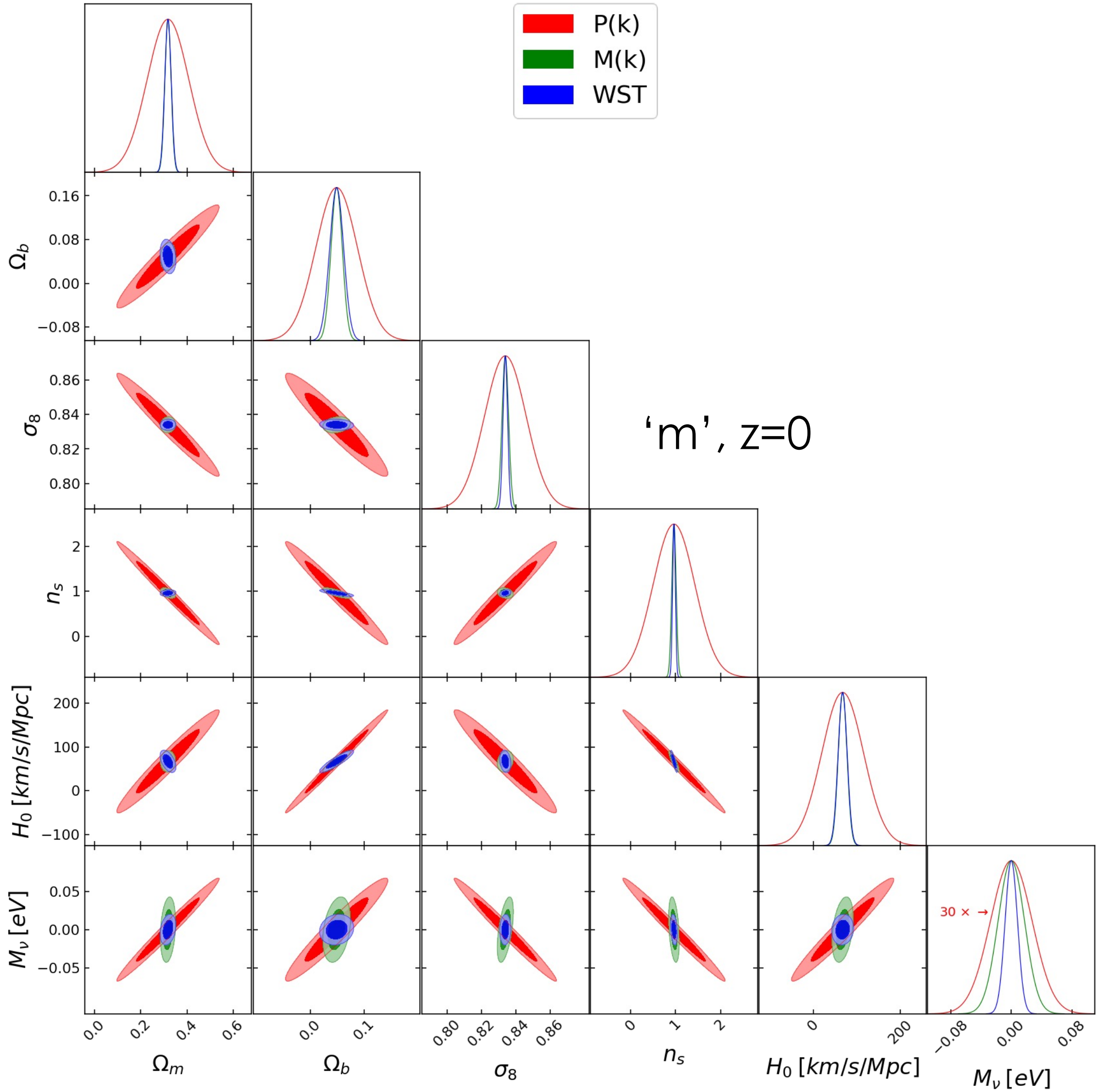
Great improvement over P(k)!

Valogiannis & Dvorkin 2021

‘cb’, z=0



‘m’, z=0



Physical explanation of results

Why does the WST work so well??

<u>WST key physical properties</u>

- Successive WST layers pick up information >2-point function ✅
  - Known to encode additional information (eg. Hahn et al. 2020 & 2021)

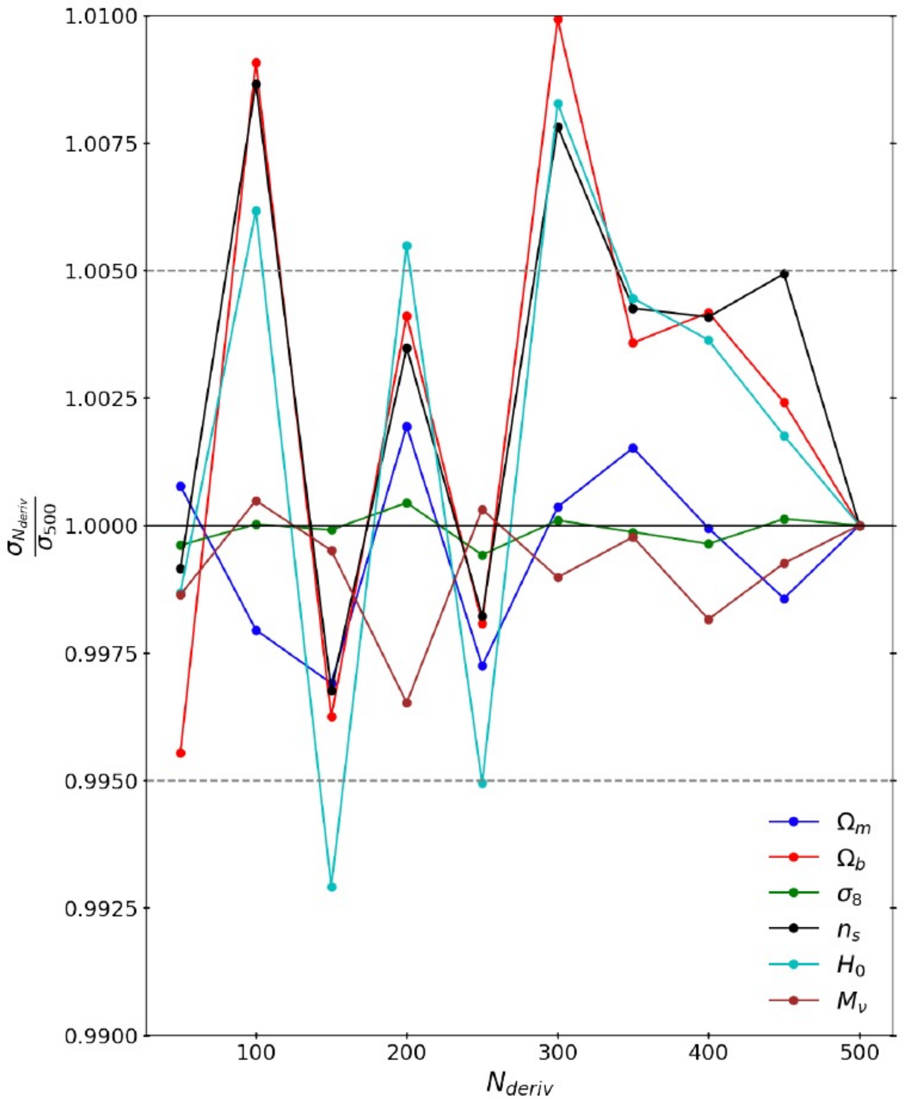
\+

- Choice of $q<1$ highlights cosmic voids (under-densities) ✅
  - Sensitive cosmological probe (eg. Massara et al, 2020)

<u>Enhanced cosmological information</u>

- Parallels to marked M(k) (Massara et al, 2020)

Convergence



- WST vector exhibits remarkable numerical convergence w.r.t. Fisher predictions

Valogiannis & Dvorkin 2021