UNIVERSIDAD DE GUANAJUATO

About the approximations for the cosmology of ultra-light axions

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Cosmology from Home 2022

Early calculations 20 years ago

Non-relativistic!

Cold and Fuzzy Dark Matter

Wayne Hu, Rennan Barkana & Andrei Gruzinov
Institute for Advanced Study, Princeton, NJ 08540
Revised February 1, 2008

Cold dark matter (CDM) models predict small-scale structure in excess of observations of the cores and abundance of dwarf galaxies. These problems might be solved, and the virtues of CDM models retained, even without postulating *ad hoc* dark matter particle or field interactions, if the dark matter is composed of ultra-light scalar particles ($m \sim 10^{-22}$eV), initially in a (cold) Bose-Einstein condensate, similar to axion dark matter models. The wave properties of the dark matter stabilize gravitational collapse providing halo cores and sharply suppressing small-scale linear power.

astro-ph/0003365

astro-ph/9910097

A New Cosmological Model of Quintessence and Dark Matter

Varun Sahni[1,*] and Limin Wang [2,†]
[1] *Inter-University Centre for Astronomy & Astrophysics, Post Bag 4, Pune 411007, India*
[2] *Department of Physics, 538 West 120th Street, Columbia University, New York NY 10027, USA*
(February 1, 2008)

We propose a new class of quintessence models in which late times oscillations of a scalar field give rise to an effective equation of state which can be negative and hence drive the observed acceleration of the universe. Our ansatz provides a unified picture of quintessence and a new form of dark matter we call *Frustrated Cold Dark Matter* (FCDM). FCDM inhibits gravitational clustering on small scales and could provide a natural resolution to the core density problem for disc galaxy halos. Since the quintessence field rolls towards a small value, constraints on slow-roll quintessence models are sa[...]y circumvented in our model.

Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\phi) = \frac{\partial V}{\partial \phi}$$

$$V(\phi) = \frac{1}{2}m_a^2\phi^2$$

(Fuzzy Dark Matter)

PHYSICAL REVIEW D, VOLUME 63, 063506

Further analysis of a cosmological model with quintessence and scalar dark matter

Tonatiuh Matos* and L. Arturo Ureña-López†
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, AP 14-740, 07000 México D.F., Mexico
(Received 1 June 2000; revised manuscript received 5 October 2000; published 20 February 2001)

We present the complete solution to a 95% scalar field cosmological model in which the dark matter is modeled by a scalar field Φ with the scalar potential $V(\Phi)=V_0[\cosh(\lambda\sqrt{\kappa_0}\Phi)-1]$ and the dark energy is modeled by a scalar field Ψ, endowed with the scalar potential $\tilde{V}(\Psi)=\tilde{V}_0[\sinh(\alpha\sqrt{\kappa_0}\Psi)]^\beta$. This model has only two free parameters, λ and the equation of state ω_Ψ. With these potentials, the fine-tuning and cosmic coincidence problems are ameliorated for both dark matter and dark energy and the model agrees with astronomical observations. For the scalar dark matter, we clarify the meaning of a scalar Jeans length and then the model predicts a suppression of the mass power spectrum for small scales having a wave number $k>k_{\mathrm{min},\Phi}$, where $k_{\mathrm{min},\Phi}\simeq 4.5h$ Mpc^{-1} for $\lambda\simeq 20.28$. This last fact could help to explain the death of dwarf galaxies and the smoothness of galaxy core halos. From this, all parameters of the scalar dark matter potential are completely determined. The dark matter consists of an ultralight particle, whose mass is $m_\Phi \simeq 1.1\times 10^{-23}$ eV and all the success of the standard cold dark matter model is recovered. This implies that a scalar field could also be a good candidate the dark matter of the Universe.

DOI: 10.1103/PhysRevD.63.063506 PACS number(s): 98.80.Cq, 95.35.+d

astro-ph/0006024

astro-ph/0105564

Quintessential Haloes around Galaxies

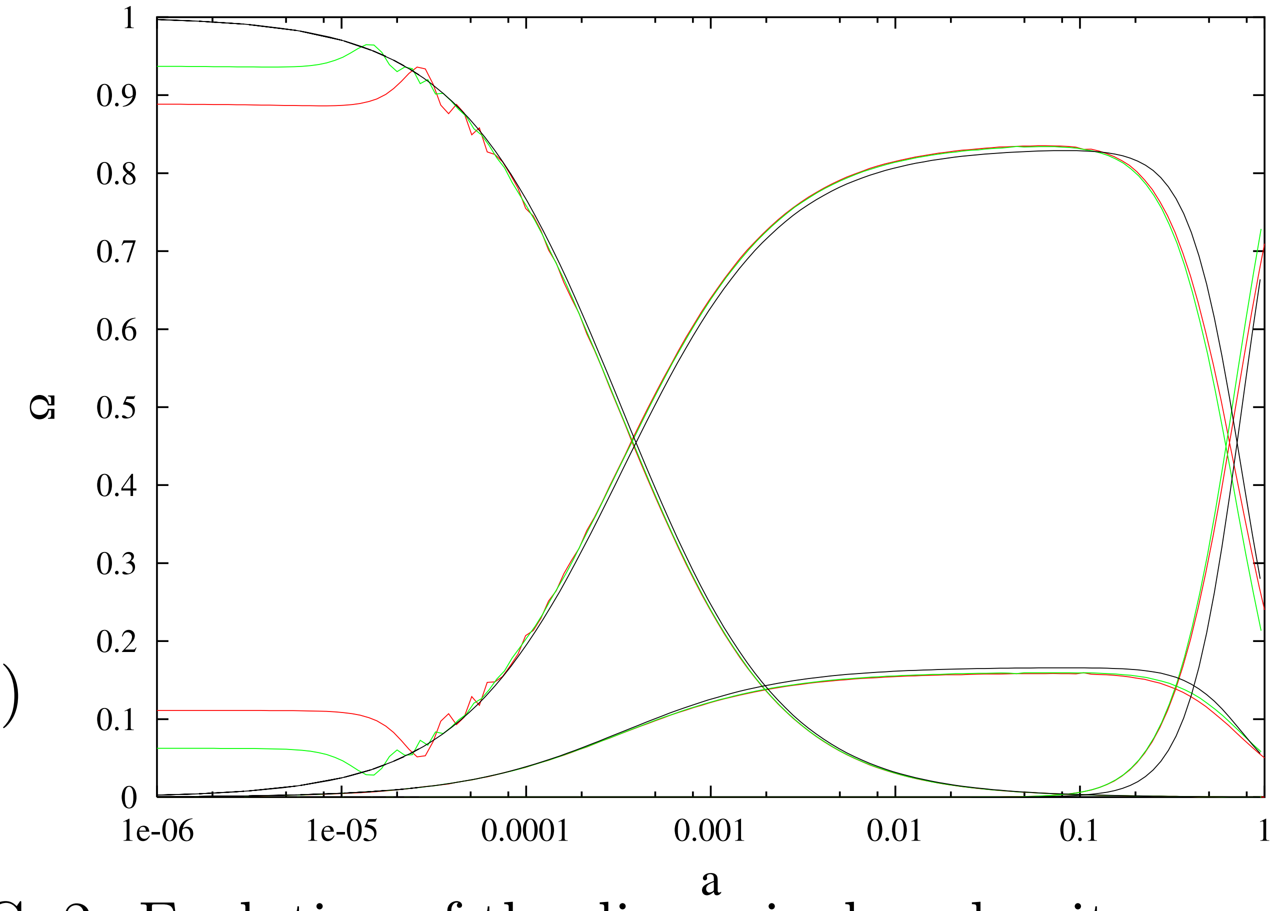
Alexandre Arbey[a,b] *, Julien Lesgourgues[a] and Pierre Salati[a,b]

a) Laboratoire de Physique Théorique LAPTH, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France.
b) Université de Savoie, B.P. 1104, F-73011 Chambéry Cedex, France.

11 September 2001

The nature of the dark matter that binds galaxies remains an open question. The favored candidate has been so far the neutralino. This massive species with evanescent interactions is now in difficulty. It would actually collapse in dense clumps and would therefore play havoc with the matter it is supposed to shepherd. We focus here on a massive and non–interacting complex scalar field as an alternate option to the astronomical missing mass. We investigate the classical solutions that describe the Bose condensate of such a field in gravitational interaction with matter. This simplistic model accounts quite well for the dark matter inside low–luminosity spirals whereas the agreement lessens for the brightest objects where baryons dominate. A scalar mass $m \sim 0.4$ to 1.6×10^{-23} eV is derived when both high and low–luminosity spirals are fitted at the same time. Comparison with astronomical observations is made quantitative through a chi–squared analysis. We conclude that scalar fields offer a promising direction worth being explored.

Old and new: ultra-light axions



. (40)

FIG. 2. Evolution of the dimensionless density parameters *vs* the scale factor a with $\Omega_{oM} = 0.30$: ΛCDM (black) and $\Psi\Phi DM$ for two values of $\lambda = 6$ (red), $\lambda = 8$ (green). The equation of state for the dark energy is $\omega_\Psi = -0.8$.

Matos and U-L, PRD 63 (2001) 063506, astro-ph/0006024. The plots show the numerical solutions of the scalar field EOM obtained from an amended version of CMBFast, the Boltzmann code available at that time. The MPS could only be calculated up to z=50, due to the difficulty to follow the rapid oscillations of the field. This was the first time that a cut-off in the MPS was obtained from a Boltzmann code.

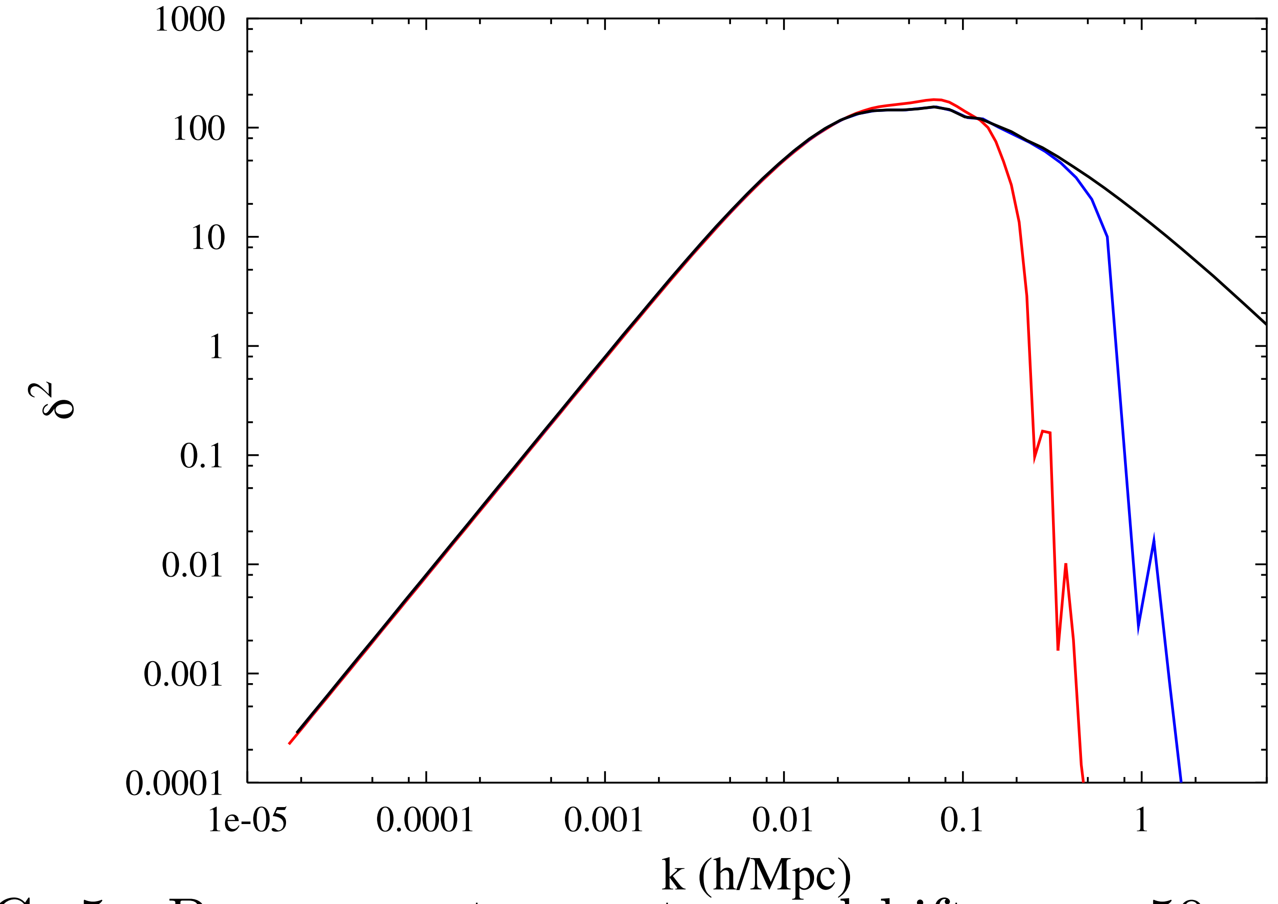
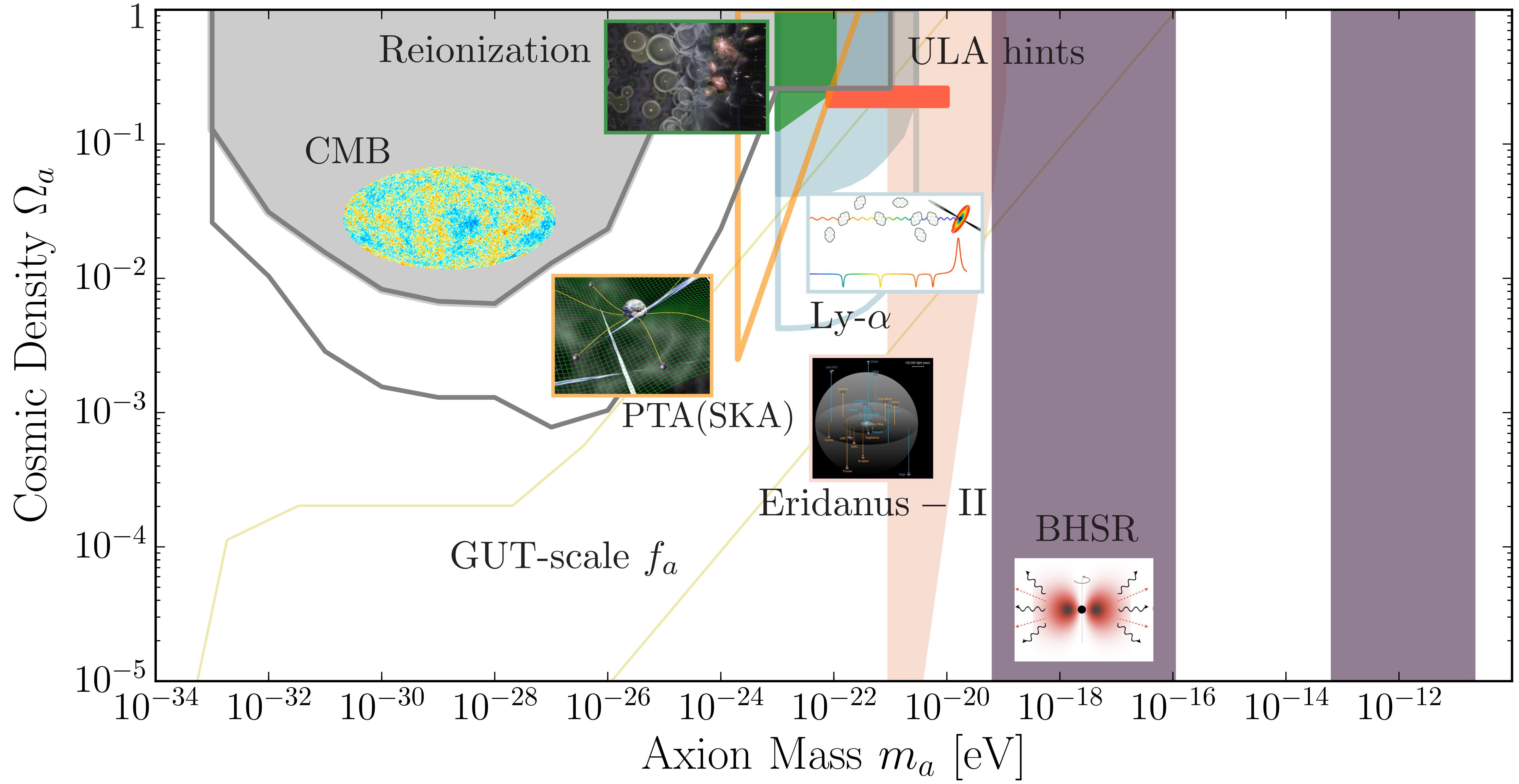


FIG. 5. Power spectrum at a redshift $z = 50$: ΛCDM (black), and ΦCDM with $\lambda = 5$ (red) and $\lambda = 10$ (blue). The normalization is arbitrary.



Astro2020 Science White Paper: Gravitational probes of ultra-light axions
Grin et al, ArXiv 1904:09003

FDM: standard approach

Hlkozek et al, PRD 91 (2015) 103512
Cookmeyer et al, PRD 101, 023501 (2020)*

Background evolution

$$\ddot{\phi}_0 + 2\mathcal{H}\dot{\phi}_0 + m_a^2 a^2 \phi_0 = 0,$$

$$m_a \approx 3H(a_{\rm osc})$$

Onset of rapid field oscillations

$$\Omega_{\rm a} = \left[\frac{a^{-2}}{2}\dot{\phi}_0^2 + \frac{m_a^2}{2}\phi_0^2\right]_{m_a=3H} a_{\rm osc}^3/\rho_{\rm crit},$$

$$\rho_{\rm a} \propto a^{-3}$$

Linear perturbations

$$\dot{\delta}_a = -\,ku_a - (1+w_{\rm a})\,\dot{\beta}/2 - 3\mathcal{H}\,(1-w_a)\,\delta_a$$
$$-9\mathcal{H}^2\left(1-c_{\rm ad}^2\right)u_{\rm a}/k,$$
$$\dot{u}_a = 2\mathcal{H}u_a + k\delta_a + 3\mathcal{H}\left(w_a - c_{\rm ad}^2\right)u_a,$$

$$\dot{\delta}_a = -ku_a - \frac{\dot{\beta}}{2} - 3\mathcal{H}c_a^2\delta_a - 9\mathcal{H}^2c_a^2u_a/k,$$
$$\dot{u}_a = -\mathcal{H}u_a + c_a^2k\delta_a + 3c_a^2\mathcal{H}u_a.$$

$$c_{\rm ad}^2 \equiv \frac{\dot{P}_a}{\dot{\rho}_a} = w_a - \frac{\dot{w}_a}{3\mathcal{H}\,(1+w_a)}.$$

Onset of rapid field oscillations

$$c_a^2 \equiv \frac{\delta P}{\delta\rho} = \frac{k^2/(4m_a^2a^2)}{1+k^2/(4m_a^2a^2)},$$

FDM: standard approach

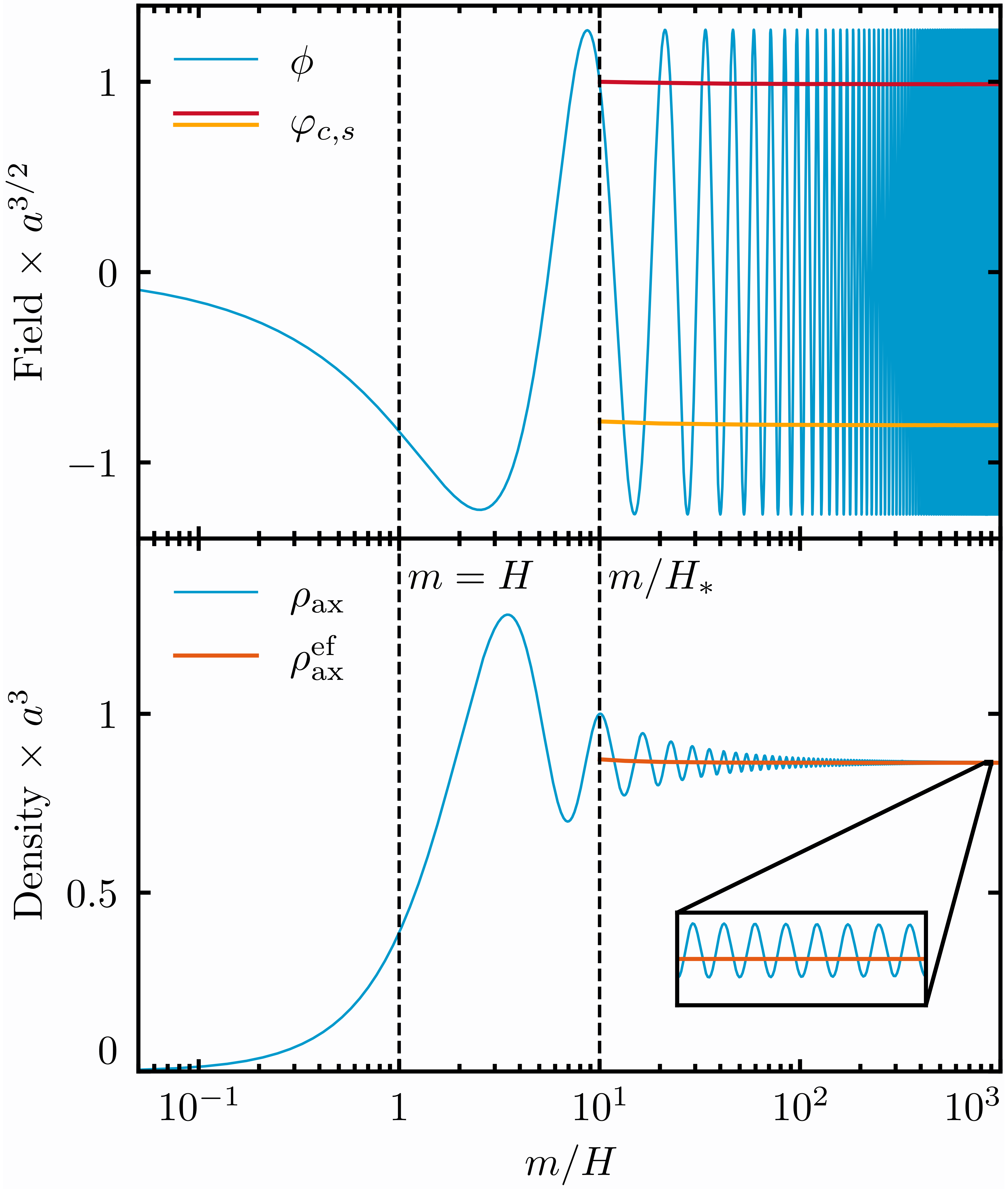
Caveats and difficulties: background

Passaglia and Hu, arXiv:2201.10238
Cookmeyer et al, PRD 101, 023501 (2020)*

$$a^3\rho_{\rm ax} \propto 1 + \mathcal{O}\left(\cos\left[2\tau\right]\left(\frac{m}{H}\right)^{-1}, \left(\frac{m}{H}\right)^{-2}\right), \quad (5)$$

$$a^3 P_{\rm ax} \propto \cos\left[2(\tau + C_2)\right] + \mathcal{O}\left(\cos\left[2\tau\right]\left(\frac{m}{H}\right)^{-1}, \left(\frac{m}{H}\right)^{-2}\right)$$

- Concern 1. The field itself, the equation of state, the density and the pressure, are all rapidly oscillating functions!
- Concern 2. The sudden switch from field to fluid, made at an arbitrary time, leads to an offset of the density to its true value from the field equations. This offset depends on the value of the switch time, and this raises accuracy concerns.
- Concern 3. It only works for fuzzy dark matter.



FDM: standard approach

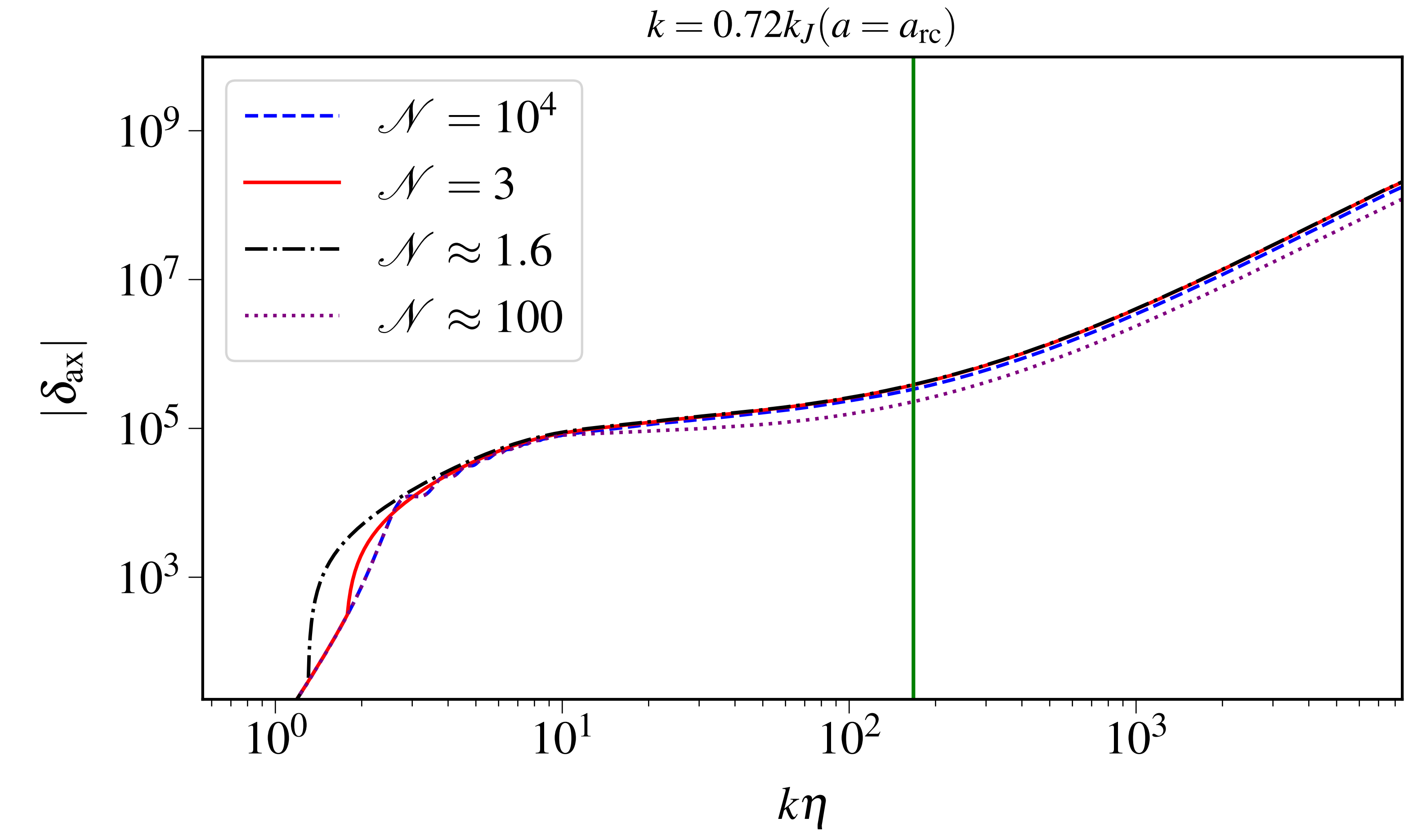
Caveats and difficulties: linear perturbations

Passaglia and Hu, arXiv:2201.10238
Cookmeyer et al, PRD 101, 023501 (2020)*

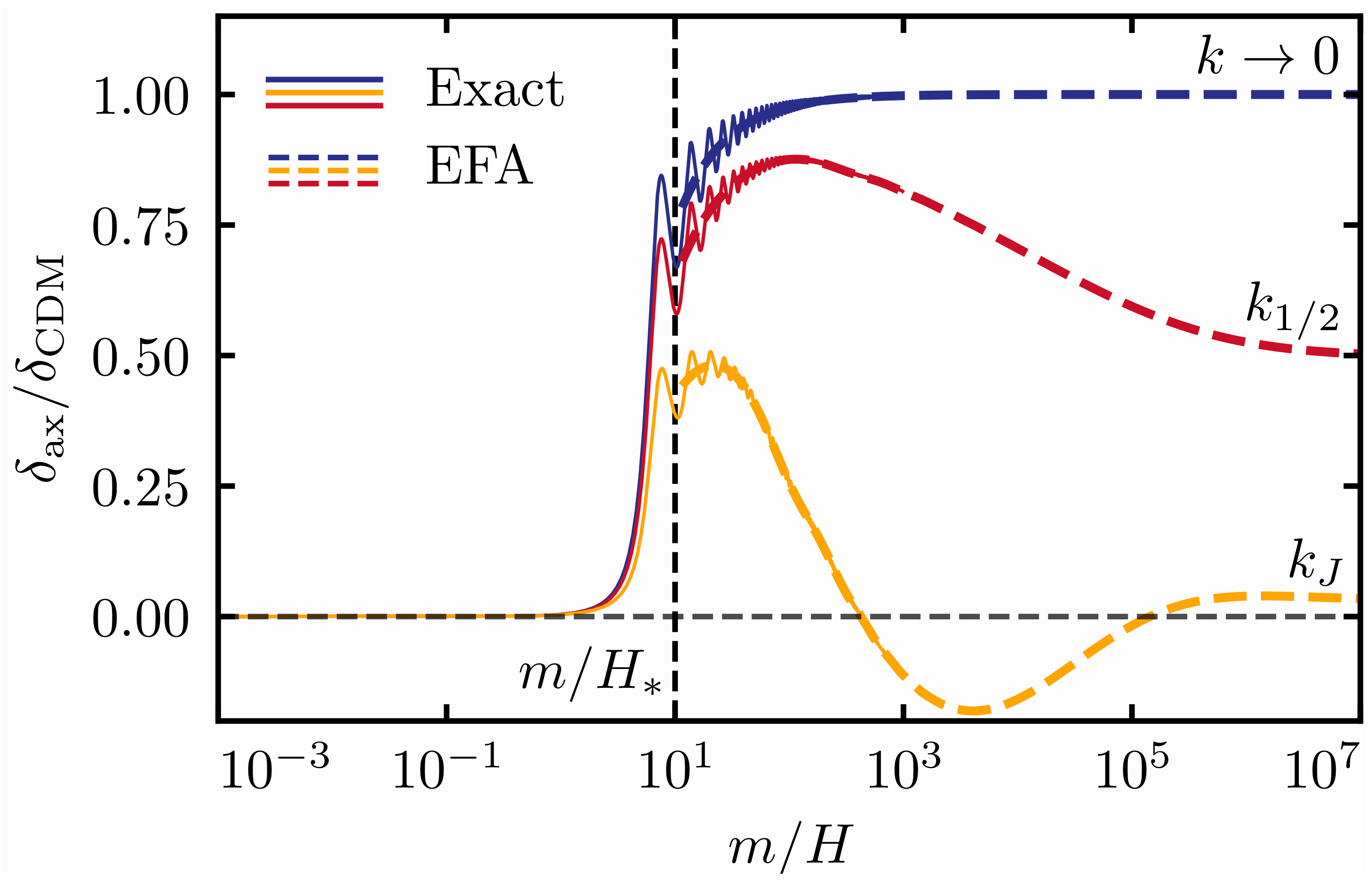
$$c_{s\phi} = \left(\frac{k}{am}\right)^{-1}\left(\sqrt{1+\left(\frac{k}{am}\right)^2} - 1\right)$$

$$c^2_{s,\mathrm{efa}} = c^2_{s\phi} + \frac{5}{4}\frac{H^2}{m^2} \quad ?$$

$$c_a^2 \equiv \frac{\delta P}{\delta \rho} = \frac{k^2/(4m_a^2a^2)}{1+k^2/(4m_a^2a^2)}, \quad ?$$



- Concern 1. What is the true sound speed?
- Concern 2. At small scales, the field density and pressure perturbations oscillate rapidly, and then the effective sound speed is not well defined.
- Concern 3. The fluid formalism has to be manually adapted to give accurate results.
- Concern 4. It only works for fuzzy dark matter.



FDM: alternative approach

U-L, González-Morales, JCAP 07 (2016) 048
Cookmeyer et al, PRD 101, 023501 (2020)*

Background

$$
\begin{aligned}
\theta' &= -3\sin\theta + y_1\,,\\
y_1' &= \frac{3}{2}\left(1+w_{tot}\right)y_1 + \frac{\lambda}{2}\Omega_\phi \sin\theta\,,\\
\Omega_\phi' &= 3(w_{tot}-w_\phi)\Omega_\phi\,.
\end{aligned}
$$

Onset of rapid field oscillations

$\cos\theta, \sin\theta \to 0$

$$
\begin{aligned}
\theta' &= [redacted]\; y_1\,,\\
y_1' &= \frac{3}{2}\left(1+w_{tot}\right)y_1\; [redacted]\,,\\
\Omega_\phi' &= 3(w_{tot}\; [redacted])\Omega_\phi\,.
\end{aligned}
$$

Fully equivalent to CDM

Linear perturbations

$$
\begin{aligned}
\delta_0' &= \left[-3\sin\theta - \frac{k^2}{k_J^2}(1-\cos\theta)\right]\delta_1 + \frac{k^2}{k_J^2}\sin\theta\,\delta_0 - \frac{\bar{h}'}{2}(1-\cos\theta)\,,\\
\delta_1' &= \left[-3\cos\theta - \frac{k_{eff}^2}{k_J^2}\sin\theta\right]\delta_1 + \frac{k_{eff}^2}{k_J^2}\left(1+\cos\theta\right)\delta_0 - \frac{\bar{h}'}{2}\sin\theta\,,
\end{aligned}
$$

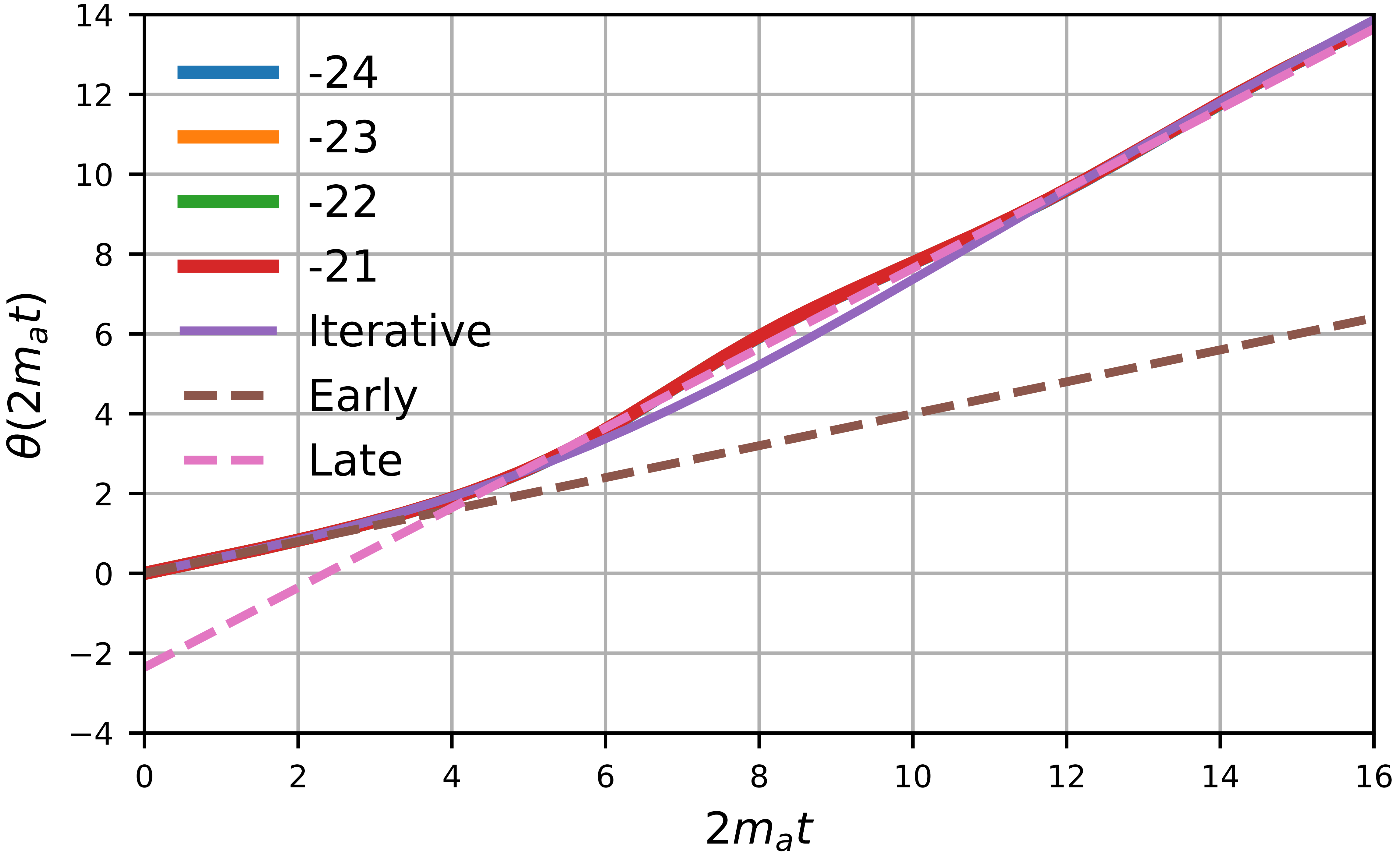
$$k_{eff}^2 \equiv k^2 - \lambda a^2 H^2 \Omega_\phi/2.$$

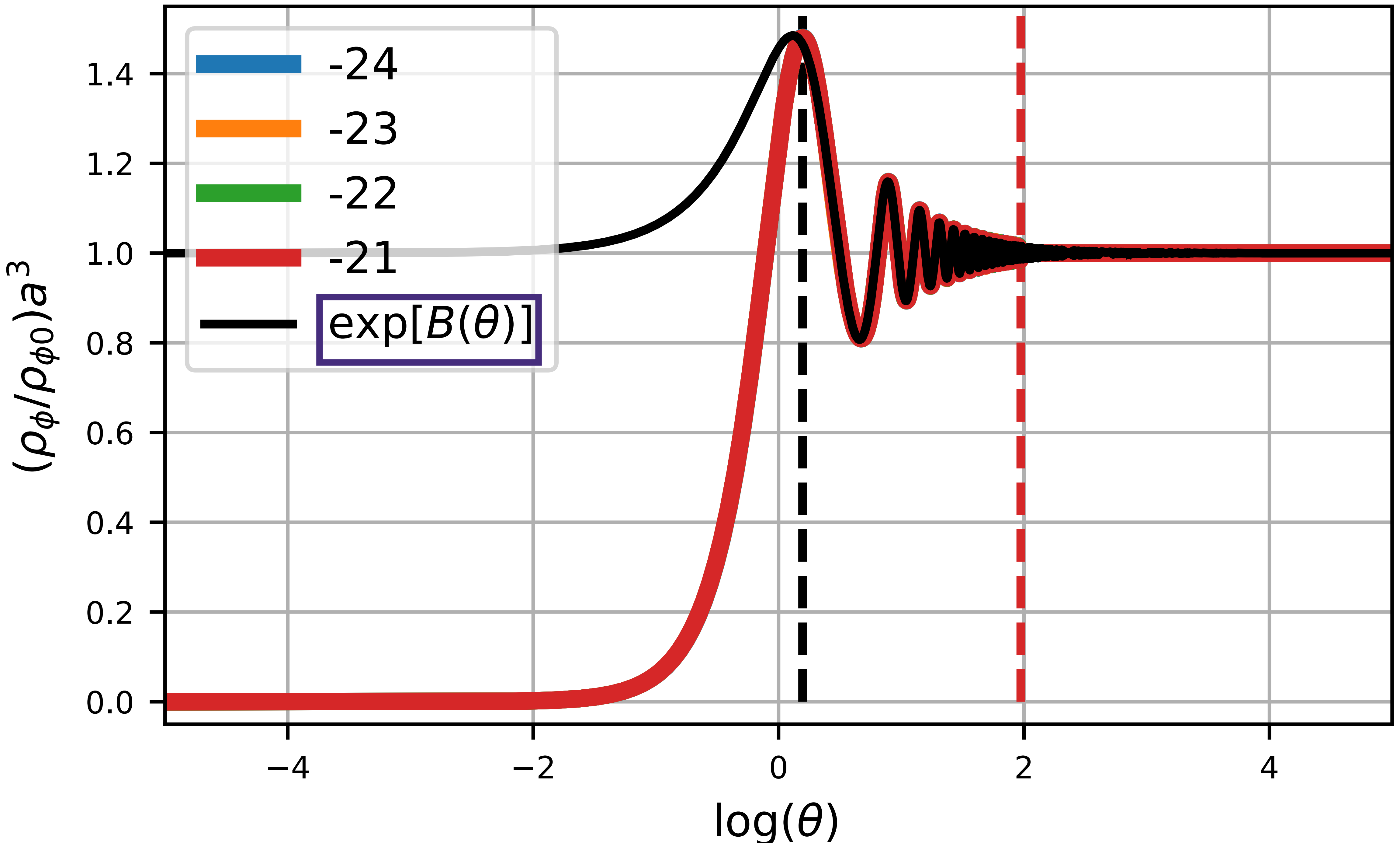
$$
\begin{aligned}
\delta_0' &= \left[[redacted]\; \frac{k^2}{k_J^2}\; [redacted]\right]\delta_1\; [redacted] - \frac{\bar{h}'}{2}\; [redacted]\\
\delta_1' &= [redacted]\; \frac{k_{eff}^2}{k_J^2}\; [redacted]\; \delta_0\; [redacted]
\end{aligned}
$$

Fully equivalent to CDM, except for some surviving terms with scale dependence that lead to a cut-off in the MPS

FDM: alternative approach

Nearly-exact solutions





- After the onset of rapid oscillations $t > t_\star$:
  - The polar angle is given by $\theta(t > t_\star) \simeq 2m_a t - 3\pi/4$. Good agreement with the numerical solution (left figure).
  - The density is given by $\rho(t > t_\star) = (\rho_0/a^3)\exp\left(\frac{3\sin\theta_\star}{2(\theta_\star + 3\pi/4)}\right)$. **The offset from neglecting rapid oscillations can be avoided if $\theta_\star = n\pi$**

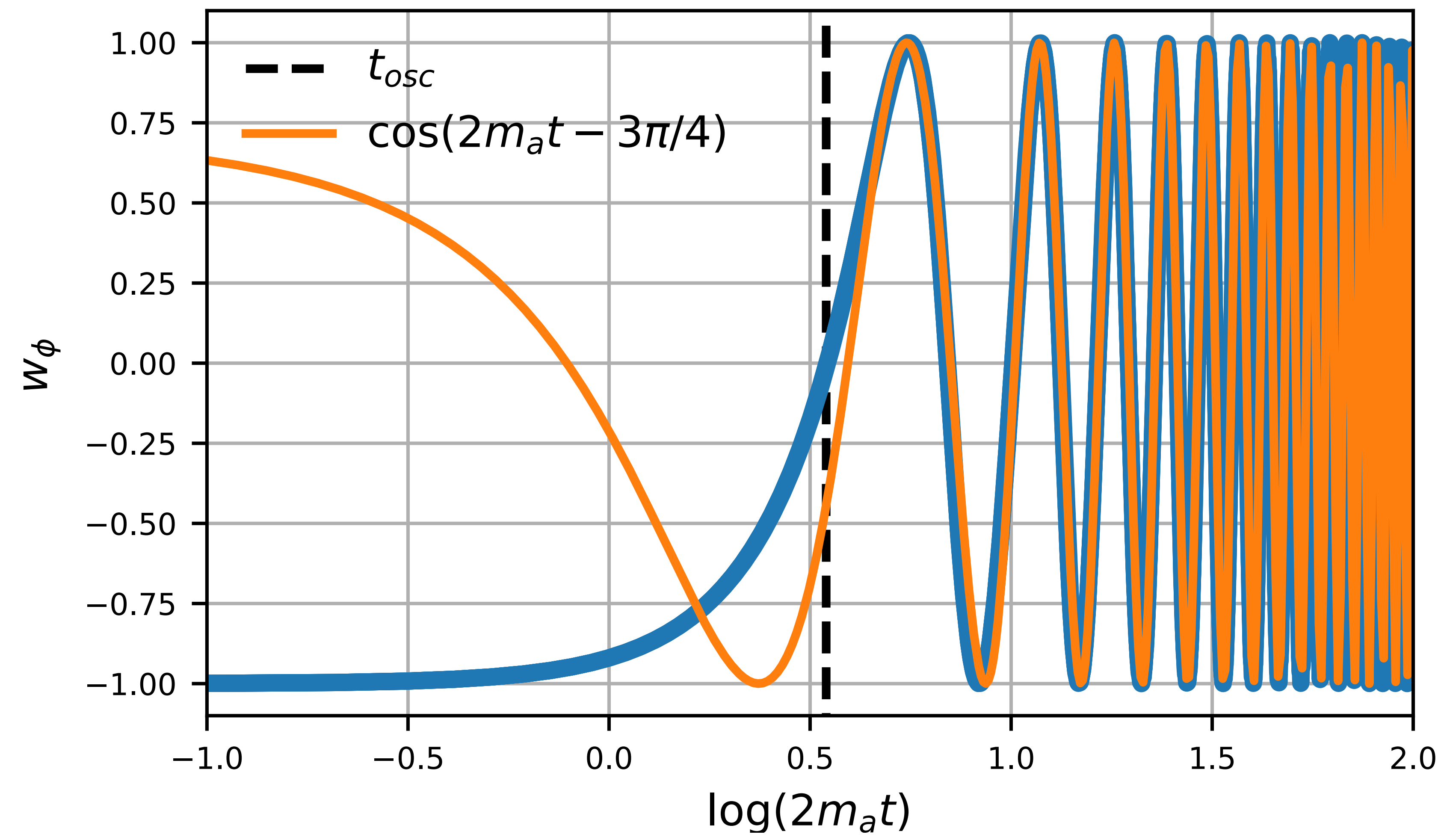
FDM: alternative approach

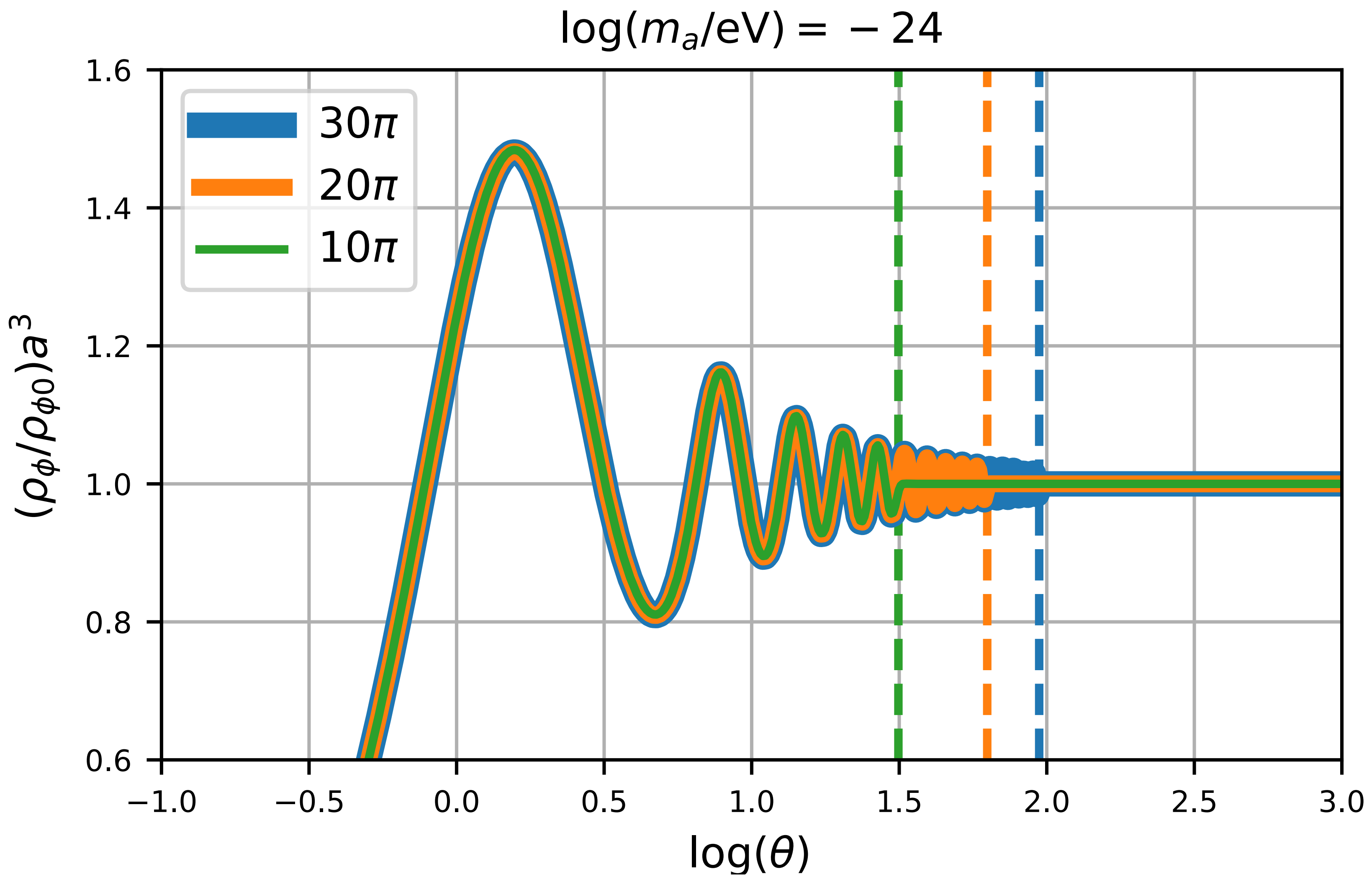
U-L, Linares, in preparation
Linares-Cedeño, González-Morales, U-L, JCAP 1, 051 (2021)
Linares-Cedeño, González-Morales, U-L et al, PRD 96 (2017) 061301(R)
U-L, González-Morales, JCAP 07 (2016) 048

Caveats and difficulties: background

$$a^3\rho_{\rm ax} \propto 1 + \mathcal{O}\left(\cos\left[2\tau\right]\left(\frac{m}{H}\right)^{-1}, \left(\frac{m}{H}\right)^{-2}\right), \qquad (5)$$

$$a^3 P_{\rm ax} \propto \cos\left[2(\tau + C_2)\right] + \mathcal{O}\left(\cos\left[2\tau\right]\left(\frac{m}{H}\right)^{-1}, \left(\frac{m}{H}\right)^{-2}\right)$$





- Concern 1. The field itself, the equation of state, the density and the pressure, are all rapidly oscillating functions! **Oscillating terms are shown explicitly in the EOM**
- Concern 2. The sudden switch from field to fluid, made at an arbitrary time, leads to an offset of the density to its true value from the field equations. This offset depends on the value of the switch time, and this raises accuracy concerns. **Accuracy for the background evolution is controlled by $\theta_\star$, and is maximized if $\theta_\star = n\pi$.**
- Concern 3. It only works for fuzzy dark matter. **It works for other cases, and can be easily adapted for a variety of cases.**

FDM: alternative approach

U-L, Linares, in preparation
Linares-Cedeño, González-Morales, U-L, JCAP 1, 051 (2021)
Linares-Cedeño, González-Morales, U-L et al, PRD 96 (2017) 061301(R)
U-L, González-Morales, JCAP 07 (2016) 048

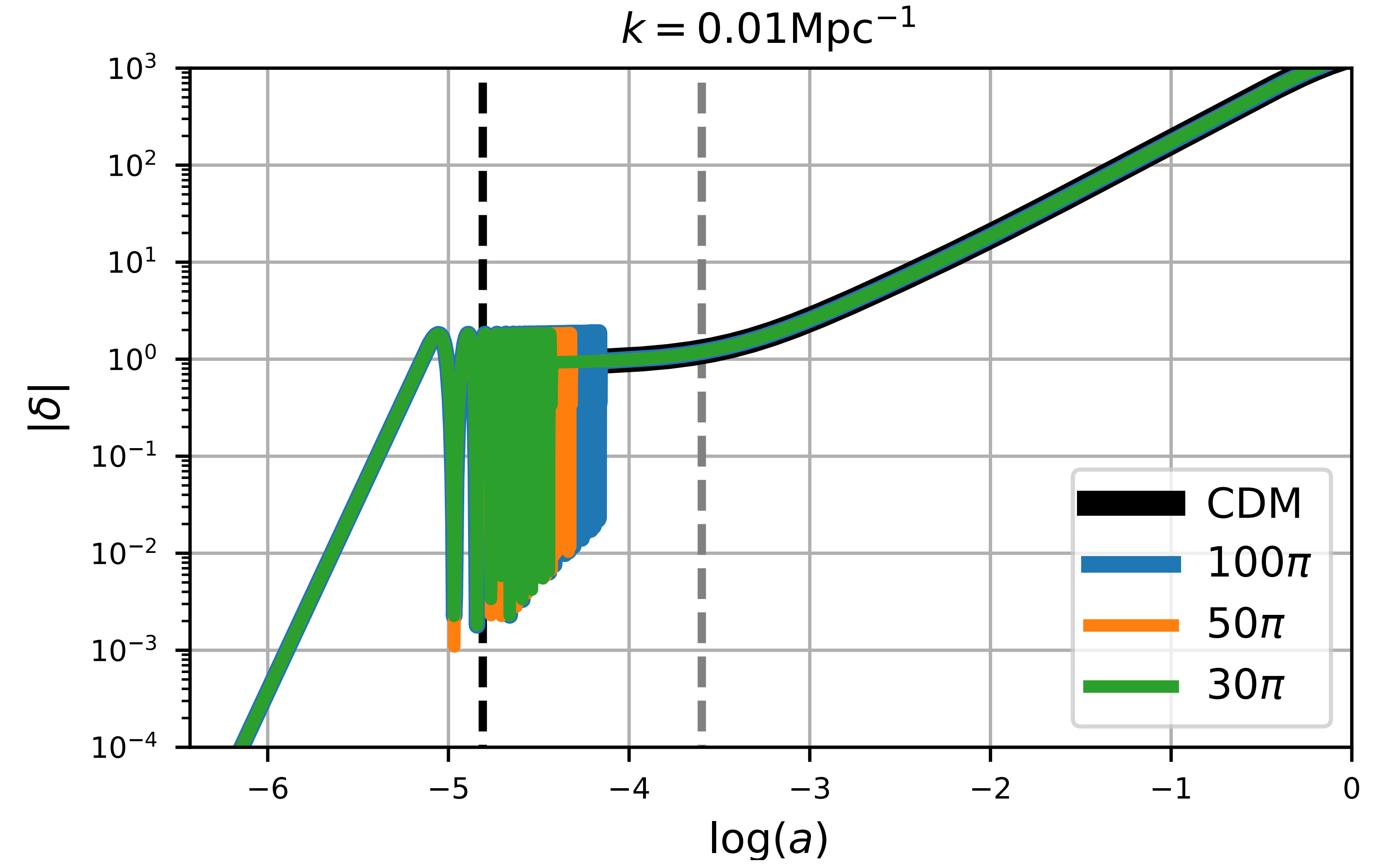
Caveats and difficulties: linear perturbations

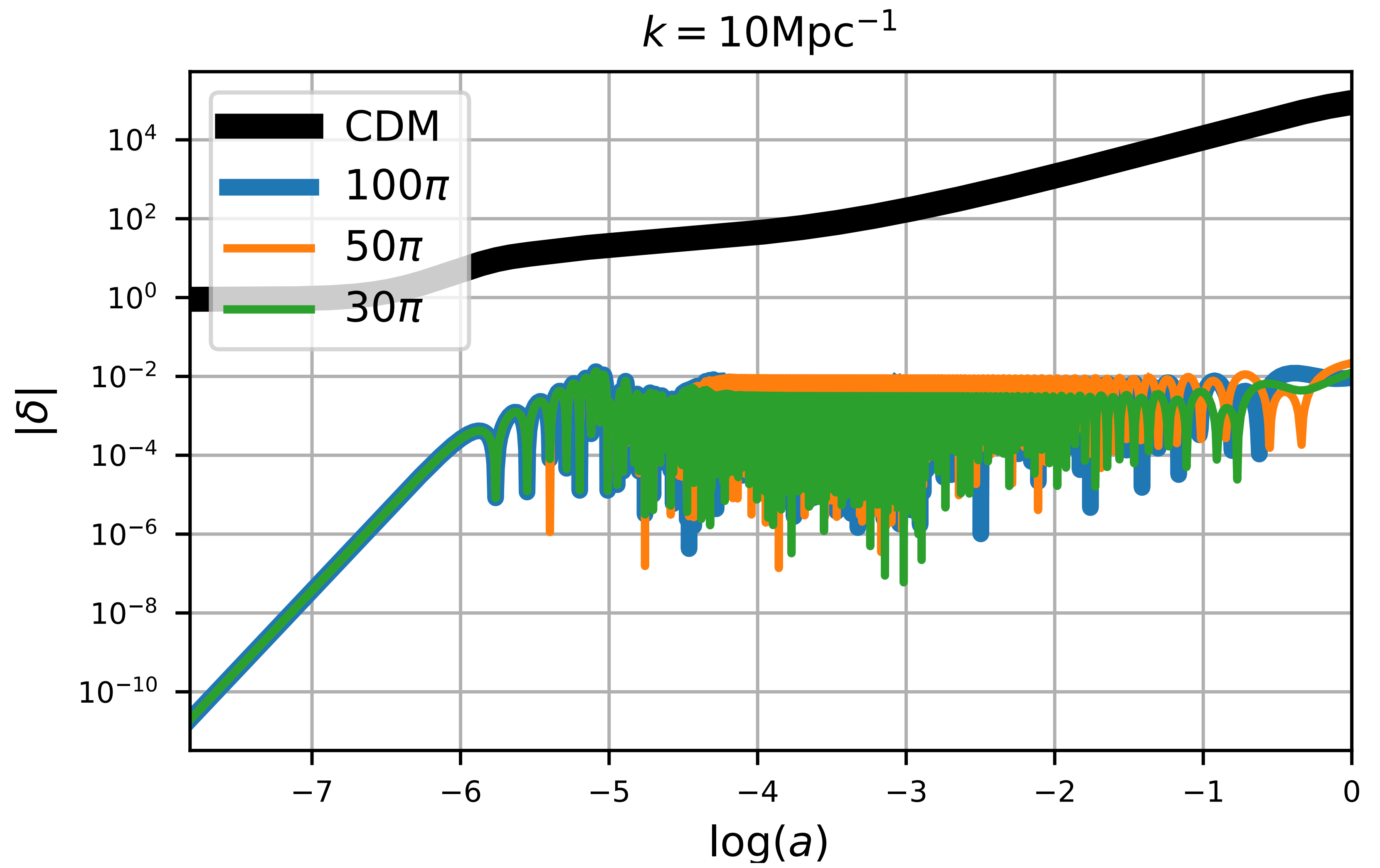
$$c_{s\phi} = \left(\frac{k}{am}\right)^{-1}\left(\sqrt{1+\left(\frac{k}{am}\right)^2}-1\right)$$

$$c^2_{s,\text{efa}} = c^2_{s\phi} + \frac{5}{4}\frac{H^2}{m^2} \quad ?$$

$$c_a^2 \equiv \frac{\delta P}{\delta \rho} = \frac{k^2/(4m_a^2a^2)}{1+k^2/(4m_a^2a^2)}, \quad ?$$

- Concern 1. What is the true sound speed? **No need to define a sound speed.**
- Concern 2. At small scales, the field density and pressure perturbations oscillate rapidly, and then the effective sound speed is not well defined. **See above.**
- Concern 3. The fluid formalism has to be manually adapted to give accurate enough results. **Accuracy for the evolution is again controlled by $\theta_\star$, and is maximized if $\theta_\star = n\pi$.**
- Concern 4. It only works for fuzzy dark matter. **It works for other cases, and can be easily adapted for a variety of cases.**





FDM: alternative approach

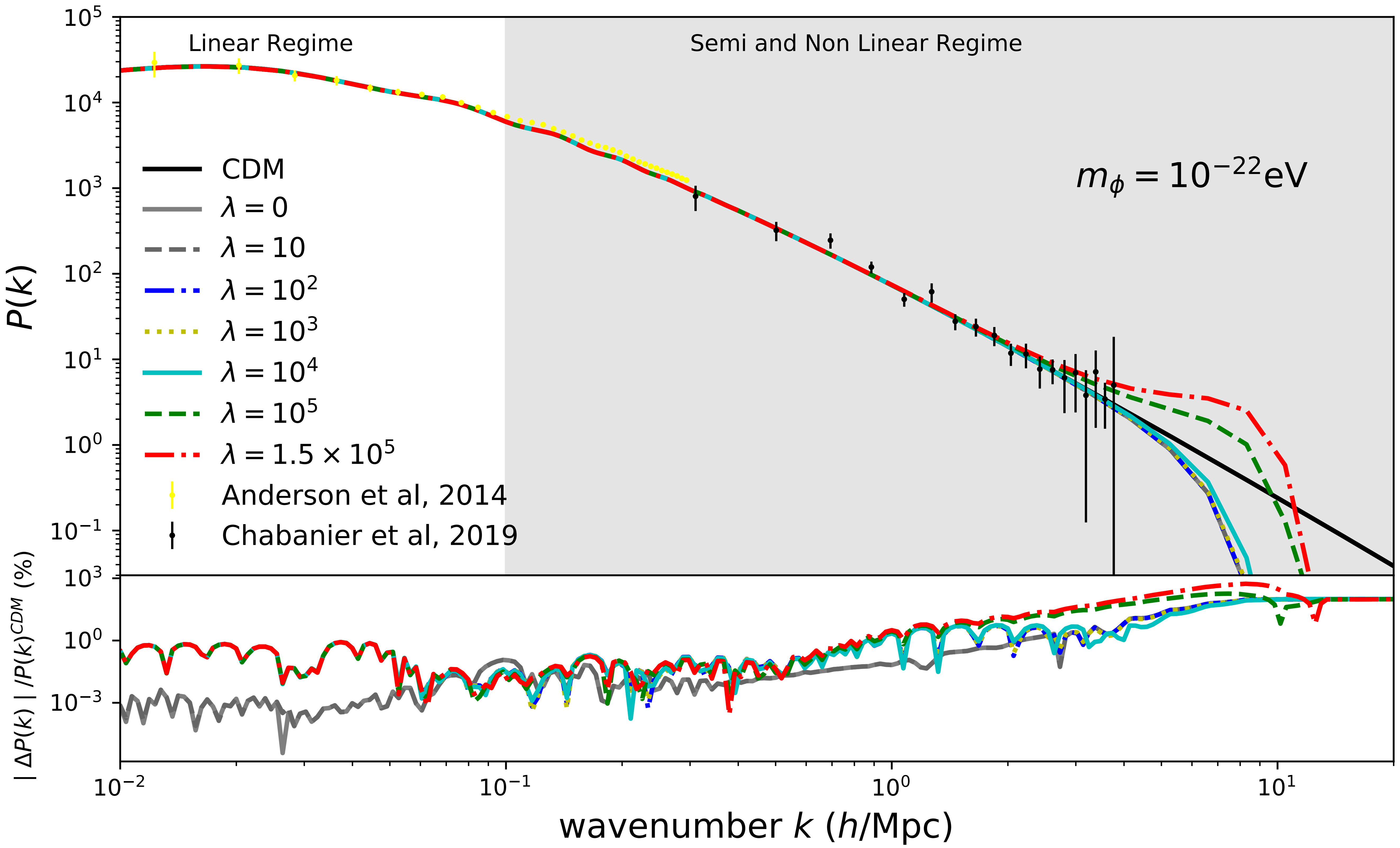
Axion-like potential

Linares-Cedeño, González-Morales, U-L, JCAP 1, 051 (2021), arXiv:2006.05037

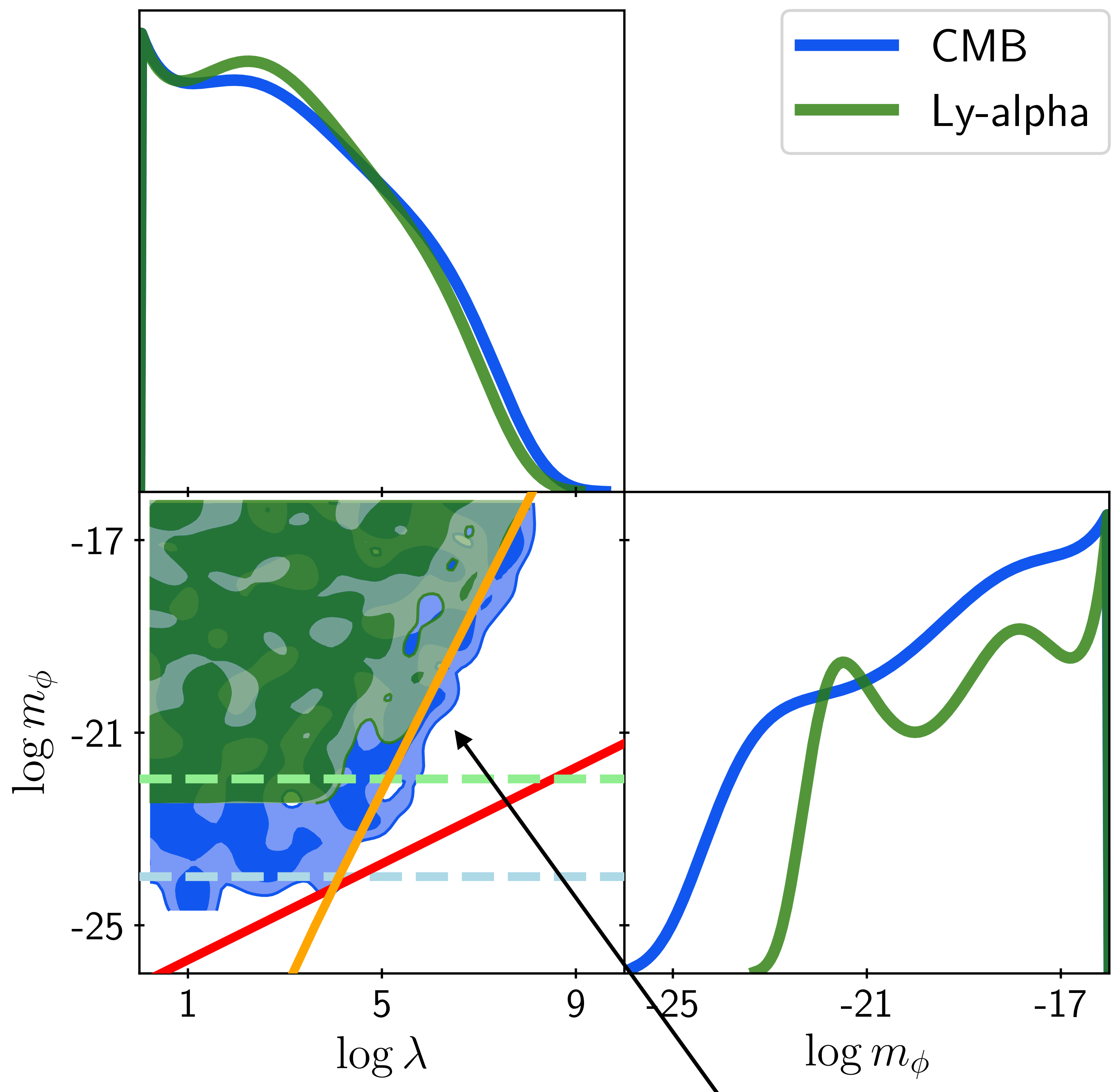
Linares-Cedeño, González-Morales, U-L et al, PRD 96 (2017) 061301(R)

$$V(\phi) = m_a^2 f_a^2 [1 - \cos(\phi/f_a)]$$

$$m_a \gtrsim 10^{-22}\,\mathrm{eV}/c^2\,(95.5\,\%\,\mathrm{CL})$$



Tachyonic instability of **linear** density perturbations



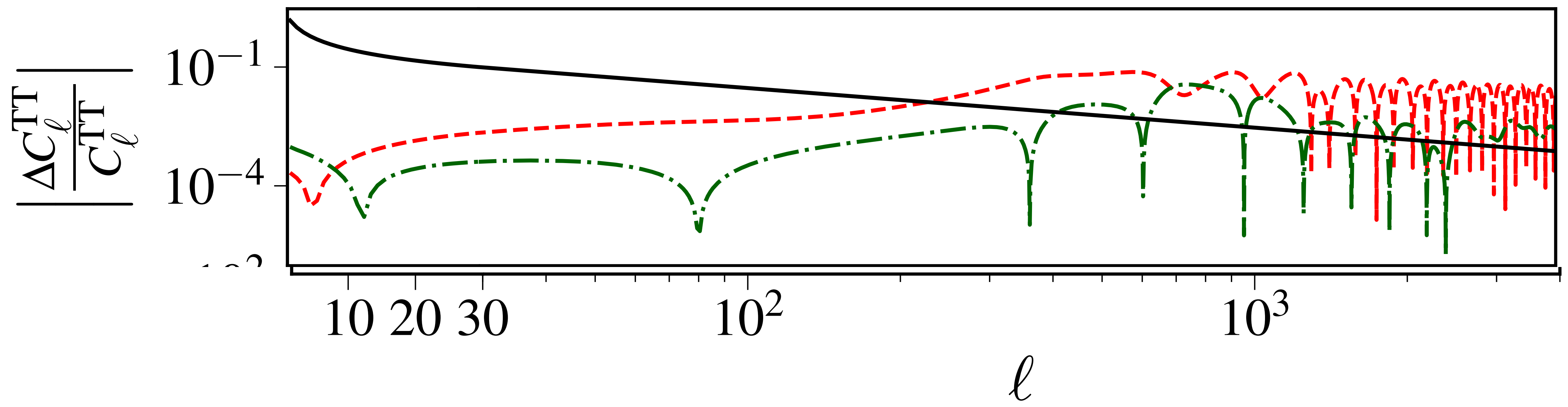
Available prior volume: **no evidence for an extra parameter!**

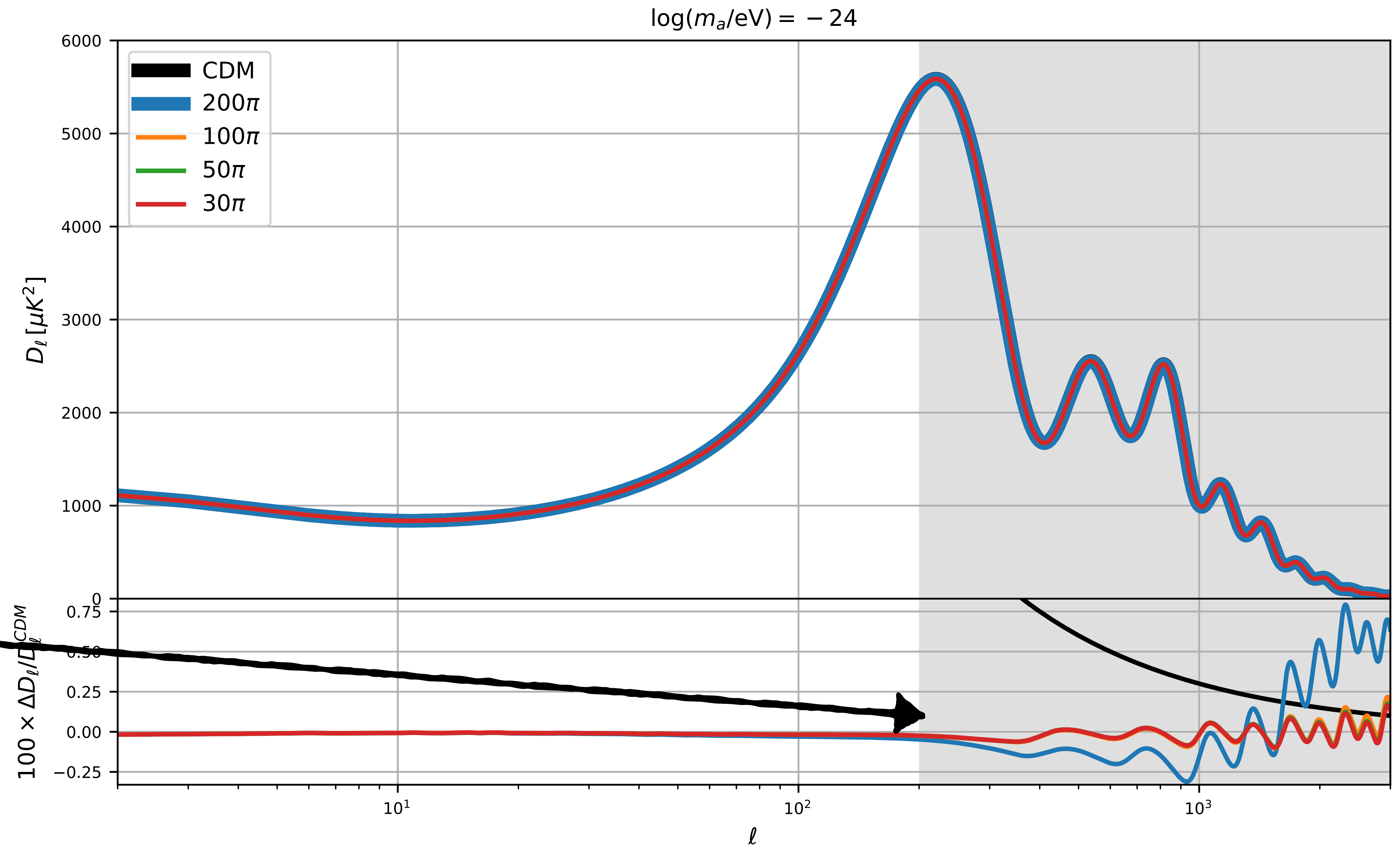
(Axion DM is the total DM budget)

Implications for statistical inference

CMB

- Cookmeyer et al, PRD 101, 023501 (2020). Rule of thumb implies that bias-free parameter inference requires $\Delta C_\ell / C_\ell \lesssim 3/\ell$.
  - This condition is violated in the fluid approximation.
- U-L, Linares, in preparation. The alternative approach keeps errors below the required accuracy already for $\theta_\star = 30\pi$





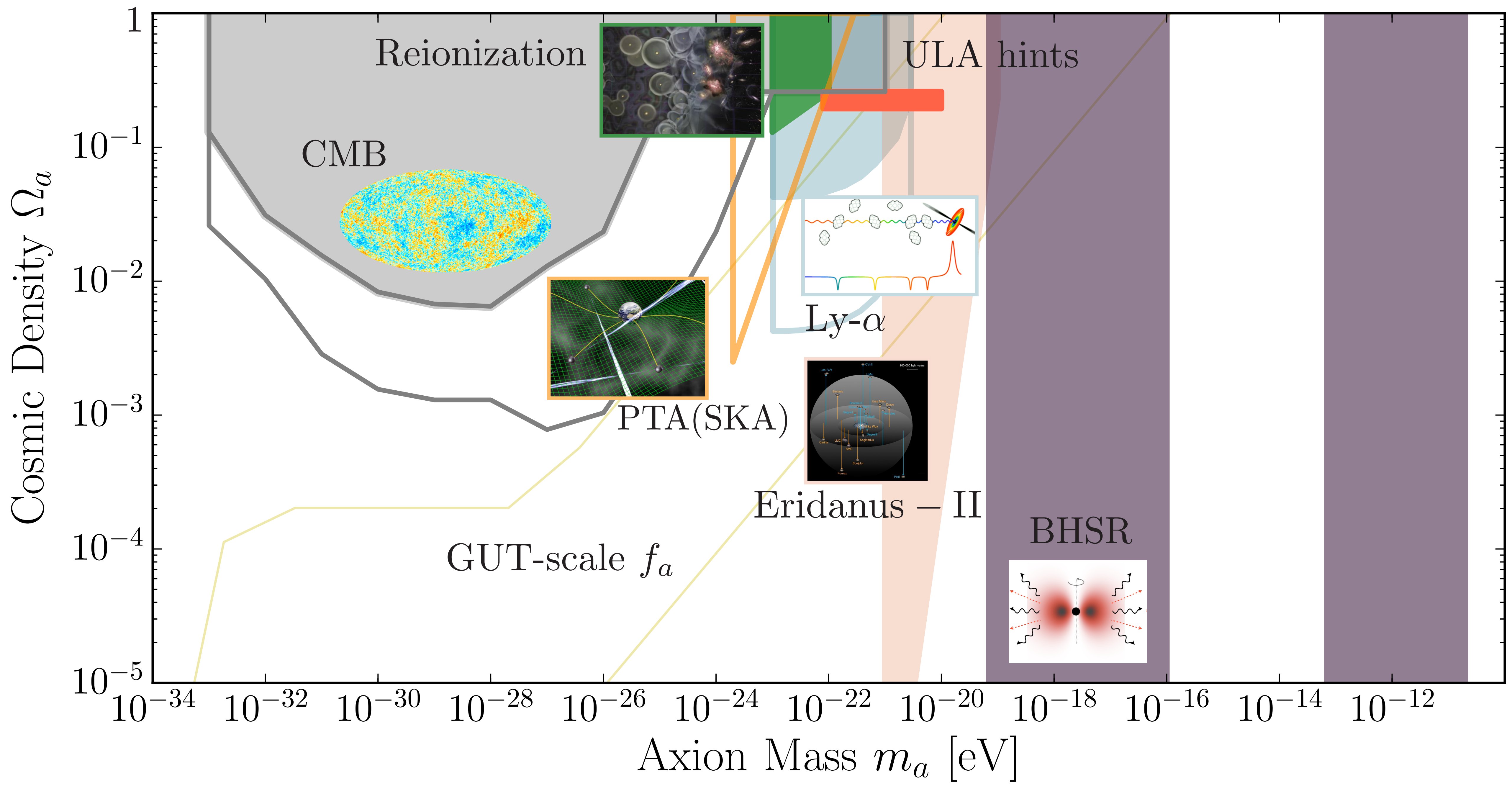
Conclusions

- Ultra-light axions have been under scrutiny for more than two decades.
- It's a good scientific model: it can be falsified by different ranges of observations.
- Accurate solutions of the cosmological EOM are required for the comparison with forthcoming observations.
- But the method should be also appropriate for its inclusion in Boltzmann codes without spoiling adaptability and speed of the numerical solutions.
- For FDM, the axion mass is the scale of reference:

$$L_C = \frac{h}{m_a c} = 0.4\, m_{a22}^{-1}\,\mathrm{pc} \qquad L_{dB} = \frac{h}{m_a v} = 400\, m_{a22}^{-1}\,\mathrm{pc}$$

- But other models may show interesting features. The alternative method can handle them easily.
- Ultimate challenge: to be sure about the different assumptions and how the affect the constraints on the model.

Astro2020 Science White Paper: Gravitational probes of ultra-light axions
Grin et al, ArXiv 1904:09003



¡Gracias! Thank you!