Deviations from slow roll inflation and its implications for magnetogenesis

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Outline

- Observational evidence for magnetic fields
- Choice of coupling functions for slow-roll inflationary models
- Construction of coupling functions for models generating features in scalar power spectra (SPS)
  - Over large scale
  - Over small scales
- Challenges in generating PMF in single field models
- Circumventing the challenges with the aid of two field model
- Imprints of PMF on CMB
- Conclusion

Observational evidence for magnetic fields

- In galaxies, strength of the observed magnetic field is $\sim 10^{-6}$ G coherent over scales of $1-10$ Kpc[1]
- In clusters of galaxies, the strength is $\sim 10^{-7}-10^{-6}$ G with coherent length of 10 Kpc $-$ 1 Mpc[2]
- In intergalactic medium(IGM) voids the strength is $\geq 10^{-16}$ G coherent on scales above 1 Mpc[3]

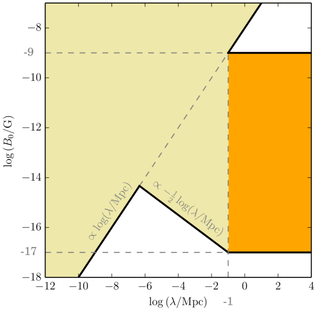
The origin of the seed magnetic field could be astrophysical or cosmological.

[1] *Beck R 2001 Space Sci. Rev. 99 243–60; Beck R and Wielebinski R 2013 Planets, Stars and Stellar Systems vol 5; ed T D Oswalt and G Gilmore (Dordrecht: Springer) p 641*

[2] *Clarke T E, Kronberg P P and Böhringer H 2001 Astrophys. J. 547 L111–4; Govoni F and Feretti L 2004 Int. J. Mod. Phys. D 13 1549–94*

[3] *A. Neronov and I. Vovk, Science **328**, 73 (2010)*

Constraints on IGMF



Constraints on B_0 , the magnetic field strength today, as a function of the comoving scale λ.[4]

The origin of these large scale magnetic fields can be explained using the processes during inflation in the early Universe.

[4] *For figure, see T. Markkanen, S. Nurmi, S. Rasanen, and V. Vennin, JCAP* ***06****, 035 (2017)*

Generation of primordial magnetic field (PMF)

The parity violating term is introduced to the actions as follows,

$$S[A^\mu] \;=\; -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,I^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$.
The equation of motion has the form

$$\mathcal{A}_k^{\sigma\,\prime\prime} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,J'}{J} - \frac{J''}{J}\right)\mathcal{A}_k^\sigma = 0.$$

where $\sigma = \pm 1$ represents positive and negative helicity.
The power spectra of the helical magnetic and electric fields[5]

$$\mathcal{P}_{\mathrm{B}}(k) \;=\; \frac{k^5}{4\,\pi^2\,a^4}\left[\left|\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^-\right|^2\right],$$

$$\mathcal{P}_{\mathrm{E}}(k) \;=\; \frac{k^3}{4\,\pi^2\,a^4}\left[\left|\mathcal{A}_k^{+\prime} - \frac{J'}{J}\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^{-\prime} - \frac{J'}{J}\mathcal{A}_k^-\right|^2\right].$$

[5] *K Subramanian, Rept.Prog.Phys.* ***79****, 076901 (2016); R. Sharma, K. Subramanian, T.R. Seshadri, Phys.Rev.D* ***97****, 083503 (2018)*

Electromagnetic (EM) power spectra

For the choice of coupling function $J(\eta) \propto a(\eta)^n$ (in de-Sitter $a = -1/H_I\eta$), we obtain scale invariant $\mathcal{P}_{_{\rm B}}(k)$ for $n = 2$.

For $\gamma = 0$ (Non-helical fields)

$$\begin{aligned} \mathcal{P}_{_{\rm B}}(k) &= \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}, \\ \mathcal{P}_{_{\rm E}}(k) &= \frac{H_{_{\rm I}}^4}{4\,\pi^2}\,(-k\,\eta_{\rm e})^2. \end{aligned}$$

For $\gamma \neq 0$ (Helical fields)

$$\begin{aligned} \mathcal{P}_{_{\rm B}}(k) &= \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,f(\gamma), \\ \mathcal{P}_{_{\rm E}}(k) &= \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,f(\gamma)\left[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\rm e}) \right. \\ &\quad \left. + \frac{1}{9}\,\left(1 + 23\,\gamma^2 + 40\,\gamma^4\right)\,(-k\,\eta_{\rm e})^2\right], \end{aligned}$$

where, $f(\gamma) = \frac{\sinh\,(4\,\pi\,\gamma)}{4\,\pi\,\gamma\,(1+5\,\gamma^2+4\,\gamma^4)}$.

For $\gamma = 1, \quad f(\gamma) \simeq 10^3$

Construction of $J(\phi)$ for slow roll (SR) models

In terms of e-folds, the coupling function is given by

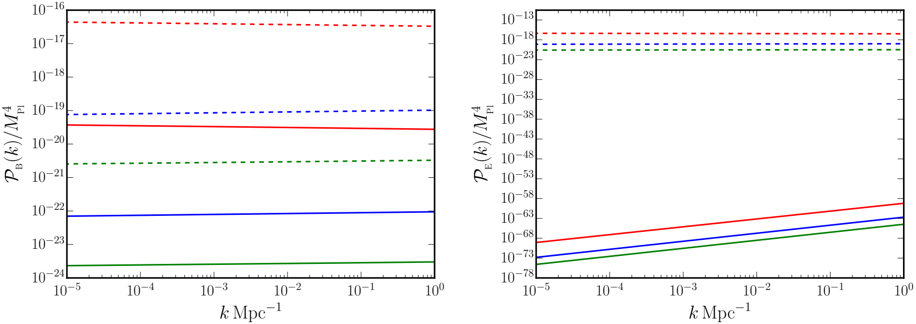
$$J(N) = \exp\left[n\left(N - N_{\mathrm{e}}\right)\right].$$

The Klein-Gordon equation for inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi} = 0.$$

SR Model	Potential	Coupling fuction $[J(\phi)]$
Quadratic potential (QP)	$\frac{m^2}{2}\phi^2$	$\exp\left[-\frac{n}{4M_{\mathrm{Pl}}^2}\left(\phi^2 - \phi_{\mathrm{e}}^2\right)\right]$
Small field model (SFM)	$V_0\left[1 - \left(\frac{\phi}{\mu}\right)^q\right]$	$\left(\frac{\phi}{\phi_{\mathrm{e}}}\right)^{n\mu^2/2M_{\mathrm{Pl}}^2} \exp\left[-\frac{n}{4M_{\mathrm{Pl}}^2}\left(\phi^2 - \phi_{\mathrm{e}}^2\right)\right]$
First Starobinsky model (FSM)	$V_0\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]^2$	$\exp\left\{-\frac{3n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\mathrm{Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\mathrm{Pl}}} - \frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right)\right]\right\}$

EM power spectra



The spectra of the magnetic (on the left) and electric (on the right) for the QP (in red), the SFM (in blue) and the FSM (in green) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[6]

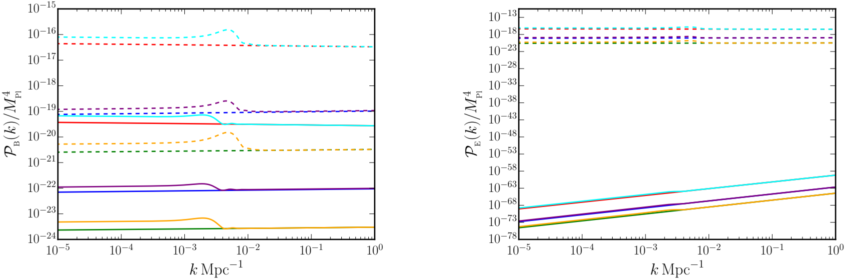
[6] ***S. Tripathy***, *D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D* ***105***, *063519 (2022)*

Models generating features in (SPS)

Features over large scales

(a) **Introducing a step by hand in the slow roll potential**[7]

$$V_{\rm step}(\phi) = V(\phi)\,\left[1+\alpha\tanh\left(\frac{\phi-\phi_0}{\Delta\phi}\right)\right]$$



The spectra of the magnetic (on the left) and electric field (on the right) for potential with step for QP (in cyan), the SFM (in purple) and the FSM (in orange) in both the non-helical (as solid lines) and helical (as dashed lines) magnetic fields.[8]

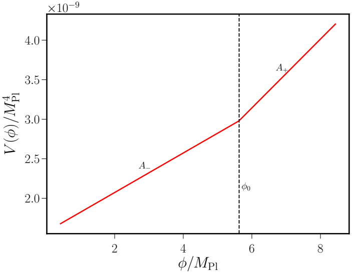
[7] *J. A. Adams, B. Cresswell, and R. Easther, Phys. Rev. D **64**, 123514 (2001).*

[8] ***S. Tripathy**, D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022)*

(b) Model with change in slope in potential

Second Starobinsky model[9],

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_-\,(\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$



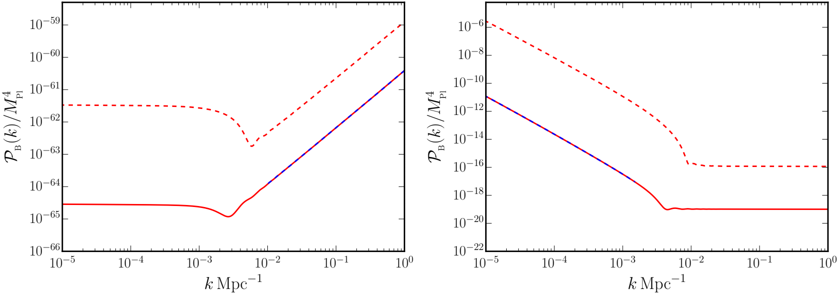
We construct the coupling functions as

$$J_+(\phi) = J_{0+} \exp\left\{-\frac{n}{2\,M_{\rm Pl}^2}\left[\left(\phi_+ - \phi_0 + \frac{V_0}{A_+}\right)^2 - \left(\phi_{\rm i} - \phi_0 + \frac{V_0}{A_+}\right)^2\right]\right\},$$

$$J_-(\phi) = J_{0-} \exp\left\{-\frac{n}{2\,M_{\rm Pl}^2}\left[\left(\phi_- - \phi_0 + \frac{V_0}{A_-}\right)^2 - \left(\frac{V_0}{A_-}\right)^2 - 2\,N_0\,M_{\rm Pl}^2\right]\right\}.$$

9 *A. A. Starobinsky, JETP Lett. **55**, 489 (1992)*

EM spectra for second Starobinsky model



The power spectra of the magnetic field arising in the second Starobinsky model for the two choices of coupling functions involving the solutions of the field in either of the slow roll regions have been plotted in the non-helical (in solid red) as well as the helical (in dashed red) magnetic fields.[10]

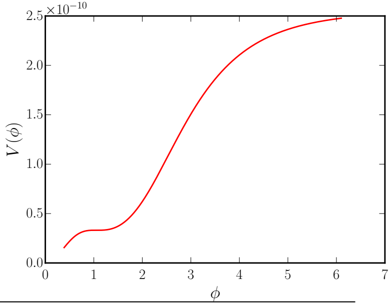
[10] ***S. Tripathy***, *D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D* ***105****, 063519 (2022)*

Potentials with point of inflection

Features over large scale in SPS

(c) **First punctuated inflation model** [11]

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{2m^2}{3\phi_0}\phi^3 + \frac{m^2}{4\phi_0^2}\phi^4$$



Features over small scales in SPS

(a) **Ultra slow roll model** [12]

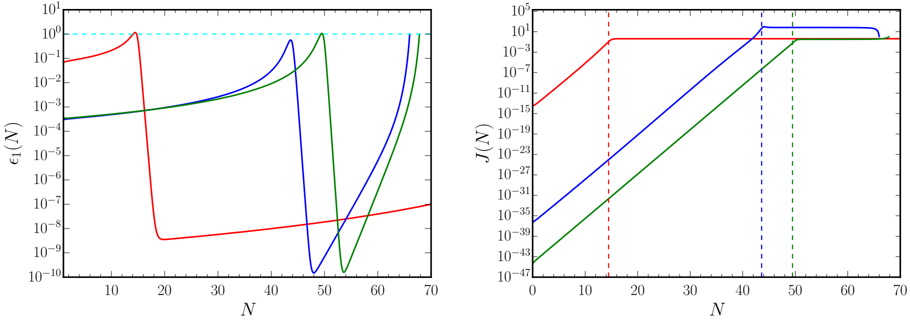
$$V(\phi) = V_0\left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + A\,\sin\left[\frac{1}{f_\phi}\tanh\left(\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right)\right]\right\}^2$$

(b) **Second punctuated inflation model** [13]

$$V(\phi) = V_0\left[c_0 + c_1\,\tanh\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + c_2\,\tanh^2\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + c_3\,\tanh^3\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right)\right]^2$$

[11] *R. K. Jain, P. Chingangbam, L. Sriramkumar, and T. Souradeep, Phys. Rev. D* ***82****, 023509 (2010)*
[12] *I. Dalianis, A. Kehagias, and G. Tringas, JCAP* ***01****, 037 (2019)*
[13] *I. Dalianis and K. Kritos, Phys. Rev. D* ***103****, 023505 (2021)*

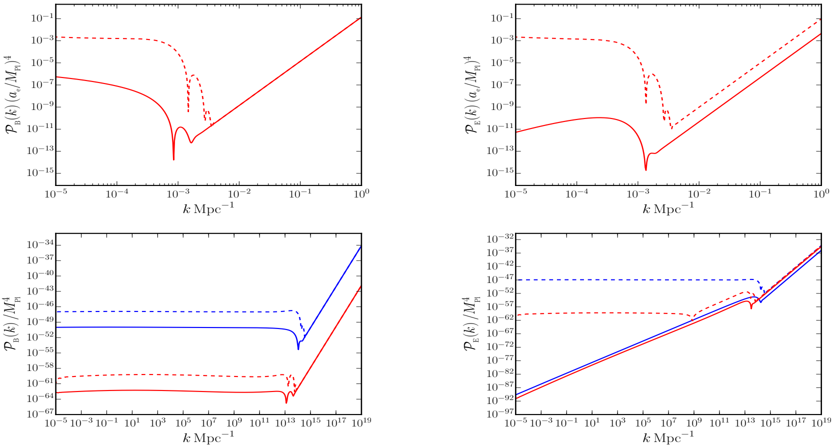
ϵ_1 and $J(\phi)$ of potentials with inflection point



The evolution of ϵ_1 and $J(N)$ for the first and second punctuated inflation model and ultra slow model (in solid red, green and blue, respectively) as a function of the e-fold N.[14]

[14] ***S. Tripathy***, *D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D* ***105***, *063519 (2022)*

EM spectra for potentials with inflection point



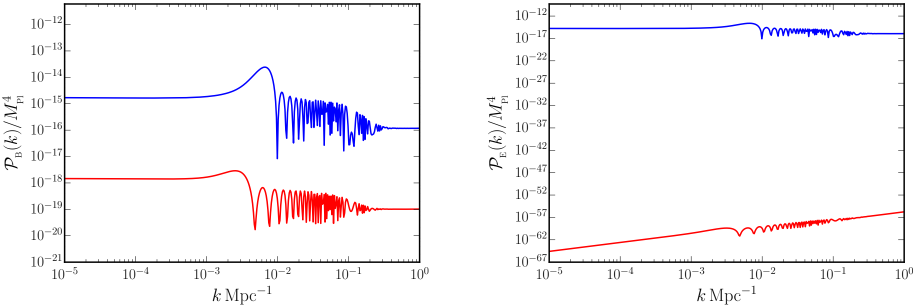
The spectra of the magnetic (on the left) and electric (on the right) fields for both the non-helical (in solid red) and helical (in dashed red) cases arising in the case of the potentials with inflection point .[15]

[15] ***S. Tripathy***, *D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D* ***105***, *063519 (2022)*

An attempt to iron out features

The remove the strong features we constructed the coupling function with the form

$$J(\phi) = \frac{J_1}{2\,J_{0+}}\left[1 + \tanh\left(\frac{\phi - \phi_0}{\Delta\phi_1}\right)\right] J_+(\phi) + \frac{J_1}{2\,J_{0-}}\left[1 - \tanh\left(\frac{\phi - \phi_0}{\Delta\phi_1}\right)\right] J_-(\phi),$$



The spectra of the magnetic (on the left) and electric (on the right) field for both the non-helical (in red) and helical (in blue) EM fields.[16]

[16] ***S. Tripathy***, *D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D* ***105***, *063519 (2022)*

Challenges in magnetogenesis in single field models

- The form of the $J(\phi)$ needs to be extremely fine tuned.
- For potentials permitting a brief phase of ultra slow roll, $\mathcal{P}_{\rm B}(k)$ has strong scale dependence.
- The amplitude of the magnetic fields are strongly suppressed on large scales.

Is there a possible way to overcome these challeges and obtain the desired shape and amplitude of $\mathcal{P}_{\rm B}(k)$?

Circumventing the challenges with the aid of two field models

- The action governing two field model is given as,

$$S[\phi, \chi] \quad = \quad \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \frac{f(\phi)}{2} \partial_\nu \chi \, \partial^\nu \chi - V(\phi, \chi) \right].$$

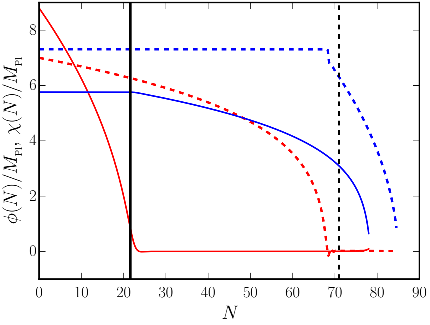
- Here the form of the non-canonical coupling $f(\phi) = \mathrm{e}^{b(\phi)}$.
- Deviations from slow-roll can be naturally achieved in two field models due to a sharp turn in the trajectory in the field space for non-zero values of $b(\phi)$.
- The equations of motion describing the evolution of the scalar fields are,

$$\begin{aligned} \ddot{\phi} + 3\,H\,\dot{\phi} + V_\phi \quad &= \quad b_\phi\,\mathrm{e}^{2\,b}\,\dot{\chi}^2, \\ \ddot{\chi} + (3\,H + 2\,b_\phi\,\dot{\phi})\,\dot{\chi} + \mathrm{e}^{-2\,b}\,V_\chi \quad &= \quad 0. \end{aligned}$$

Two field models generating features in SPS

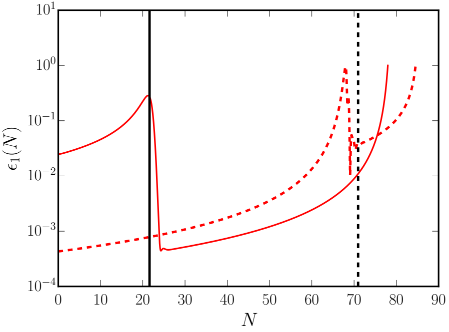
The potential that leads to suppression in SPS over large scales has the form[17]

$$V(\phi,\chi) = \frac{m_\phi^2}{2}\,\phi^2 + V_0\,\frac{\chi^2}{\chi_0^2+\chi^2}.$$



The potential that leads to enhancement in SPS over small scales has the form[18]

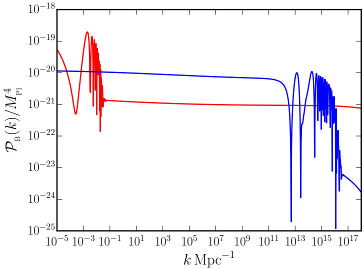
$$V(\phi,\chi) = V_0\,\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{m_\chi^2\,\chi^2}{2}.$$

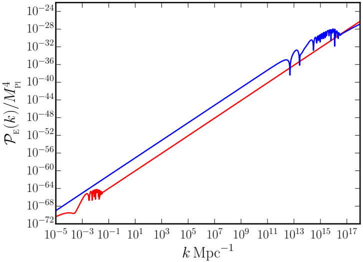


[17] *R. Kallosh, A. Linde, and Y. Yamada, JHEP 01, 008 (2019); M. Braglia, D. K. Hazra, L. Sriramkumar, and F. Finelli, JCAP **08**, 025 (2020)*

[18] *M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar, and A. A. Starobinsky, JCAP **08**, 001 (2020)*

EM power spectra for two field models





We obtain the power spectra for magnetic field (on the left) and electric field (on the right) of expected strength and shape for the two models.[19]

The magnitude of magnetic field is constrained through the quantity B_λ which is related to $\mathcal{P}_{\rm B}(k)$ as

$$B_\lambda^2 = \frac{1}{4\pi}\int {\rm d}^3\boldsymbol{k}\, {\rm e}^{-k^2\lambda^2}\, \frac{\mathcal{P}_{\rm B}(k)}{k^3}.$$

where $\lambda = 1$ Mpc is the coherence length.

For the two field models of interest the estimates of B_λ^0 turn out to be $\mathcal{O}(10^{-1})$ nG, well within the constraint of $B_\lambda^0 < 1.2$ nG.[20]

[19] ***S. Tripathy***, *D. Chowdhury, H.V. Ragavendra, R.K. Jain, L. Sriramkumar, manuscript under preparation*

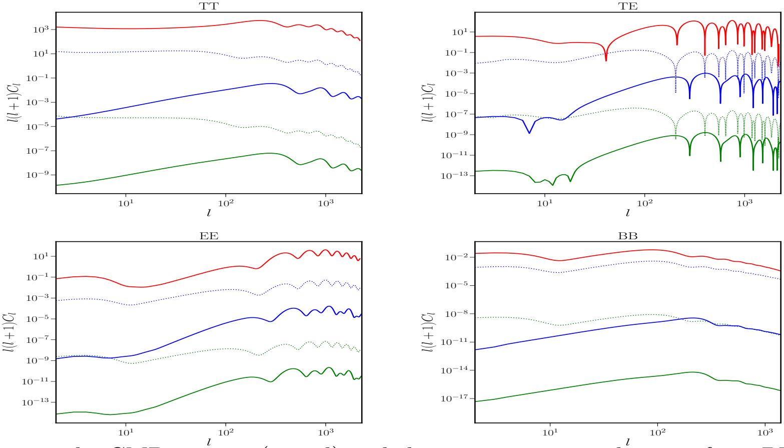
[20] *A. Zucca, Y. Li, L. Pogosian, Phys. Rev. D* ***95****, 063506 (2017)*

Imprints of PMF on CMB

- MagCAMB: A code to compute the contributions of PMF to the CMB angular spectra[21].
- It assumes as power law form for the magnetic power spectrum and arrives at the corresponding C_ℓs due to both compensated and passive modes.
- We estimated such an angular spectrum for the two field model generating features over small scales in SPS.

[21] *J. R. Shaw and A. Lewis, Phys. Rev. D **81**, 043517 (2010); A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. **538**, 473 (2000); A. Zucca, Y. Li, L. Pogosian, Phys. Rev. D **95**, 063506 (2017)*

The CMB spectra



We present the CMB spectra (in red) and the respective contributions from PMF arising from the model that generates features over small scales (in green) due to compensated (solid) and passive (dotted) modes. We also present the PMF contributions corresponding to upper bound on B_λ^0 (in blue) for reference[22].

[22] ***S. Tripathy**, D. Chowdhury, H. V. Ragavendra, R. K. Jain, L. Sriramkumar, manuscript under preparation*

Conclusions

- The features in the SPS, which are generated due to deviations from slow roll, can improve the fit to the CMB data.
- When strong departures from slow roll arise, these deviations also led to features in the spectra of electromagnetic fields.
- In the case of single field inflationary scenarios, it is also possible that the strengths of the magnetic fields are considerably suppressed on large scales and the spectrum is scale dependent on small scales.
- The features were ironed out in a specific model, but it is achieved at the terrible cost of extreme fine-tuning.
- Using two field models along with suitable choices of coupling functions, we could obtain EM spectra of desired strength and shape.

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Thank You