Gravitational-wave lensing as a probe of dark matter halos

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Cosmology From Home
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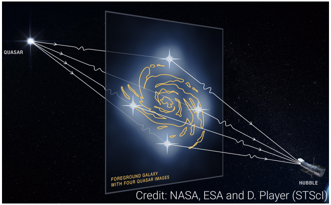
Outline

- Introduction on Gravitational lensing
- Lensing signals: methods and lens models
- Forecasts for gravitational wave (GW) detectors
- Applications to Dark Matter (DM) models
- Conclusions and outlooks

Gravitational lensing

Lensing of EM waves

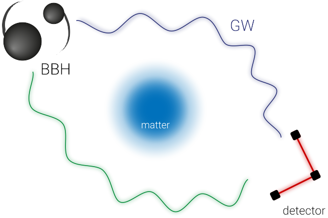
- Established probe at very different scales
- Powerful insights on matter distribution



Credit: NASA, ESA and D. Player (STScI)

Lensing of GWs can soon become reality

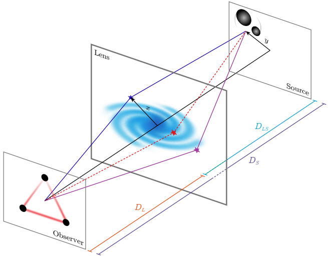
- Coherence and low frequencies: probe of diffraction regime
- Sensitivity to $1/r$ instead of $1/r^2$
- No absorption: probe of dense DM regions



Lensing of GWs

- $g_{\mu\nu} = g_{\mu\nu}^{\rm FRW} + h_{\mu\nu}, \quad \Box h_{\mu\nu} = 0$
- Amplification factor: $F(f)$

$$F(w) = h^L(f)/h^0(f)$$
$$= \frac{w}{2\pi i} \int \mathrm{d}^2x \, e^{iw\phi(\boldsymbol{x},\boldsymbol{y})}$$



[Schneider, Gravitational Lenses '92]

- $\boldsymbol{x}$, $\boldsymbol{y}$ dimensionless distances in units of the Einstein's radius
- Fermat potential: $\phi(\boldsymbol{x},\boldsymbol{y}) \propto$ time delay

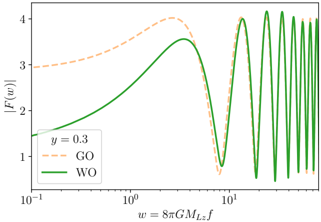
$$\phi(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{2}|\boldsymbol{x}-\boldsymbol{y}|^2 - \psi(\boldsymbol{x})$$

- Lensing potential: $\psi(\boldsymbol{x})$, sourced by the projected mass distribution
- Dimensionless frequency: $w \equiv 8\pi G M_{Lz} f \simeq \frac{M_{Lz}}{10^7\,M_\odot} \cdot \frac{f}{\rm mHz}$,
  $M_{Lz} \equiv$ redshifted lens mass

Computing $F(w)$

$$F(w) = \frac{w}{2\pi i}\int \mathrm{d}^2x\; e^{iw\phi(\boldsymbol{x},\boldsymbol{y})}$$

$$w \equiv 8\pi G M_{Lz} f \simeq \frac{M_{Lz}}{10^7\, M_\odot}\cdot\frac{f}{\mathrm{mHz}}$$



- $w \ll 1$: wave does not feel the lens $F(w) \simeq 1$
- Geometric optics (GO) $w \gg 1$: stationary-phase approx. (lens eq.)

  $$\nabla_{\boldsymbol{x}}\phi(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{x} - \boldsymbol{y} - \nabla_{\boldsymbol{x}}\psi(\boldsymbol{x}) = 0$$

  solutions: images J with magnification μ_J, time delay ϕ_J and Morse phase $n_J = 0, 1/2, 1$

  $$F(w) \simeq \sum_J |\mu_J|^{1/2} e^{iw\phi_J - i\pi n_J}$$

- Wave optics (WO) $w \sim 1$: no analytic expansion for $F(w)$.
  Carries more info about the lens: opportunity for GW lensing

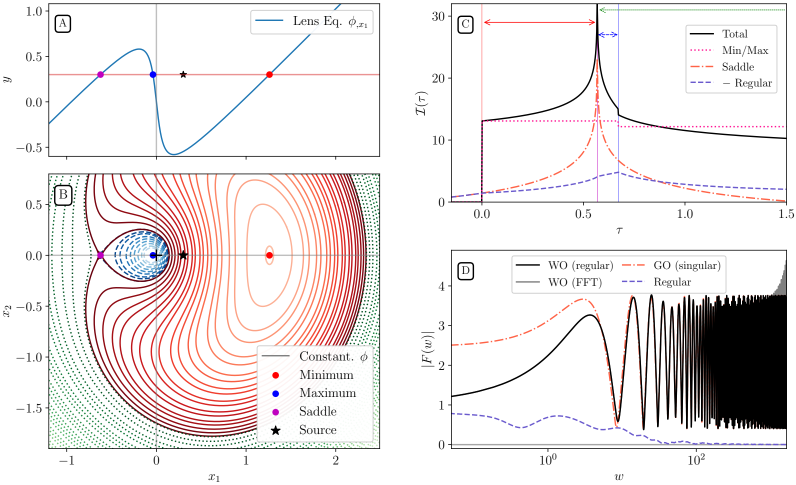
Computing $F(w)$: contour method

- Standard numerical integration is troublesome: highly oscillatory integral
- We implemented a "contour method": [A. Ulmer, J. Goodman, '94]
  evaluate the time-domain signal $\mathcal{I}(\tau)$, then use inverse Fourier transform

$$\mathcal{I}(\tau) = \int \mathrm{d}w\, e^{-iw\tau} \frac{F(w)}{(-iw)} = \int \frac{\mathrm{d}w}{2\pi} \int \mathrm{d}^2x\, e^{iw(\phi(\boldsymbol{x},\boldsymbol{y})-\tau)}$$
$$= \int \mathrm{d}^2x\, \delta\left(\phi(\boldsymbol{x},\boldsymbol{y}) - \tau\right) = \sum_k \oint_{\gamma_k} \frac{\mathrm{d}s}{|\boldsymbol{\nabla}\phi(\boldsymbol{x}(\tau,s),\boldsymbol{y})|}$$

- Reduced to a 1D integral over contours γ_k of constant $\phi(\boldsymbol{x},\boldsymbol{y}) = \tau$.
  The sum $\sum_k$ is over stationary points (images), where the contours end.

Computing $F(w)$: contour method



Lens models

- For simplicity, we focus on axially-symmetric lenses $\psi(\boldsymbol{x}) = \psi(x)$ that model DM halos
- DM halos roughly described by the *Singular Isothermal Sphere* (SIS)

$$\rho = \frac{\sigma_v^2}{2\pi G r^2}, \quad \psi(x) = x$$

  In GO gives two images (minimum and saddle)

- We study deformations from the SIS, motivated by DM models
- The presence of a core modelled by the *Cored Isothermal Sphere* (CIS)

$$\rho = \rho_0 \frac{r_c^2}{r^2 + r_c^2}, \quad \psi(x) = \sqrt{x^2 + x_c^2} + x_c \log\left(\frac{2x_c}{x_c + \sqrt{x^2 + x_c^2}}\right)$$

- One additional central image (maximum) with finite magnification
- Specific DM models (e.g. Fuzzy DM) predict cores [L. Hiu+, '16]

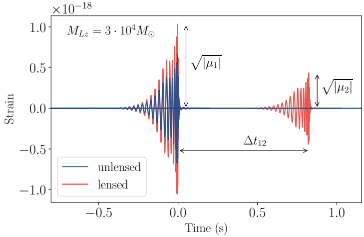
Lensing of GW: results and forecasts

- Lensing features are investigated in current detectors

  [L.Dai+, ’20; LIGO, Virgo, ’21]

- Previous analyses mostly focused on singular lenses

  [R. Takahashi+ ’03; P. Cremonese+, ’21; H. G. Choi+, ’21; ...]



- We focus on distinguishing between different lens features:
  cored vs. singular DM distribution
  Evaluate sensitivities on lens parameters (core size x_c for LISA)

Lensing of GW: results and forecasts

- We perform a *Fisher matrix forecast* on source and lens parameters for LISA

  [M. Vallisneri, '07]

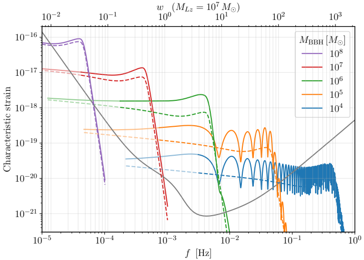
  $$\theta_i = \{D_L,\ \phi_0,\ M_{Lz},\ y,\ x_c\}$$
  $$\mathcal{F}_{ij} \equiv (\partial_i h_L | \partial_j h_L), \quad \partial_i \equiv \partial/\partial\theta_i$$
  $$\sigma_i^2 = (\mathcal{F}^{-1})_{ii}, \quad \text{marginalized posteriors}$$

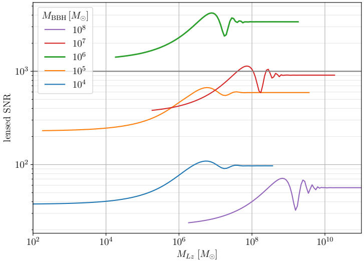
- GW sources with equal mass, non spinning and fixed orientation, using **PhenomD** waveforms

  [S. Husa+, '15, S. Khan, '15]

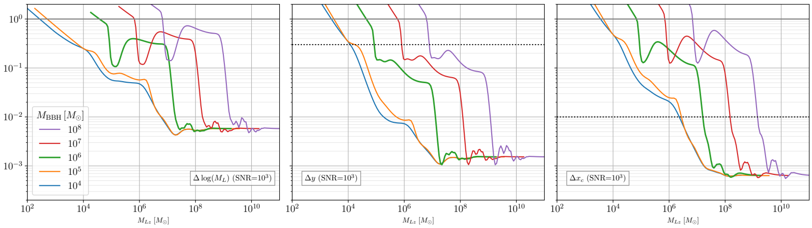
- Focus on strong-lensing regime (multiple images)

- Fiducial lens parameters: $M_{Lz} = 10^7\, M_\odot$, $y = 0.3$, $x_c = 10^{-2}$





Results and forecasts: dependence on source mass

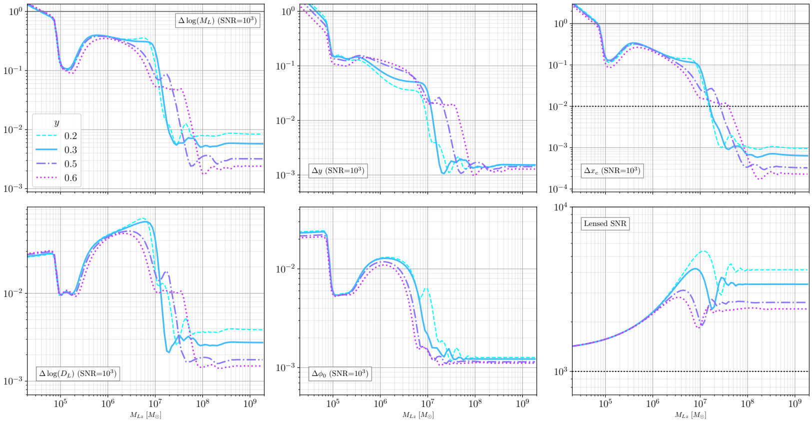


- High M_{Lz} dominated by GO regime (results saturate). Low M_{Lz} gives no lensing (lens parameters cannot be reconstructed)

- SNR is peaked at the Innermost Stable Circular Orbit (ISCO), with $f_{\rm ISCO} \sim 1/M_{\rm BBH}$

- Lighter BBH give better constraints at small M_{Lz}: easier to have larger w at ISCO
  $w_{\rm ISCO} \sim M_{Lz}/M_{\rm BBH}$

Results and forecasts: dependence on y



Larger y improves the constraints

- M_{Lz} is probed in GO through the time delays, that increase for large y
- x_c: magnification of the third image increases with y

Application: Ultra-light DM

- Forecast results on lens parameters have implications for constraints on DM models

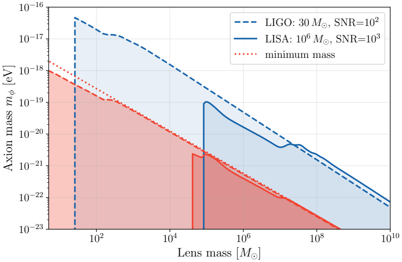
- Models of Ultra-light DM predict cores with a minimum size and mass

$$r_{1/2} \geq 0.33\,\mathrm{kpc}\,\frac{10^9 M_\odot}{M_L}\left(\frac{10^{-22}\mathrm{eV}}{m_\phi}\right)^2$$

$$M_L \gtrsim 1.4 \cdot 10^7\, M_\odot \left(\frac{10^{-22}\mathrm{eV}}{m_\phi}\right)$$

[L. Hiu+, '16]

- A non detection of core features or of small M_{Lz} would imply bounds on DM mass, assuming halos can be described by the CIS lens



Conclusions and outlooks

- GW lensing is a very promising tool for DM characterization
- We implemented fast, accurate and flexible methods to evaluate lensing signals in the WO regime
- Lensed LISA ans LIGO events could test DM-halos features, such as the presence of cores

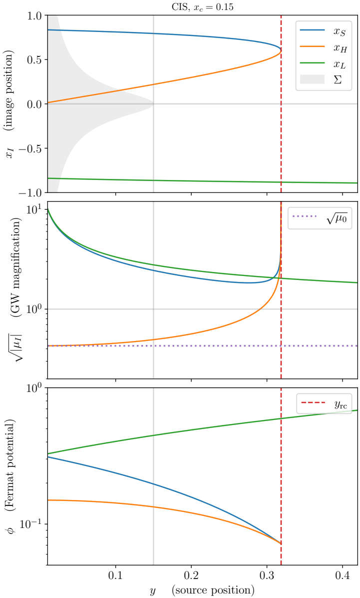
Future directions

- Investigation of the weak-lensing regime (single image): WO effects give more information about the lens model
- Include more GW parameters (e.g. LIGO/LISA antenna pattern, spins ecc..) to provide more robust lensing forecasts
- Study of more complicated lens models and configurations

Backup slides

Lens models: Cored Isothermal Sphere

- Central image has a finite minimum magnification $\mu_H > \mu_0 = 4x_c^2/(1-2x_c)^2$
- Time delays between images can be of order of days $\Delta T \simeq (1\,\mathrm{day})\left(M_\mathrm{v}/10^{11}M_\odot\right)^{4/3}\Delta\phi$
- Potential for GW observations: for $x_c \neq 0$ an additional GW signal can be detected



Results and forecasts: correlations

- For high M_{Lz}, precision on lens parameters saturates
- In this limit, we are sensitive to linear combinations of the parameters: their accuracy increases and the parameters become almost degenerate
- Precision could drastically improve if some parameters are independently measured (e.g. EM counterparts)