A general Bayesian framework for high fidelity interferometric calibration with applications to 21 cm cosmology

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Cosmology from Home, 2022

21 cm cosmology

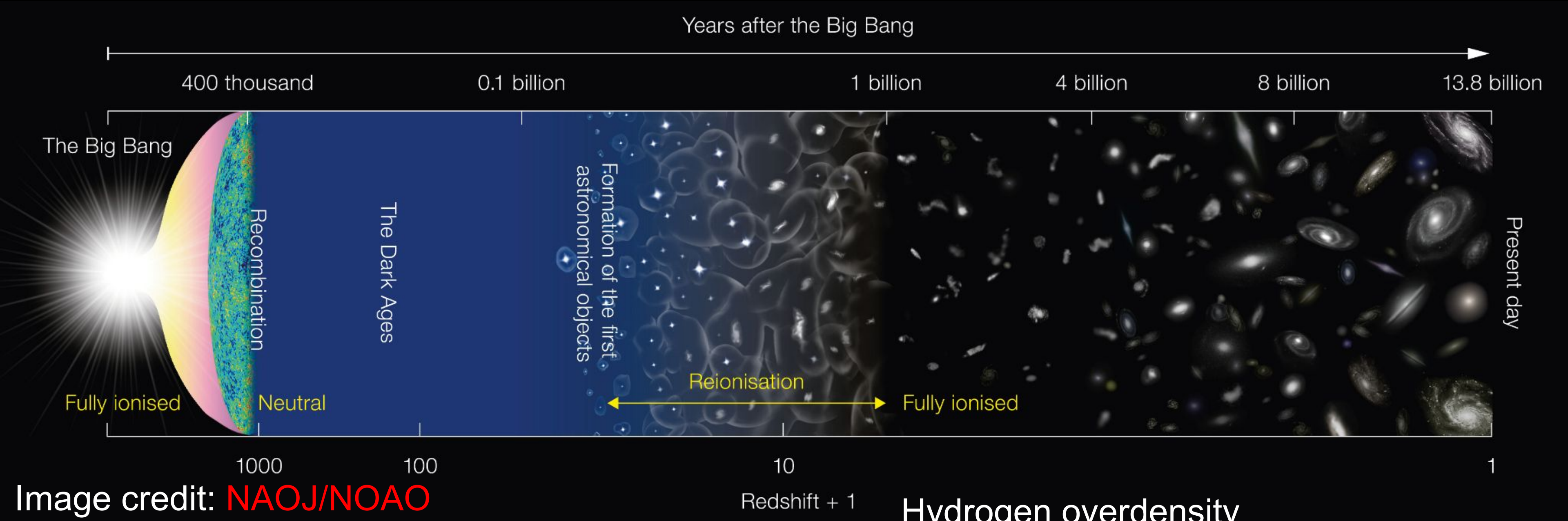
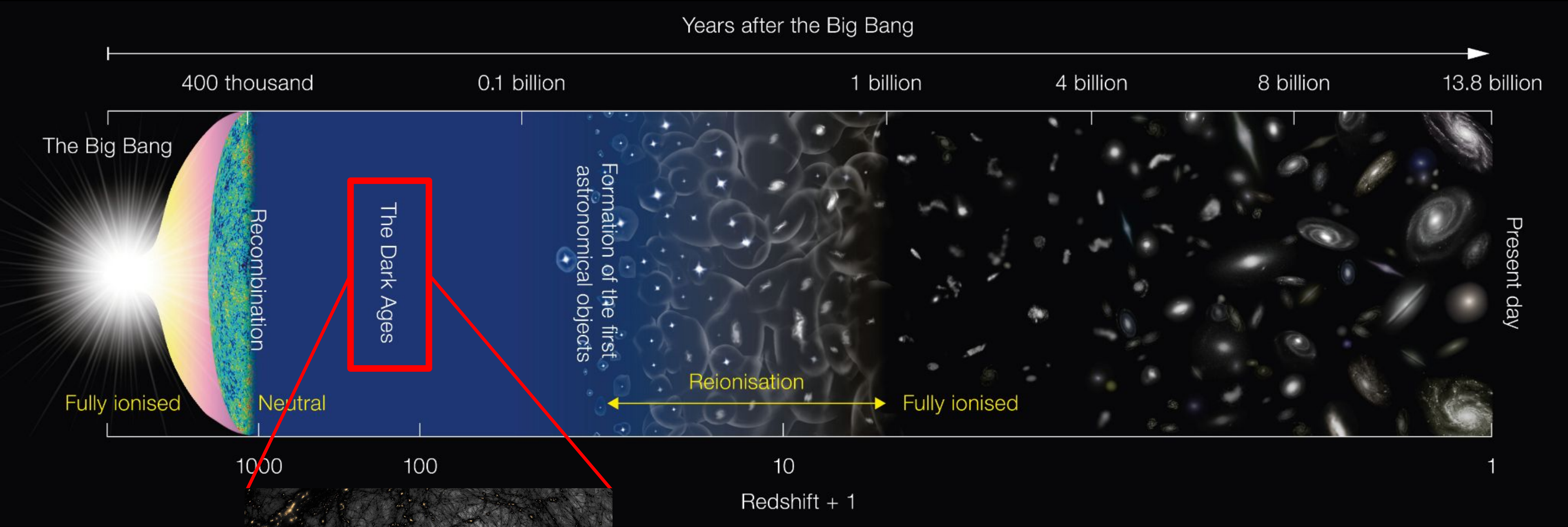
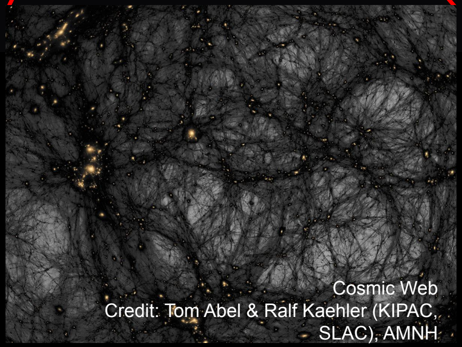


Image credit: NAOJ/NOAO

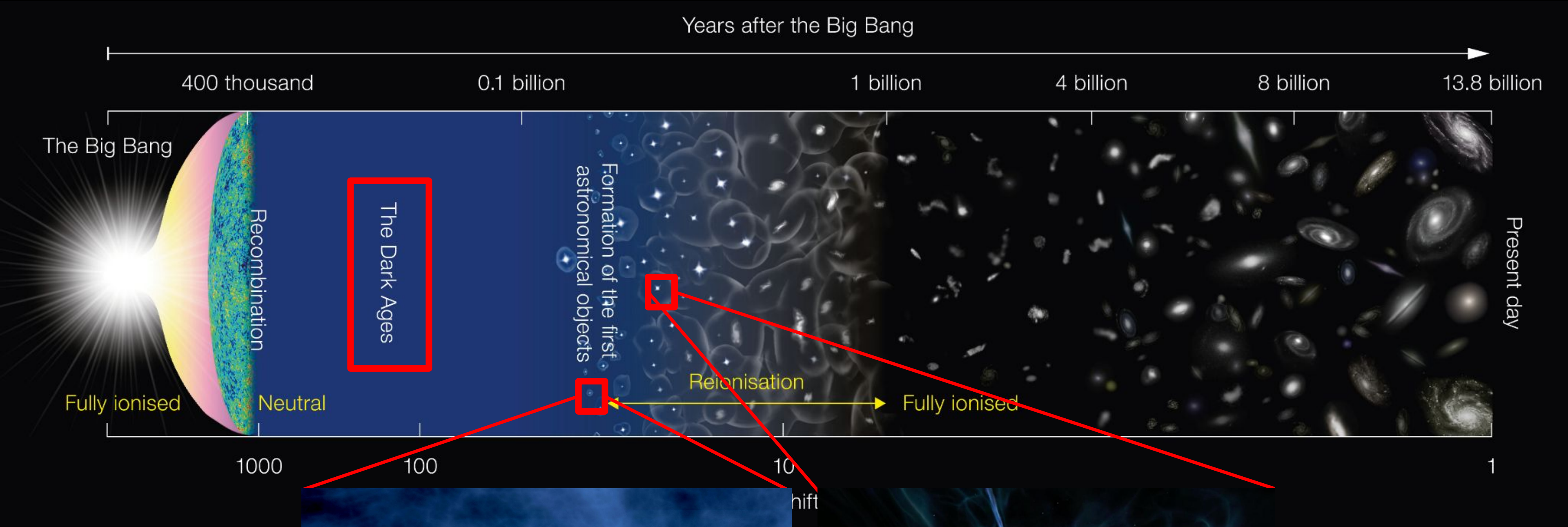
$$\underbrace{\delta T_{21\mathrm{cm}}(\nu)}_{\text{21 cm brightness temperature}} \propto \underbrace{x_{\mathrm{HI}}}_{\text{Hydrogen neutral fraction}}(1+\overbrace{\delta)}^{\text{Hydrogen overdensity}}\left[1-\frac{T_\gamma}{\underbrace{T_{\mathrm{s}}}_{\text{Hydrogen spin temperature}}}\right]$$

21 cm cosmology





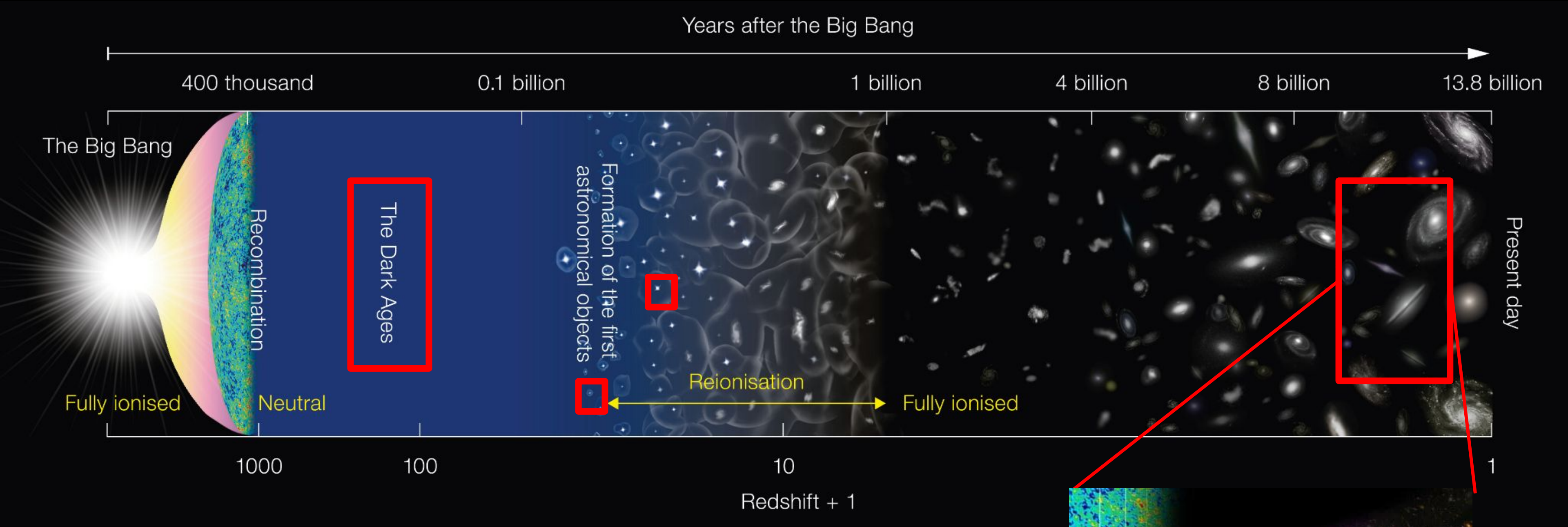
21 cm cosmology

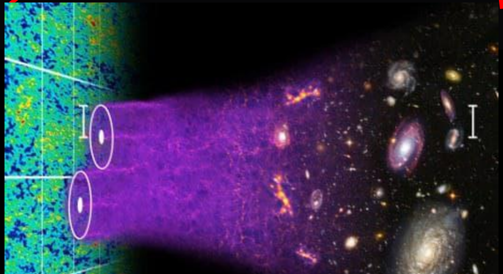


Artist impression of first stars
Credit: NASA/WMAP Science Team

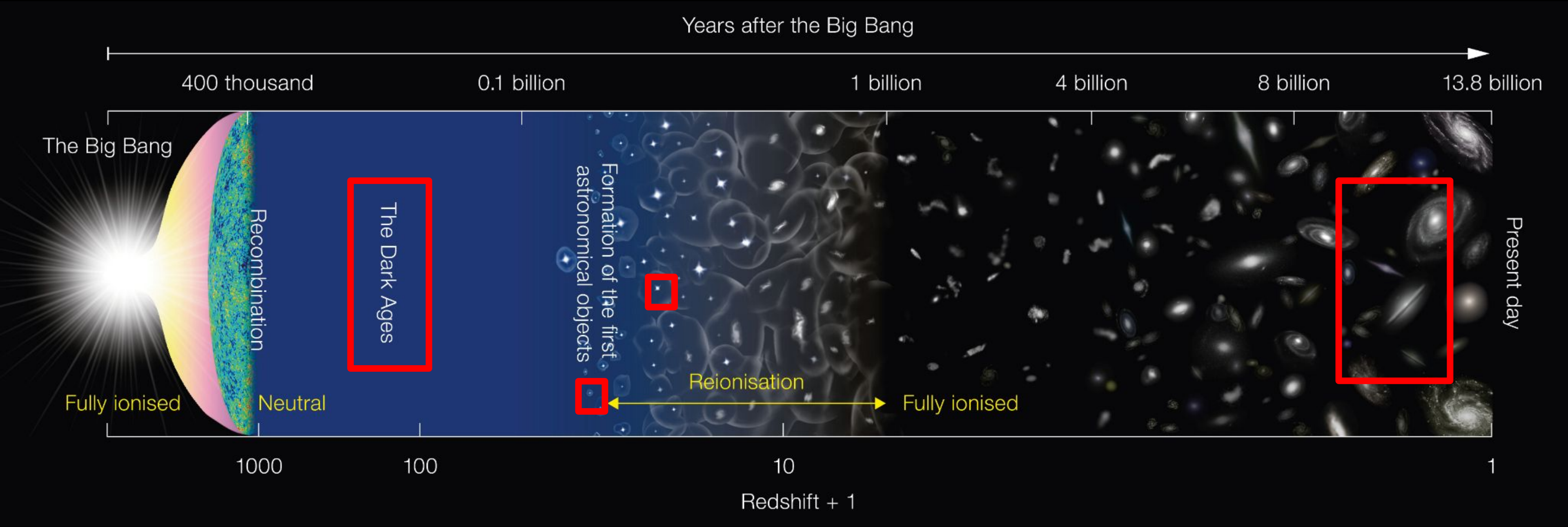
Artist impression of a very distant quasar
Credit: ESA/Hubble, NASA, M. Kornmesser

21 cm cosmology



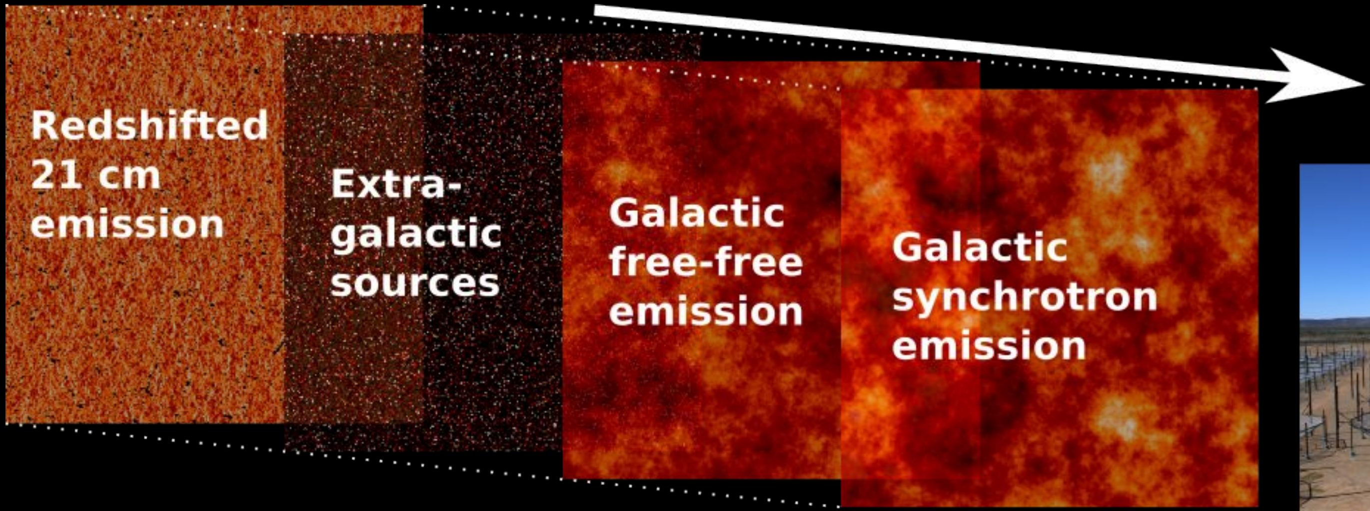

Baryon acoustic oscillations imprinted in the low-*z* galaxy distribution measured via their neutral hydrogen content
Credit: Chris Blake and Sam Moorfield

21 cm cosmology



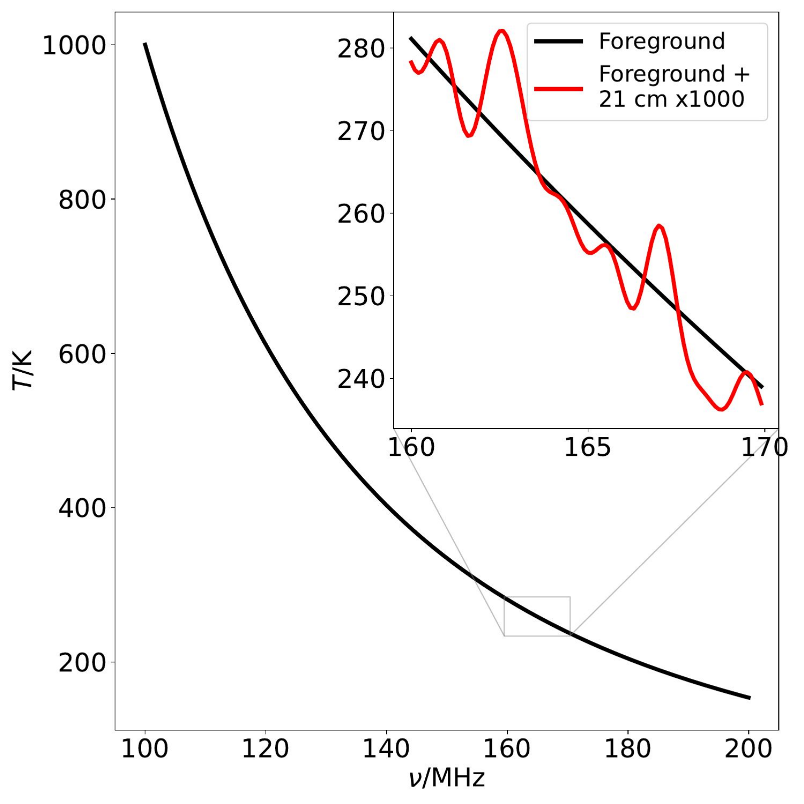
Each of these measurements is reliant on exquisite instrumental calibration, requiring modelling of the instrument bandpass at a spectral fidelity of 1 part in 10^3-10^6

21 cm cosmology data analysis

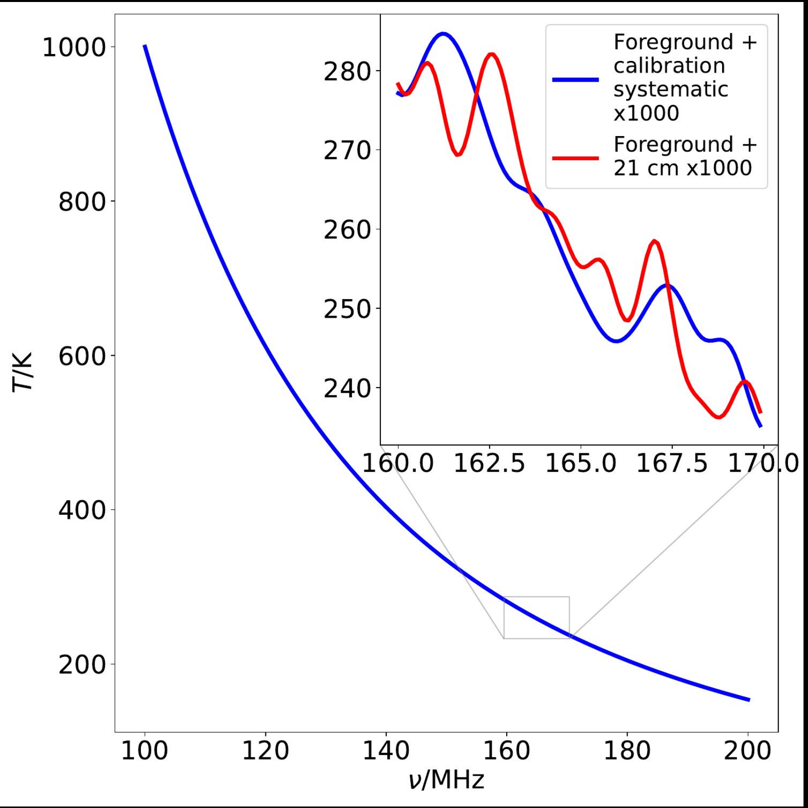


- The goal: extract the redshifted 21 cm signal from data dominated by Galactic synchrotron and bremsstrahlung radiation and extragalactic radio sources that, combined, are 3 - 6 orders of magnitude brighter

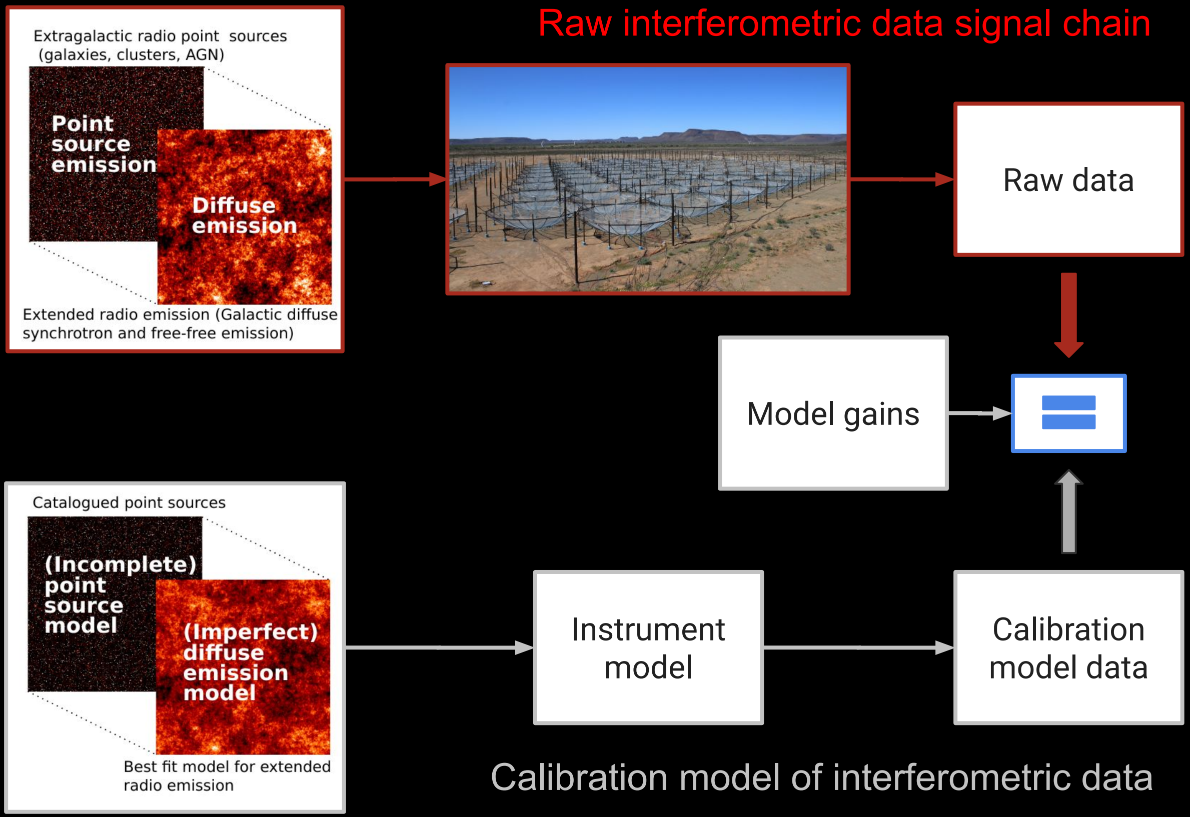
21 cm cosmology data analysis



The calibration challenge

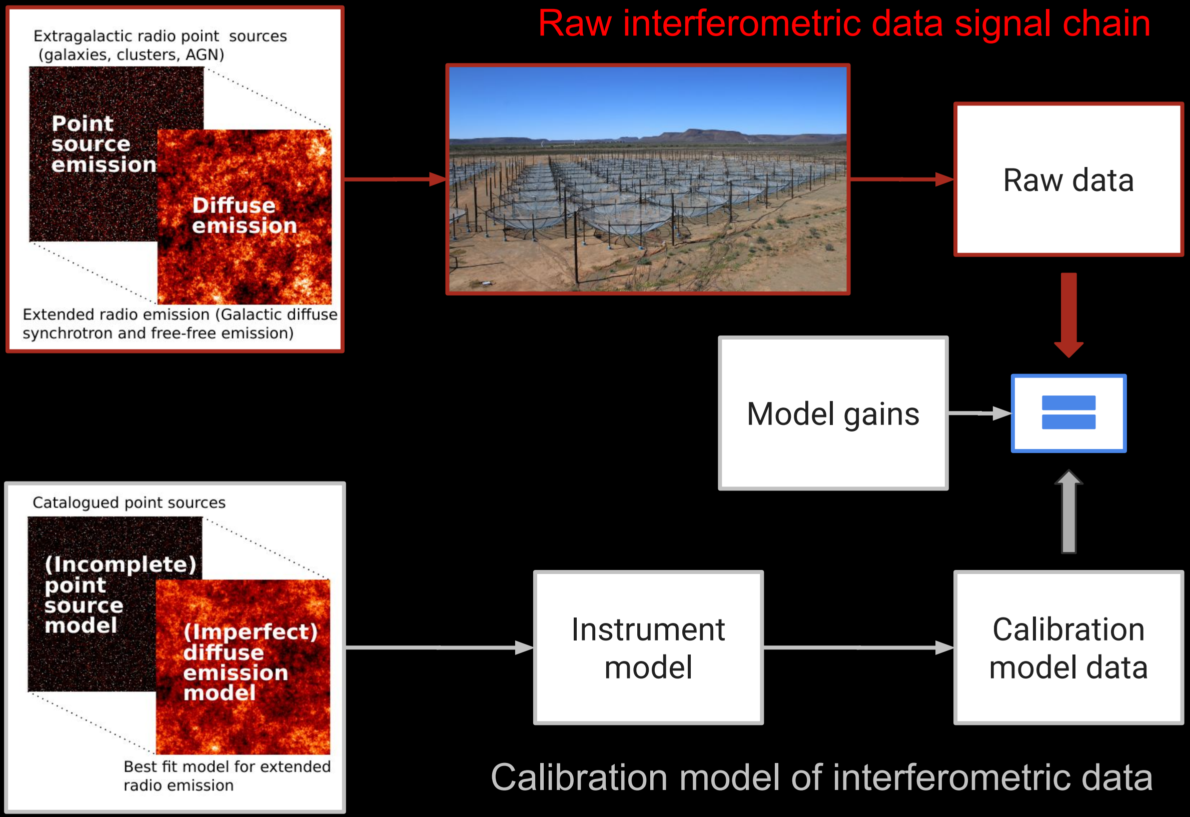


How interferometers are calibrated



- Interferometers are calibrated by fitting the raw data with a calibration model of the data comprised of 1) a known sky model, 2) a known instrument model and 3) a set of unknown per-antenna complex gains to be determined in the fit

The sky model incompleteness problem



- Errors and incompleteness in the calibration sky model, due to uncertainties in low-frequency sky maps and flux-density limited catalogues of extragalactic sources, translate to biased per-antenna complex gains parameters, resulting in spurious spectral structure in the data

The sky model incompleteness problem

Define a complex Gaussian likelihood for sky calibration:

Gain parameters fit during calibration

Model visibility data

$$\Pr(\boldsymbol{V}^{\mathrm{obs}} \mid \boldsymbol{g}) \propto \exp\left[-\left(\boldsymbol{V}^{\mathrm{obs}} - \mathbf{G}^{\mathrm{m}}\boldsymbol{V}^{\mathrm{sim}}\right)\mathbf{N}^{-1}\left(\boldsymbol{V}^{\mathrm{obs}} - \mathbf{G}^{\mathrm{m}}\boldsymbol{V}^{\mathrm{sim}}\right)^{\dagger}\right]$$

Observed visibility data

Model gain matrix (a function of ***g***)

The sky model incompleteness problem

Define a complex Gaussian likelihood for sky calibration:

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$$\Pr(\boldsymbol{V}^{\mathrm{obs}} \mid \boldsymbol{g}) \propto \exp\left[-\left(\boldsymbol{V}^{\mathrm{obs}} - \mathbf{G}^{\mathrm{m}}\boldsymbol{V}^{\mathrm{sim}}\right)\mathbf{N}^{-1}\left(\boldsymbol{V}^{\mathrm{obs}} - \mathbf{G}^{\mathrm{m}}\boldsymbol{V}^{\mathrm{sim}}\right)^{\dagger}\right]$$

Observed visibility data

Model gain matrix (a function of ***g***)

$$\boldsymbol{V}^{\mathrm{obs}} = \mathrm{G}^{\mathrm{true}}\boldsymbol{V}^{\mathrm{true}} + \boldsymbol{n}$$

If $\boldsymbol{V}^{\mathrm{sim}} = \boldsymbol{V}^{\mathrm{true}}$, then $\lim_{\boldsymbol{n}\to\mathbf{0}} \mathrm{G}^{\mathrm{m}} = \mathrm{G}^{\mathrm{true}}$

The sky model incompleteness problem

Define a complex Gaussian likelihood for sky calibration:

Gain parameters fit during calibration

Model visibility data

$$\Pr(\boldsymbol{V}^{\rm obs} \mid \boldsymbol{g}) \propto \exp\left[-\left(\boldsymbol{V}^{\rm obs} - \mathbf{G}^{\rm m}\boldsymbol{V}^{\rm sim}\right)\mathbf{N}^{-1}\left(\boldsymbol{V}^{\rm obs} - \mathbf{G}^{\rm m}\boldsymbol{V}^{\rm sim}\right)^{\dagger}\right]$$

Observed visibility data

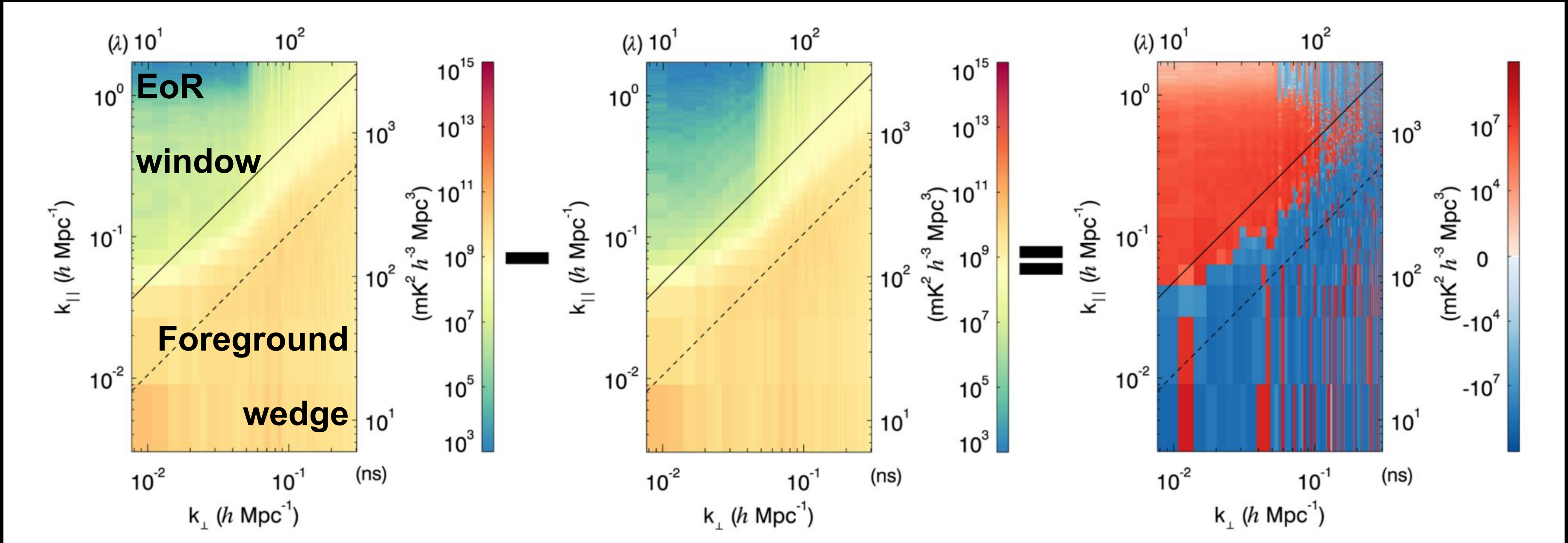
Model gain matrix (a function of ***g***)

$$\boldsymbol{V}^{\rm obs} = \mathrm{G}^{\rm true}\boldsymbol{V}^{\rm true} + \boldsymbol{n}$$

Problem: if the simulated calibration model is incomplete (sources missing, diffuse emission missing or imperfectly modelled),

$\boldsymbol{V}^{\rm sim} \neq \boldsymbol{V}^{\rm true}$ then, $\lim_{\boldsymbol{n}\to\boldsymbol{0}} \mathbf{G}^{\rm m} \neq \mathbf{G}^{\rm true}$

The sky model incompleteness problem



The residual power spectrum derived using an incomplete sky model to calibrate the data has excess power in the 'EoR window' Figure adapted from: Barry et al. 2016

Solutions to the sky model incompleteness problem

1. Better sky models

Solutions to the sky model incompleteness problem

1. Better sky models
2. A very smooth instrument bandpass

Solutions to the sky model incompleteness problem

1. Better sky models
2. A very smooth instrument bandpass
3. Novel calibration algorithms

BayesCal

- Fully Bayesian calibration framework to jointly estimate, with the calibration solutions, a statistical model of the incomplete component of the calibration sky model constrained by a prior on the expected angular power spectrum of this sky model component, to eliminate sky-model incompleteness bias
- First two papers currently in review (**Sims**, Pober & Sievers 2022a,b)

A Bayesian approach to high fidelity interferometric calibration I: mathematical formalism

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A Bayesian approach to high fidelity interferometric calibration II: demonstration with simulated data

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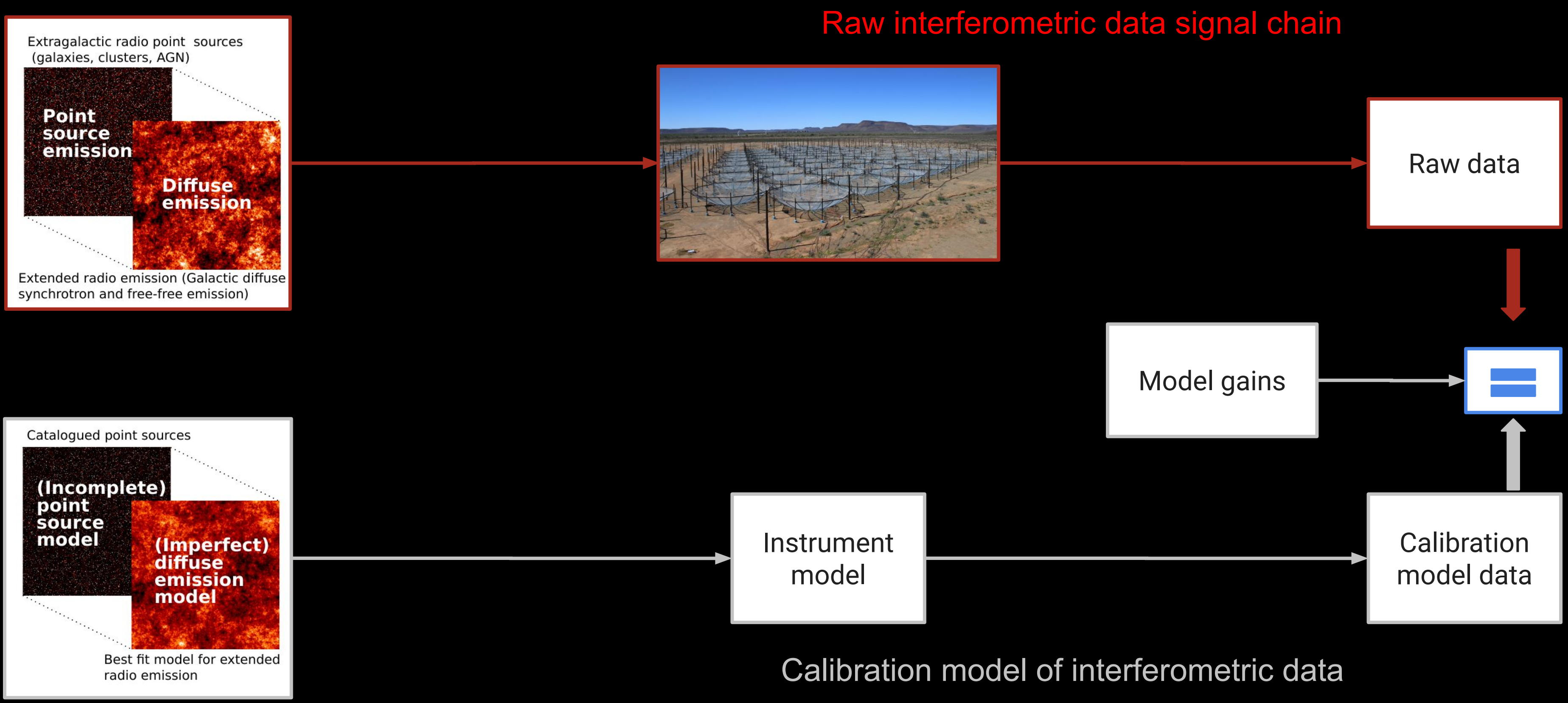
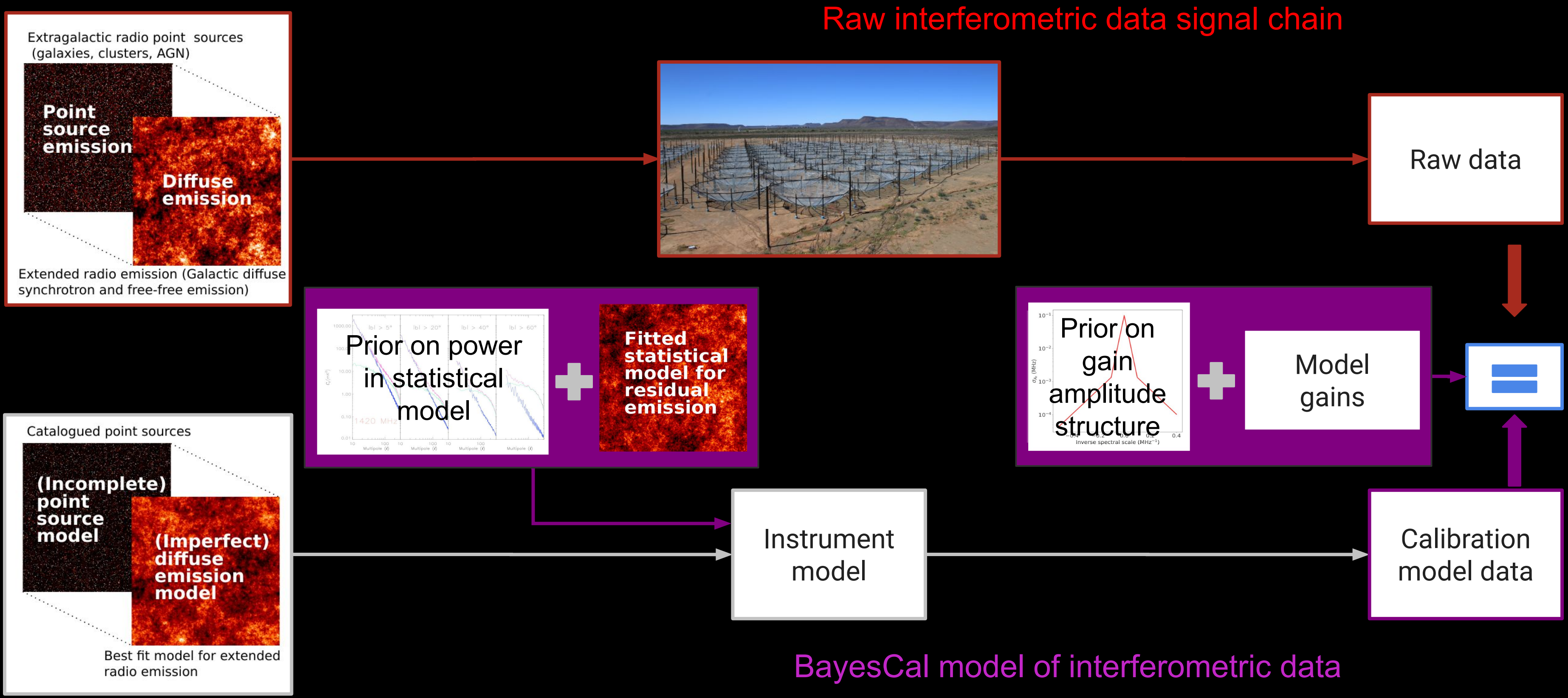
[3]*Department of Physics, Brown University, Providence, RI 02912, USA*

BayesCal framework



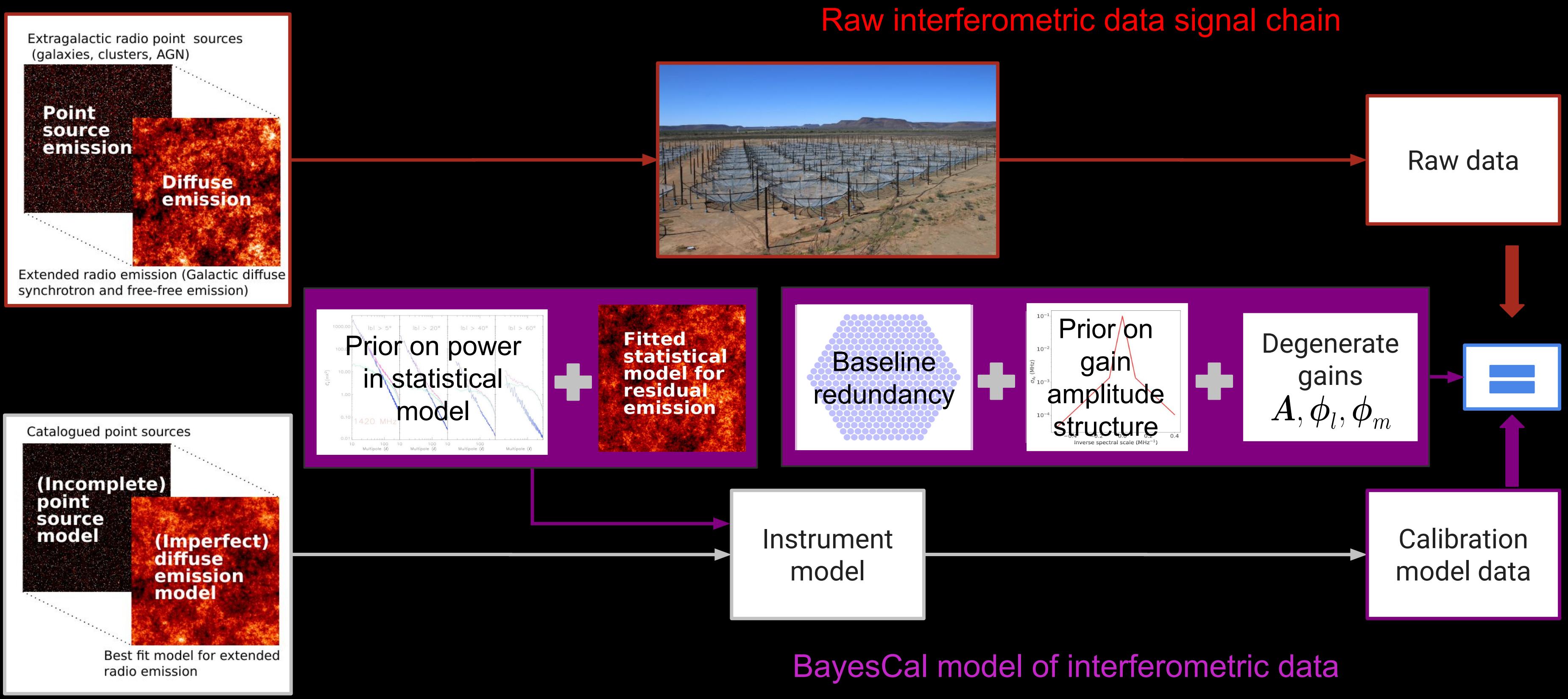
Sims, Pober & Sievers 2022a

BayesCal framework



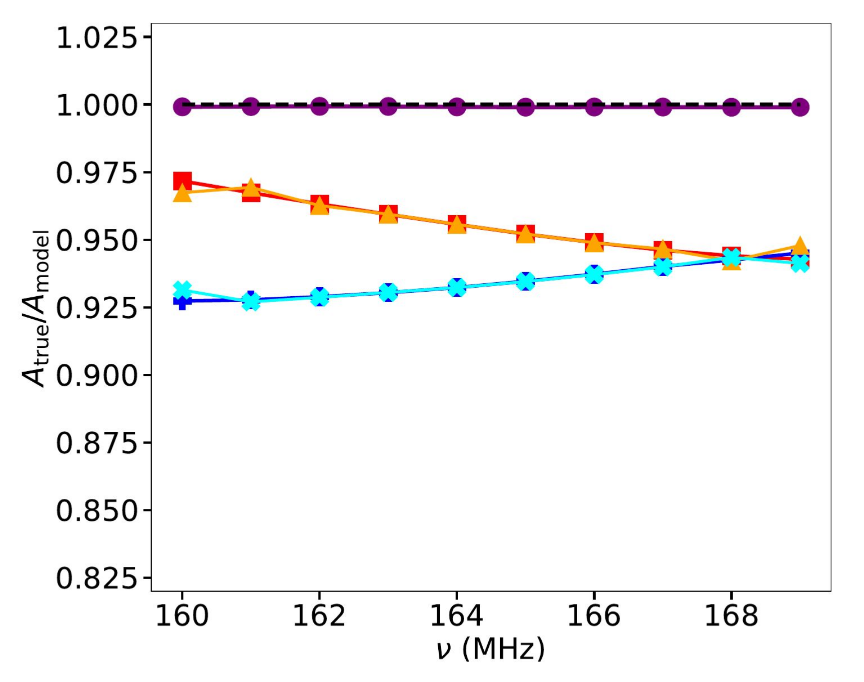
Sims, Pober & Sievers 2022a

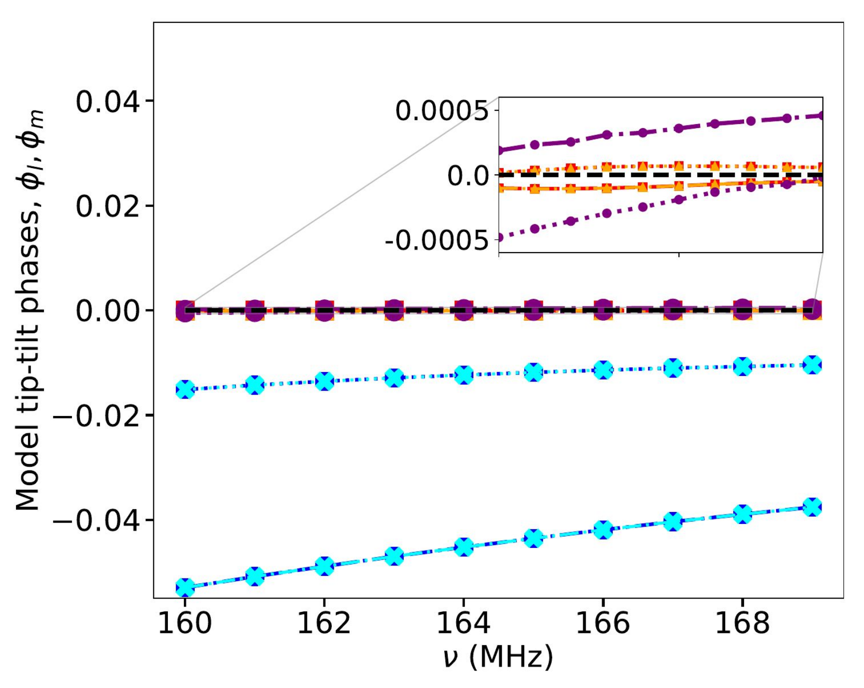
BayesCal framework



Sims, Pober & Sievers 2022a

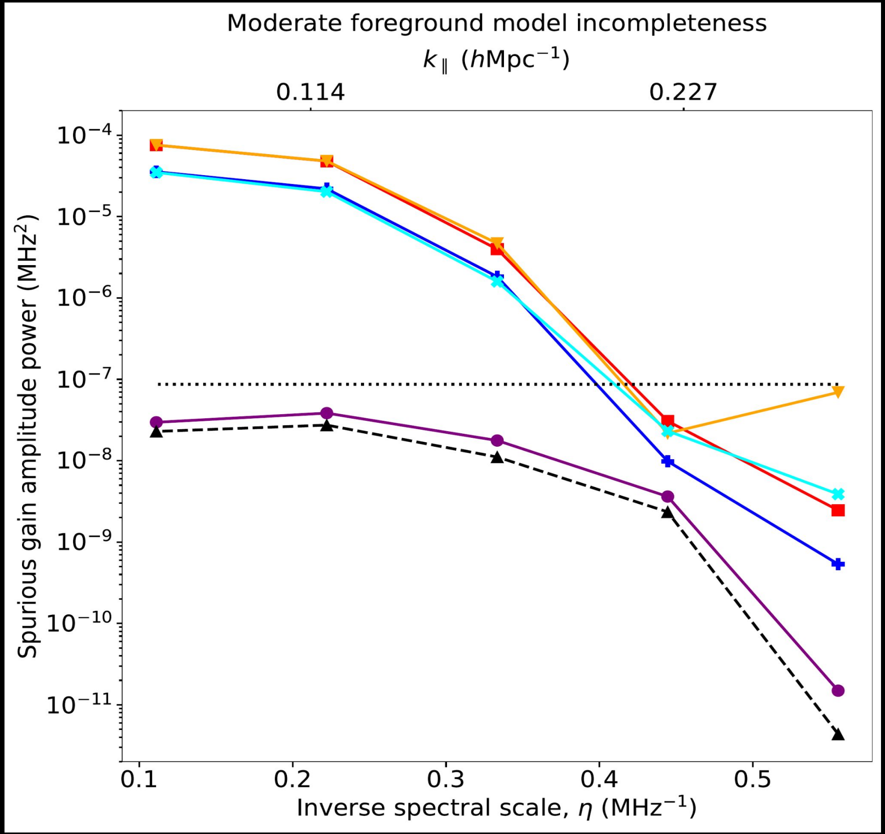
BayesCal calibration solutions (average bias mitigation)





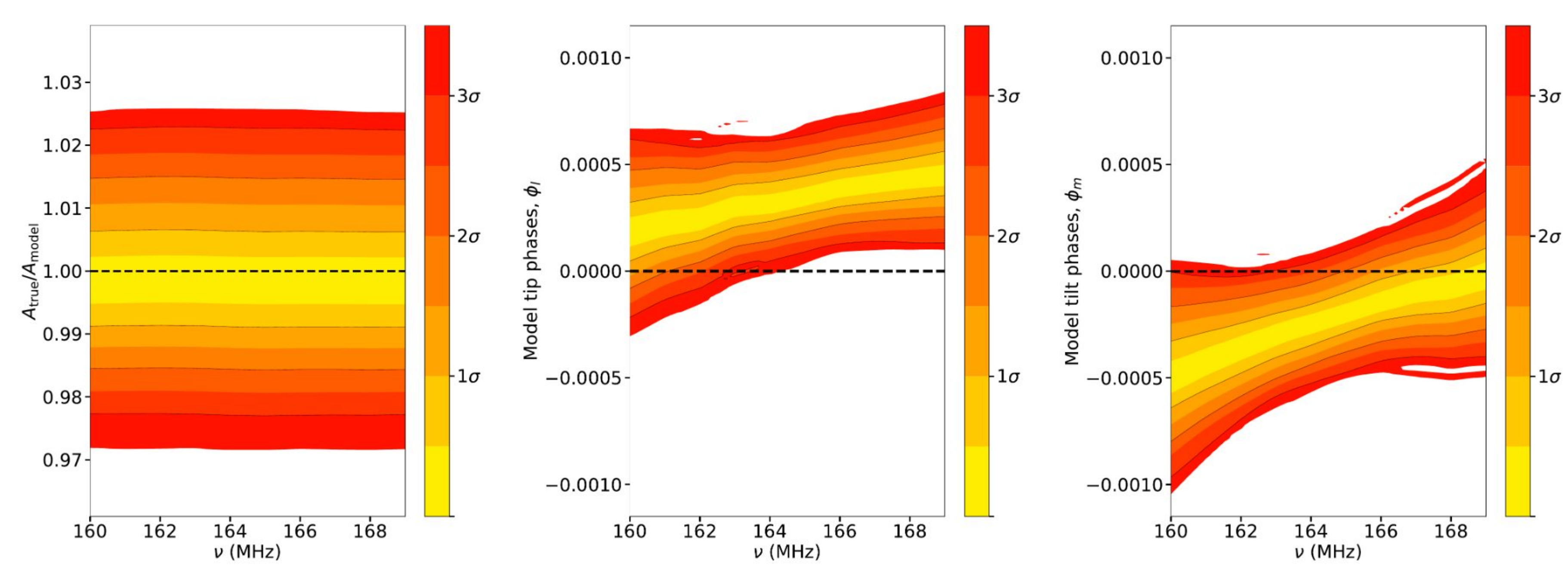
Sims, Pober & Sievers 2022b

BayesCal calibration solutions (spurious power mitigation)



Sims, Pober & Sievers 2022b

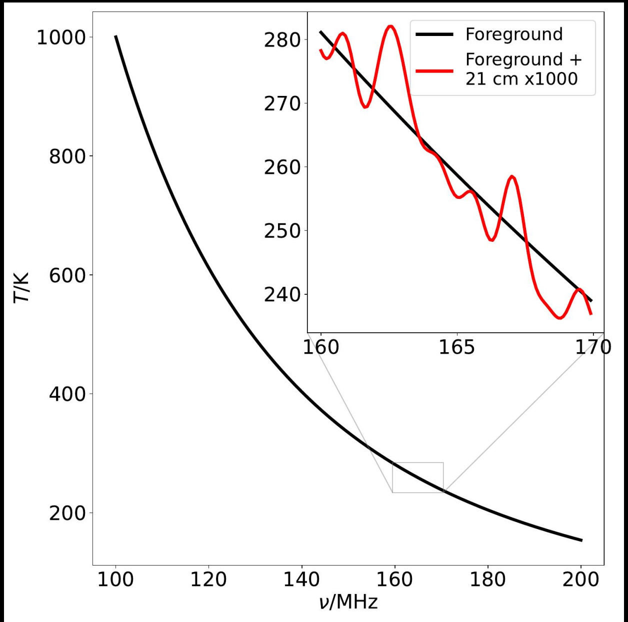
BayesCal calibration solutions (full posteriors)



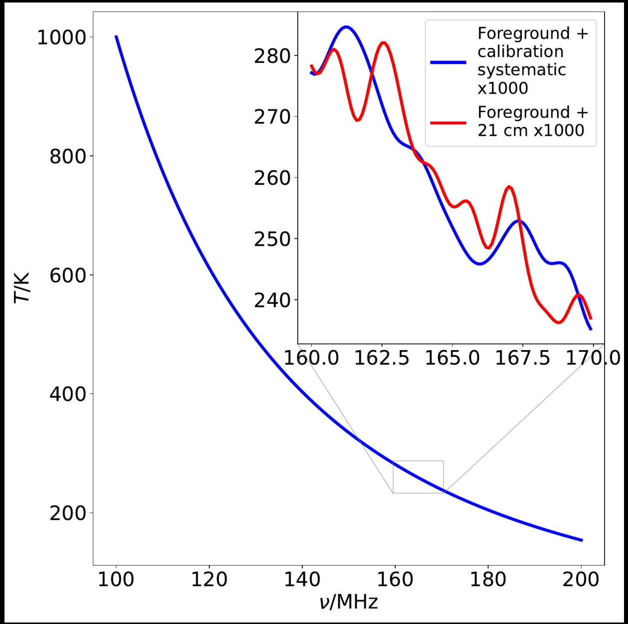
Sims, Pober & Sievers 2022b

Thank you for listening!

- Intrinsic spectral structure (no instrumental effects)



- Spectral structure after standard calibration



- Spectral structure after calibration using BayesCal