Rip Cosmological Models in Extended Symmetric Teleparallel Gravity

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Cosmology From Home (2022)

Topic Outlines

- Introduction
- Overview of $f(Q, T)$ gravity
- Mathematical formalism
- Three rip cosmological models
- Analysis of the model
- Results and Conclusion

Introduction

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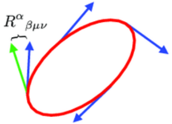
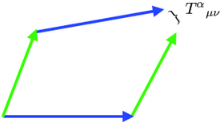
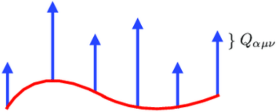


The rotation of a vector transported along a closed curve is given by the curvature: General Relativity.



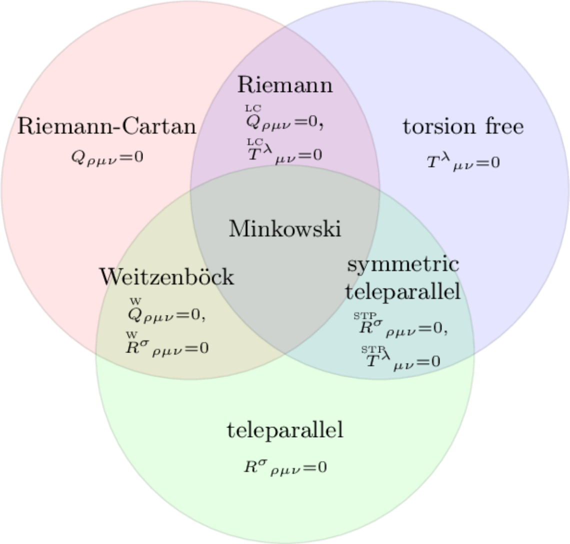
The non-closure of parallelograms formed when two vectors are transported along each other is given by the torsion: Teleparallel Equivalent of General Relativity.



The variation of the length of a vector as it is transported is given by the non-metricity: Symmetric Teleparallel Equivalent of General Relativity. [1]

[1] J. B. Jiménez, L. Heisenberg, T. M. Koivisto, *Universe*, **5**, 173, (2019)

Conts..



[2]

[2] L. Järv, M. Rünkla, etal., *PHYSICAL REVIEW D*, **97**, 124025, (2018)

Overview of $f(Q,T)$ gravity

• General relativity is basically a geometric theory, which is formulated in the Riemann metrical space and it has a great role within modified theories of gravity and also it helps to describe the gravitational field.[3]

• Even though Einstein's general relativity currently regarded as one of the most effective theories, there appear to be some limitations on standard GR in describing those phenomena in the wake of current observational advances in cosmology.

• We propose an extension of the symmetric teleparallel gravity in which the gravitational action L is given by an arbitrary function f, of the non-metricity Q and the trace of the matter-energy momentum tensor T, so that [4] $L = f(Q,T)$.

• We imposed the cosmological model which is the functional form of $f(Q,T)$:

$f(Q,T) = aQ^m + bT$

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[3] Y. Xu, T. Harko, et al.,*Eur. Phys. J. C*, **80**, 449 (2020).

[4] Y. Xu, G. Li,T. Harko, S. Liang, *Eur. Phys. J. C*, **79**, 708 (2019).

Mathematical Formalism

The gravitational action is:

$$S = \int \left[\frac{1}{16\pi} f(Q,T) + L_M\right] \sqrt{-g} d^4x \tag{1}$$

By varying the above gravitational action:

$$-\frac{2}{\sqrt{-g}} \nabla_\alpha (f_Q \sqrt{-g} P^\alpha_{\ \mu\nu}) - \frac{1}{2} f g_{\mu\nu} + f_T (T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q (P_{\mu\alpha\beta} Q_\nu^{\ \alpha\beta} - 2Q^{\alpha\beta}_{\ \ \mu} P_{\alpha\beta\nu}) = 8\pi T_{\mu\nu} \tag{2}$$

The traces of the non-metricity:

$$Q_\alpha = Q_\alpha{}^\mu{}_\mu \qquad \tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu} \tag{3}$$

Where,

$$p^\alpha_{\mu\nu} = -\frac{1}{2} L^\alpha_{\mu\nu} + \frac{1}{4}(Q^\alpha - \tilde{Q}^\alpha) g_{\mu\nu} - \frac{1}{4} \delta^\alpha_{(\mu} Q_{\nu)} \tag{4}$$

The disformation tensor:

$$L^\alpha_{\beta\gamma} = -\frac{1}{2} g^{\alpha\lambda} (Q_{\gamma\beta\lambda} + Q_{\beta\lambda\gamma} - Q_{\lambda\beta\gamma}) \tag{5}$$

Also given

$$Q_{\lambda\mu\nu} = -\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + g_{\nu\sigma}\hat{\Gamma}^{\sigma}_{\mu\lambda} + g_{\sigma\mu}\hat{\sigma}^{\sigma}_{\nu\lambda}$$

$$\Gamma^{\lambda}_{\mu\nu} = -L^{\lambda}_{\mu\nu} \tag{6}$$

$$Q \equiv -g^{\mu\nu}(L^{\alpha}_{\beta\mu}L^{\beta}_{\nu\alpha} - L^{\alpha}_{\beta\alpha}L^{\beta}_{\mu\nu}) \tag{7}$$

Assume a flat FLRW space time:

$$ds^2 = a^2(t)(dx^2 + dy^2 + dz^2) - N^2(t)dt^2$$

$$H = \frac{\dot{a}}{a}, \qquad \tilde{T} \equiv \frac{\dot{N}}{N}$$

By solving equation(7) we get

$$Q = 6\frac{H^2}{N^2}$$

The energy momentum tensor is given by

$$T^{\mu}_{\nu} = diag(-\rho, p, p, p)$$

Also,

$$\Theta^{\mu}_{\nu} = diag(2\rho + p, -p, -p, -p)$$

By using FLRW metric from the field equation we can easily find

$$\begin{aligned} \frac{f}{2} - 6F\frac{H^2}{N^2} &= 8\pi\tilde{G}(\rho + p) && (8) \\ \frac{f}{2} - \frac{2}{N^2}\left[\left(\dot{F} - F\tilde{T}\right)H + F\left(\dot{H} + 3H^2\right)\right] &= -8\pi p && (9) \end{aligned}$$

Next we consider the standard case when $N = 1$ which is the case of standard FLRW geometry.Thus we get

$$Q = 6H^2 \qquad (10)$$

and the generalized Friedmann equations reduces to

$$\rho = \frac{1}{8\pi}\left[\frac{f}{2} - 6FH^2 - 2\frac{\tilde{G}}{1+\tilde{G}}(\dot{F}H + F\dot{H})\right] \qquad (11)$$

$$p = -\frac{1}{8\pi}\left[\frac{f}{2} + 6FH^2 + 2(\dot{F}H + F\dot{H})\right] \qquad (12)$$

$$F \equiv f_Q \qquad and \quad 8\pi\tilde{G} \equiv f_T \qquad (13)$$

Three Rip Cosmological Models

Our considered model is, $f(Q,T) = aQ^m + bT$

$$p = \frac{-(1-2m)aQ^m + 2\dot{\chi}[2+\kappa-\kappa\kappa_1]}{4\pi\left[(2+\kappa)(2+3\kappa)-3\kappa^2\right]} \quad (14)$$

$$\rho = \frac{(1-2m)aQ^m + 2\dot{\chi}[3\kappa-(2+3\kappa)\kappa_1]}{4\pi\left[(2+\kappa)(2+3\kappa)-3\kappa^2\right]} \quad (15)$$

$$\omega = \frac{-(1-2m)aQ^m + 2\dot{\chi}[2+\kappa-\kappa\kappa_1]}{(1-2m)aQ^m + 2\dot{\chi}[3\kappa-(2+3\kappa)\kappa_1]} \quad (16)$$

Little Rip

• In 2011, Frampton, Ludwick and Sherrer has given some crucial concept about little rip along with some description and conditions of future singularities.

• The little rip is a cosmological abrupt event predicted by some phantom dark energy models that could describe the future evolution of our Universe.

• Only phantom energy with improbable physical attributes is capable of experiencing a sudden rip singularity. Physically, in the little rip, the scale factor and the density are never infinite at a finite time.

- The little rip scale factor is taken as:

$$R = R_0 Exp^{\left[\frac{A}{\lambda}(e^{\lambda t} - e^{-\lambda t_0})\right]}$$

- $H = H_0 e^{\lambda t}$
- $q = -1 - \frac{\lambda e^{-\lambda t}}{A}$

The dynamical parameters are represented as:

$$\begin{aligned} p &= -\frac{a(6)^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{8\pi(1+2\kappa)}\left[3Ae^{\lambda t} + m\lambda(2+\kappa-\kappa\kappa_1)\right], \\ \rho &= \frac{a(6)^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{8\pi(1+2\kappa)}\left[3Ae^{\lambda t} - m\lambda(3\kappa-2\kappa_1-3\kappa\kappa_1)\right], \\ \omega &= -1 - \frac{2m\lambda(1-\kappa_1+2\kappa-2\kappa\kappa_1)}{3Ae^{\lambda t} - m\lambda(3\kappa-2\kappa_1-3\kappa\kappa_1)} \end{aligned}$$

- P. H. Frampton, K. J. Ludwick, R. J. Scherrer, *Phys. Rev. D*, **84**, 063003 (2011).
- P. H. Frampton, K. J. Ludwick, R. J. Scherrer, *Phys. Rev. D*, **85**, 083001 (2012).

Analysis of the LR graph

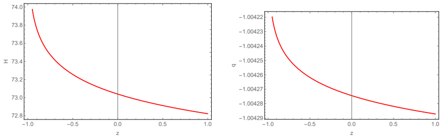

Figure 1: Behaviour of Hubble parameter (left panel) and deceleration parameter (right panel) in redshift, (A = 25.11, λ = 0.3122, t_0 = 3.42).

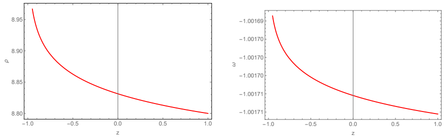

Figure 2: Behaviour of energy density (left panel) and EoS parameter (right panel) in redshift, (a = -4.4, b = 0.01, m = 0.6, A = 25.11, λ = 0.3122, t_0 = 3.42).

• The EoS parameter value from observational sources, Supernova data[5]. $\omega = -1.084 \pm 0.063$, WMAP[6]. $\omega = -1.073 \pm^{0.090}_{0.089}$ favours ΛCDM.

[5] R. Amanullah*et al.*, *Astrophy. J.*, **712**, 716 (2010)

[6] G. Hinshaw *et al*, *Astrophys. J. Suppl. Ser.*, **19**, 208 (2013)

Big Rip

• The most well-known sort of finite time future singularity is the Big Rip singularity, which is linked to phantom evolution.

• The density of the dark energy increases with increasing scale factor, and both the scale factor and the phantom energy density can become infinite at a finite time t, is known as big rip.

• In the big rip, the scale factor and density diverge in a singularity at a finite future time.

• The big rip scale factor is

$$R(t) = R_0 + \frac{1}{(t_s - t)^{\alpha}}$$

• $H(t) = \frac{\alpha}{(t_s - t)^{\alpha}}$

• $q = -\frac{(\alpha+1)(1+R_0(t_s-t)^{\alpha})}{\alpha}$

$$\begin{aligned}
p &= -\frac{a(6)^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 + m(2+\kappa-\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right],\\
\rho &= \frac{a(6)^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right],\\
\omega &= -1 - \frac{2m(1-\kappa_1+2\kappa-2\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}}{3\left(\frac{\alpha}{t_s-t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}}
\end{aligned}$$

- P. Ray *et al.*, *Fortschr. Phys.*, **69**, 2100086 (2021).

Analysis of the BR graph

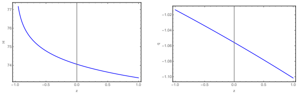

Figure 3: The behaviour of Hubble parameter (left panel) and deceleration parameter (right panel) vs redshift, H_0=74.31, t_s=13.8, α=12.7

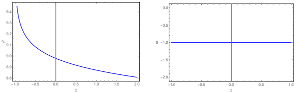

Figure 4: Behaviour of energy density (left panel) and EoS parameter (right panel) vs redshift, a=-4.4, b=0.01, m=0.6, t_s=13.8, α=12.7

• According to the present observational value of the deceleration parameter q = -1.08, the present value of the Hubble parameter H_0 for the BR model can be $74.33 Kms^{-1} Mpc^{-1}$. [7]

[7] D Camarena, V. Marra, *Phys. Rev. D*, **2**, 013028 (2020).

Pseudo Rip

- The cosmos begins in the infinite past from a phase where the scale factor was zero, but the Hubble parameter was a constant. This situation is known as the early phase Pseudo-bang as the characteristics of this are similar to the fate of Pseudo Rip(PR).
- The scale factor of PR model is

$$\mathcal{R} = R_1 exp\left[H_0 t + H_1 \frac{1}{\eta} e^{\eta t}\right]$$

- $H = H_0 - \frac{H_1}{e^{\eta t}}$
- $q = -1 - \frac{\eta H_1 e^{-\eta t}}{(H_0 - H_1 e^{\eta t})^2}$

Now the dynamical parameters can be obtained as,

$$p = -\frac{a(6)^{m-1}(1-2m)\left(H_0 - H_1 e^{-\eta t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(H_0 - H_1 e^{-\eta t}\right)^2 + m(2+\kappa-\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}\right], \quad (17)$$

$$\rho = \frac{a(6)^{m-1}(1-2m)\left(H_0 - H_1 e^{-\eta t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(H_0 - H_1 e^{-\eta t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}\right], \quad (18)$$

$$\omega = -1 - \frac{2m(1-\kappa_1+2\kappa-2\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}}{3\left(H_0 - H_1 e^{-\eta t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}} \quad (19)$$

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[8] W. EI. Hanafy, E. N. Saridakis, *Journal of Cosmology and Astroparticle Physics, **09**, 019 (2021).*

Analysis of the PR graph

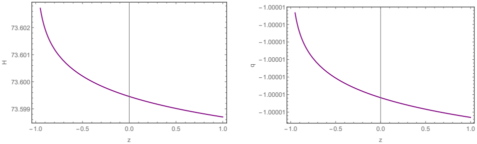

Figure 5: The behaviour of Hubble parameter (left panel) and deceleration parameter (right panel) vs redshift, H_0=74.31, H_1=1, η=0.3011

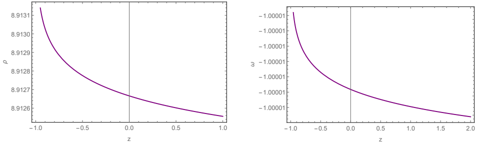

Figure 6: Behaviour of energy density (left panel) and EoS parameter (right panel)wrt to redshift, a=-4.4, b=0.01, m=0.6, H_0=74.31, H_1=1, η=0.3011

Energy Conditions for LR Model

$$\rho + p = -\frac{a6^{m-1}m(1-2m)\lambda(Ae^{\lambda t})^{2m-1}}{4\pi}[1-\kappa_1], \textbf{NEC}$$

$$\rho + 3p = -\frac{a6^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{4\pi(1+2\kappa)}[3Ae^{\lambda t} + m\lambda(3+3\kappa-\kappa_1-3\kappa\kappa_1)], \textbf{SEC}$$

$$\rho - p = \frac{a6^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{4\pi(1+2\kappa)}[3Ae^{\lambda t} + m\lambda(1-\kappa+\kappa_1+\kappa\kappa_1)], \textbf{DEC}$$

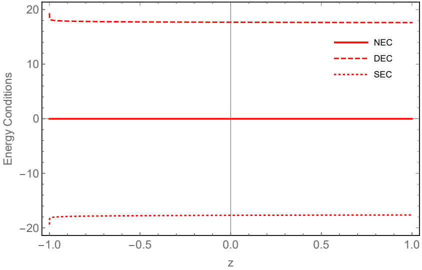

Figure 7: Behaviour of energy Conditions w.r.t to redshift in LR model, a=-4.4, b=0.01, m=0.6, A = 25.11, $\lambda = 0.3122$, $t_0 = 3.42$).

Energy Conditions for BR Model

$$\rho + p = -\frac{a6^{m-1}m(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}\frac{\alpha}{(t_s-t)^2}}{4\pi}\left[1-\kappa_1\right], \textbf{NEC}$$

$$\rho + 3p = -\frac{a6^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{4\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 + m(3+3\kappa-\kappa_1-3\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right],$$

$$\rho - p = \frac{a6^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{4\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 + m(1-\kappa+\kappa_1+\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right], \textbf{DE}$$

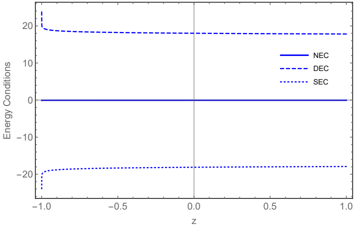

Figure 8: Behaviour of energy Conditions w.r.t to redshift in BR model, a=-4.4, b=0.01, m=0.6, t_s=13.8, α=12.7.

Energy Conditions for PR Model

$$\rho + p = -\frac{a6^{m-1}(1-2m)\left(H_0 - H_1 e^{\eta t}\right)^{2m-2}}{4\pi}[m(1-\kappa_1)(\eta H_1 e^{-\eta t})], \textbf{NEC}$$

$$\rho + 3p = -\frac{a6^{m-1}(1-2m)\left(H_0 - H_1 e^{\eta t}\right)^{2m-2}}{4\pi(1+2\kappa)}[3\left(H_0 - H_1 e^{\eta t}\right)^2 + m(3+3\kappa-\kappa_1-3\kappa\kappa_1)(\eta H_1 e^{-\eta t})], \textbf{SE}$$

$$\rho - p = \frac{a6^{m-1}(1-2m)\left(H_0 - H_1 e^{\eta t}\right)^{2m-2}}{4\pi(1+2\kappa)}[3\left(H_0 - H_1 e^{\eta t}\right)^2 + m(1-\kappa+\kappa_1+\kappa\kappa_1)(\eta H_1 e^{-\eta t})], \textbf{DEC}$$

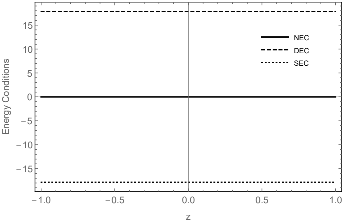

Figure 9: Behaviour of energy Conditions w.r.t to redshift in PR model, a=-4.4, b=0.01, m=0.6, H_0=74.31, H_1=1, η=0.3011.

Results and Conclusion

• The motivation behind an LR model is to avoid singularity at finite time scale and therefore, in this model, we cannot demonstrate the transit behaviour of the Universe from a decelerated phase of expansion to an accelerated one.

• The EoS parameter in case of LR model through shows a phantom type behaviour but remains very close to the ΛCDM line whereas in BR model it entirely remain on the ΛCDM line.

• At the same time in PR model, it show similar behaviour as in LR except the fact that in PR model it remains in a future narrow range.

• As required in the modified theories of gravity, here also in all three models violation of SEC and satisfaction of DEC are obtained. The interesting behaviour we noticed is that the NEC appears just below the null line. It indicates that the contribution from NEC is almost negligible in these models.

• Finally, based on our model we can conclude that no singularity scenario appear in the accelerating models, so the study in $f(Q,T)$ gravity may give new insight in to resolving the singularity issue.


Thank
you