Bispectrum and finite volume effects: window-convolution

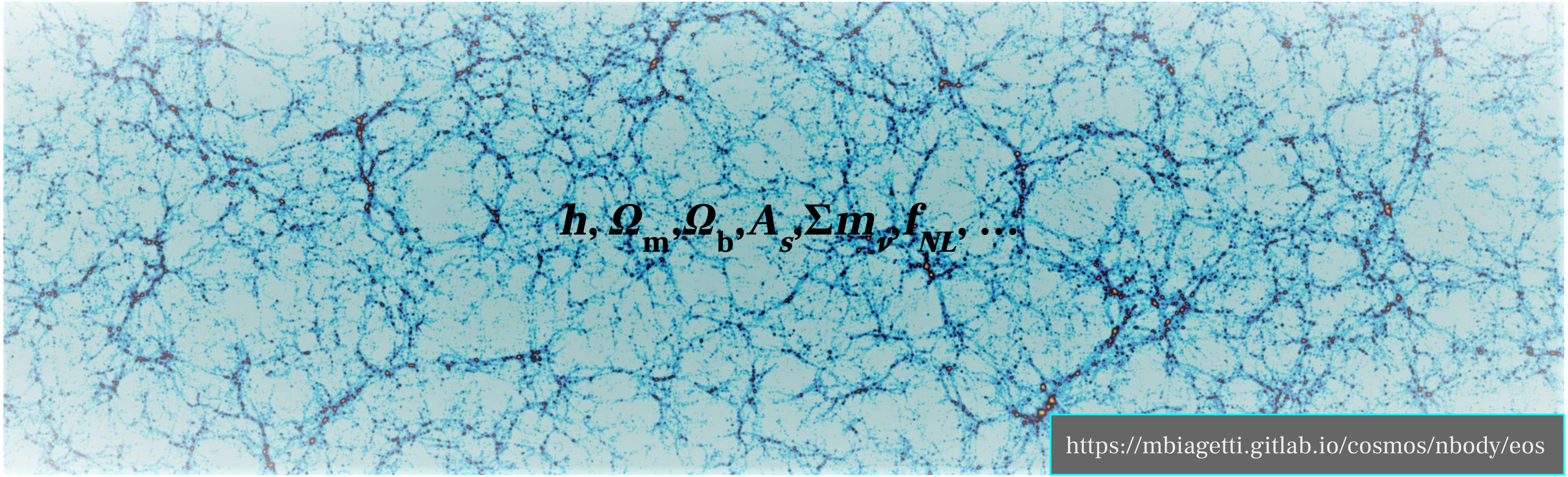
Cosmology from Home 2022

Kevin Pardede

PhD Student @ AstroParticle Physics, SISSA

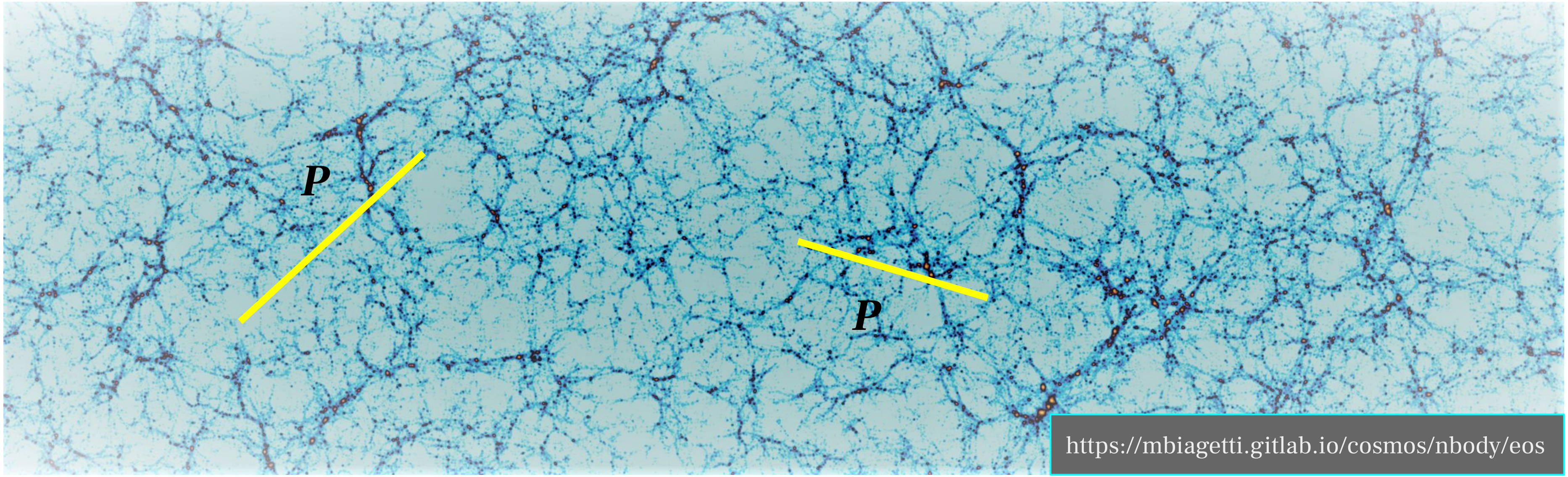
Galaxy clustering

Galaxy distribution as an independent probe for the cosmological parameters



Characterized by its summary statistics

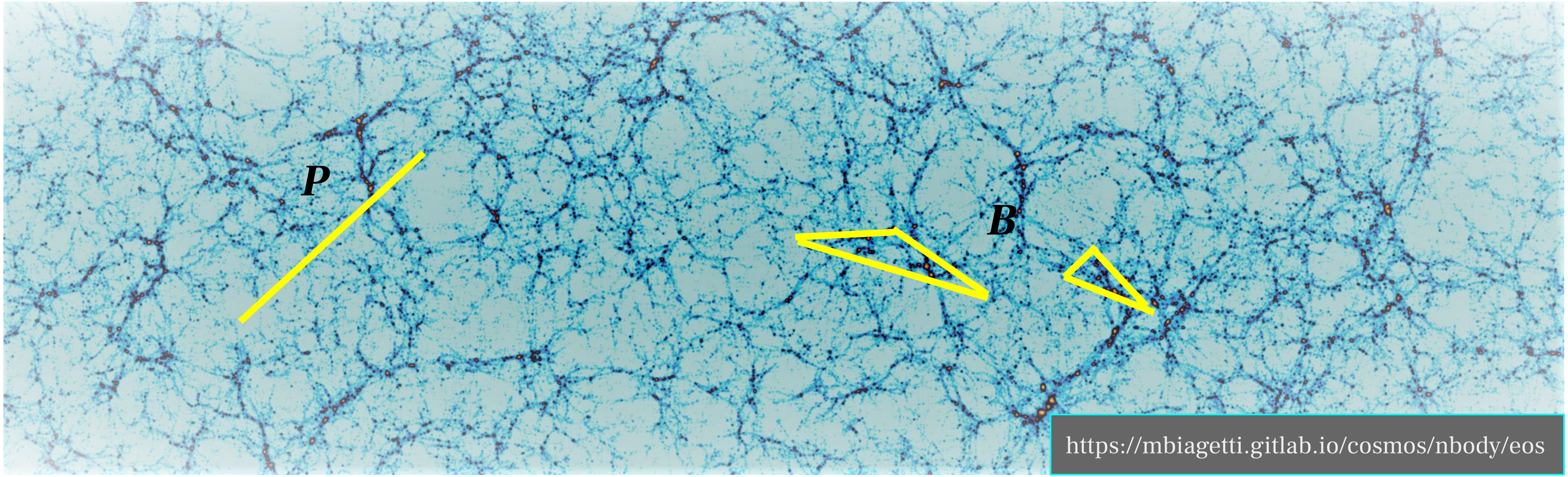
Power Spectrum (P) + Bispectrum (B) + …



$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P(\mathbf{k}_1)$$

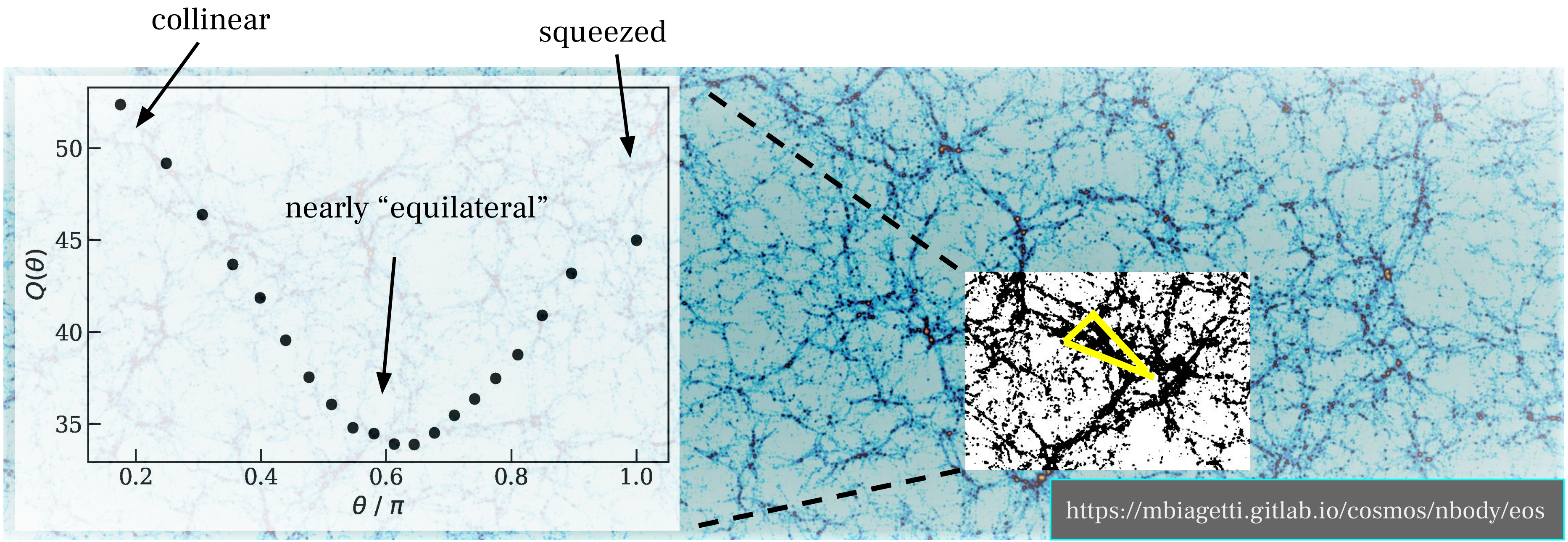
Bispectrum captures non-Gaussianity

Power Spectrum (P) + **Bispectrum** (B) + …



$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(\mathbf{k}_1,\mathbf{k}_2)$$

… the filamentary structure



$$Q(k_1, k_2, k_3) \equiv \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$

Why bispectrum?

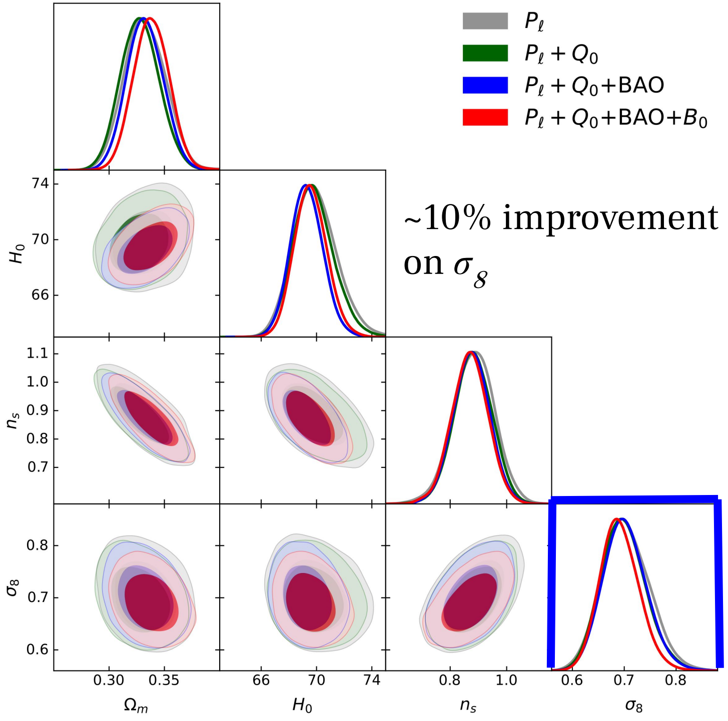
Bispectrum improves*
bias/cosmological parameters constraints

Sefusatti+06, Gagrani+18, Yankelevich+18, Karagiannis+18, Chudaykin+19, Hahn+19, Oddo+21, ...

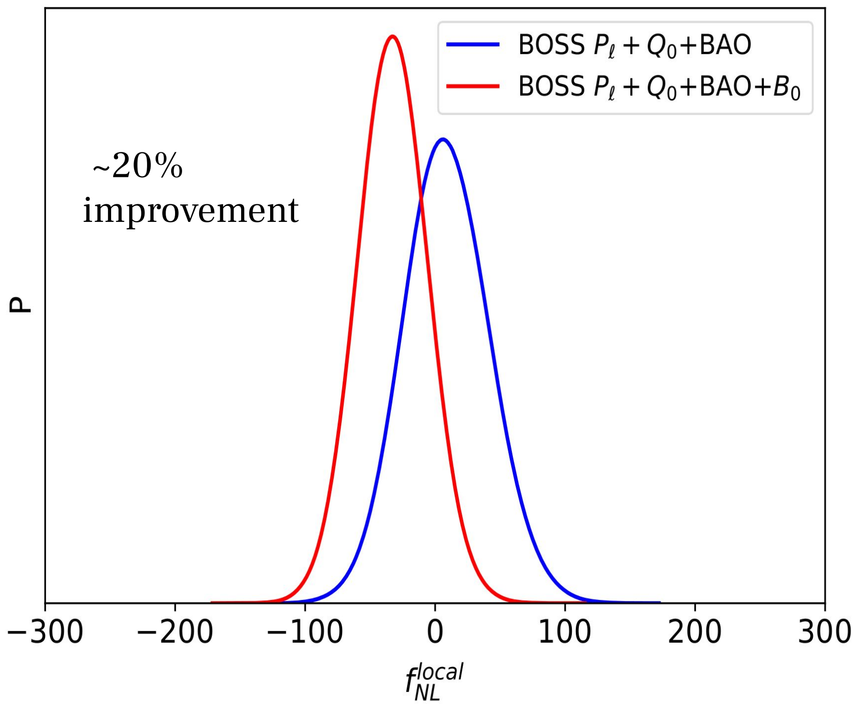
*break degeneracies, tighten constraints, ...

Including bispectrum monopole (BOSS DR12)

Constraint on cosmological params:



Constraint on primordial non-Gaussianity:



Philcox&Ivanov21 (also: D'Amico+19)

also one-loop bispectrum: Philcox+22, D'Amico+22b

Cabass+22b (also: D'Amico+22a)

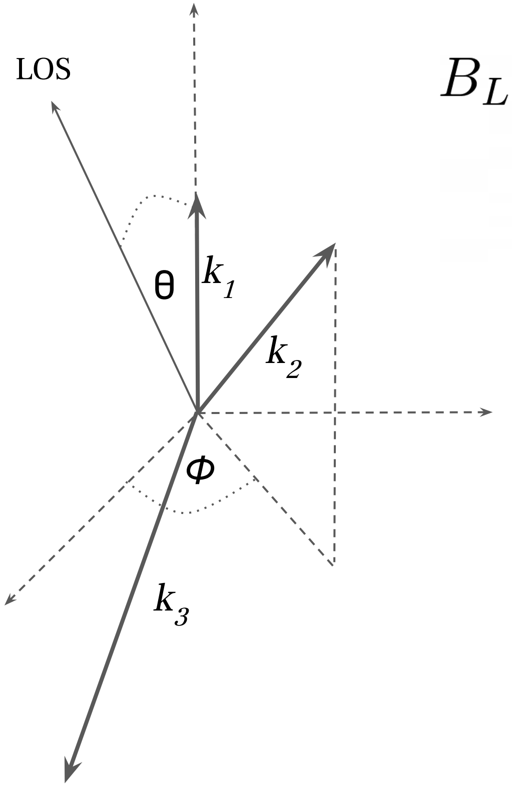
non-local PNG: Cabass+22a, bispectrum is **necessary**

Including bispectrum multipoles?

Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

incl. P_ℓ: Moretti, Rizzo, Pardede+ (in prep.)

The bispectrum multipoles



$$B_L(k_1,k_2,k_3) = \frac{2L+1}{4\pi}\int d\cos\theta \int d\phi B(k_1,k_2,k_3,\theta,\phi)\,\mathcal{L}_L(\cos\theta)\,.$$

PT (perturbation theory) model

angles w.r.t line of sight

- galaxies are not in their rest frame
- $m\neq0$ contains negligible information Gagrani+16
- **tree-level: only even multipoles exist B_0, B_2, B_4, …**

Tree-level bispectrum

$$B(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = B^{(\mathrm{det})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) + B^{(\mathrm{stoch})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}})$$

$$B^{(\mathrm{det})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = 2Z_2(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}})Z_1(\mathbf{k}_1, \hat{\mathbf{x}})Z_1(\mathbf{k}_2, \hat{\mathbf{x}})P_L(k_1)P_L(k_2) + \mathrm{cyc.}$$

$$B^{(\mathrm{stoch})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = \frac{1}{\bar{n}}[(1+\alpha_1)b_1 + (1+\alpha_3)f(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{x}})^2]Z_1(\mathbf{k}_1, \hat{\mathbf{x}})P_L(k_1) + \mathrm{cyc.} + \frac{1+\alpha_2}{\bar{n}^2}$$

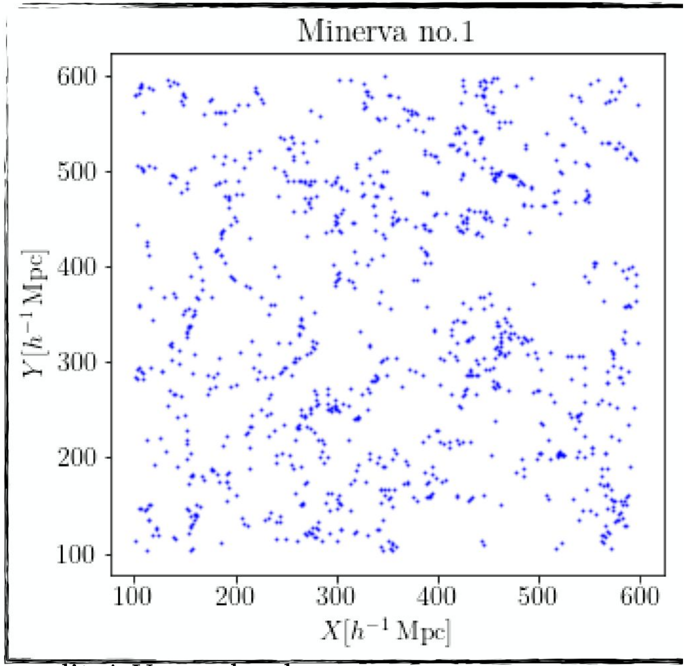
$$Z_1(\mathbf{k}, \hat{\mathbf{x}}) = b_1 + f(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{x}})^2$$

$$Z_2(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_{\mathcal{G}_2} S(\mathbf{k}_1, \mathbf{k}_2)$$
$$f(\hat{\mathbf{k}}_{12} \cdot \hat{\mathbf{x}})^2 G(\mathbf{k}_1, \mathbf{k}_2) + \frac{f(\mathbf{k}_{12} \cdot \hat{\mathbf{x}})}{2}\left[\frac{\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{x}}}{k_1} Z_1(\mathbf{k}_2, \hat{\mathbf{x}}) + \frac{\hat{\mathbf{k}}_2 \cdot \hat{\mathbf{x}}}{k_2} Z_1(\mathbf{k}_1, \hat{\mathbf{x}})\right]$$

$$\mathbf{k}_{12} \equiv \mathbf{k}_1 + \mathbf{k}_2$$

Dark matter halo catalogs

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) Monaco+02



credit: A.Veropalumbo

@$z = 1$

ΛCDM cosmology

$L_{box} = 1500$ Mpc/h

$V_{eff} \simeq 1000\ (\mathrm{Gpc}/h)^3$

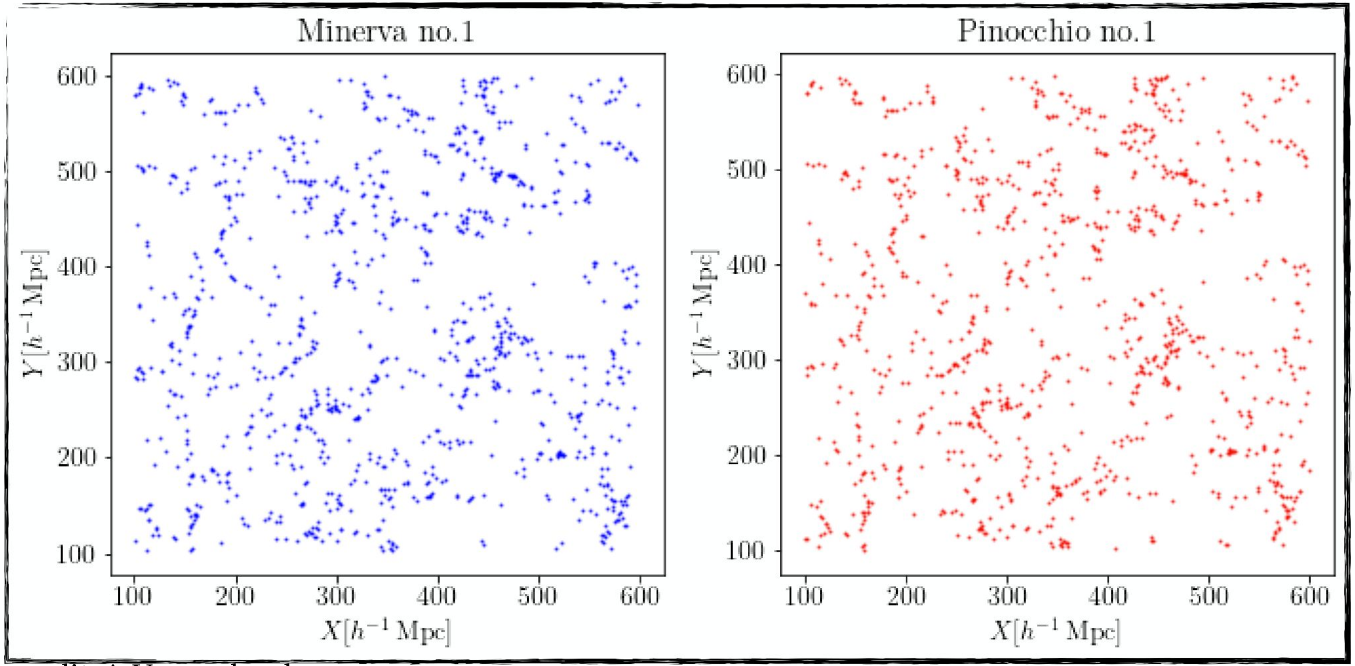
$\simeq$ 2x volume in Nishimichi+20

Dark matter halo catalogs + numerical covariance

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) Monaco+02

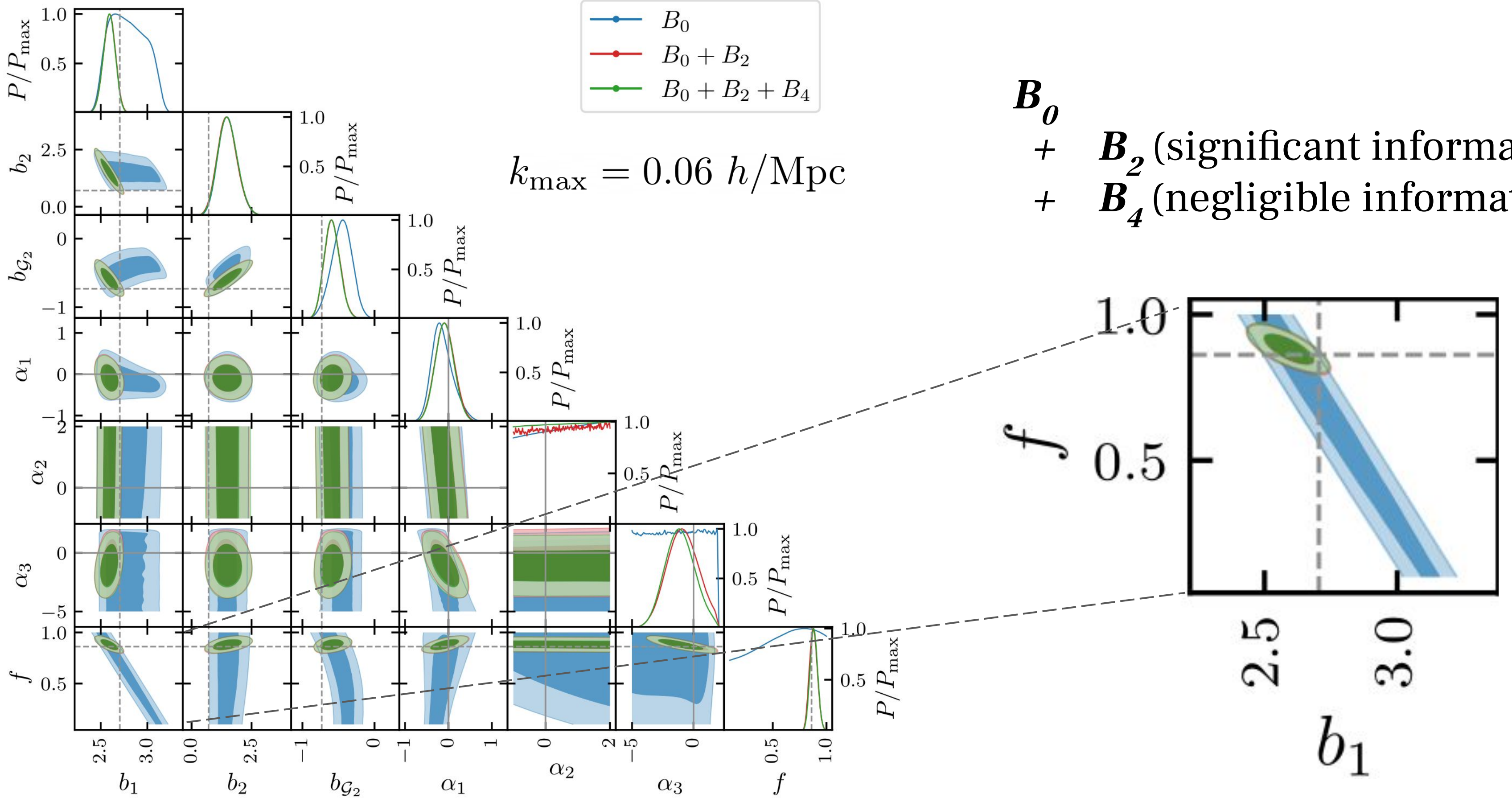
- approx. based on Lagrangian pert. theory
- relatively fast and accurate

provide a robust estimate of the covariance



credit: A.Veropalumbo

Including the multipoles



B_0

\+ ***B_2*** (significant information)

\+ ***B_4*** (negligible information)

How to measure the bispectrum?

Scoccimarro estimator Scoccimarro +15

FFT-based!

$$\hat{B}_L(k_1,k_2,k_3) = \frac{2L+1}{V_B}\int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123})\tilde{\delta}_L(\mathbf{q}_1)\tilde{\delta}(\mathbf{q}_2)\tilde{\delta}(\mathbf{q}_3)$$

binning operator

choose one side as the **LOS** $\tilde{\delta}_L(\mathbf{q}) \equiv \int d^3x\ \tilde{\delta}(\mathbf{x})\mathcal{L}_L(\mathbf{q}\cdot\mathbf{x})e^{-i\mathbf{q}\cdot\mathbf{x}}$

window function $\tilde{\delta}(\mathbf{x}) = W(\mathbf{x})\delta(\mathbf{x})$

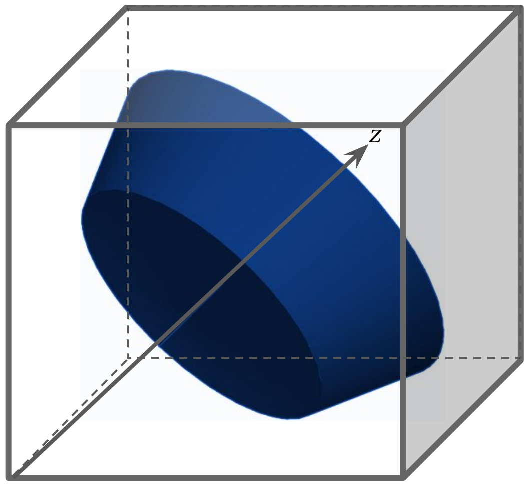
$$\int_{k_1} d^3q_1 \equiv \int_{|k_1-\Delta k/2|\leq k_1 \leq |k_1+\Delta k/2|} d^3q_1 \qquad V_B \equiv \int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123})$$

Survey window effects in bispectrum

Pardede, Rizzo, Biagetti, Castorina, Sefusatti, Monaco (arXiv: 2203.04174)

Estimator is biased by window function

We need to Fourier transform $\delta(\mathbf{x})$



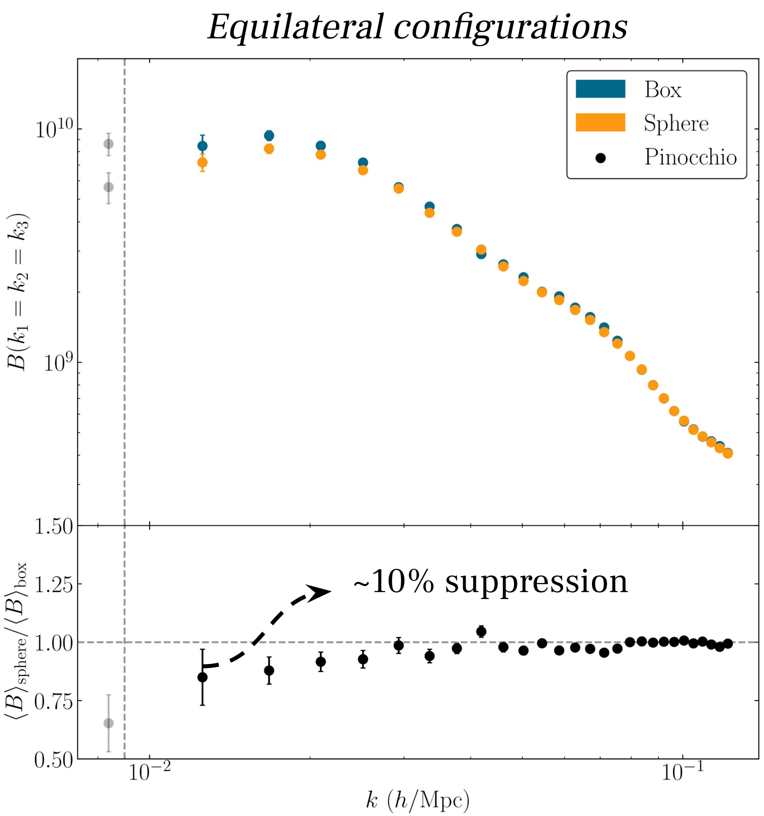
Baumgart, Fry 1991

Defined over a periodic box

$$\tilde{\delta}(\mathbf{x}) = \underbrace{W(\mathbf{x})}_{\text{window function}}\delta(\mathbf{x})$$

$$\Rightarrow \tilde{\delta}(\mathbf{k}) = \int \frac{d^3k'}{(2\pi)^3} W(\mathbf{k}-\mathbf{k}')\delta(\mathbf{k}')$$

In bispectrum …



window convolution will mix modes

10000 Pinocchio sphere catalogue

Note: this is a huge volume ≈ 3500 [Gpc/h]^3

… main effect is on large scale

To include window effect in bispectrum

Schematically (monopole with no binning operator)

$$\tilde{B}(\vec{k}_1, \vec{k}_2) = \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} B_W(\vec{k}_1 - \vec{p}_1, \vec{k}_2 - \vec{p}_2) B(\vec{p}_1, \vec{p}_2)$$

- 6D integral
- Time ~ **hours**/evaluation
- Not feasible for likelihood analysis

An approximation

$$\tilde{B}[P_L] \simeq B[\tilde{P}_L]$$

1DFFTLog-approx

$$\tilde{B}(\mathbf{k}_1, \mathbf{k}_2) \simeq Z(\mathbf{k}_1, \mathbf{k}_2)\tilde{P}_L(k_1)\tilde{P}_L(k_2) + \text{cyc.}$$

- Reduced to power spectrum-window convolution
  see e.g. Wilson+15, Castorina+17, d'Amico+19
- BOSS DR 11/12 Gil-Marin+14a, b and +16a, b
- Recently used in d'Amico +19,+22
- Doesn't work for squeezed triangles

Computed via (1D) FFTLog

Exact bispectrum window convolution*

Taking $\langle \hat{B}_L \rangle = \tilde{B}_L$

$$\tilde{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B} \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \, \delta_D(\vec{q}_{123})$$
$$\times \int d^3 x_3 \int d^3 x_{13} \int d^3 x_{23} \, e^{-i\vec{q}_1 \cdot \vec{x}_{13}} e^{-i\vec{q}_2 \cdot \vec{x}_{23}} \zeta(\vec{x}_{13}, \vec{x}_{23}, \hat{x}_3)$$
$$\times W(\vec{x}_1) \, W(\vec{x}_2) \, W(\vec{x}_3) \, \mathcal{L}_L(\hat{q}_1 \cdot \hat{x}_3)$$

*window-free estimator Tegmark+97 has been revived recently: Philcox20 (power spectrum), Philcox21 (bispectrum)

As a matrix multiplication

We showed that bispectrum-window convolution
can be casted into a 1D integral

2DFFTLog

$$\tilde{B}_\ell[T_i] = \sum_{j,\ell'} \mathcal{M}_{\ell\ell'}[T_i, T_j'] B_{\ell'}[T_j']$$

Mixing matrix

Computable via (2D) FFTLog

e.g. `2D-FFTLog` (Fang+20)

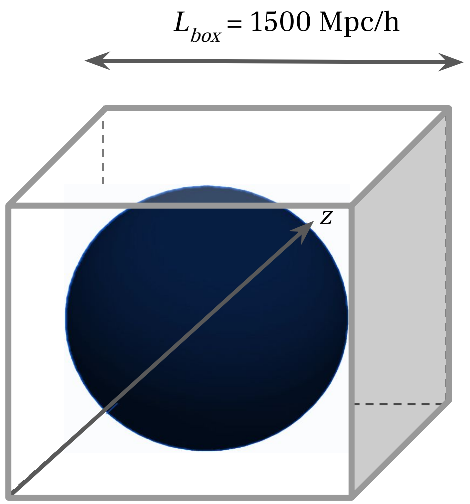
of the window 3PCF multipoles

Bispectrum

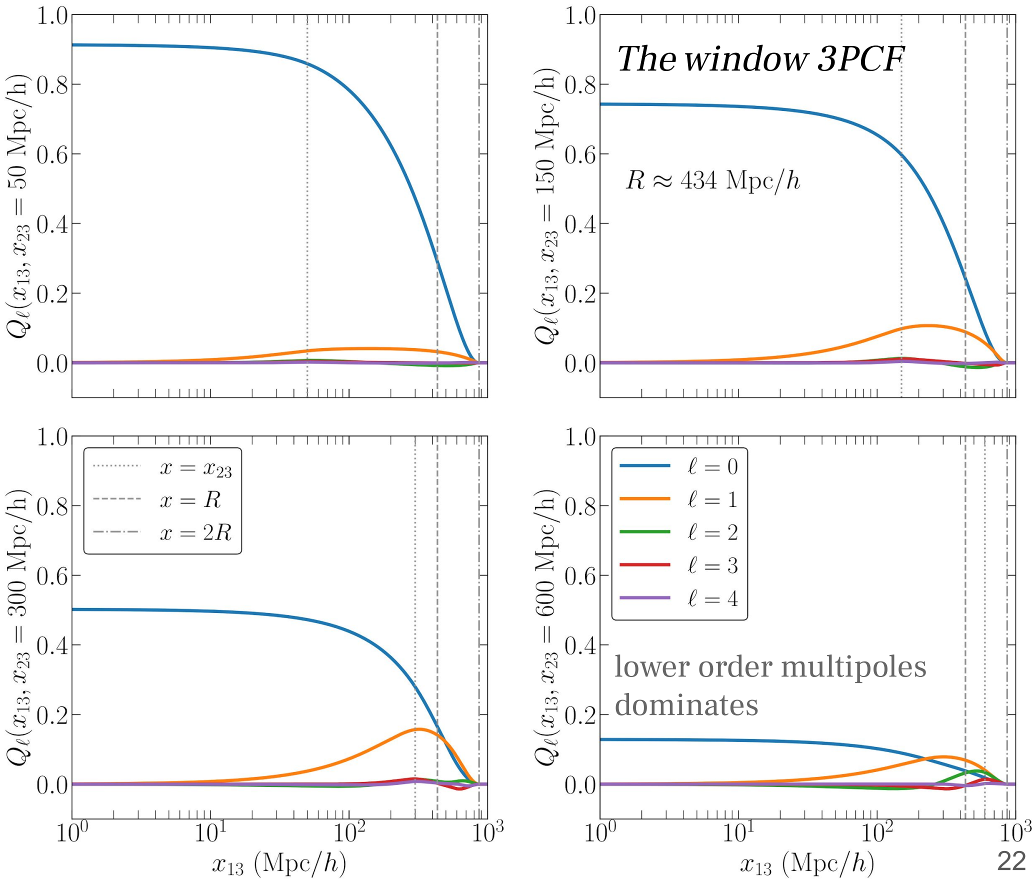
Function of three sides (k_1, k_2, k_3)

Spherical window convolution in real-space

Sphere catalogue:
Minerva/Pinocchio carved on a sphere of $R \sim 434$ Mpc/h

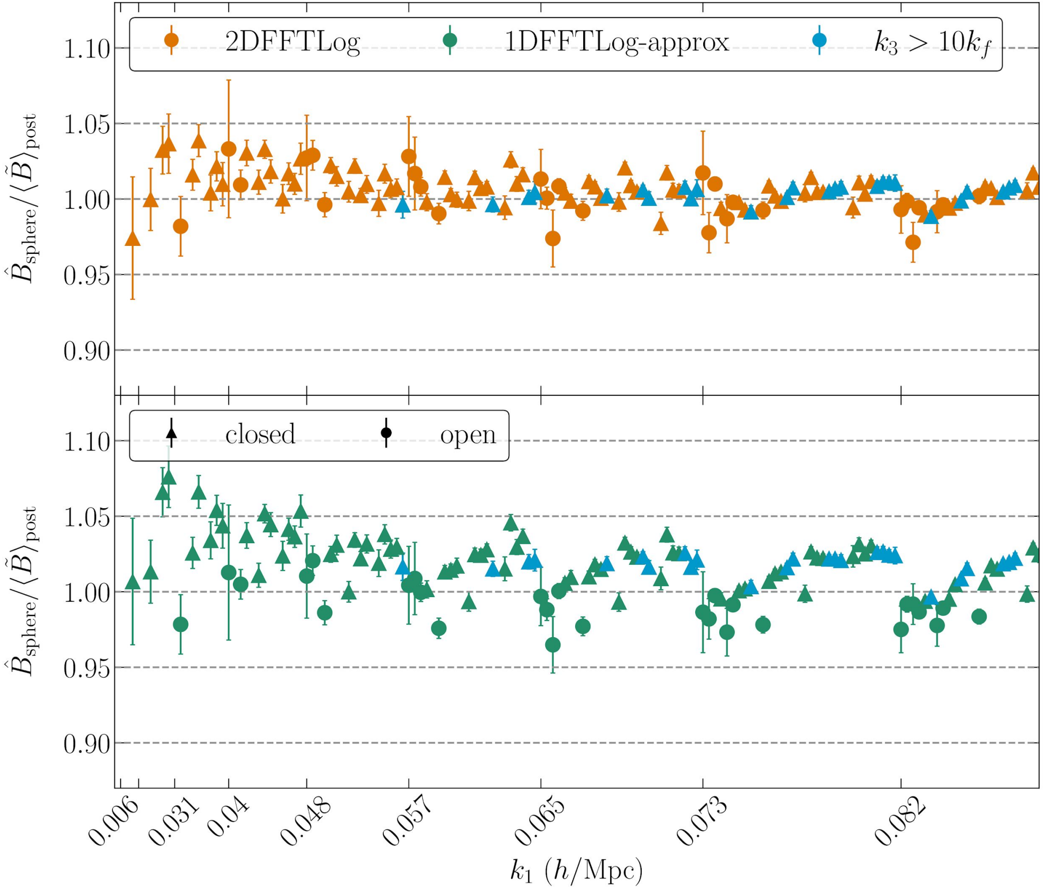


Total vol = 700^3 (Mpc/h)]^3



Full-set triangles

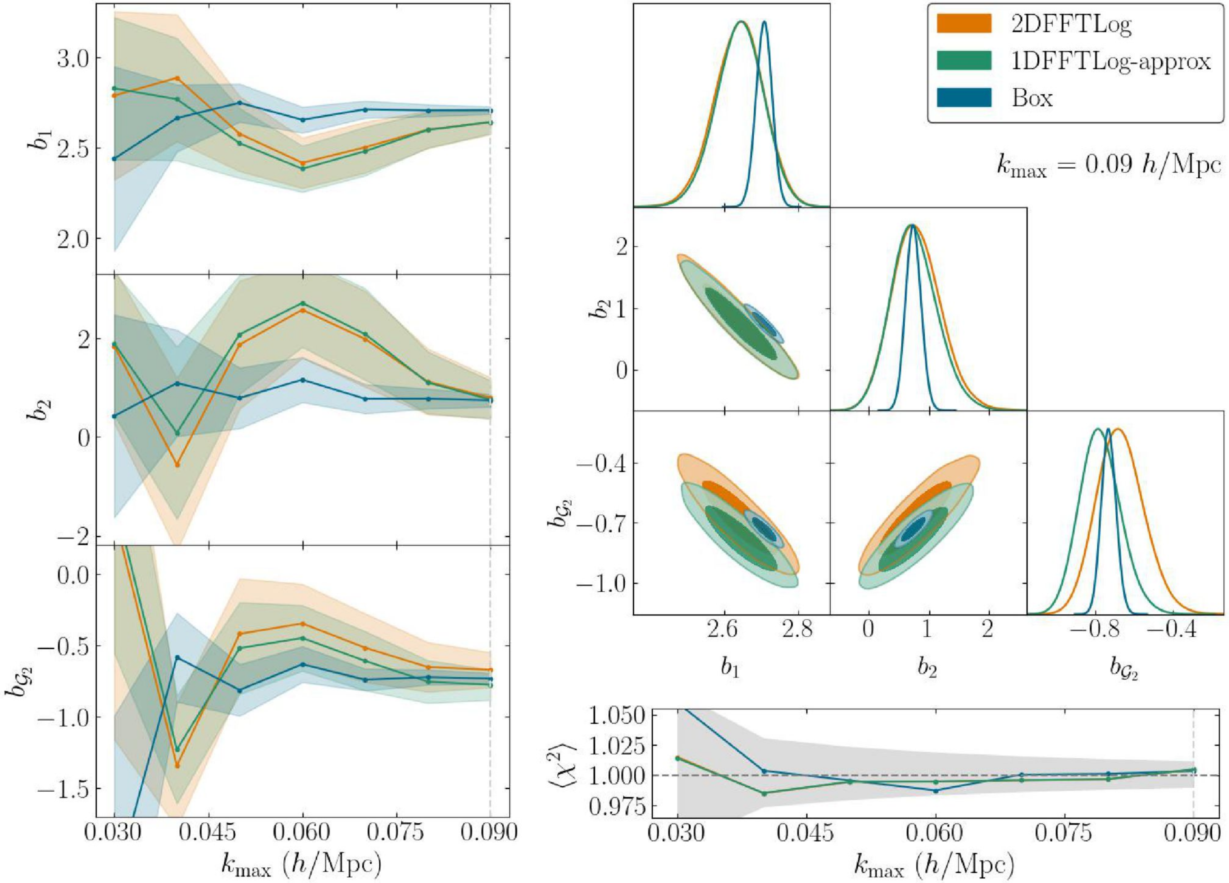
Fit on **Pinocchio** mocks



Recovering bias parameters

Analysis on **Minerva** data

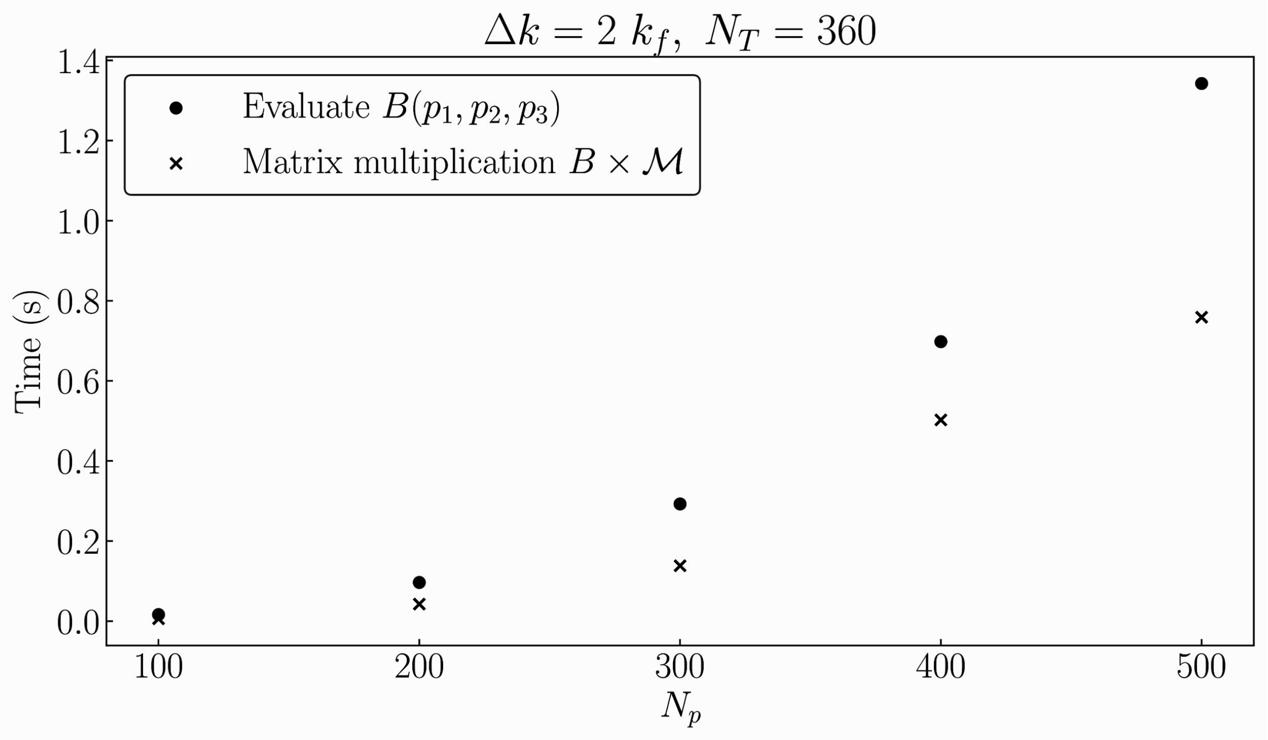
≈ ¼ times volume in Nishimichi+20
≈ 10 times $z \in [1.5, 1.8]$ *Euclid* volume



Window convolution computation time

Takes ~ **2 seconds**

⇒ comparable to a typical Boltzmann solver call



Conclusions

- Including bispectrum multipoles analysis is important but come with extra modelling complexity, ex: survey window effects
- We gave an efficient formulation for bispectrum window convolution
- We tested the formulation in ideal case of spherical window convolution in real space
- Useful in future surveys when you want to extract signal, free from systematic effects

Thank you!