# Canonical and Non-canonical Inflation: In light of recent BICEP/Keck Results
## Cosmology from Home 2022

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# Based on –

**Canonical and Non-canonical Inflation in the light of the recent BICEP/Keck results**

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# Prologue

- Developments in the past two decades $\Rightarrow$ standard scenario of Cosmology (it e.g **flat $\Lambda$CDM model** as a toy model).
- Evolution of the Universe from about 1 sec to 13.8 billion years (background and perturbation level)

**A reasonably successful theory!!**

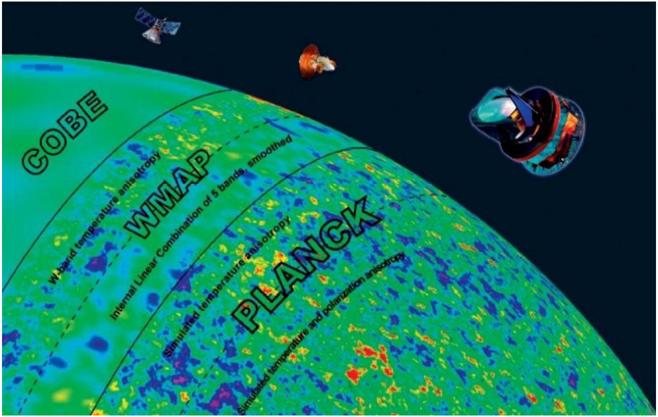
**Assumptions on Composition**

1. **Dark Matter $\longrightarrow$ CDM / ?**
2. **Dark Energy $\longrightarrow$ $\Lambda$/ ?**

**Assumptions on Initial Conditions**

1. **Homogeneity and Isotropy on large length scales**
2. **Spatial flatness**
3. **Almost scale invariant, nearly Gaussian and adiabatic initial density fluctuations**

# What creates the seed fluctuations?




**COSMIC INFLATION : a transient period of rapid accelerated expansion of space**

**Sets Natural Initial Conditions for the Hot Big Bang Phase**

# Cosmic Inflation

The standard scenario of inflation has two aspects –

1. **Background Evolution/Dynamics**

   (Solves/addresses the horizon and flatness problem)

2. **Linear Perturbations** due to Quantum Fluctuations during an epoch of accelerated expansion of space.

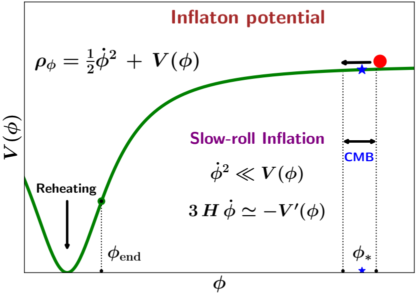
   (Provides the origin/seed fluctuations for the hot Big Bang inhomogeneities)

**Origin of CMB fluctuations and LSS in the Universe**

# Inflationary Background Dynamics

**A single scalar field minimally coupled to Gravity**

# Inflationary Dynamics of a Scalar Field




**Density** $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

**Pressure** $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Friedmann Equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi,$$

Motion of the scalar field is governed by

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

Condition for Inflation $\dot{\phi}^2 < V(\phi)$

$$\epsilon_H = \frac{1}{2m_p^2}\frac{\dot{\phi}^2}{H^2} = -\frac{\dot{H}}{H^2} < 1$$

Prolonged period of inflation

$$|\eta_H| = |-\frac{\ddot{\phi}}{H\dot{\phi}}| < 1$$

# Slow-roll Inflation

**Slow-roll Regime of Inflation**

$$\epsilon_{_H} = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2m_p^2 H^2} \ll 1 \ , \quad |\eta_{_H}| = \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

Cosmological Equations become

$$H^2 \simeq \frac{1}{3m_p^2} V(\phi) \ , \qquad \dot{\phi} = -\frac{V'(\phi)}{3H}$$

$\Rightarrow$ **Quasi de Sitter Expansion** $a(t) = a_i\, e^{H\,t}$

# Perturbations during Inflation

**Light fields** $m_\phi < H$ in **quasi de Sitter space** receive quantum fluctuations on macroscopically large length scales (super-Hubble).

# Full System during Inflation

$$\boxed{\textbf{System} = \textbf{Gravity}\ (g_{\mu\nu}) + \textbf{Scalar Field}\ (\phi)}$$

$$S[g_{\mu\nu}, \phi] = \int \mathrm{d}^4x \sqrt{-g} \left( \frac{m_p^2}{2} R - \frac{1}{2} \partial_\mu \phi \, \partial_\nu \phi \, g^{\mu\nu} - V(\phi) + ... \right)$$

Particularly two (gauge invariant) light fields are guaranteed to exist –

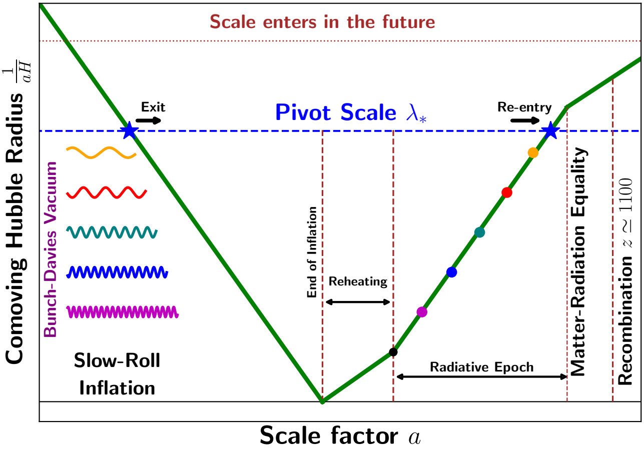
1. **Comoving Curvature Perturbation**

$$-\zeta(t, \vec{x}) = \frac{1}{\sqrt{2\epsilon_H}} \frac{\delta\phi}{m_p} + \Psi$$

   (Later becomes **density and temperature fluctuations**)

2. **Tensor Perturbation** (Transverse, traceless $h_{ij}(t, \vec{x})$ – **relic Gravitational Waves**)

# Behaviour of fluctuations




**Input $\longrightarrow$ Comp-I $\longrightarrow$ Comp-II $\longrightarrow$ Output**

# Inflationary SR Power-spectrum

During slow-roll $\epsilon_H, \eta_H \ll 1$ Primordial Power-spectra at least at large scales –

$$\mathcal{P}_\zeta = A_s \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$$\mathcal{P}_\mathcal{T} = A_t \left(\frac{k}{k_*}\right)^{n_T}$$

CMB pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$

$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_H^*}$$

$$A_\mathcal{T} = \frac{2}{\pi^2}\left(\frac{H_*}{m_p}\right)^2$$

Scalar and Tensor Spectral Index

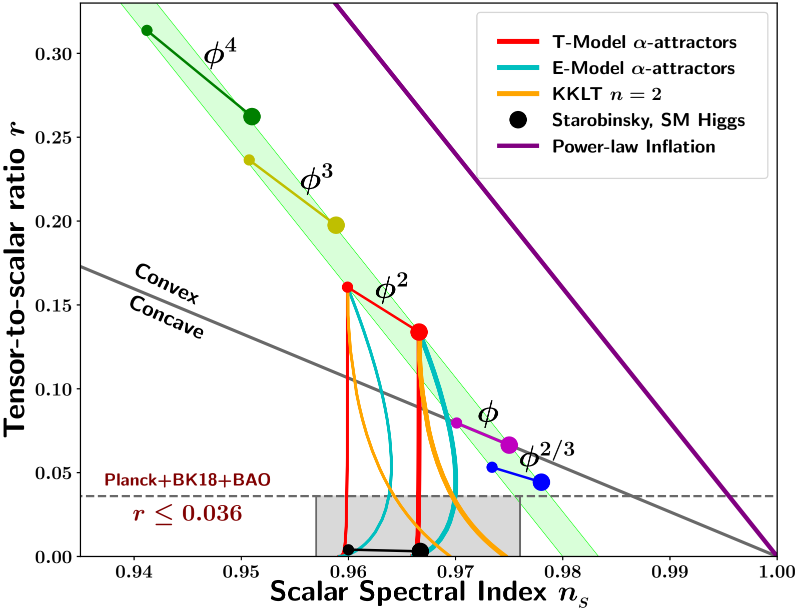
$$\boxed{n_S = 1 + 2\eta_H(k_*) - 4\epsilon_H(k_*)}$$

$$\boxed{n_T = -2\epsilon_H}$$

Tensor to Scalar Ratio

$$\boxed{r = 16\epsilon_H(k_*)}$$

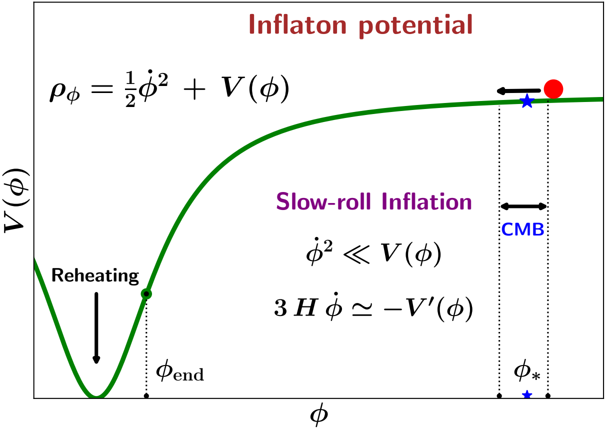
# Implications of CMB Observations



T-Model α-attractors
E-Model α-attractors
KKLT n = 2
Starobinsky, SM Higgs
Power-law Inflation
$\phi^4$
$\phi^3$
$\phi^2$
$\phi$
$\phi^{2/3}$
Convex
Concave
Planck+BK18+BAO
$r \leq 0.036$
Tensor-to-scalar ratio $r$
Scalar Spectral Index $n_s$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.94
0.95
0.96
0.97
0.98
0.99
1.00

# Shallow/Asymptotically flat potentials

**Whole family of Monomial potential $V(\phi) \sim \phi^p$ is disfavoured**



# What we know from Observations

$$\boxed{r \leq 0.036\ , \quad n_{_S} \in [0.957, 0.976]}$$

$$H_{\rm inf} \leq 1.93 \times 10^{-5}\ m_p = 4.6 \times 10^{13}\ {\rm GeV}$$

$$\boxed{\Rightarrow R \simeq 12\ \left(\frac{H_{\rm inf}}{m_p}\right)^2 \leq 4.7 \times 10^{-9}}$$

$$\epsilon_V = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2\ ,\ \eta_V = m_p^2\frac{V''}{V}$$

$$\boxed{\epsilon_V \simeq \epsilon_{_H} \leq 0.00225} \quad \text{and} \quad \boxed{w_\phi \leq -0.9985}$$

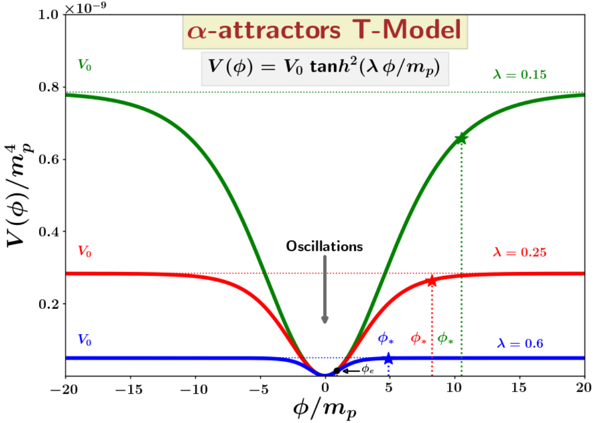
$$\boxed{|\eta_V| \simeq 0.02}$$

$$\boxed{\frac{\Delta\phi}{m_p} \leq 5 \times \left(\frac{N_*}{60}\right)}$$

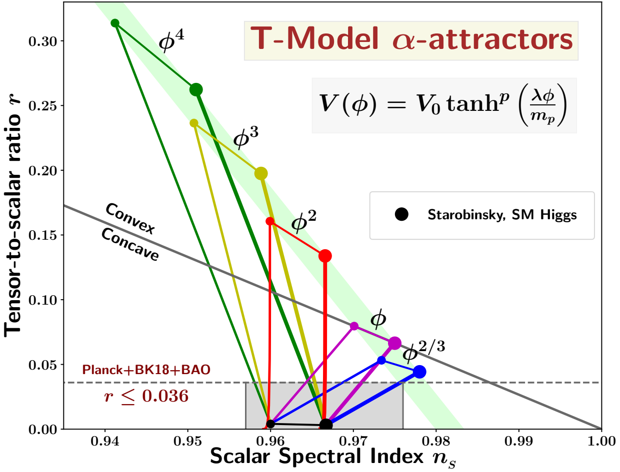
# T-model $\alpha$-attractors

Potential

$$V(\phi) = V_0 \tanh^p\left(\lambda \frac{\phi}{m_p}\right)$$



# Predictions of T-model $n_s = 1 - \frac{2}{N_*}\,, \quad r = \frac{2}{\lambda^2 N_*^2}$



# Non-canonical Inflation

# Non-canonical Lagrangian

$$\boxed{\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)}, \qquad X = \frac{1}{2}\dot{\phi}^2$$

Density $$\boxed{\rho_{\phi} = (2\alpha-1)\,X\left(\frac{X}{M^4}\right)^{\alpha-1} + V(\phi)}$$

**Pressure** $$\boxed{p_{\phi} = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)}$$

**EOM** $$\boxed{\ddot{\phi} + \frac{3\,H\dot{\phi}}{2\alpha-1} + \left(\frac{V'(\phi)}{\alpha(2\alpha-1)}\right)\left(\frac{2\,M^4}{\dot{\phi}^2}\right)^{\alpha-1} = 0}$$

# Non-canonical Monomial Potential $V(\phi) \sim \phi^p$

Spectral index $\quad n_S = 1 - 2\left(\frac{\gamma + p}{2\,\gamma\,N_* + p}\right)$

**Tensor-to-scalar ratio** $\quad r = \frac{1}{\sqrt{2\alpha - 1}}\left(\frac{16\,p}{2\,\gamma\,N_* + p}\right)$

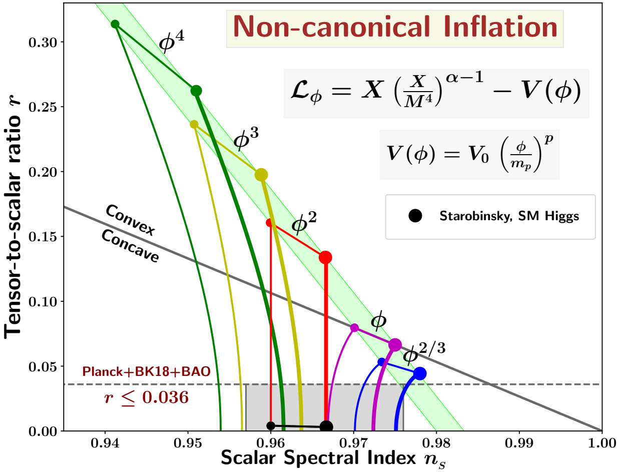
With $\quad \gamma = \frac{2\,\alpha + p(\alpha - 1)}{2\,\alpha - 1}$

Consistency relation $\quad \boxed{r = -\frac{8}{\sqrt{2\alpha - 1}}\,n_T}$

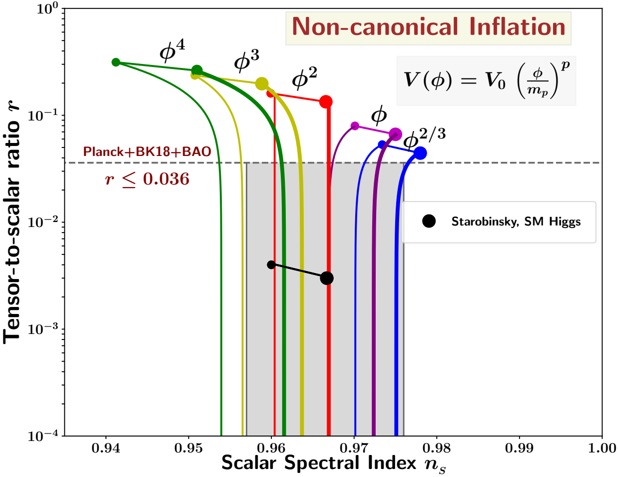
For $\alpha \gg 1$, we obtain asymptotic solution

$$\boxed{n_S = 1 - \frac{3\,p + 2}{(p + 2)\,N_* + p}}\,, \qquad \boxed{r = \frac{1}{\sqrt{2\alpha - 1}}\left(\frac{16\,p}{(p + 2)\,N_* + p}\right)}$$

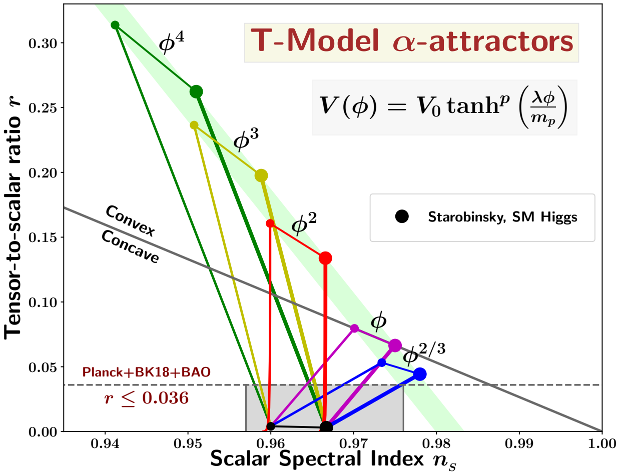
# Predictions of Non-canonical Monomial potential



# Predictions of Non-canonical Monomial potential



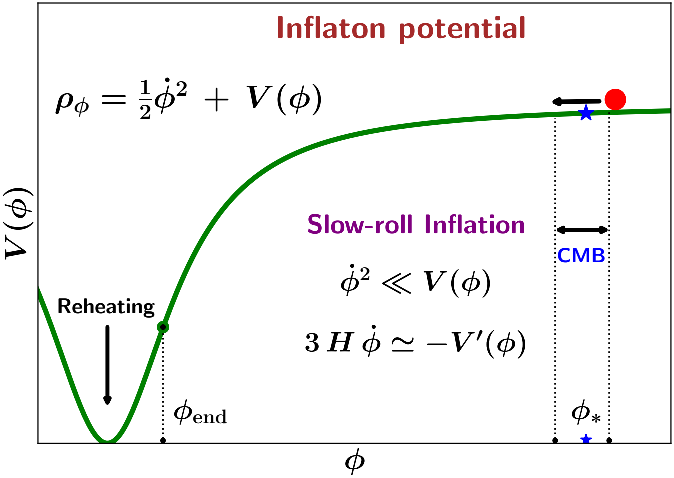
# Contrasting with T-model predictions



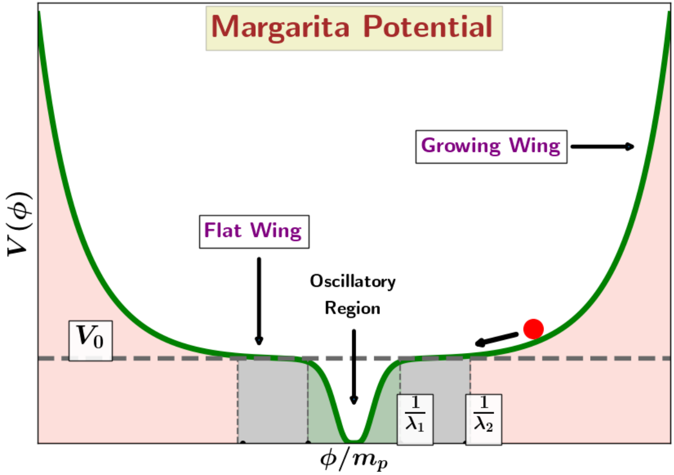
# Plateau Potentials:

# Issue of Initial conditions

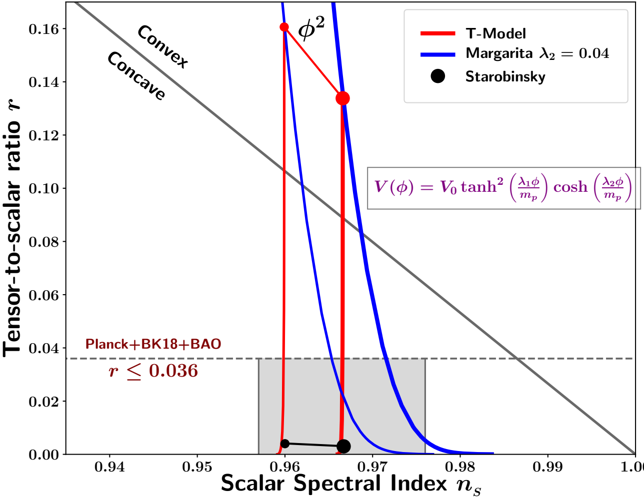
# Shallow/Asymptotically flat potentials



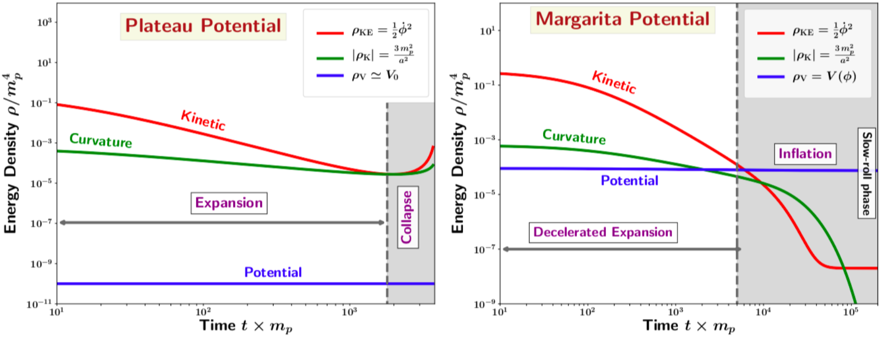
# Margarita Potential



# CMB predictions of Margarita Potential



# Addressing the problem of initial conditions




(a) (b)