Matter bounce scenario in nonmetricity f(Q) gravity

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Outline of Presentation

- Introduction and Problem statement
- $f(Q)$ Gravity Field Equations
- $f(Q)$ Gravity in Matter Bounce Scenario
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- Scalar Perturbation
- Conclusion

Introduction

- Observational evidences suggest that the Universe had undergone an exponential expansion phase in the early Universe, known as the inflation phase [1].

- During the inflationary phase, the Universe grew exponentially, expanded rapidly and in a short span of time attained an immense size.

- Geometrically, the expansion rate along the spatial directions can be obtained through the scale factor $a(t)$ and the evolution of Hubble parameter is based on the scale factor as, $H = \dot{a}(t)/a(t)$. So, there are two possibilities:

  - the scale factor attains a value zero, that leads to the big bang singularity or the space time curvature singularity.

  - the bouncing behaviour i.e. without attain the singularity, the evolution would increase again, which is an early Universe era. Since the scale factor never zero, the space time singularity would never occur. The bounce happens when H vanishes and $\dot{H} > 0$.

Introduction

- Bouncing cosmology can be derived as a cosmological solution of loop quantum cosmology (LQC)[2].
- The matter bounce scenario generates an almost scale-invariant primordial power spectrum and leads to a matter-dominated epoch during the late phase of expansion[3].
- In this scenario, the Universe formed from an epoch in the contracting era with enormous negative time where primordial space time perturbations are generated far inside the comoving Hubble radius.
- The comoving Hubble radius, $r_h = 1/(aH)$ rises monotonically over time and eventually diverges to infinity in the far future. This has be resulted in the deceleration stage at the late expansion phase.
- The comoving Hubble radius in most of the bouncing models based on the modified theories of gravity grows with the cosmic time.

Introduction

- Realizing a bouncing cosmological model is not straightforward because the null energy condition contained in the phenomenological models, which needs to be negative when the Hubble rate to grow and the bounce to happen[4].

- An exact matter bounce scenario with a single scalar field leads to an essentially scale-invariant power spectrum[5]. The matter bounce scenario is suffering from two important flaws:

  - BKL (Belinski–Khalatnikov–Lifshitz) instability, i.e, the space time anisotropic energy density increases faster than that of the bouncing agent during the contracting phase. As a result the background evolution became unstable

  - In the perturbation evolution, large tensor to scalar ratio implying the scalar and tensor perturbations have similar amplitudes.

Problem Statement

- The extended symmetric teleparallel gravity, namely $f(Q)$ gravity is another geometrical modified theories of gravity that has been recently formulated using the non-metricity approach[6].
- The matter bounce scenario motivated with the loop quantum cosmology in $f(Q)$ gravity would be investigated.

The Field Equations

The action of $f(Q)$ gravity,

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} f(Q) + \mathcal{L}_M \right), \tag{1}$$

The field equations of $f(Q)$ gravity,

$$\frac{2}{\sqrt{-g}} \nabla_\alpha \left(\sqrt{-g} f_Q P^\alpha{}_{\mu\nu} \right) + \frac{1}{2} g_{\mu\nu} f + f_Q \left(P_{\mu\alpha\beta} Q_\nu{}^{\alpha\beta} - 2 Q_{\alpha\beta\mu} P^{\alpha\beta}{}_\nu \right) = -T_{\mu\nu} \tag{2}$$

The energy momentum tensor,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g} \mathcal{L}_m}{\delta g^{\mu\nu}} \tag{3}$$

The homogeneous and isotropic FLRW space time,

$$ds^2 = -N^2(t) dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{4}$$

The field equations,

$$6 f_Q H^2 - \frac{1}{2} f = \rho \tag{5}$$

$$\left(12 H^2 f_{QQ} + f_Q \right) \dot{H} = -\frac{1}{2} (\rho + p) \tag{6}$$

$f(Q)$ Gravity in Matter Bounce Scenario

In the geometrical modified theories of gravity the bouncing models are reconstructed based on gravitational theory in this case, the nonmetricity based gravitational theory. The focus would be mainly to reconstruct a model for which the value of Hubble squared parameter would be,

$$H^2 = \frac{\rho_m(\rho_c - \rho_m)}{3\rho_c} \tag{7}$$

This is to mention here that the same equation can be realised from the holonomy corrected Friedmann equations in the context of LQC for a matter-dominated Universe[7]. The matter energy density and critical energy density are represented respectively as ρ_m and ρ_c. Also, the critical energy density,

$$\rho_c = (c^2\sqrt{3})/(32\pi^2\gamma^3 G_N l_p^2), \tag{8}$$

where, $\gamma = 0.2375$ and $l_p = \sqrt{\hbar G_N/c^3}$ are respectively the Barbero-Immirzi parameter and the Planck length. We use the Planck units, $c = \hbar = G_N = 1$.

$f(Q)$ Gravity in Matter Bounce Scenario

From eqn.(7), it can be inferred that when the matter energy density reaches to its critical value, $H^2 = 0$, which shows the occurrence of a bounce. Now, in the matter bounce scenario with zero pressure, the continuity equation and the energy density can be written as,

$$\dot{\rho}_m = -3H\rho_m \qquad \text{and} \qquad \rho_m = \rho_{m0}a^{-3} \tag{9}$$

Motivated from the LQC, the bounce cosmology has been appealing in the sense that it can produce as a cosmological solution to the LQC theory. Now, the scale factor, $a(t) \propto t^{2/3}$ for the matter dominated case.

$$\rho_m = \frac{\rho_c}{\left(\frac{3}{4}\rho_c t^2 + 1\right)}, \qquad H(t) = \frac{2\rho_c t}{3\rho_c t^2 + 4}, \qquad a(t) = \left(\frac{3}{4}\rho_c t^2 + 1\right)^{\frac{1}{3}} \tag{10}$$

$f(Q)$ Gravity in Matter Bounce Scenario

$$H^2 = \frac{\rho_c}{3}\left(\frac{1}{a^3} - \frac{1}{a^6}\right) \tag{11}$$

Using the relation between the e-folding parameter and the scale factor, $e^{-N} = \frac{a_0}{a}$, eqn. (11) becomes,

$$H^2 = \frac{\rho_c}{3a_0^3}\left(e^{-3N} - \frac{e^{-6N}}{a_0^3}\right) \tag{12}$$

We assume following quantities,

$$A = \frac{\rho_c}{3a_0^3}, \qquad b = \frac{1}{a_0^3}. \tag{13}$$

From eqn. (12), the nonmetricity scalar in the form of e-folding parameter as,

$$Q = 6A\left[e^{-3N} - be^{-6N}\right] \tag{14}$$

On solving,

$$N = -\frac{1}{3}Log\left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{6Ab}\right) \tag{15}$$

$f(Q)$ Gravity in Matter Bounce Scenario

In addition, we assume that the matter energy density (5) is of form,

$$\rho_m = \sum_i \rho_{i0} a_0^{-3(1+\omega_i)} e^{-3N(1+\omega_i)} \tag{16}$$

By setting $S_i = \rho_{i0} a_0^{-3(1+\omega_i)}$, the matter energy density becomes

$$\rho_m = \sum_i S_i \left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{6Ab} \right)^{(1+\omega_i)} \tag{17}$$

Substituting eqn. (17) in eqn. (5), we get

$$Qf_Q - \frac{1}{2} f - \sum_i S_i \left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{2Ab} \right)^{(1+\omega_i)} = 0 \tag{18}$$

We consider the Universe is filled with dust fluid only, from eqn. (9), the value of matter-energy density at $t = 0$ as,

$$Qf_Q - \frac{1}{2} f - \left(\frac{\rho_c + \sqrt{\rho_c(\rho_c - 2Q)}}{2} \right) = 0 \tag{19}$$

$f(Q)$ Gravity in Matter Bounce Scenario

On solving, we get

$$f(Q) = -\sqrt{\rho_c(\rho_c - 2Q)} - \sqrt{2\rho_c Q}\,\arcsin\left(\frac{\sqrt{2}\sqrt{Q}}{\sqrt{\rho_c}}\right) - \rho_c, \tag{20}$$

- The above form of $f(Q)$ produces the matter bounce evolution of the Universe.
- The late-time acceleration of the Universe epoch is ensured by the diminishing trend of the cosmic Hubble radius as shown in FIG. 1.

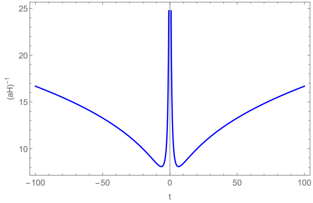
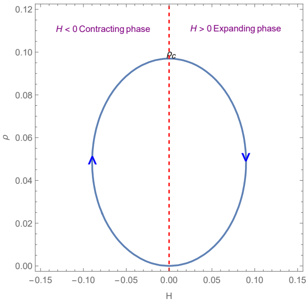


Figure: Hubble radius in cosmic time.



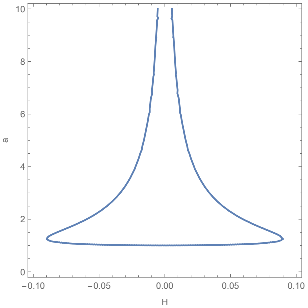


Figure: Energy density (left panel) and scale factor in Hubble parameter (right panel).

Phase Space Analysis

The general form of $f(Q)$ as $Q + \psi(Q)$ and accordingly,

$$3H^2 = \rho + \frac{\psi}{2} - Q\psi_Q \tag{21}$$

$$2\dot{H} + 3H^2 = -p - 2\dot{H}(2Q\psi_{QQ} + \psi_Q) + \left(\frac{\psi}{2} - Q\psi_Q\right) \tag{22}$$

The density parameters for the matter dominated, radiation dominated and dark energy phase are respectively denoted as, $\Omega_m = \frac{\rho_m}{3H^2}$, $\Omega_r = \frac{\rho_r}{3H^2}$ and $\Omega_{de} = \frac{\rho_{de}}{3H^2}$ with $\Omega_m + \Omega_r + \Omega_{de} = 1$. Hence the effective equation of state parameter and the equation of state due to dark energy take the form.

$$\omega_{eff} = -1 + \frac{\Omega_m + \frac{4}{3}\Omega_r}{2Q\psi_{QQ} + \psi_Q + 1} \tag{23}$$

$$\omega_{de} = -1 + \frac{4\dot{H}(2Q\psi_{QQ} + \psi_Q)}{\psi - 2Q\psi_Q} \tag{24}$$

Phase Space Analysis

We consider the dimensionless variables,

$$x = \frac{\psi - 2Q\psi_Q}{6H^2} \qquad\qquad y = \frac{\rho_r}{3H^2}. \tag{25}$$

The autonomous dynamical system,

$$x' = -2\frac{\dot{H}}{H^2}(\psi_Q + 2Q\psi_{QQ} + x) \tag{26}$$

$$y' = -2y\left(2 + \frac{\dot{H}}{H^2}\right), \tag{27}$$

With an algebraic manipulation, we can obtain the relation,

$$\frac{\dot{H}}{H^2} = -\frac{1}{2}\left(\frac{3 - 3x + y}{2Q\psi_{QQ} + \psi_Q + 1}\right) \tag{28}$$

Phase Space Analysis

If we compare the $f(Q)$, then $\psi(Q)$ can be represented as,

$$\psi(Q) = -\sqrt{\rho_c(\rho_c - 2Q)} - \sqrt{2\rho_c Q}\arcsin\left(\frac{\sqrt{2}\sqrt{Q}}{\sqrt{\rho_c}}\right) - \rho_c - Q \tag{29}$$

and

$$2Q\psi_{QQ} + \psi_Q = -\frac{\rho_c}{\sqrt{\rho_c(\rho_c - 2Q)}} - 1 \tag{30}$$

Now the dimensionless variables can be represented as,

$$x' = x\,(3(x-1) - y) \tag{31}$$

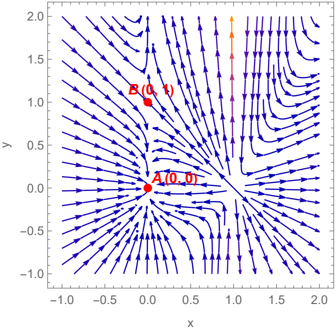
$$y' = -\frac{y(x(-3x + y + 4) + y - 1)}{x - 1} \tag{32}$$

The effective EoS and deceleration parameter in terms of the dynamical variables,

$$\omega_{eff} = -1 + \frac{(x+1)(3x - y - 3)}{3(x-1)} \tag{33}$$

$$q = -1 + \frac{(x+1)(3x - y - 3)}{2(x-1)} \tag{34}$$

Phase Space Analysis



(x,y)	Ω_m	Ω_r	Ω_{de}	ω_{eff}	(q)	Eigenvalues	Stability
$A(0,0)$	1	0	0	0	$1/2$	{-3,-1}	Stable Node
$B(0,1)$	0	1	0	$1/3$	1	{-4,1}	Unstable Node

Scalar Perturbation

The first order perturbation in the FLRW background with the perturbation geometry functions $\delta(t)$ and matter functions $\delta_m(t)$ can be expressed as,

$$H(t) \rightarrow H_b(t)(1+\delta(t)), \qquad \rho(t) \rightarrow \rho_b(t)(1+\delta_m(t)) \tag{35}$$

The perturbation of the function $f(Q)$ and f_Q can be calculated as,

$$\delta f = f_Q \delta Q, \qquad \delta f_Q = f_{QQ}\delta Q, \tag{36}$$

Neglecting higher power of $\delta(t)$, the Hubble parameter becomes,

$$6H^2 = 6H_b^2(1+\delta(t))^2 = 6H_b^2(1+2\delta(t)) \tag{37}$$

and subsequently

$$Q(2Qf_{QQ}+f_Q)\delta = \rho\delta_m, \tag{38}$$

which gives the relation between the matter and geometric perturbation and the perturbed Hubble parameter can be realised from eqn.(37). Now, to obtain the analytical solution to the perturbation function, we consider the perturbation continuity equation as,

$$\dot{\delta}_m + 3H(1+\omega)\delta = 0 \tag{39}$$

Scalar Perturbations

and from eqns. (38)-(39), the first order differential equation can be obtained,

$$\dot{\delta}_m + \frac{3H(1+\omega)\rho}{Q(2Qf_{QQ}+f_Q)}\delta_m = 0 \tag{40}$$

Further using the tt-component field equation and eqn. (**??**), the simplified relation can be obtained,

$$\dot{\delta}_m - \frac{\dot{H}}{H}\delta_m = 0, \tag{41}$$

which provides $\delta_m = C_1 H$, where C_1 is the integration constant. Subsequently from eqn. (39), we obtain

$$\delta = C_2 \frac{\dot{H}}{H} \tag{42}$$

where, $C_2 = -\frac{C_1}{3(1+\omega)}$. The evolution behaviour of δ and δ_m are given in FIG. 3.

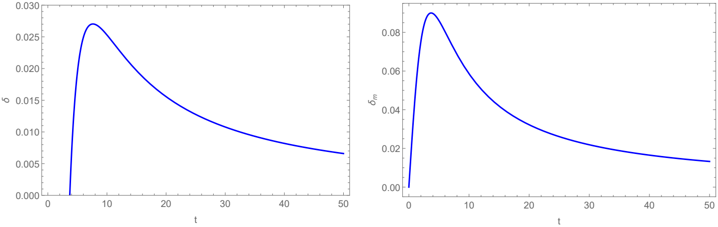


Figure: Evolution of Hubble parameter and the energy density in cosmic time.

At the beginning both the deviations, $\delta(t)$ and $\delta_m(t)$, have some increment before declining through time and approaching zero at late times. As a result, we can say that though at the beginning the model shows unstable behaviour for a brief period, but in most of the time it shows stable behaviour under the scalar perturbation approach.

Conclusion

- The matter bounce scenario of the Universe has been reconstructed in an extended symmetric teleparallel gravity; a specific form of $f(Q)$ has been obtained that shows the matter bounce scenario.
- As expected, the model fails to explain the dark energy era, which has been observed from the dynamical stability analysis.
- From the critical points, the eigenvalues and the corresponding cosmology are obtained. Two critical points are obtained, one provides stable node and the other one unstable. The positive deceleration parameters show the decelerating Universe, occurred at early Universe.
- To check the stability of the reconstructed model, the deviation of the Hubble parameter and the energy density in cosmic time, it has been observed that both the deviations (i.e., $\delta(t)$ and $\delta_m(t)$) approaching zero at late times.
- Further study can be carried out on the reconstructed form of $f(Q)$, which may give some more results on the bouncing scenario.

Paper in arXiv

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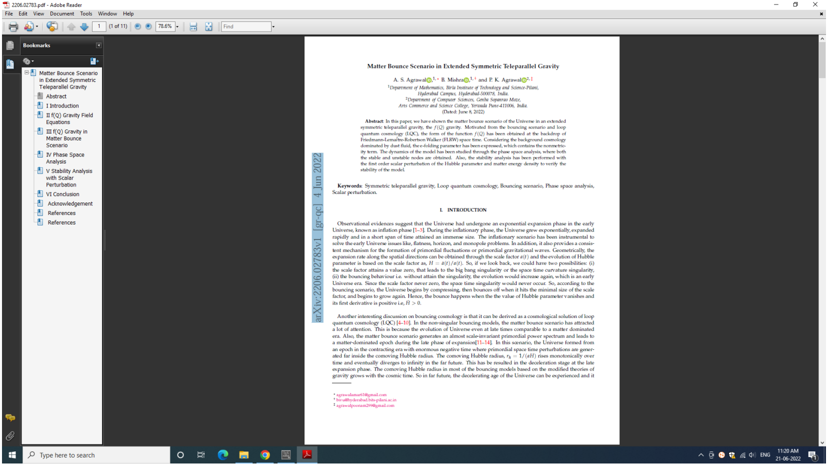

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Matter Bounce Scenario in Extended Symmetric Teleparallel Gravity

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Abstract: In this paper, we have shown the matter bounce scenario of the Universe in an extended symmetric teleparallel gravity, the $f(Q)$ gravity. Motivated from the bouncing scenario and loop quantum cosmology (LQC), the form of the function $f(Q)$ has been obtained at the backdrop of Friedmann-Lemaître-Robertson Walker (FLRW) space time. Considering the background cosmology dominated by dust fluid, the e-folding parameter has been expressed, which contains the nonmetricity term. The dynamics of the model has been studied through the phase space analysis, where both the stable and unstable nodes are obtained. Also, the stability analysis has been performed with the first order scalar perturbation of the Hubble parameter and matter energy density to verify the stability of the model.

Keywords: Symmetric teleparallel gravity, Loop quantum cosmology, Bouncing scenario, Phase space analysis, Scalar perturbation.

I. INTRODUCTION

Observational evidences suggest that the Universe had undergone an exponential expansion phase in the early Universe, known as inflation phase [1–3]. During the inflationary phase, the Universe grew exponentially, expanded rapidly and in a short span of time attained an immense size. The inflationary scenario has been instrumental to solve the early Universe issues like, flatness, horizon, and monopole problems. In addition, it also provides a consistent mechanism for the formation of primordial fluctuations or primordial gravitational waves. Geometrically, the expansion rate along the spatial directions can be obtained through the scale factor $a(t)$ and the evolution of Hubble parameter is based on the scale factor as, $H = \dot{a}(t)/a(t)$. So, if we look back, we could have two possibilities: (i) the scale factor attains a value zero, that leads to the big bang singularity or the space time curvature singularity, (ii) the bouncing behaviour i.e. without attain the singularity, the evolution would increase again, which is an early Universe era. Since the scale factor never zero, the space time singularity would never occur. So, according to the bouncing scenario, the Universe begins by compressing, then bounces off when it hits the minimal size of the scale factor, and begins to grow again. Hence, the bounce happens when the the value of Hubble parameter vanishes and its first derivative is positive i.e, $\dot{H} > 0$.

Another interesting discussion on bouncing cosmology is that it can be derived as a cosmological solution of loop quantum cosmology (LQC) [4–10]. In the non-singular bouncing models, the matter bounce scenario has attracted a lot of attention. This is because the evolution of Universe even at late times comparable to a matter dominated era. Also, the matter bounce scenario generates an almost scale-invariant primordial power spectrum and leads to a matter-dominated epoch during the late phase of expansion[11–14]. In this scenario, the Universe formed from an epoch in the contracting era with enormous negative time where primordial space time perturbations are generated far inside the comoving Hubble radius. The comoving Hubble radius, $r_h = 1/(aH)$ rises monotonically over time and eventually diverges to infinity in the far future. This has be resulted in the deceleration stage at the late expansion phase. The comoving Hubble radius in most of the bouncing models based on the modified theories of gravity grows with the cosmic time. So in far future, the decelerating age of the Universe can be experienced and it

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Thank You
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Attention