Cosmological Bootstrap

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Outline

- Unitarity: Cosmological Optical Theorem
- Locality: Manifest Locality Test

What is unitarity?

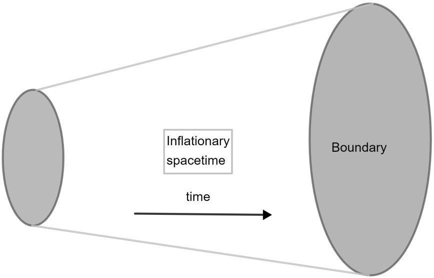
- Evolution of states described by a time evolution operator:

$$|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle.$$

- Unitarity means:

$$U^\dagger U = 1.$$

- Norm of a state is conserved.
- But we don't live inside the inflationary spacetime!



How to see unitarity?

- Suppose we specify the Hamiltonian of a field theory
- Time evolution operator:

$$U = T\exp(-i\int d\eta H_{int}(\eta))$$
$$U^\dagger U = 1$$

- Perturbation theory:

$$U = 1 + \delta U$$
$$\delta U + \delta U^\dagger = -\delta U \delta U^\dagger .$$

- Non-linear relation, relates different orders in perturbation theory.

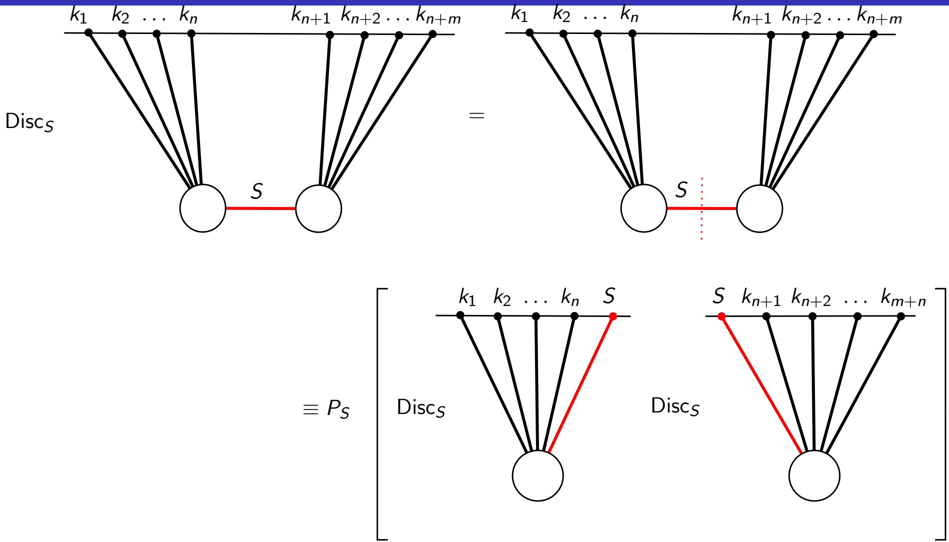
Discontinuity

- Define discontinuity:

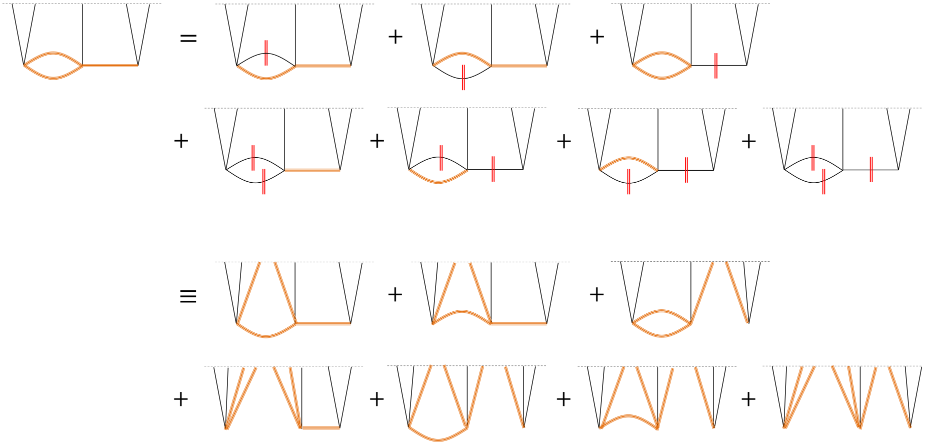
$$\underset{\boldsymbol{k}_1,\ldots\boldsymbol{k}_j}{\text{Disc}} f(k_1,\ldots k_n;\{\boldsymbol{k}\}) = f(k_1,\ldots k_j, k_{j+1}\ldots k_n;\{\boldsymbol{k}\}) \\ - f^*(k_1,\ldots k_j, -k^*_{j+1}\cdots - k^*_n; -\{\boldsymbol{k}\}).$$

- $\underset{S}{\text{Disc}}$ of a wavefunction coefficient is expressed as the product of $\underset{S}{\text{Disc}}$ of simpler wavefunctions.
- Works for fields with any mass/integer spin and any flat FLRW spacetime!

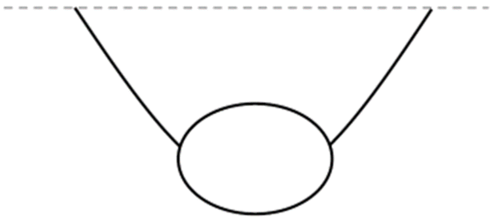
Cutting rules: single cut



Cutting rules: general cutting rule



Cutting rules: loop



- Can calculate the disc of this graph in EFToI to be:

$$i\text{Disc}\left[i\psi_{k_1 k_2}^{\text{1-loop}}\right] = \frac{H^2}{f_\pi^4}\frac{ik^3}{480\pi}\frac{(1-c_s^2)^2}{c_s^4}\left[(4\tilde{c}_3+9+6c_s^2)^2+15^2\right]$$

- The exact form of the constant is first found using cutting rule!

Manifest Locality

- Interaction must be built from fields and their derivatives

Example

The following is manifestly local:

$$\phi(\partial\phi)^2$$

while the following is not manifestly local:

$$\dot{\zeta}^2\nabla^{-2}\dot{\zeta}$$

- More restrictive than bulk locality.

Manifest locality

- For massless scalar and graviton, we have a manifest locality test:

$$\frac{\partial}{\partial k_c}\psi_n\Big|_{k_c=0} = 0$$

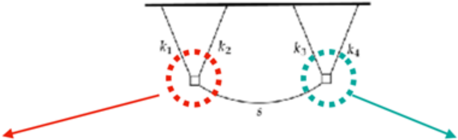
- Completely fixes the three point function of a massless scalar (with Bose symmetry and scale invariance):

$$\begin{aligned}\psi_3 &= \sum_p C_p \frac{\text{Poly}_{3+p}(k_T, e_2, e_3)}{k_T^p} \\ k_T &= k_1 + k_2 + k_3 \\ e_2 &= k_1 k_2 + k_1 k_3 + k_2 k_3 \\ e_3 &= k_1 k_2 k_3\end{aligned}$$

- Can be extended to graviton

Combining unitarity and locality

- Example: Constructing ψ_4 from two ψ_3
- Locality fixes ψ_3.
- Unitarity tells us how to glue them together to make a ψ_4.



$$E_L = k_1 + k_2 + s, \qquad s = |\mathbf{k}_1 + \mathbf{k}_2| \qquad E_R \equiv k_3 + k_4 + s,$$

- Recursively construct tree level correlators!

Unitarity+Locality