The universality of small-scale structures in dark matter

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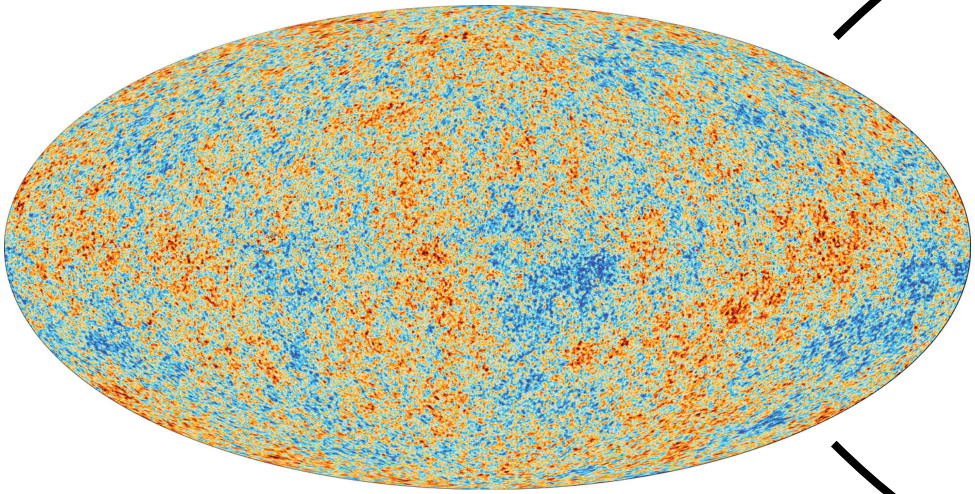
ITP

STRUCTURES CLUSTER OF EXCELLENCE

How do cosmic structures grow?



ESA and the Planck Collaboration

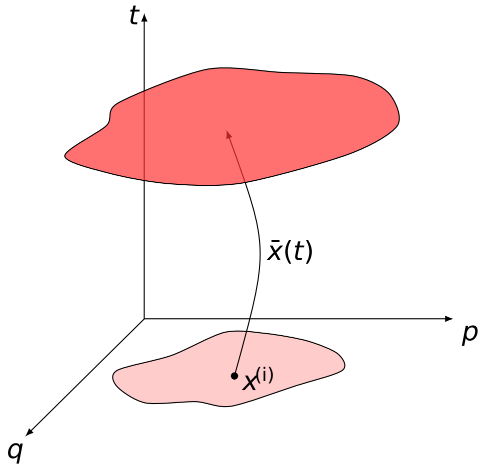
2MASS redshift survey

Particles in classical phase space: Kinetic Field Theory (KFT) picture

- Full 6N dimensional phase space: $\mathbf{x}(t) = (\mathbf{q}(t), \mathbf{p}(t))$
- Initial conditions: $P(\mathbf{q}^{(\mathrm{i})}, \mathbf{p}^{(\mathrm{i})})$
- Particle interactions
- expanding space-time

} Hamiltonian $\mathcal{H}$

$\Rightarrow$ equations of motion



$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} + m\varphi(\mathbf{q})$$

m time-dependent due to expanding background

$\bar{\mathbf{x}}(t)$: solution of Hamilton's equations in expanding background.

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$$

$$\bar{\mathbf{x}}(t) = \bar{\mathbf{x}}_0(t) + \mathbf{x}_I(t)$$

"free motion", known solution

contribution from interactions

Zel'dovich ansatz

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} + m\varphi(\mathbf{q}) \longrightarrow \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \longrightarrow \bar{\mathbf{x}}(t) = \bar{\mathbf{x}}_0(t) + \mathbf{y}(t)$$

Choose the linear growth factor D_+ as time coordinate

Choose $\mathcal{H}_0$ such that

$$\left.\begin{aligned} \bar{\mathbf{q}}_0(t) &= \mathbf{q}^{(\mathrm{i})} + t\mathbf{p}^{(\mathrm{i})} \\ \bar{\mathbf{p}}_0(t) &= \mathbf{p}^{(\mathrm{i})} \\ \bar{\mathbf{q}}(t) &= \bar{\mathbf{q}}_0(t) - \int_0^t \mathrm{d}t' g_H(t,t') m(t') \nabla_q \phi \end{aligned}\right\}$$

This ansatz is exact.

No approximation is made here.

$$\mathcal{H}_I = \mathcal{H} - \mathcal{H}_0 \quad \Rightarrow \quad \nabla^2\phi = A(t)\left(\delta - \delta^{(\mathrm{lin})}\right)$$

Zel'dovich takes gravitational interactions from large scale structures into account!

Zel'dovich power spectrum

$$\mathcal{P}(k,t) = \mathrm{e}^{-\frac{\sigma_1^2}{3}k^2t^2}\int \mathrm{d}^3q\ \left(\mathrm{e}^{t^2\vec{k}^\intercal \hat{C}_{pp}(\vec{q})\vec{k}} - 1\right)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

$$\hat{C}_{pp}(\vec{q}) =: -\frac{\vec{q}\otimes\vec{q}}{q^2}a_2(q) - \mathbb{I}_3 a_1(q)$$

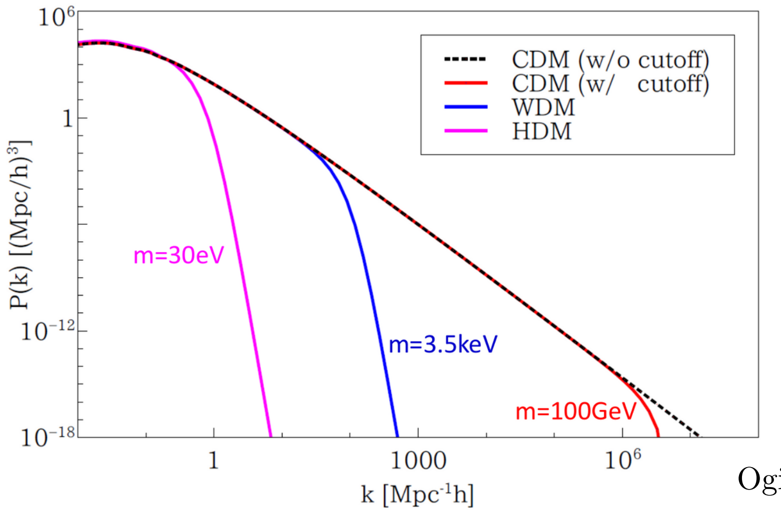
$$a_1(q) := -\frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\ P_\delta^{(\mathrm{i})}(k)\frac{j_1(kq)}{kq}$$

$$a_2(q) := \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\ P_\delta^{(\mathrm{i})}(k)j_2(kq)$$

Introduce a small-scale smoothing:

$$P_\delta^{(\mathrm{i})}(k) = Ak^{n_s}T_{\mathrm{CDM}}^2(k)\varphi(k/k_s)$$

$$\sigma_n^2 := \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\ k^{2n-2}P_\delta^{(\mathrm{i})}(k) < \infty$$



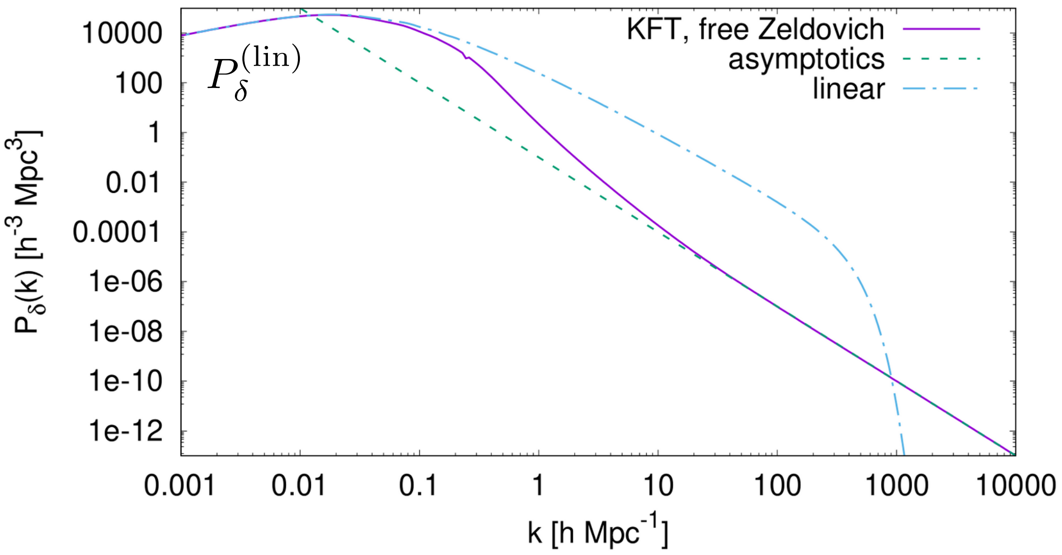
Ogiya 2017

Zel'dovich power spectrum

$$\mathcal{P}(k,t) = \mathrm{e}^{-\frac{\sigma_1^2}{3}k^2t^2} \int \mathrm{d}^3q \, \left(\mathrm{e}^{-t^2\left[\frac{(\vec{k}\cdot\vec{q})^2}{q^2}a_2(q) + k^2 a_1(q)\right]} - 1 \right) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

Asymptotics:

As $k \to 0 : \mathcal{P}(k,t) \sim P_\delta^{(\mathrm{lin})}(k,t)$



Zel'dovich power spectrum

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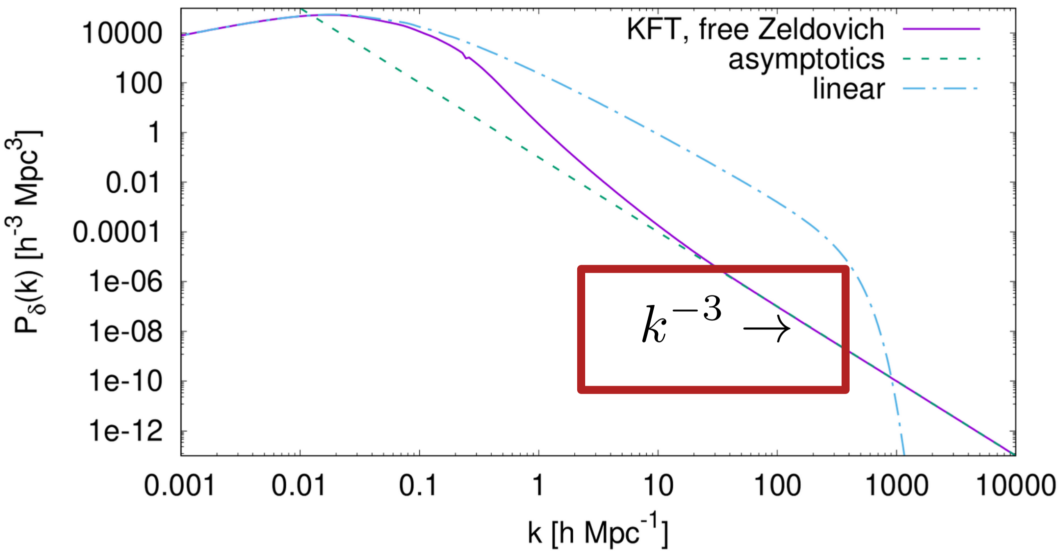
Asymptotics:

As $k \to 0 : \mathcal{P}(k,t) \sim P_\delta^{(\mathrm{lin})}(k,t)$

As $k \to \infty : \mathcal{P}(k,t) \sim \sum_{m=0}^{\infty} \frac{\mathcal{P}^{(m)}(t)}{k^{3+2m}}$



Zel'dovich power spectrum

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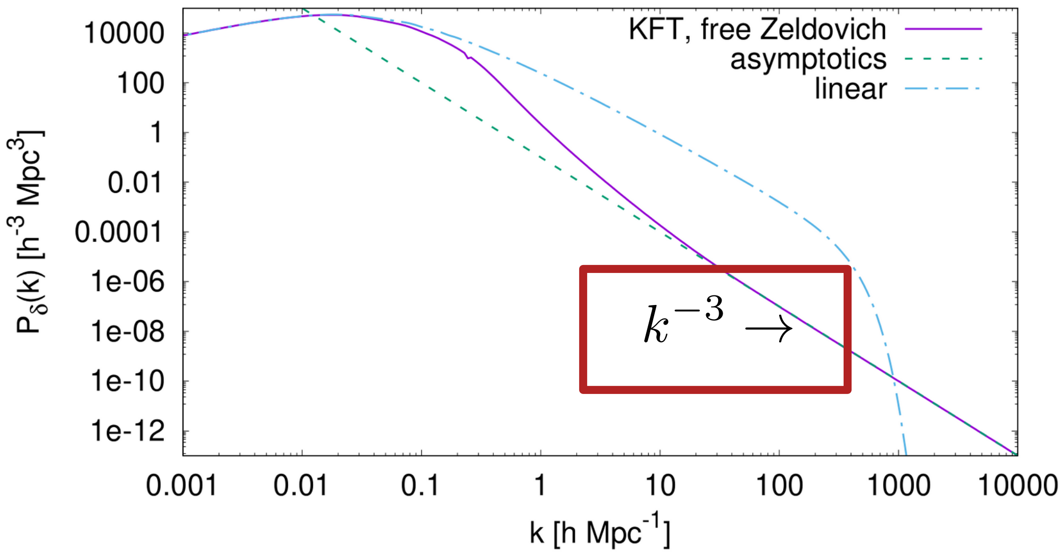
Asymptotics:

As $k \to 0 : \mathcal{P}(k,t) \sim P_\delta^{(\mathrm{lin})}(k,t)$

As $k \to \infty : \; \mathcal{P}(k,t) \sim \sum_{m=0}^{\infty} \frac{\mathcal{P}^{(m)}(t)}{k^{3+2m}}$

$$a_1(q) \sim \sum_{n=0}^{\infty} a_n \sigma_{n+1}^2 q^{2n} = -\frac{\sigma_1^2}{3} + \frac{\sigma_2^2}{30} q^2 + \mathcal{O}(k^4)$$

$$a_2(q) \sim \sum_{n=1}^{\infty} 2n \; a_n \sigma_{n+1}^2 q^{2n} = \frac{\sigma_2^2}{15} q^2 + \mathcal{O}(k^4)$$



Asymptotic amplitude $\mathcal{P}^0$

Time evolution of small structures in Zel'dovich

$$\mathcal{P}^{(0)}(t) = 3(4\pi)^{3/2}\left(\frac{5}{2\sigma_2^2 t^2}\right)^{3/2} \exp\left(-\frac{5}{2\sigma_2^2 t^2}\right)$$

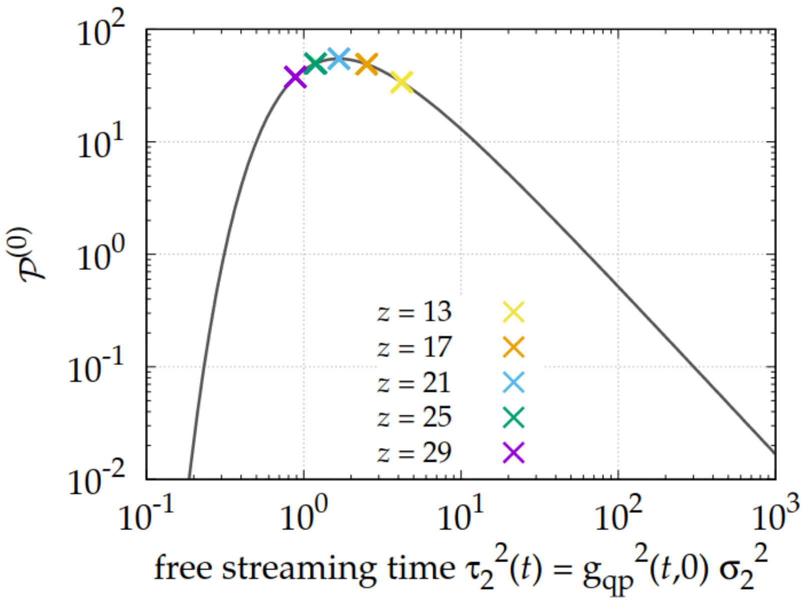
Small-scale stream crossing: maximal amplitude

$$\mathcal{P}^{(0)}(t_{\max}) = 3\left(\frac{6\pi}{\mathrm{e}}\right)^{3/2} \approx 54.78$$

universal

when $\sigma_2^2 t_{\max}^2 = \frac{5}{3}$

or $t_{\max} = \sqrt{\frac{5}{3\sigma_2^2}}$ cutoff dependent



Initial correlations - no UV-cutoff

Consider: $P_\delta^{(\mathrm{i})}(k) \sim \sum_{m=0}^{\infty} k^{-r_m} \sum_{n=0}^{N(m)} c_{mn} \log^n k, \quad \text{as } k \to +\infty$

$$\mathcal{P}(k,t) = \mathrm{e}^{-\frac{\sigma_1^2}{3}k^2t^2} \int \mathrm{d}^3q \left(\mathrm{e}^{-t^2\left[\frac{(\vec{k}\cdot\vec{q})^2}{q^2}a_2(q)+k^2a_1(q)\right]} - 1 \right) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

$$\text{with} \quad a_1(q) = -\frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\ P_\delta^{(\mathrm{i})}(k)\frac{j_1(kq)}{kq}$$
$$a_2(q) = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\ P_\delta^{(\mathrm{i})}(k)j_2(kq)$$

Taylor expanding the j_n does not work now to get the $q \to 0$ asymptotics of a_1, a_2 due to divergences of the resulting integrals.

Applying the Mellin transform technique

$\to$ Search for poles in the analyt. cont. of the Mellin transforms of $P_\delta^{(\mathrm{i})}$ and j_n and compute residua

Initial correlations - no UV-cutoff

Consider: $$P_\delta^{(\mathrm{i})}(k) \sim \sum_{m=0}^{\infty} k^{-r_m} \sum_{n=0}^{N(m)} c_{mn} \log^n k, \quad \text{as } k \to +\infty$$

Applying the Mellin transform technique leads to

$$\begin{aligned} a_1(q) \sim & -\frac{1}{2\pi^2} \sum_{m=0}^{\infty} q^{2m} \frac{(-1)^m}{(2m+3)(2m+1)!} \, \mathcal{M}[P_\delta^{(i)}; z]\Big|_{z=1+2m} \\ & -\frac{1}{2\pi^2} \sum_{m=0}^{\infty} q^{r_m-1} \sum_{n=0}^{N(m)} c_{mn} \sum_{j=0}^{n} \binom{n}{j} (-\log q)^j \, \mathcal{M}^{(n-j)}[j_1; z]\Big|_{z=-r_m} , \\ a_2(q) \sim & \frac{1}{2\pi^2} \sum_{m=1}^{\infty} q^{2m} \frac{(-1)^{m+1}\, 2m}{(2m+3)(2m+1)!} \, \mathcal{M}[P_\delta^{(i)}; z]\Big|_{z=1+2m} \\ & +\frac{1}{2\pi^2} \sum_{m=0}^{\infty} q^{r_m-1} \sum_{n=0}^{N(m)} c_{mn} \sum_{j=0}^{n} \binom{n}{j} (-\log q)^j \, \mathcal{M}^{(n-j)}[j_2; z]\Big|_{z=1-r_m} . \end{aligned}$$

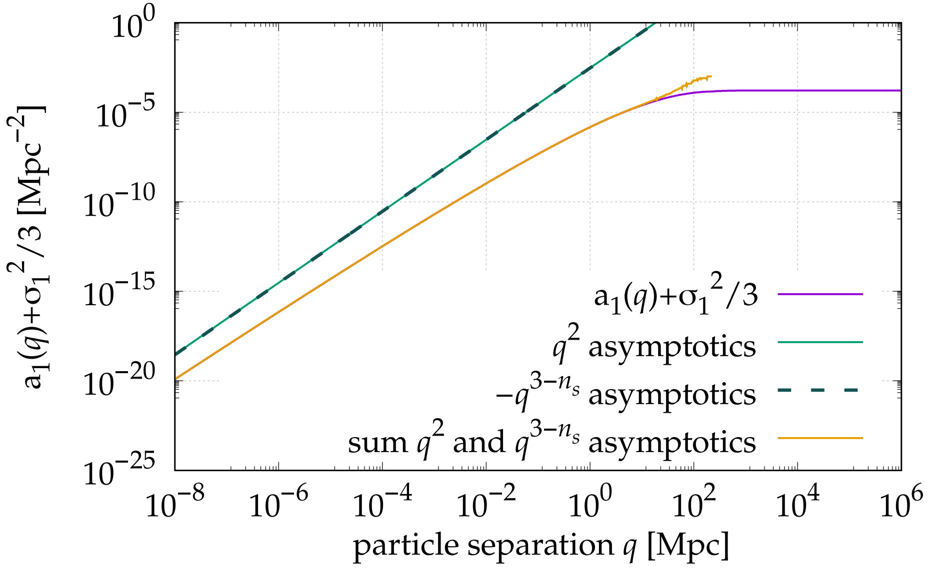
Initial correlations - no UV-cutoff

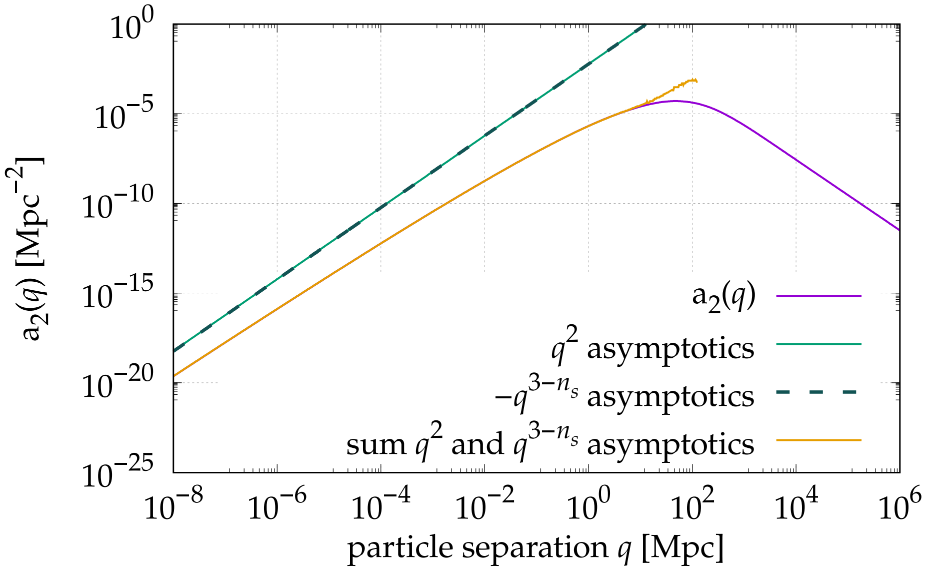
$$P_\delta^{(\mathrm{i})}(k) \sim \sum_{m=0}^{\infty} k^{n_s-4-m} \sum_{n=0}^{N(m)} c_{mn} \log^n k, \quad \text{as } k \to +\infty$$

Initial correlations - no UV-cutoff

$$a_1(q) \sim -\frac{\sigma_1^2}{3} + \frac{\sigma_2^2}{30} q^2 + q^{3-n_s} \xi_1(\log q, n_s) + \mathcal{O}(q^{4-n_s})$$

$$a_2(q) \sim \frac{\sigma_2^2}{15} q^2 + q^{3-n_s} \xi_2(\log q, n_s) + \mathcal{O}(q^{4-n_s})$$





Zel'dovich power spectrum - no UV-cutoff

$$\mathcal{P}(k,t) = \mathrm{e}^{-\frac{\sigma_1^2}{3}k^2t^2} \int \mathrm{d}^3q \, \left(\mathrm{e}^{-t^2\left[\frac{(\vec{k}\cdot\vec{q})^2}{q^2}a_2(q)+k^2a_1(q)\right]} - 1 \right) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

$$\text{with} \quad a_1(q) \sim -\frac{\sigma_1^2}{3} + \frac{\sigma_2^2}{30}q^2 + q^{3-n_s}\xi_1(\log q, n_s) + \mathcal{O}(q^{4-n_s})$$

$$a_2(q) \sim \frac{\sigma_2^2}{15}q^2 + q^{3-n_s}\xi_2(\log q, n_s) + \mathcal{O}(q^{4-n_s})$$

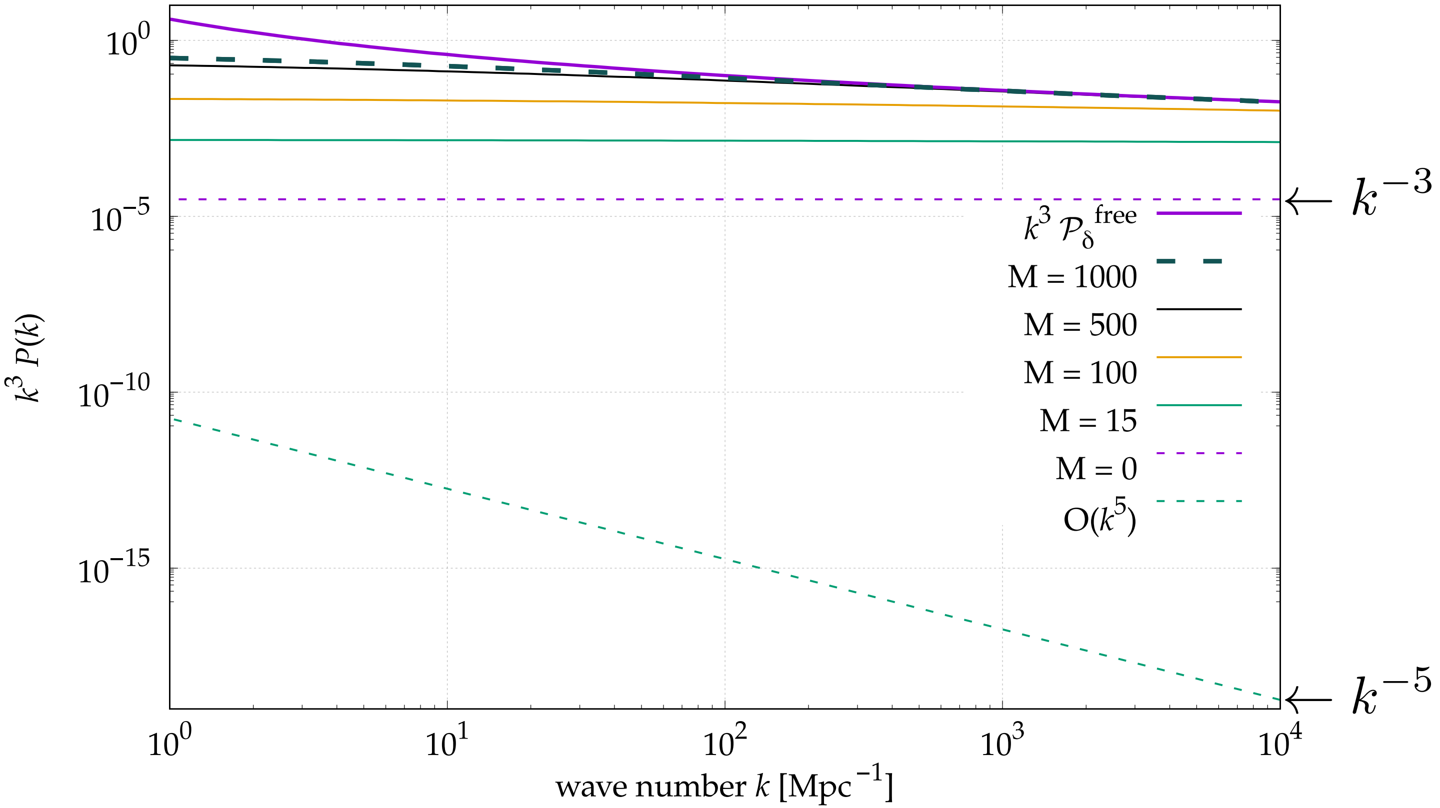
Plug asymptotics of a_1, a_2 into the integral of $\mathcal{P}$

$$\mathcal{P}(k,t) \sim \int \mathrm{d}^3q \; \mathrm{e}^{-t^2k^2q^2\left[\frac{\sigma_2^2}{30}\left(2\mu^2+1\right)+q^{1-n_s}\left(2\mu^2\xi_2+\xi_1\right)\right]} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

$$\sim \int \mathrm{d}^3q \; \mathrm{e}^{-t^2k^2q^2\frac{\sigma_2^2}{30}\left(2\mu^2+1\right)} \sum_{m=0}^{\infty} \frac{\left[t^2k^2q^{3-n_s}\left(2\mu^2\xi_2+\xi_1\right)\right]^m}{m!} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

Zel'dovich power spectrum - no UV-cutoff

$$\mathcal{P}(k,t) \sim \frac{1}{(kt)^3} \sum_{m=0}^{\infty} \frac{1}{(kt)^{m(1-n_s)}} \sum_{n=0}^{2m} \mathcal{P}_{mn}(t) \log^n(kt) + \mathcal{O}(k^{-5})$$



Non-linear power spectrum

$$\langle \rho^2 \rangle = \hat{\rho}^2 Z = \mathrm{e}^{\mathrm{i}\hat{S}_I} \hat{\rho}^2 Z_0 \quad \longrightarrow \quad P_\delta(k,t) = \langle \mathrm{e}^{-\mathrm{i}\vec{k}\cdot[\vec{q}_1(t)+\vec{q}_2(t)]} \rangle$$

Mean-field ansatz $P_\delta^{(\mathrm{nl})}(k) = \mathrm{e}^{\langle S_I \rangle (k)} \mathcal{P}(k)$

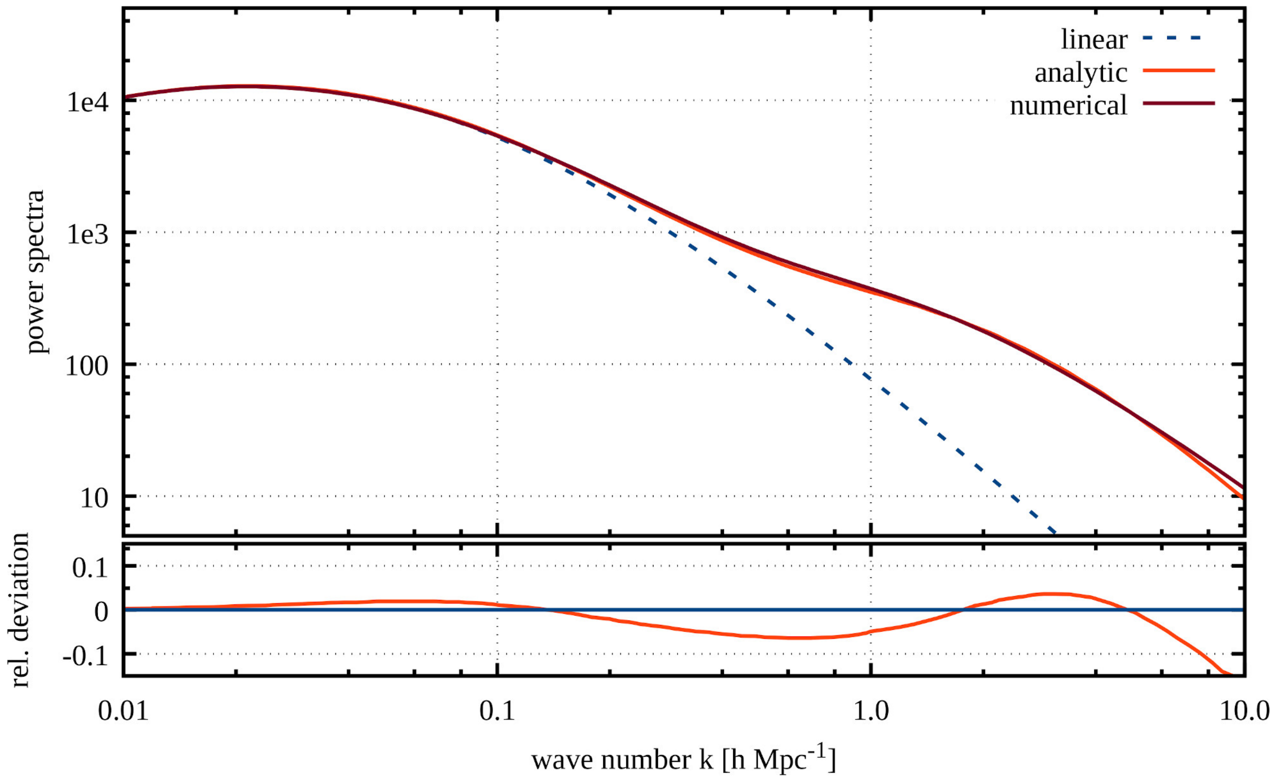
$$P_\delta^{(\mathrm{nl})}(k) = \mathrm{e}^{\langle -\mathrm{i}\vec{k}\cdot\left[\vec{q}_1^{\,I}(t)+\vec{q}_2^{\,I}(t)\right]\rangle_{\mathrm{corr}}} \underbrace{\langle \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\left[\vec{q}_1^{\,Z}(t)+\vec{q}_2^{\,Z}(t)\right]} \rangle}_{\mathcal{P}(k,t)}$$

$$\boxed{\langle S_I \rangle (k) = \langle -\int_0^t \mathrm{d}t' g_H(t,t') m(t') \vec{k} \cdot \left(\vec{\nabla}_1 - \vec{\nabla}_2 \right) \phi \rangle_{\mathrm{corr.}}}$$

$$\boxed{\langle S_I \rangle (k) \to \text{const.} \quad \text{as} \quad k \to \infty}$$

Non-linear power spectrum

$$P_\delta^{(\mathrm{nl})}(k) = \mathrm{e}^{\langle S_I \rangle (k)} \mathcal{P}(k)$$



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What to take home?

KFT can be used without extensive assumptions to investigate the universality of cosmic structures

- · Zel'dovich power spectrum $\sim k^{-3}$
- · Mean-field power spectrum $\sim k^{-3}$

These results are independent of the initial power spectra!

They are a consequence of the number of spatial dimensions.

Time evolution of small scale structures is sensitive to initial power spectrum.

This opens new possibilities to investigate the formation of small structures in dark matter vs. baryons, depending on the stream crossing time scale in dark matter vs. cooling time scale of baryons

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