A MINIMAL SCALAR FIELD MODEL TO TACKLE THE COSMOLOGICAL CONSTANT PROBLEM

THE UNIVERSITY of EDINBURGH

Arnaz Khan

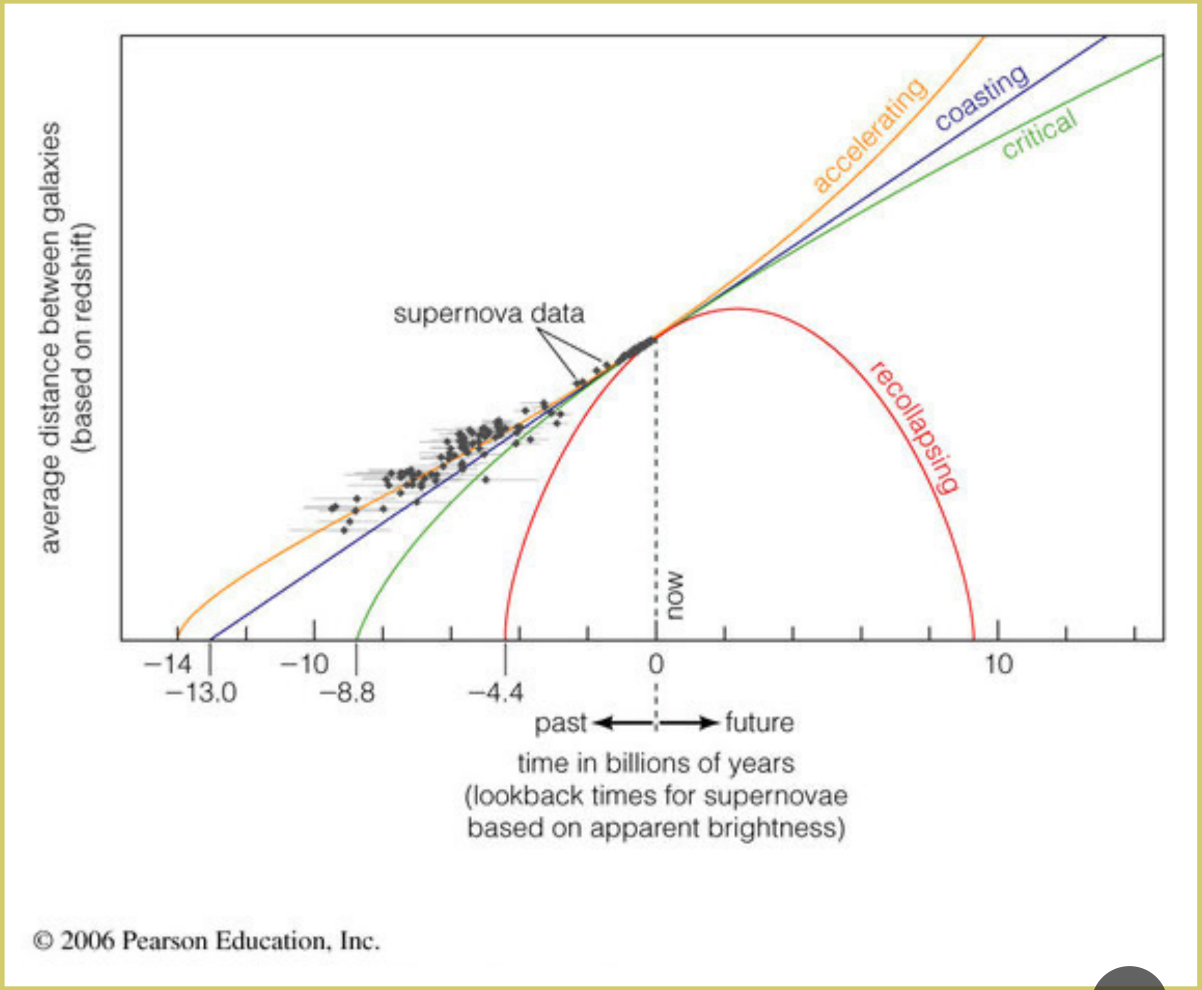
Khan & Taylor (2022) arxiv:2201.09016

THE COSMOLOGICAL CONSTANT PROBLEM

- The Universe is expanding, and its expansion is accelerating.
  (Riess et. al. 1998, Perlmutter et. al. 1999)

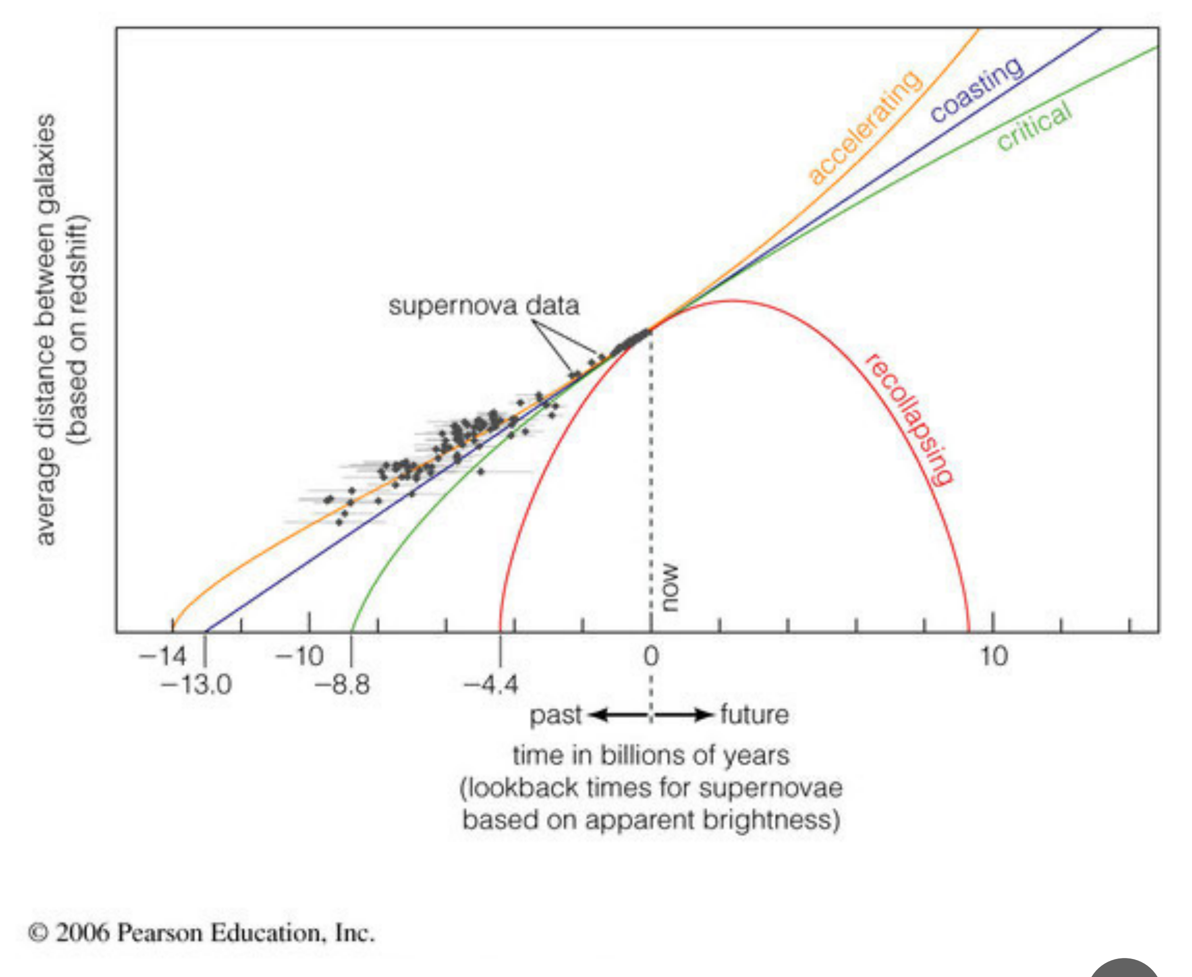
- The Cosmological Constant can model this expansion,
  $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \kappa T_{\mu\nu}$.

- But Λ receives quantum corrections from the vacuum
  $\Lambda = \Lambda_B + \rho_\Lambda$.

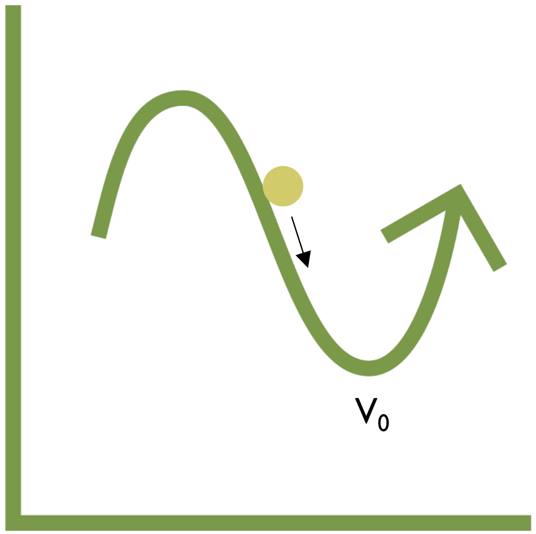


THE COSMOLOGICAL CONSTANT PROBLEM

- The corrections are very large ~ 60 orders of magnitude larger than observations imply.
- Crucially, vacuum energy suffers from issues of fine-tuning due to sensitivity from unknown high-energy physics.
- This is the Cosmological Constant Problem.
- What is the simplest way to tackle the problem using a scalar field?



WEINBERG'S NO-GO THEOREM



- Consider a scalar field potential $V(\varphi)$ with a minimum at V_0.
- Solve the Cosmological Constant Problem with a scalar field that *relaxes* at V_0 and cancels ρ_Λ to the required degree.
- Weinberg's No-Go theorem blocks such solutions as V_0 will also need fine-tuning.
- But there's a loop-hole:

  Relax Weinberg's assumptions and *dynamically* cancel the large vacuum energy.
- How much freedom does a single scalar field grant us?

HORNDESKI THEORY

Gregory Horndeski and his artwork
www.horndeskicontemporary.com

- Horndeski theory is the most general scalar-tensor theory of gravity in 4 dimensions that is stable.

- First proposed in 1974, it was revived in 2012 to find self-tuning solutions to solve the Cosmological Constant Problem (Charmousis et. al. 2012).

- A complicated gravitational action, with four Lagrangian density terms

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\, \mathcal{L}_i$$

HORNDESKI THEORY

$$\mathcal{L} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$

$$+ G_5(\phi, X)G_{\mu\upsilon}\nabla^{\mu}\nabla^{\upsilon}\phi$$

$$+ 2G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\upsilon\phi)(\nabla^\mu\nabla^\nu\phi)]$$

$$+ \frac{1}{3}G_{5X}(\phi, X)\big[(\Box\phi)^3 - 3(\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)\Box\phi$$

$$+ 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\sigma\nabla^\nu\phi)(\nabla_\sigma\nabla^\mu\phi)\big].$$

where $X = \frac{\dot{\phi}^2}{2}$

- The full Horndeski modifies the speed of gravitational waves, and generally tunes away any matter present.

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- Gravitational wave speed measurements constrain Horndeski to 3 terms.
  (Lombriser & Taylor 2016, Baker et. al. 2017, Ezquiaga & Zumalacárregui 2017)

HORNDESKI THEORY

$$\mathcal{L} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + M_{pl}^2/2\,R$$

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- Further pressures on G_4 (Lombriser & Taylor 2016, Lombriser & Lima 2017, Noller & Nicola 2019) leave behind Kinetic Gravity Braiding as the surviving subclass of Horndeski theory.

THE MINIMAL MODEL

$$G_2(\phi, X) = k(X) - V(\phi)$$

$$G_3(\phi, X) \propto \sqrt{2X}$$

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Friedmann $\quad 3M_{pl}^2 H^2 = \rho_\Lambda + \rho_m + \rho_\phi \,,$

Acceleration $\quad -\left(3M_{pl}^2 H^2 + 2M_{pl}^2 \dot{H}\right) = -\rho_\Lambda + p_\phi \,,$

Scalar field $\quad \left(\ddot{\phi} + 3H\dot{\phi}\right)(H - H_{ds}) + \dot{H}\dot{\phi} + V'(\phi) = 0,$

where $H = H_{ds}$ is the de Sitter Hubble attractor.

THE MINIMAL MODEL

$$G_2(\phi, X) = k(X) - V(\phi)$$

- A non-canonical kinetic term $k(X) \propto -H_{ds}X$ introduces the Hubble attractor H_{ds} in the equations.
- A linear ϕ potential removes vacuum energy.

$$G_3(\phi, X) \propto \sqrt{2X}$$

- Simply $\dot{\phi}$, introduces H-dependence in $\rho_\phi \propto \frac{1}{2}\dot{\phi}^2(2H - H_{ds}) + V(\phi)$.
- Shift-symmetry $\phi \to \phi + c$ of the Lagrangian density aids in controlling quantum corrections (Padilla 2015, Appleby & Linder 2018).

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Acceleration	$-\left(3M_{pl}^2 H^2 + 2M_{pl}^2 \dot{H}\right) = -\rho_\Lambda + p_\phi$,
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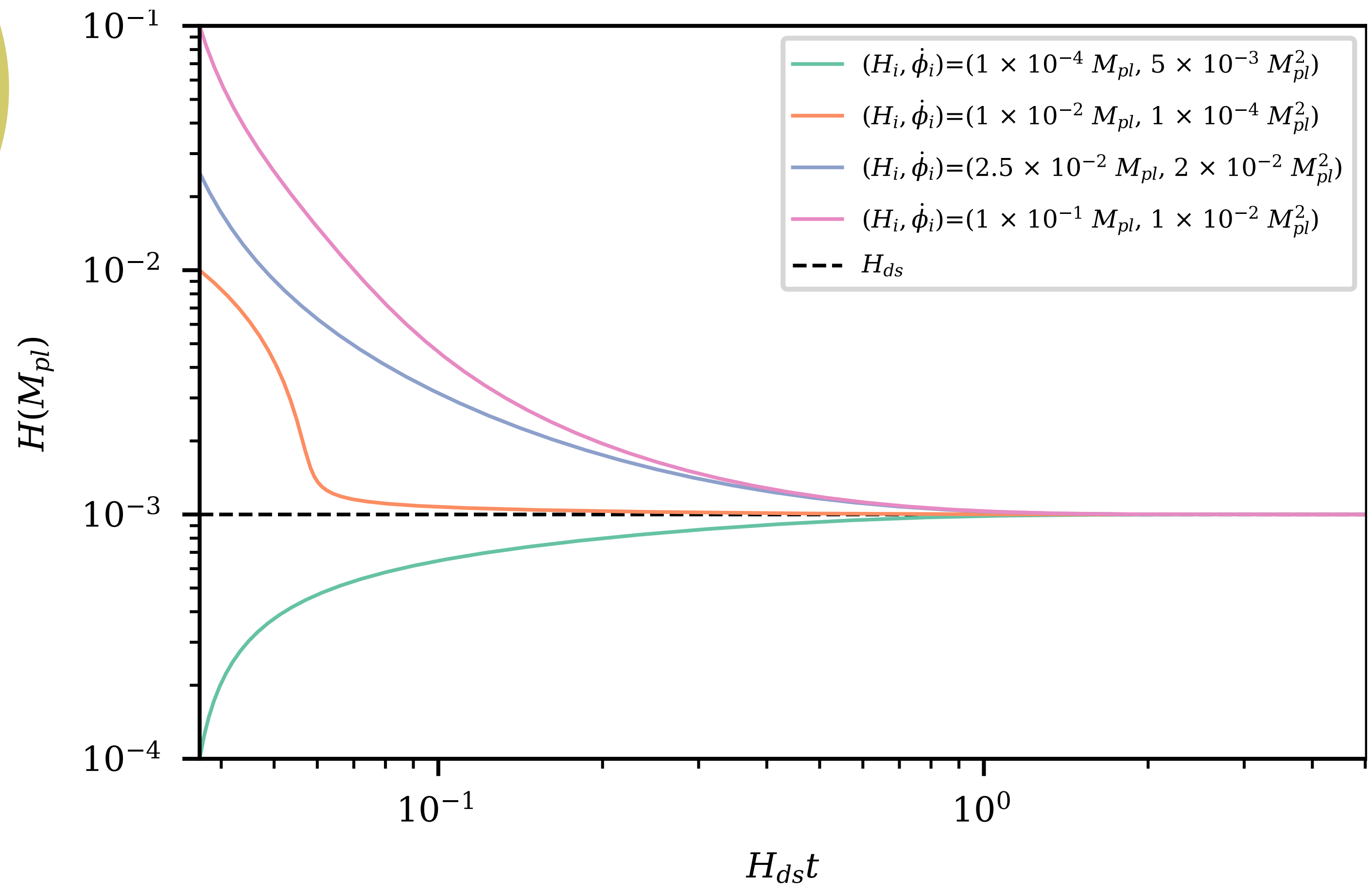
ATTRACTOR BEHAVIOUR



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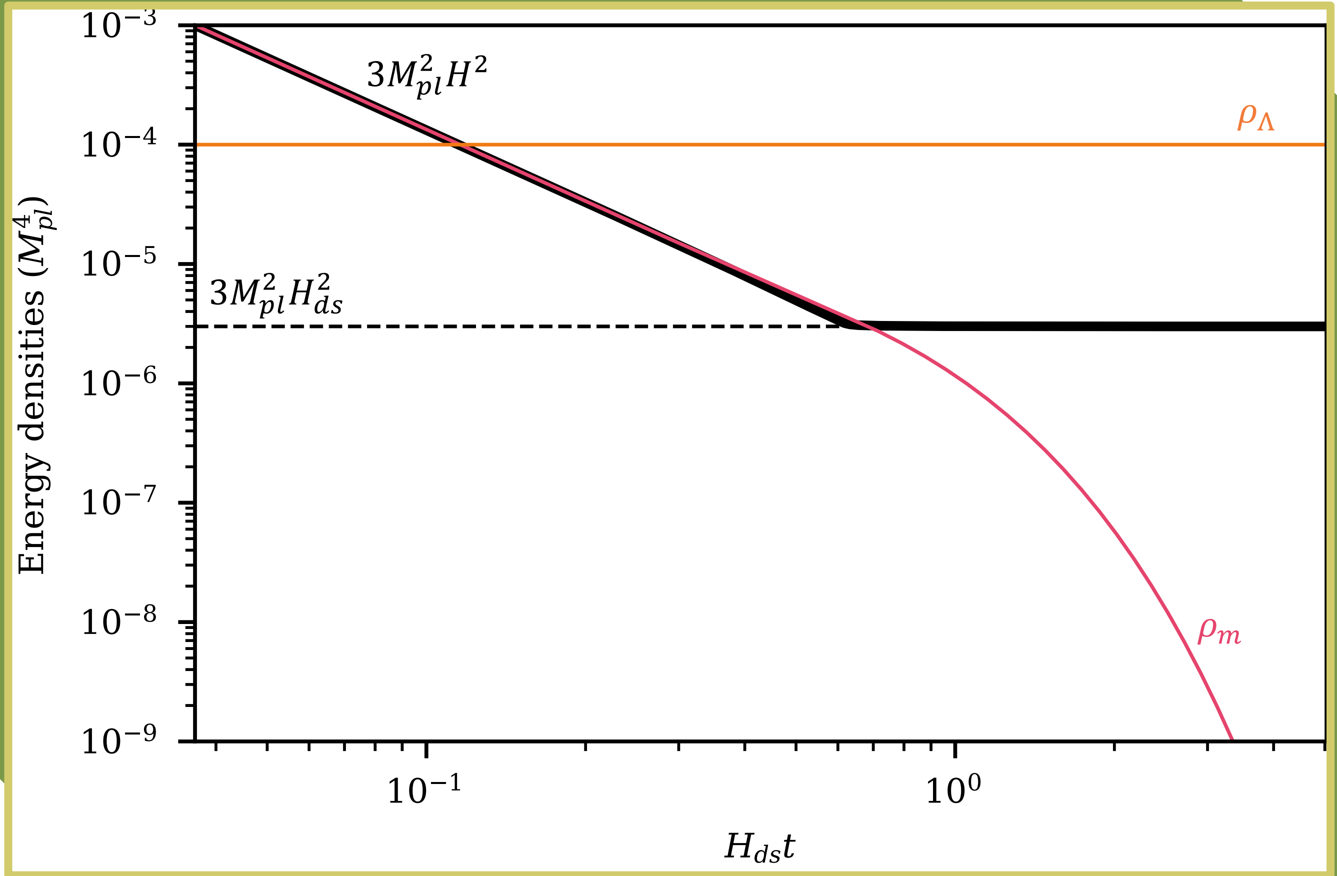
MODEL MECHANISM

- The total energy $3M_{pl}^2H^2$ follows $\rho_m \propto t^{-2}$ at early times, despite the presence of ρ_Λ .
- The potential energy removes ρ_Λ while the field is in slow-roll.
- H asymptotes to the de Sitter attractor at the end of matter domination.

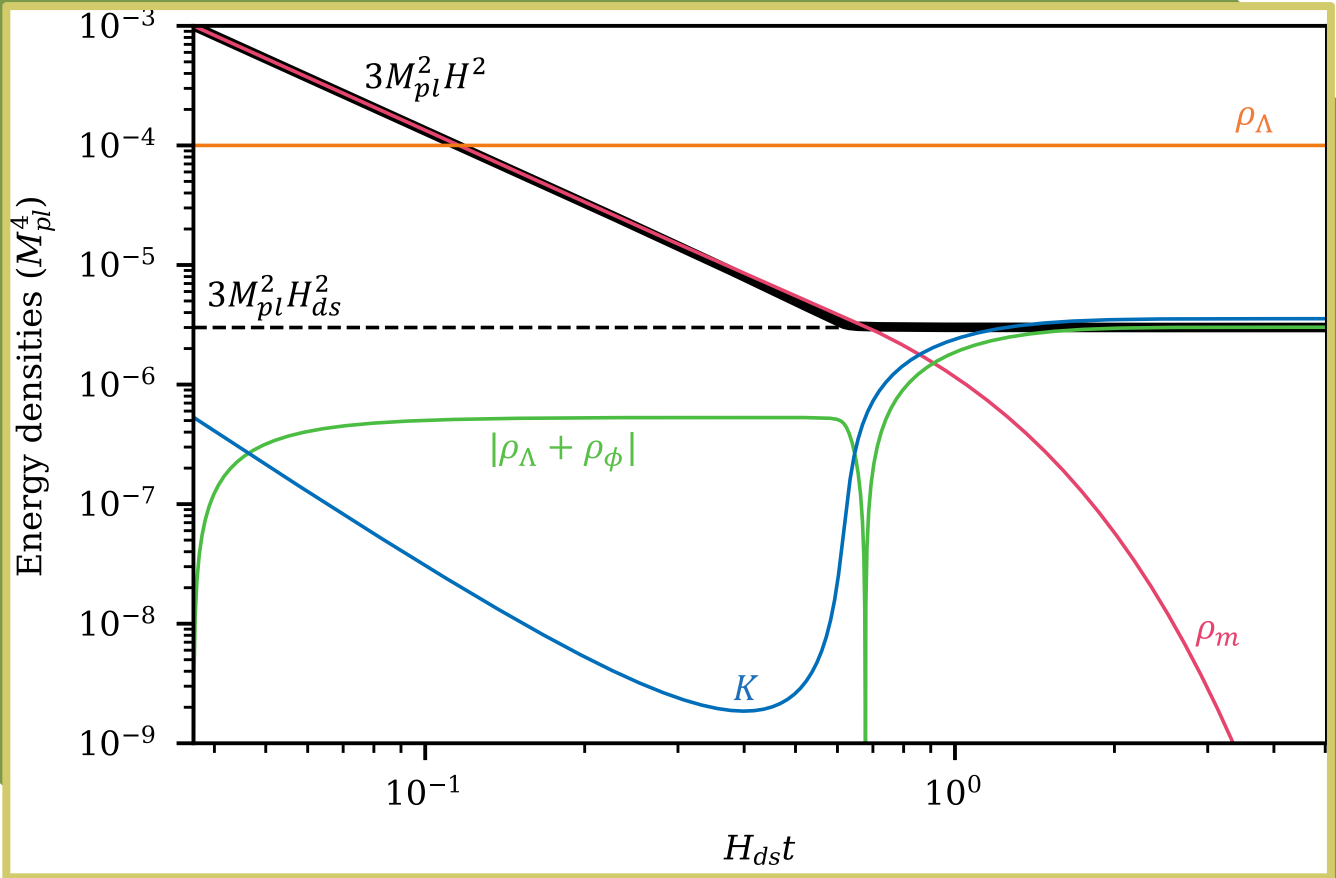


MODEL MECHANISM

- The potential provides a driving force to K at late times, which is triggered near the attractor.

- Attractor solution maintained by the scalar field dynamically driving effective dark energy $|\rho_\Lambda + \rho_\phi|$ to attractor value $\rho_{atr} = 3M_{pl}^2 H_{ds}^2$.

- We also find that the attractor is stable under a phase transition in the value of ρ_Λ .



DISCUSSION

- The mechanism is novel (Khan & Taylor 2022, Appleby & Bernardo 2022) and shows the existence of a wider class of self-tuning models than previously assumed.
- The model can be scaled such that $\rho_{atr} = \rho_{crit}$, giving a very light scalar field of mass $\sim 10^{-33}$ eV corresponding to a low-energy particle physics scale.
- The model may be a lower-energy manifestation of a high-energy theory.

SUMMARY

- We show there exists a simple model and mechanism to remove a large vacuum energy density and give acceleration at a much lower energy scale.
- The model is theoretically viable and passes gravitational wave speed constraints.
- The field preserves a matter dominated era and is stable under a vacuum energy phase transition.