Dynamical features of $f(T, B)$ cosmology

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(Cosmology From Home - 2022)

- Introduction and field equations in $f(T, B)$ gravity.
- Dynamical parameters.
- Energy conditions.
- Geometrical parameters.
- Stability of Model.
- Results and conclusions.

The Einstein field equations are best description of how space-time behaves on macroscopic scales.

The cosmic acceleration may have arise due to the repulsive gravity of DE.

From the observational and experimental points of view GR is no longer capable of addressing galactic, extra-galactic and cosmic dynamics.

Einstein first proposed the model to unify electromagnetism and gravity known as teleparallel gravity.

The dynamical object in teleparallel gravity are the four linearly independent vierbein (tetrad) fields.

The action for $f(T, B)$ gravity can be written as

$$S_{f(T,B)} = \frac{1}{\kappa^2} \int d^4x \;\; e \, f(T, B) + S_m \,, \tag{1}$$

The flat FLRW space time is given by

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \,, \tag{2}$$

The tetrad e_i^{μ} where i, μ running over 0,1,2,3 index and relates with the metric through the equation

$$g_{\mu\nu} = \eta_{ij} e^i_{\mu} e^j_{\nu} \,, \tag{3}$$

The Weitzenböck connection is defined as

$$\hat{\Gamma}^{\lambda}_{\mu\nu} = e_i^{\lambda} \partial_{\nu} e^i_{\mu} = -e^i_{\mu} \partial_{\nu} e_i^{\lambda} \,, \tag{4}$$

The torsion scalar T in teleparallel gravity is given by

$$T = S_{\sigma}{}^{\mu\nu} T^{\sigma}{}_{\mu\nu} \,. \tag{5}$$

We can derive torsion scalar from equation (5) as $T = 6H^2$.

Boundary term can be defined as $B = (\frac{2}{e})\partial_i e T^i$.
From above expression it can be derived as $B = 6(3H^2 + \dot{H})$.
Which reproduce Ricci scalar $R = B - T = 6(2H^2 + \dot{H})$.

Field equations under these conditions as fallow,

$$\begin{aligned} \kappa^2 \rho &= 9H^2 f_B + 6f + 6H^2 f_T - 3\dot{f}_B H + 3\dot{H} f_B - \frac{1}{2} f, \\ \kappa^2 p &= -(3H^2 - \dot{H})(3f_B + 2f_T) - 2H\dot{f}_T + \ddot{f}_B. \end{aligned} \tag{6}$$

Here we studied the scale factor, $a(t) = \sqrt{\eta} cosh\gamma t$.
[1]

The model considered during the study is $f(T, B) = \alpha T log(\frac{T}{T_0}) + \beta B$, where T_0 is an arbitrary constant.
The dynamical parameters expressions can be calculated as follow,

$$
\begin{aligned}
p(z) &= \alpha\gamma^2 \left(6 - \left(\eta(z+1)^2 - 3\right) \log\left(-\frac{6\gamma^2\left(\eta(z+1)^2 - 1\right)}{T_0}\right)\right), \qquad (7)\\
\rho(z) &= 3\alpha\gamma^2 \left(\eta(z+1)^2 - 1\right) \left(\log\left(-\frac{6\gamma^2\left(\eta(z+1)^2 - 1\right)}{T_0}\right) + 2\right), \\
\omega(z) &= \frac{6 - \left(\eta(z+1)^2 - 3\right) \log\left(-\frac{6\gamma^2\left(\eta(z+1)^2-1\right)}{T_0}\right)}{3\left(\eta(z+1)^2 - 1\right)\left(\log\left(-\frac{6\gamma^2\left(\eta(z+1)^2-1\right)}{T_0}\right) + 2\right)}.
\end{aligned}
$$

[1] Fakhereh Md. Esmaeili and B. Mishra 2018 *J. Astrophys. Astr.*

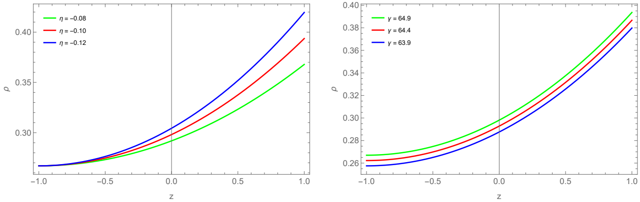

Figure 1: Plots of energy density vs redshift for varying η and μ. The other parameters value are, $\alpha = 0.05,\ T_0 = 27\pi$.

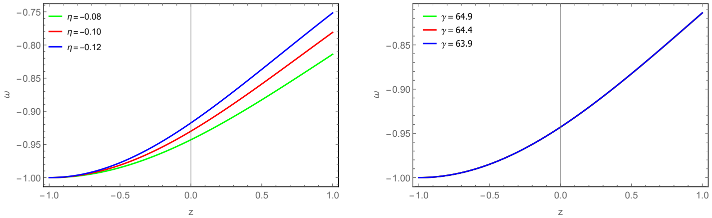

Figure 2: Plots of EoS parameter vs redshift for varying η and μ. The other parameters value are, $\alpha = 0.05,\ T_0 = 27\pi$.

From SN1a data gives ω_0=-1.084$\pm$0.063.
We have obtained $\omega_0 = -0.93 \pm 0.3$ from graph.

The general equations for energy conditions are:

1) NEC: $\rho + p \geq 0$, 2) WEC: $\rho + p \geq, \rho \geq 0$, 3) SEC: $\rho + 3p \geq 0$, 4) DEC: $\rho - p \geq 0$.

[2]

$$\begin{aligned}
\rho(z) + p(z) &= 2\alpha\gamma^2\eta(z+1)^2\left(\log\left(-\frac{6\gamma^2\left(\eta(z+1)^2-1\right)}{T_0}\right)+3\right), \\
\rho(z) + 3p(z) &= 6\alpha\gamma^2\left(\log\left(-\frac{6\gamma^2\left(\eta(z+1)^2-1\right)}{T_0}\right)+\eta(z+1)^2+2\right), \\
\rho(z) - p(z) &= 2\alpha\gamma^2\left(\left(2\eta(z+1)^2-3\right)\log\left(-\frac{6\gamma^2\left(\eta(z+1)^2-1\right)}{T_0}\right)+3\eta(z+1)^2\right).
\end{aligned} \tag{8}$$

[2] S. Capozziello, *Phys. Lett. B*, **730**, 280 (2014).

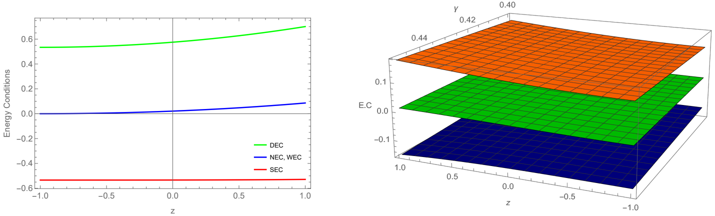

Figure 3: 2D plot (left pannel) with η=-0.23 and 3D plot (right pannel) of energy conditions vs redshift with varing γ in the range 0.4-0.45. The other parameters value are, $\eta = -0.23$, α=0.05, $T_0 = 27\pi$.

The general formula to calculate deceleration parameter can be written as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1,$$

[3] Deceleration parameter for hyperbolic scale factor can be calculated as

$$q(z) = \frac{1}{\eta(z+1)^2 - 1}$$

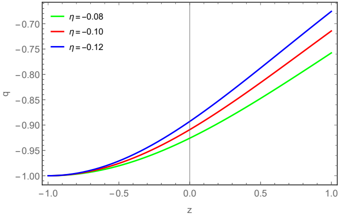

Figure 4: Plotting of deceleration parameter as a function of redshift for η=-0.23 and γ= 0.45.

[3] J. R. Garza et al., *Eur. Phys. J. C*, **79**, 890 (2019).

The general formula to calculate jerk and snap parameter is as follow

$$j = -2 - 3q + \frac{d^2(H)}{dt^2}\frac{1}{H^3}, \qquad (9)$$

$$s = 6 + 4j + 3q(q+4) + \frac{d^3(H)}{dt^3}\frac{1}{H^4}.$$

$$j(z) = -\frac{1}{\eta(z+1)^2 - 1},$$

$$s(z) = \frac{1}{(\eta(z+1)^2 - 1)^2}.$$

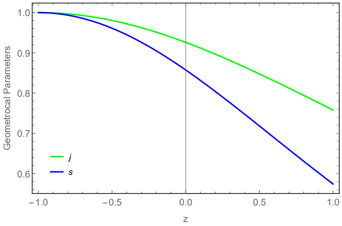

Figure 5: Plot of jerk and snap parameter for the parametric values η-0.23, γ=0.45.

We consider the perturbation of Hubble parameter and the energy density respectively as,

$$H(t) = H_a(1 + \delta(t))\,, \tag{8}$$

and

$$\rho(t) = \rho_a(1 + \delta_m(t))\,. \tag{9}$$

The functional $f(T,B) = \alpha T log\left(\frac{T}{T_0}\right) + \beta B$ can be expanded in powers of T_a and B_a as,

$$f(T,B) = f_a + \left(\alpha log\left[\frac{T_a}{T_0} + \alpha\right]\right)(T - T_a) + \beta(B - B_a) + \mathcal{O}^2\,. \tag{10}$$

On solving, we obtain the perturbation parameters $\delta(t)$ and $\delta_m(t)$ as,

$$\delta(t) = \tau \exp\left\{\frac{3\alpha\left(\beta \log\left(2\alpha + 3\beta - 2(\alpha + 2\beta)\cosh^2(\gamma t)\right) - 4(\alpha + 2\beta)\log(\cosh(\gamma t))\right)}{8(\alpha + 2\beta)(2\alpha + 3\beta)}\right\},$$

$$\delta_m(t) = \frac{\delta(t)}{6\alpha\gamma^3}\left\{\frac{\coth^3(\gamma t)\left(24\alpha\gamma^2\tanh^2(\gamma t) + 48\beta\gamma^2\tanh^2(\gamma t) + 12\beta\gamma^2\text{sech}^2(\gamma t)\right)}{6\alpha\gamma^3}\right\}. \tag{11}$$

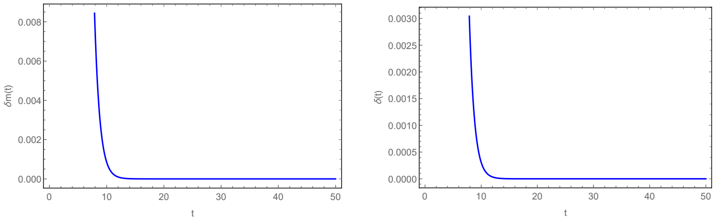

Figure 6: Plots of perturbation in the energy density ($\delta_m(t)$) (left panel) and Hubble parameter ($\delta(t)$) (right panel) in cosmic time. Other parameter values remain same as before.

- We have studied dynamical parameters with hyperbolic scale factor in the context of $f(T, B)$ gravity.
- The model shows the accelerating behaviour, with energy density lies in the positive region. The equation of state parameter approaches to -1 at late time hence coincides with the ΛCDM model.
- The energy conditions are investigated to check viability of model, violation of strong energy condition supports to the accelerating model in the extended teleparallel gravity.
- The deceleration parameter lies in the negative region confirms accelerating behaviour. The jerk and snap parameters approaches to 1 at late time and show increasing behaviour from early to late time.
- As the energy density and Hubble parameter perturbation shows decreasing behaviour and approach to zero for increasing values of cosmic time t, the stability of the model has been confirmed.

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Thank You!