Dynamical mass generation for a massless minimal scalar with quartic plus cubic self interaction in de Sitter spacetime

(S. Bhattacharya and N. Joshi, Manuscript under preparation.)

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Cosmology from Home; 4th July to 15 July, 2022

Motivation

- Our interest $\rightarrow$ to study the physics of the very early universe. Corresponding study involves cosmic inflation (a theory of exponential expansion of spacetime in the early universe.), decoherence and entanglement in open quantum systems.
- Such study might provide us insight about the initial state as well as the geometry of the early inflationary universe.
- It might provide a clear mechanism on how inflation ended into radiation dominated era and small value of cosmological constant is reached.
- There can be various approaches towards inflation such as stochastic and field theoretic and it may also involve investigations like the decoherence, entanglement entropy, various other measures of quantum entanglement.

Introduction & overview

- In this talk $\rightarrow$ stochastic approach to inflation for a massless minimally coupled quantum scalar field with an asymmetric (quartic plus cubic) self interaction, the generation of dynamical mass and the comparison with field theoretic results.
- Potential choice $\rightarrow$ Why this potential $V(\phi) = \frac{\lambda\phi^4}{4!} + \frac{\beta\phi^3}{3!}$?.
- Stochastic method $\rightarrow$ where we split the quantum fields into long and short wavelength modes, and viewing the former as classical objects evolving stochastically in an environment provided by quantum fluctuations of shorter wavelengths. (A. A. Starobinsky and J. Yokoyama, *Equilibrium state of a self interacting scalar field in the De Sitter background*, Phys. Rev. D **50**, 6357-6368 (1994) [arXiv:astro-ph/9407016 [astro-ph]]).
- Langevin equation $\rightarrow$ a stochastic differential equation that experiences a particular type of random force.

Introduction & overview

- Fokker-Planck equation $\rightarrow$ is a partial differential equation describing the time evolution of the probability density distribution function.
- Dynamical mass $\rightarrow$ means the mass that's get generated due to the self interaction or due to the radiative process even though originally field was massless. Dynamical mass contribution comes from loops which makes the particle massive.
- Backreaction $\rightarrow$ is a shift in the cosmological constant Λ due to the cosmological perturbation.

Potential choice

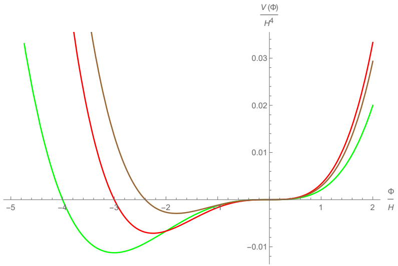

Figure: The Green, red and brown curves correspond to λ and β/H values : 0.01; 0.01; 0.02; 0.015; and 0.02; 0.012 respectively.

- From above Fig. one can expect that the late time vacuum expectation value of $V(\phi)$ may be negative. [1]
- For $\lambda = 0$, the potential becomes unbounded from below and system will roll away to $-\infty$ and for $\beta = 0$, the potential's contribution will be positive but field's contribution will be vanishing.

[1] (S. Bhattacharya, *Massless minimal quantum scalar field with an asymmetric self interaction in de Sitter spacetime*, [arXiv:2202.01593 [hep-th]]).

Stochastic approach to inflation

- The basic idea of the stochastic formulation is that the long wave-length behaviour can be treated classically and the short wavelength modes provide an effective stochastic term to the equation of motion for ϕ.
- For interaction potential $V(\phi)$, $\Box\phi + V'(\phi) = 0$, we obtain the following equation for the coarse-grained part $\bar{\phi}$:

  $$\dot{\bar{\phi}}(t, \vec{x}) = -\frac{1}{3H} V'(\bar{\phi}) + f(t, \vec{x})$$

- Above equation can be regarded as the Langevin equation for the stochastic quantity with a stochastic noise term $f(t, \vec{x})$
- The long-wavelength part of the quantum field $\phi(\vec{x}, t)$ in de Sitter spacetime can be modeled by an auxiliary classical stochastic variable with a probability distribution $\rho(\bar{\phi}(\vec{x}, t) = \phi)$ that satisfies the Fokker-Planck equation

  $$\frac{\partial \rho}{\partial t} = \frac{H^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \phi^2} + \frac{1}{3H} \frac{\partial}{\partial \phi} \left(\frac{\partial V}{\partial \phi} \rho(t, \phi) \right)$$

Stochastic approach to inflation

- The general solution of the Fokker-Planck equation is,
  $\rho(\phi, t) = \exp(-\nu(\phi)) \sum_{n=0}^{\infty} a_n \Phi_n(\phi) e^{-\Lambda_n (t-t_0)}$
  where $\Phi_n(\phi)$ is the eigenfunctions of the Schrodinger-type equation and
  $\nu(\phi) = \frac{4\pi^2}{3H^4} V(\phi)$
- At late times any solution of the above equation approaches the static equilibrium solution
  $\rho_{\rm eq}(\phi) = N^{-1} \exp\left(-\frac{8\pi^2}{3H^4} V(\phi)\right)$
- where N is the normalization fixed by the condition
  $\int_{-\infty}^{\infty} \rho_{\rm eq}(\phi)\, d\phi = 1$
- If N is finite, then Φ exists, but if it diverges, then no such equilibrium solution exists.

ϕ, ϕ^2 and $V(\phi)$ Averages

- The distribution can be used to compute the fluctuation of ϕ, i.e. the expectation value of ϕ and ϕ^2 given as

  $\langle\phi\rangle = \int_{-\infty}^{\infty} \phi \rho_{\rm eq}(\phi) d\phi$

  $= N^{-1} \int_{-\infty}^{\infty} \phi \exp\left[-\frac{8\pi^2}{3H^4}\left(\frac{\lambda}{4!}\phi^4 + \frac{\beta}{3!}\phi^3\right)\right] d\phi$

- for ϕ^2

  $\langle\phi^2\rangle = \int_{-\infty}^{\infty} \phi^2 \rho_{\rm eq}(\phi) d\phi$

  $= N^{-1} \int_{-\infty}^{\infty} \phi^2 \exp\left[-\frac{8\pi^2}{3H^4}\left(\frac{\lambda}{4!}\phi^4 + \frac{\beta}{3!}\phi^3\right)\right] d\phi$

- And the $V(\phi)$ given as

  $\langle V(\phi)\rangle = N^{-1} \int_{-\infty}^{\infty} V(\phi) \exp\left[-\frac{8\pi^2}{3H}\left(\frac{\lambda}{4!}\phi^4 + \frac{\beta}{3!}\phi^3\right)\right] d\phi$

Plots for ϕ average

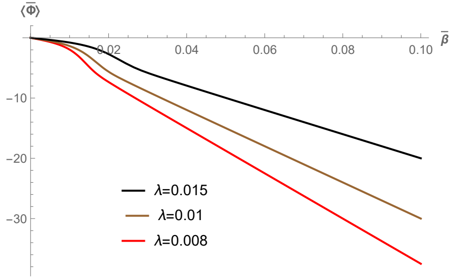

Figure: A 2-D plot for the expectation value of $\overline{\phi} = \frac{\phi}{H}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ.

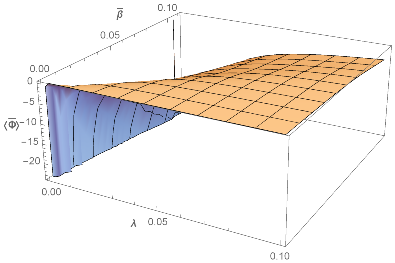

Figure: A 3-D plot for the expectation value of $\overline{\phi} = \frac{\phi}{H}$, λ and $\overline{\beta} = \frac{\beta}{H}$.

Plots for ϕ^2 average

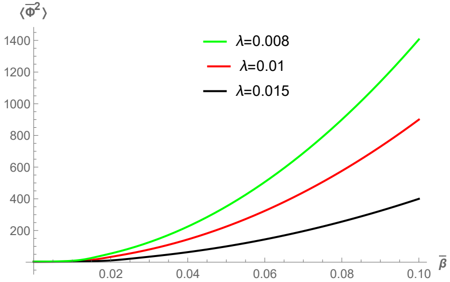

Figure: A 2-D plot for the expectation value of $\overline{\phi}^2 = \frac{\phi^2}{H^2}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ.

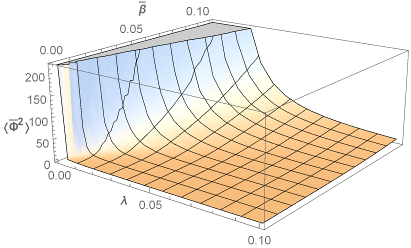

Figure: A 3-D plot for the expectation value of $\overline{\phi}^2 = \frac{\phi^2}{H^2}$, λ and $\overline{\beta} = \frac{\beta}{H}$.

Plots for $V(\phi)$ average

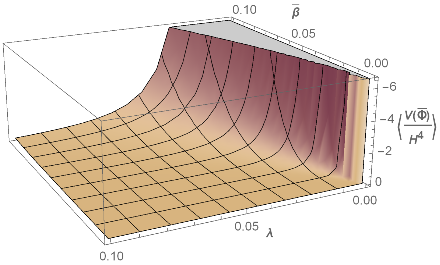

Figure: A 3-D plot for the expectation value of $\frac{V(\bar{\phi})}{H^4}$, λ and $\overline{\beta} = \frac{\beta}{H}$.

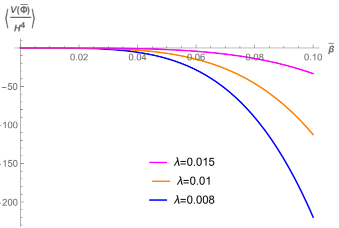

Figure: A 2-D plot for the expectation value of $\frac{V(\bar{\phi})}{H^4}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ.

Field theoretic results

- The final form of $\langle\phi\rangle$ after the renormalisation and resummation look like, where $\overline{\beta} = \beta/H$ and $\overline{\phi} = \phi/H$ are dimensionless is [2]
$\langle\bar{\phi}\rangle = -\frac{9\bar{\beta}}{22\lambda}$

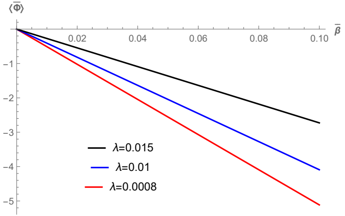

Figure: A 2-D plot for the expectation value of $\overline{\phi} = \frac{\phi}{H}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ upto $\mathcal{O}(\beta)$ and $\mathcal{O}(\lambda\beta)$.

[2](S. Bhattacharya, *Massless minimal quantum scalar field with an asymmetric self interaction in de Sitter spacetime*, [arXiv:2202.01593 [hep-th]]).

Field theoretic results

- We have the perturbative expansion for the expectation value of average ϕ^2, where $\overline{\beta} = \beta/H$ and $\overline{\phi}^2 = \phi^2/H^2$ are dimensionless.

$$\langle \overline{\phi}^2 \rangle = \frac{1}{4\pi^2} \ln a - \frac{(\lambda - \frac{\overline{\beta}^2}{2})}{2^4 \times 9\pi^4} \ln^3 a + \frac{\lambda^2}{2^6 \times 27\pi^6} \frac{\ln^5 a}{5} - \frac{\lambda \overline{\beta}^2}{2^7 \times 9\pi^6} \frac{\ln^6 a}{36}$$

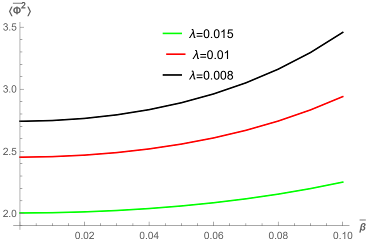

Figure: A 2-D plot for the expectation value of $\overline{\phi}^2 = \frac{\phi^2}{H^2}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ upto $\mathcal{O}(\beta^2)$ and $\mathcal{O}(\lambda^2)$.

Dynamical mass generation

- In d dimension expression of dynamical mass can be written as (R. L. Davis, Phys. Rev. D **45**, 2155 (1992))

$$m_{dyn}^2 = \frac{\Gamma(\frac{d+1}{2})H^d}{2\pi^{\frac{d+1}{2}}\langle\phi^2\rangle}$$

- Comparison $\rightarrow$ For λ and β/H values: (0.008; 0.03); (0.01; 0.03); and (0.015; 0.03), stochastic $\frac{m_{dyn}^2}{H^2}$ vs. field theoretic $\frac{m_{dyn}^2}{H^2}$ values are (0.0003; 0.13618); (0.00047; 0.1532); and (0.0011; 0.1879) respectively.

Approximate shift in Cosmological constant Λ

- The backreaction to Λ in the Einstein equation will be,

  $$\Lambda \to \Lambda(1 + 16\pi G\gamma\langle\phi\rangle) \approx \Lambda\left(1 - 1.5 \times 10^5 \times \frac{L_P^2}{L_C L_\gamma}\right)$$

  where $H^{-1} = L_C$ is the length scale of the cosmological event horizon, whereas $L_\gamma = \gamma^{-1}$ is the length scale associated with γ (γ is a constant of length dimension -1) and L_p is the planck length.

- Hence the approximate shift in the inflationary Λ is given by,

  $$\Lambda\left(1 - 10^0 \times \frac{L_C}{L_\gamma}\right)$$

  for $\bar{\beta}/\lambda \sim \mathcal{O}(10^3)$

- Similarly the approximate shift in the inflationary Λ due to the potential $\langle V(\phi)\rangle$ is given by, for $\bar{\beta}/\lambda \sim \mathcal{O}(10^3)$

  $$\Lambda\left(1 - 10^{-1} \times \frac{L_C}{L_\gamma}\right)$$

Discussion & Future plans

- In this work, we have considerd stochastic approach to inflation for a massless minimally coupled quantum scalar field with an asymmetric self interaction, the generation of dynamical mass and compared our result with the field theoretic results.
- We note that nature of plots for $\langle \phi \rangle$ are the same for both approaches but the negative value of average ϕ stochastically is greather than field theoretically.
- We also note that upto $\mathcal{O}(\beta^2)$ and $\mathcal{O}(\lambda^2)$, nature of plots for $\langle \phi^2 \rangle$ is similar for both approaches but there is huge mismatch appear in average ϕ^2 value as we increase β parameter and due to this reason there is miss match in dynamically generated mass as well.
- Even though we have not shown here but we have also calculated $\langle \phi^2 \rangle$ upto $\mathcal{O}(\lambda\beta^2)$, in this case our results do not match for both approaches.
- We find a shift in Λ is of $\mathcal{O}(10^0)$ owing to $\langle \phi \rangle$ and $\mathcal{O}(10^{-1})$ owing to $\langle V(\phi) \rangle$.
- We hope to resolve the issue of mismatch in numereric values for both approaches for $\langle \phi \rangle$ and $\langle \phi^2 \rangle$ in near future.

Acknowledgment

- IIT Ropar
- Department of Physics
- Dr. Sourav Bhattacharya (Supervisor)
- Family and friends

Thank You