Cosmological bootstrap in slow motion and the low speed collider

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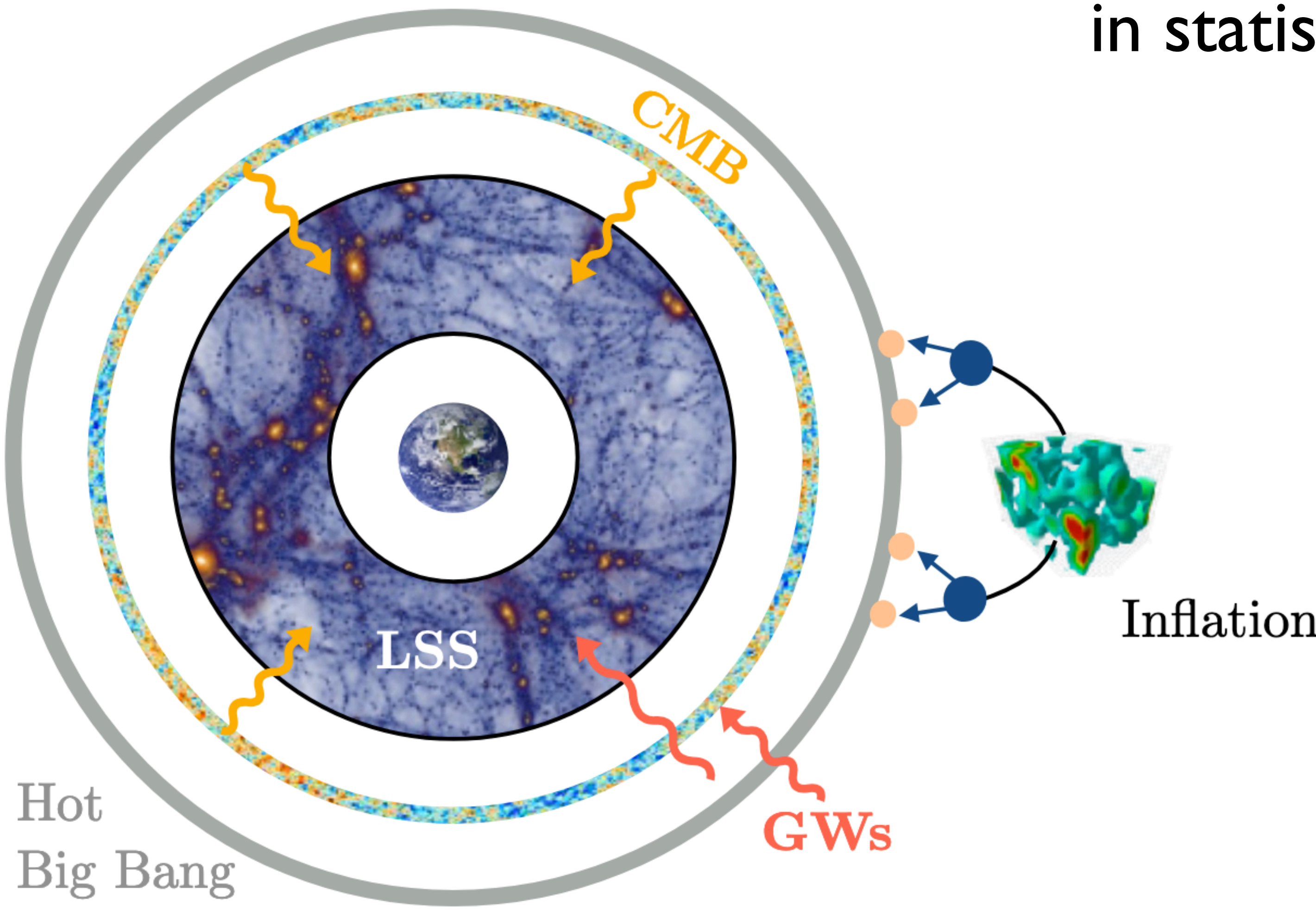
Conference Cosmology from Home
July 2022

based on arXiv: 2205.10340 [hep-th]

Probing inflation



Physics at extremely high-energies encoded in statistical properties of primordial fluctuations

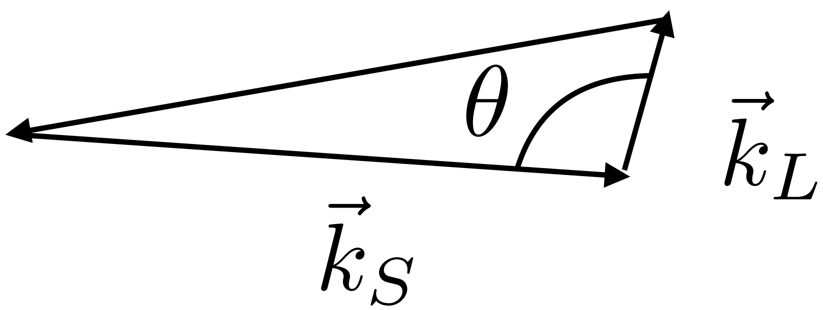
density fluctuations & gravitational waves

2pt, 3pt, n-pt
Even full pdf

Treasure of information to extract

Inflation as a cosmological collider

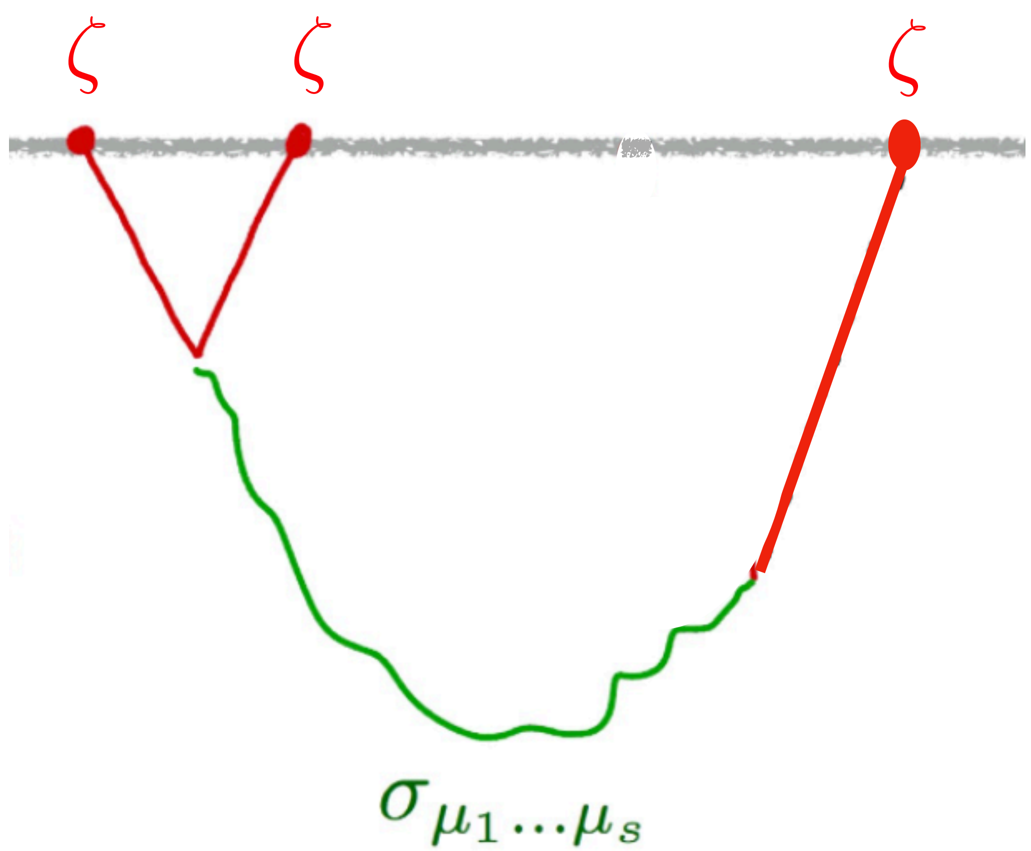
3pt



$$\lim_{k_L \to 0} \langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle \propto \left(\frac{k_L}{k_S} \right)^{3/2} \cos \left[\frac{M}{H} \ln \left(\frac{k_L}{k_S} \right) + \delta \right] P_S(\cos\theta)$$

Mass & Spin

of heavy exchanged particle



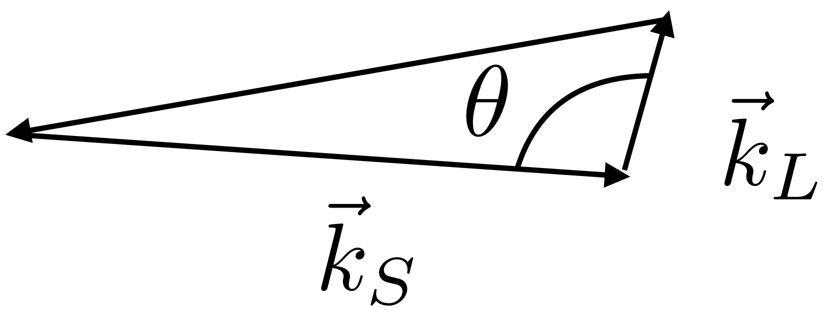
Which mass actually? Inflationary flavor oscillations and cosmic spectroscopy

Pinol, Aoki, Renaux-Petel, Yamaguchi, 2112.05710

Chen, Wang 2009
Baumann Green 2011
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
Arkani-Hamed, Baumann, Lee, Pimentel 2018
+ many works

Inflation as a cosmological collider

3pt



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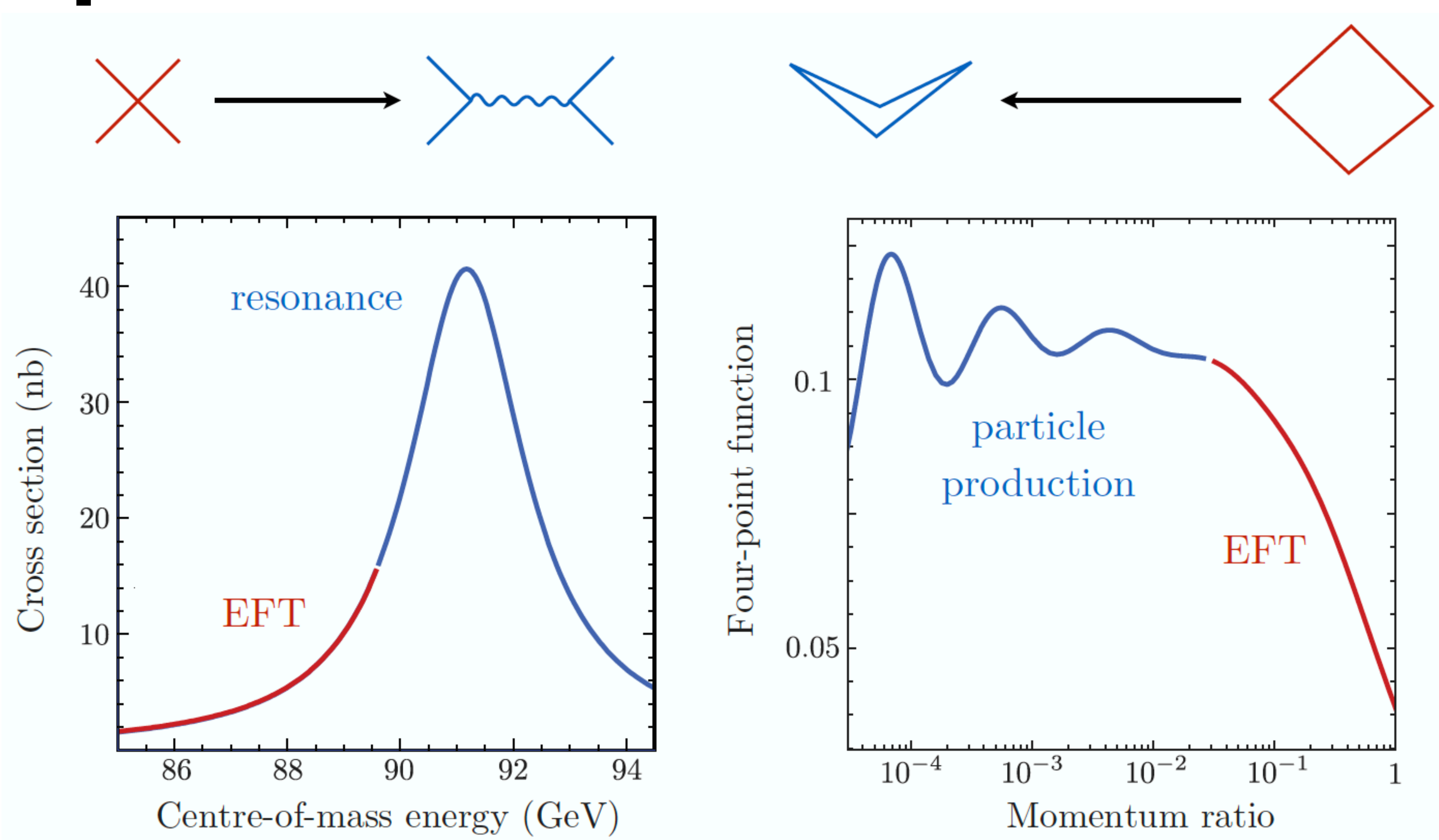
Mass & Spin

of heavy exchanged particle

4pt



From 1811.00024

Chen, Wang 2009
Baumann Green 2011
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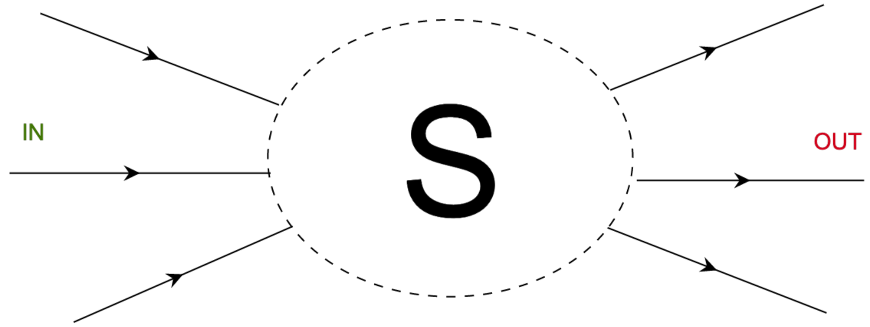
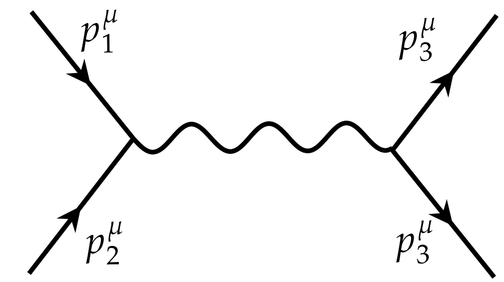
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
Arkani-Hamed, Baumann, Lee, Pimentel 2018
+ many works

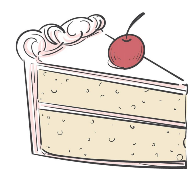
Time-dependent perturbation theory is hard!

Flat space



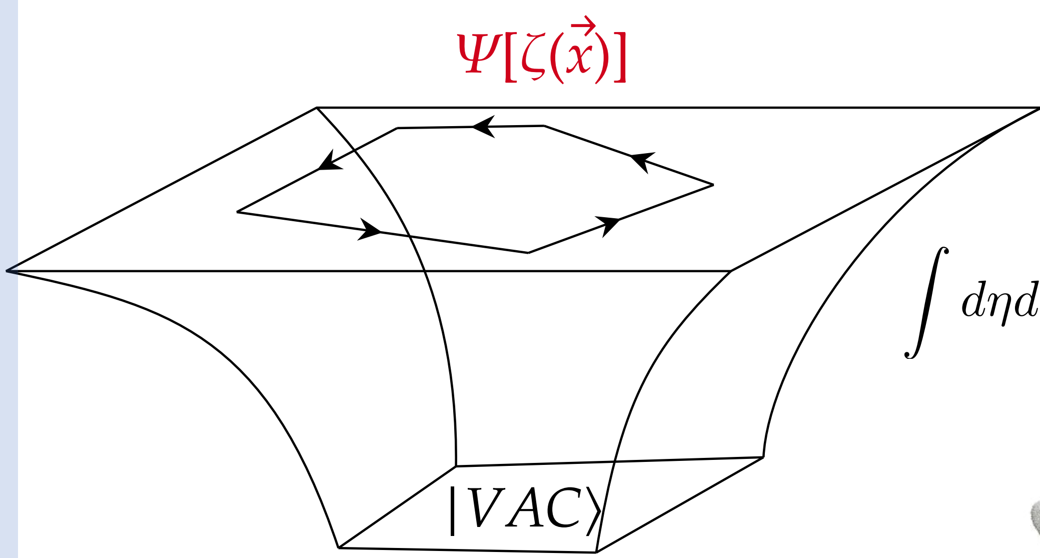
$$\int dt dt' \, e^{iE_1 t} e^{iE_2 t'} \subset$$



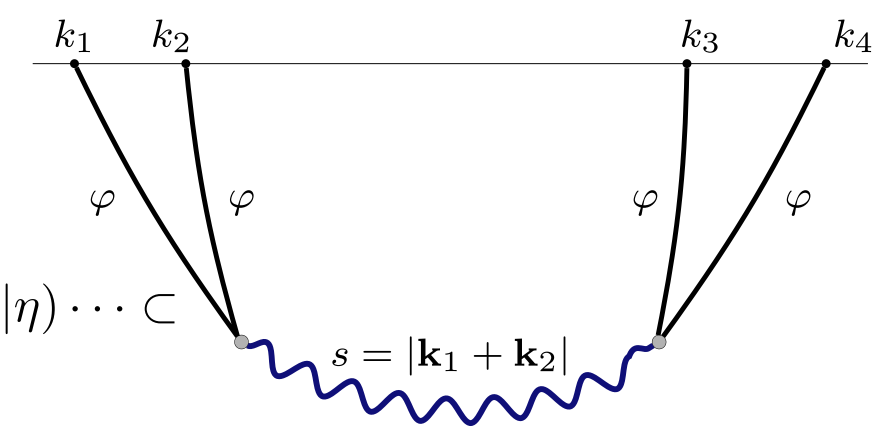


$$\mathcal{A}_{1,2\to3,4} = \frac{1}{(p_1+p_2)^2 - m^2} + t - u \text{ channels}$$

Cosmology



$$\int d\eta d\eta' a^p(\eta) \times e^{i|\mathbf{k}|\eta} \times H_{i\mu}(-|\mathbf{k}|\eta) \cdots \subset$$



$$\Psi[\zeta(\mathbf{x}), t = -\infty] = \text{Gaussian}$$

Not until **2018** by Arkani-Hamed et al

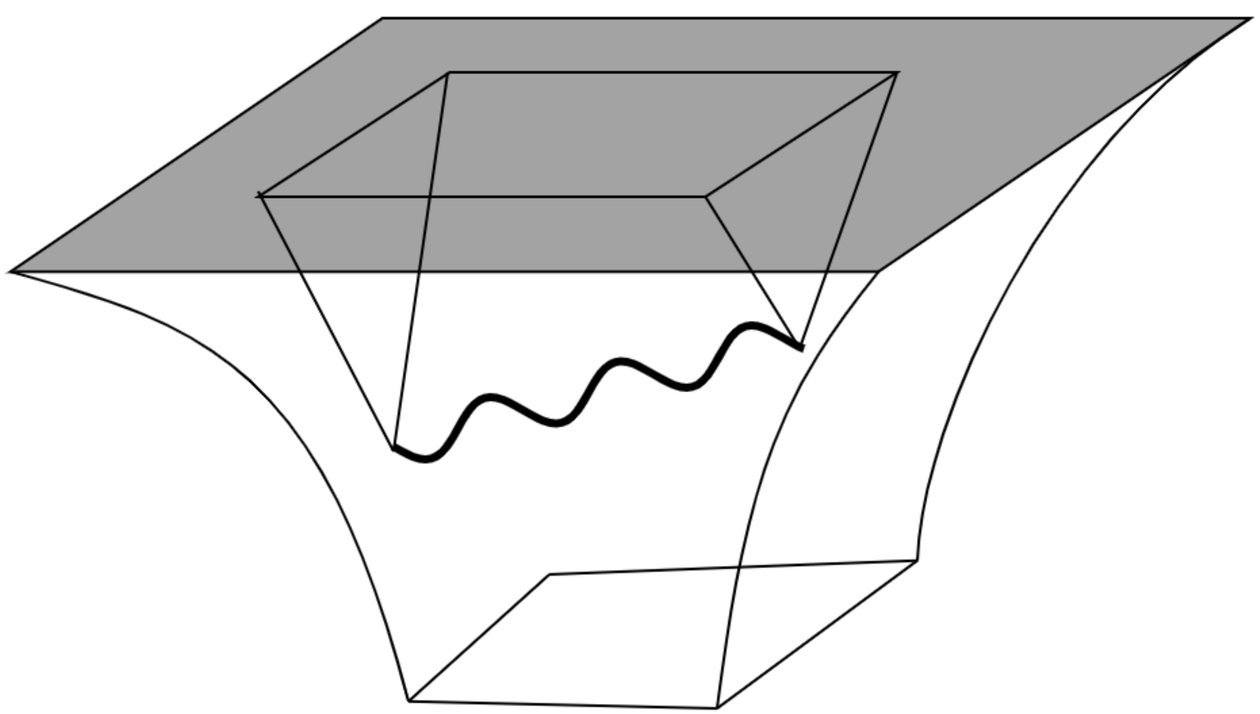
Cosmological bootstrap

Shifting the perspective on cosmological correlators: finding them without directly following the bulk time evolution. Active field.

Recent review, Baumann et al, 2203.08121

2017-2022: Arkani-Hamed, Baumann, Benincasa, Duaso Pueyo, Goodhew, Gorbenko, Jazayeri, Joyce, Lee, Meltzer, Melville, Pajer, Penedones, Pimentel, Renaux-Petel, Sleight, Salehi-Vaziri, Stefanyszyn, Tarona ….

Earlier works: Bzowski et al (2011,2012, 2013), Raju (2012), Kundo et al (2013, 2015), Maldacena and Pimentel(2011)



Boundary Rules

↑

Locality, Unitarity, Analyticity, Symmetries

Our work

Cosmological collider + cosmological bootstrap + breaking dS boosts

Production of heavy particles

Exact solution from first principles

Different propagation speeds

↙ ↘

Observational consequences and physical understanding

Theoretical methods

I Low speed collider

II Cosmological bootstrap in slow motion

The setup

- Curvature perturbation $\zeta = -H\pi$ with a reduced sound speed c_s

$$S_\pi = \int d\eta\, d^3\boldsymbol{x}\, a^2 \epsilon H^2 M_{\rm Pl}^2 \left[\frac{1}{c_s^2}\left(\pi'^2 - c_s^2(\partial_i\pi)^2\right) - \frac{1}{a}\left(\frac{1}{c_s^2}-1\right)\left(\pi'(\partial_i\pi)^2 + \frac{A}{c_s^2}\pi'^3\right) + \dots\right]$$

- Additional relativistic heavy field

$$S_\sigma^{(2)} = \int d\eta d^3\boldsymbol{x}\, a^2\left(\frac{1}{2}\sigma'^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{1}{2}m^2a^2\sigma^2\right) \qquad \mu = \sqrt{\frac{m^2}{H^2}-\frac{9}{4}} > 0$$

- Quadratic and cubic couplings

$$S_{\pi\sigma} = \int d\eta d^3\mathbf{x}\, a^2\left(\rho a \pi_c' \sigma + \frac{1}{\Lambda_1}\pi_c'^2\sigma + \frac{c_s^2}{\Lambda_2}(\partial_i\pi_c)^2\sigma\right) \qquad \pi_c = \sqrt{2\epsilon} H M_{\rm Pl} c_s^{-1}\pi$$

Low speed collider

Qualitative picture

Two characteristic times in dynamics

Event 1: sound horizon crossing for pi $k/a = H/c_s$

Event 2: mass crossing for the heavy field $k/a = m$

For $m > H/c_s$ event 2 is before 1, not qualitatively different from cs=1

For $m < H/c_s$ event 1 is before 2, unusual:

Between 1 and 2, curvature perturbation outside sound horizon quantum-mechanically interacts with sigma still in the Bunch-Davies vacuum

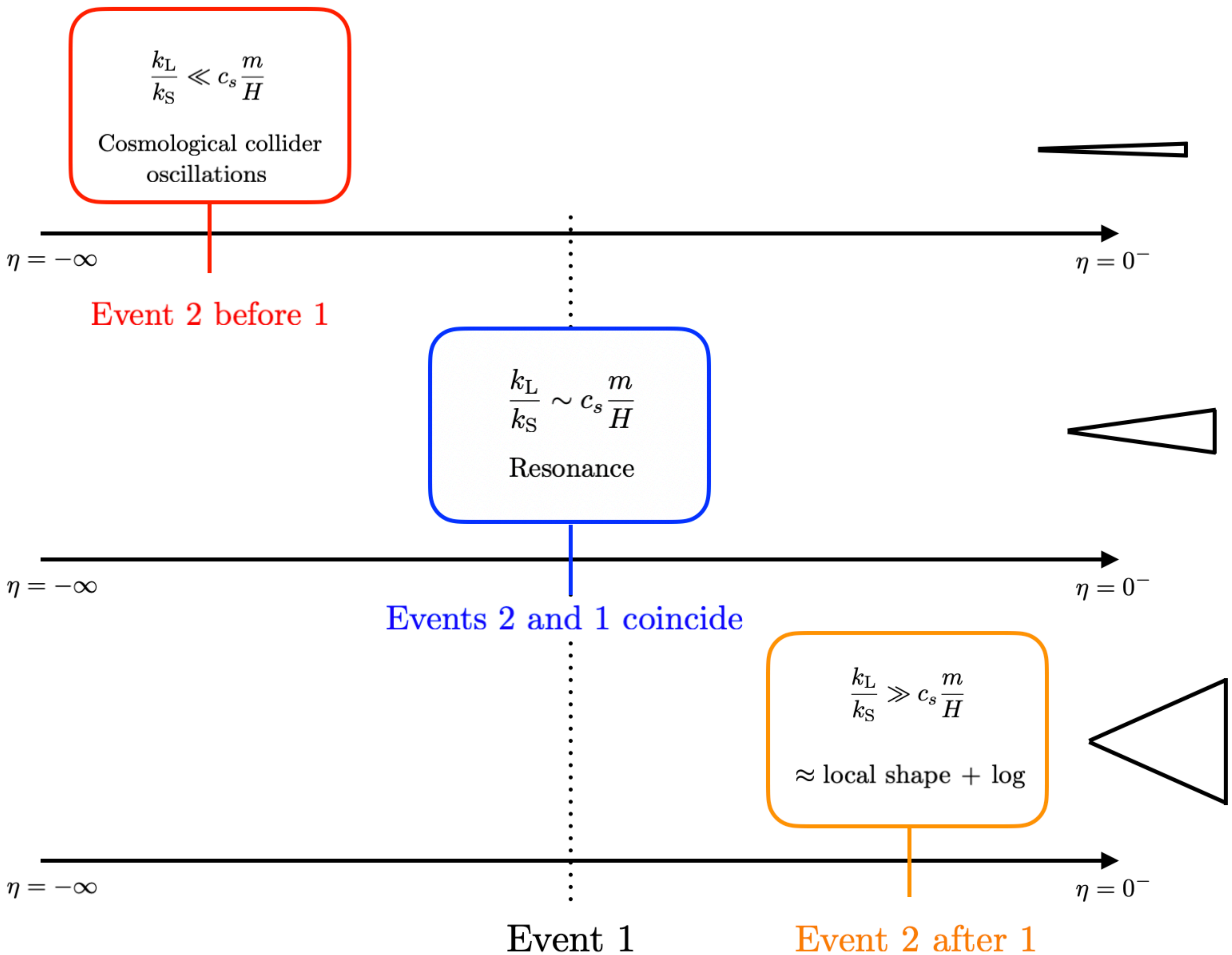
$\longrightarrow$

Growth of curvature power spectrum during $-\log(c_s m/H)$ e-folds

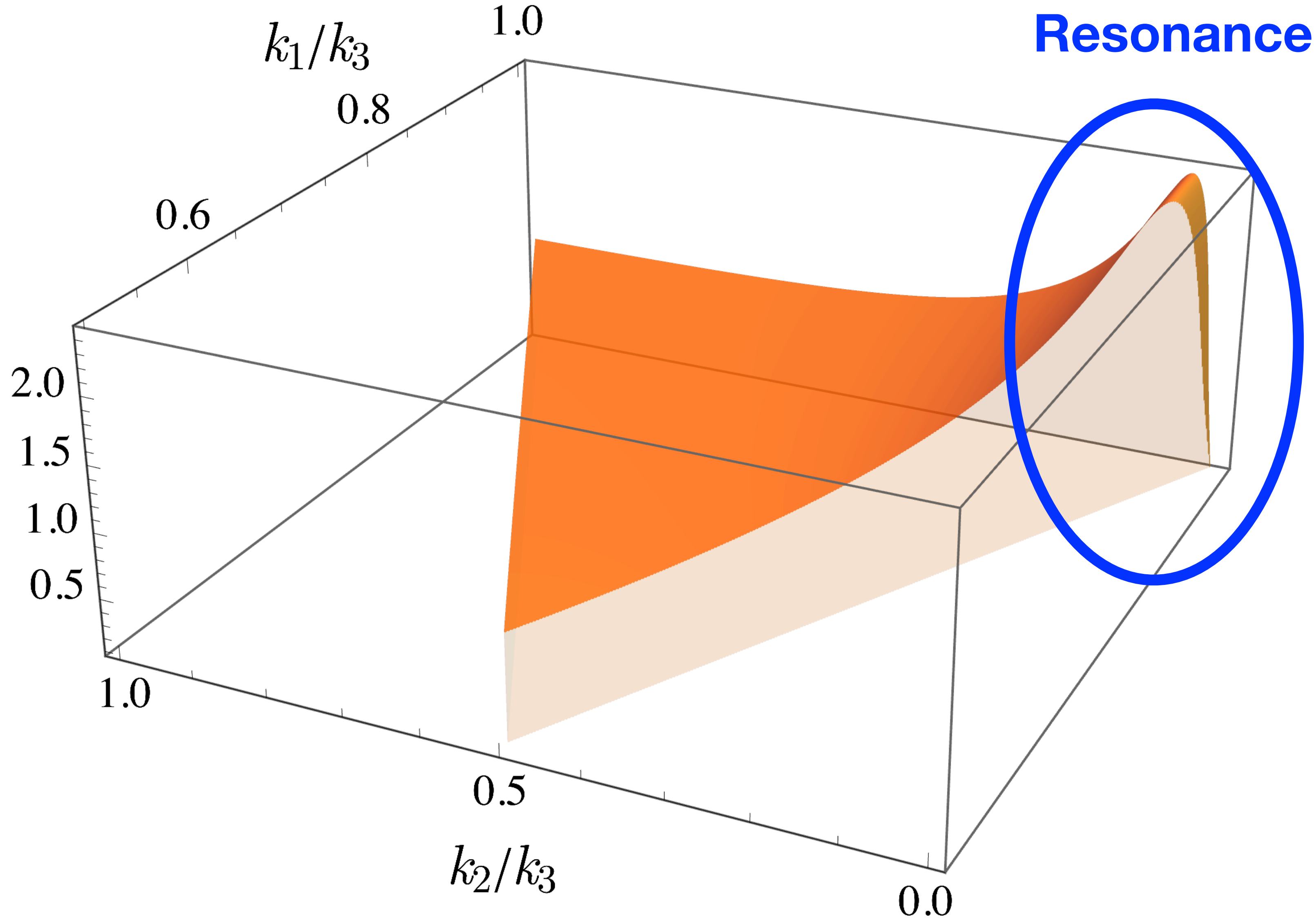
Qualitative picture (bispectrum)

Event 1: sound horizon crossing for pi's short mode $k_{\rm S}/a(t_1) = H/c_s$

Event 2: mass crossing of long mode $k_{\rm L}/a(t_2) = m$



New signature of heavy fields: the low speed collider



Bispectrum shape (normalised)

$$c_s = 0.03, \mu = 2$$

$$f_{\rm NL}^{\rm eq} \sim \left(\frac{\rho}{H}\right)^2 \lesssim \left(\frac{m}{H}\right)^2$$

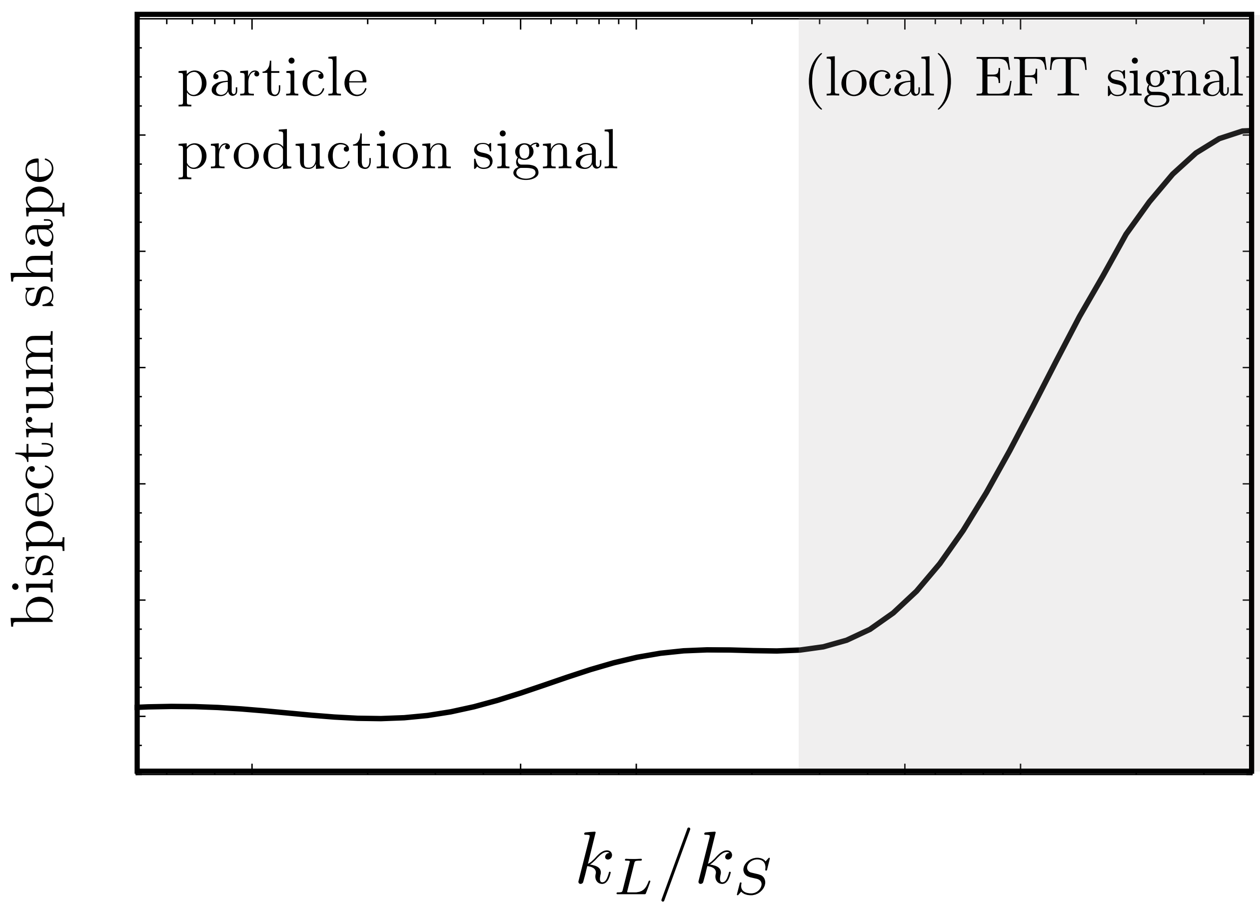
weak mixing

New signature of heavy fields: the low speed collider

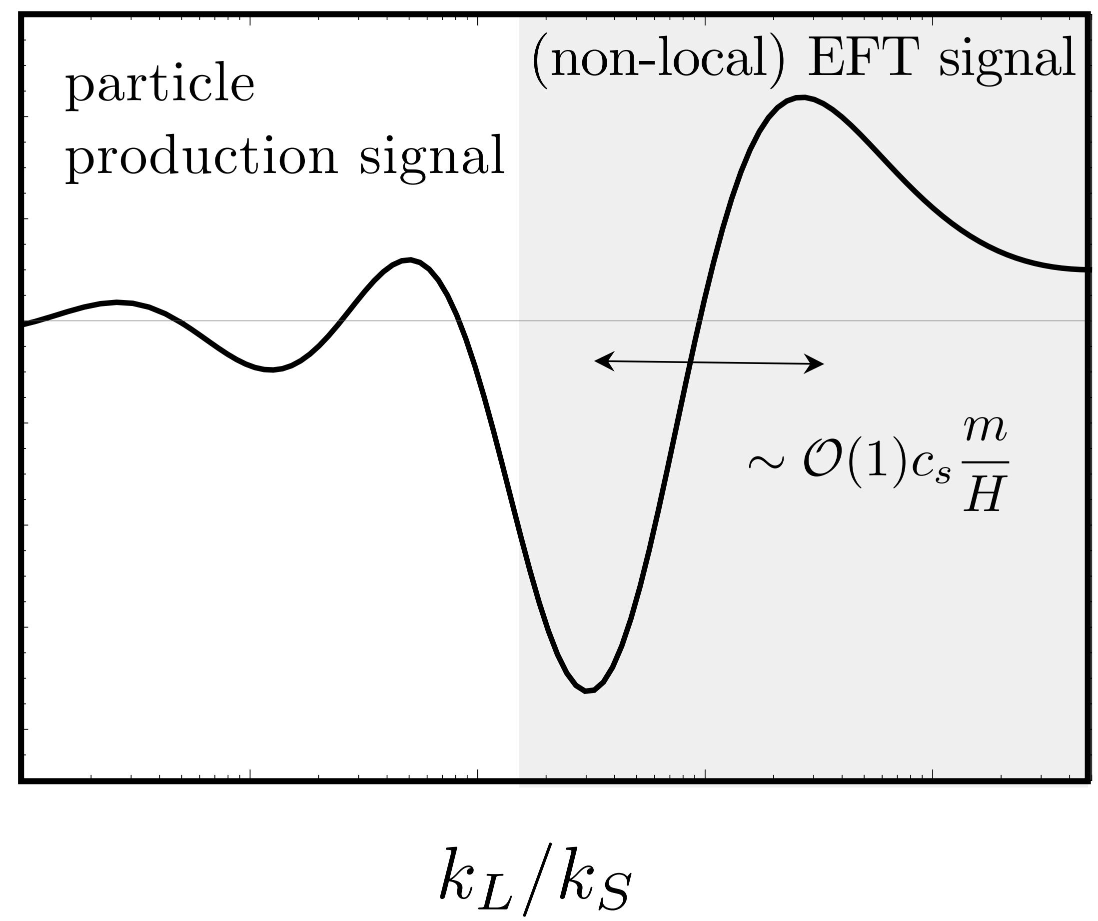
$c_s = 1,\, m \gg H$

$c_s \ll 1,\, H \ll m \ll H/c_s$

de Sitter Invariant Collider



Low Speed Collider



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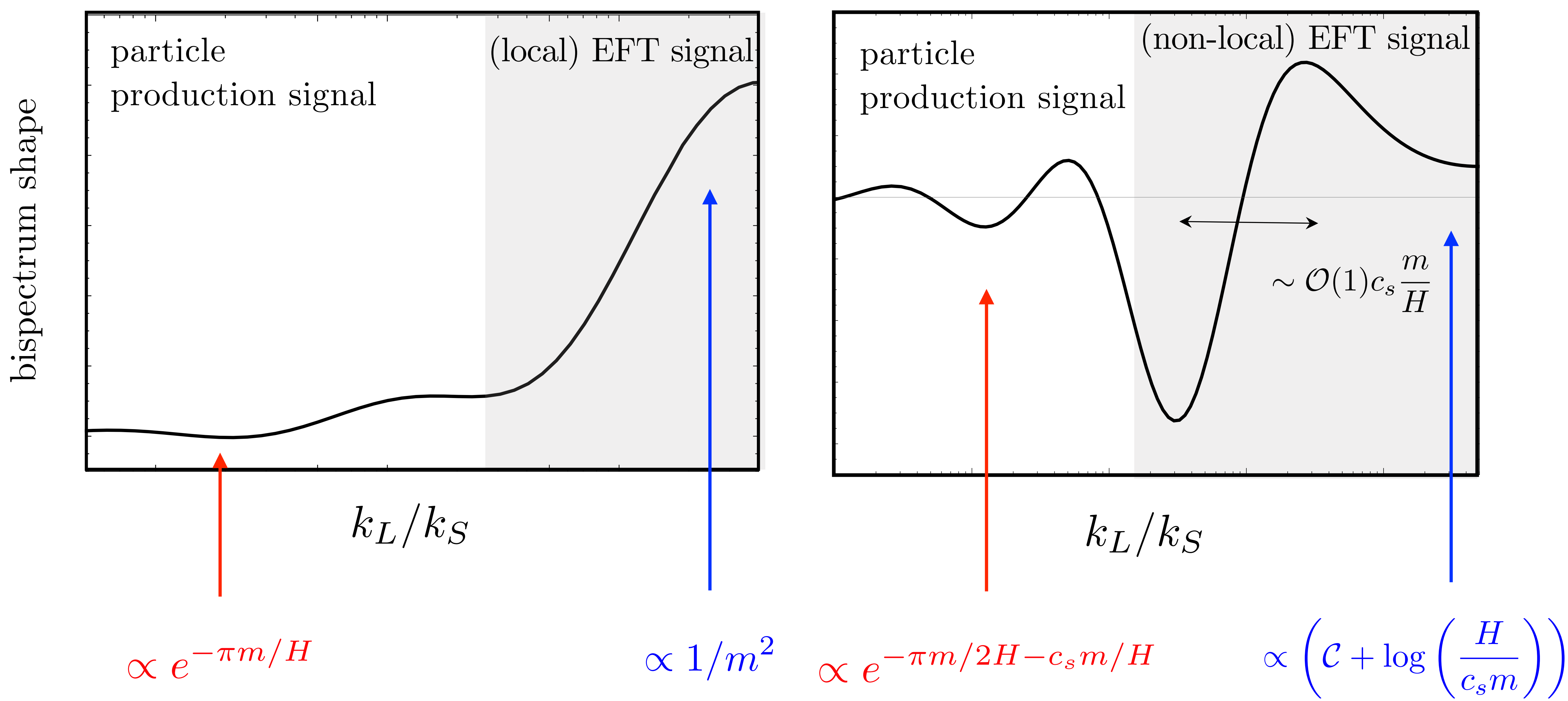
de Sitter Invariant Collider $\qquad$ Low Speed Collider



Non-local single-field EFT

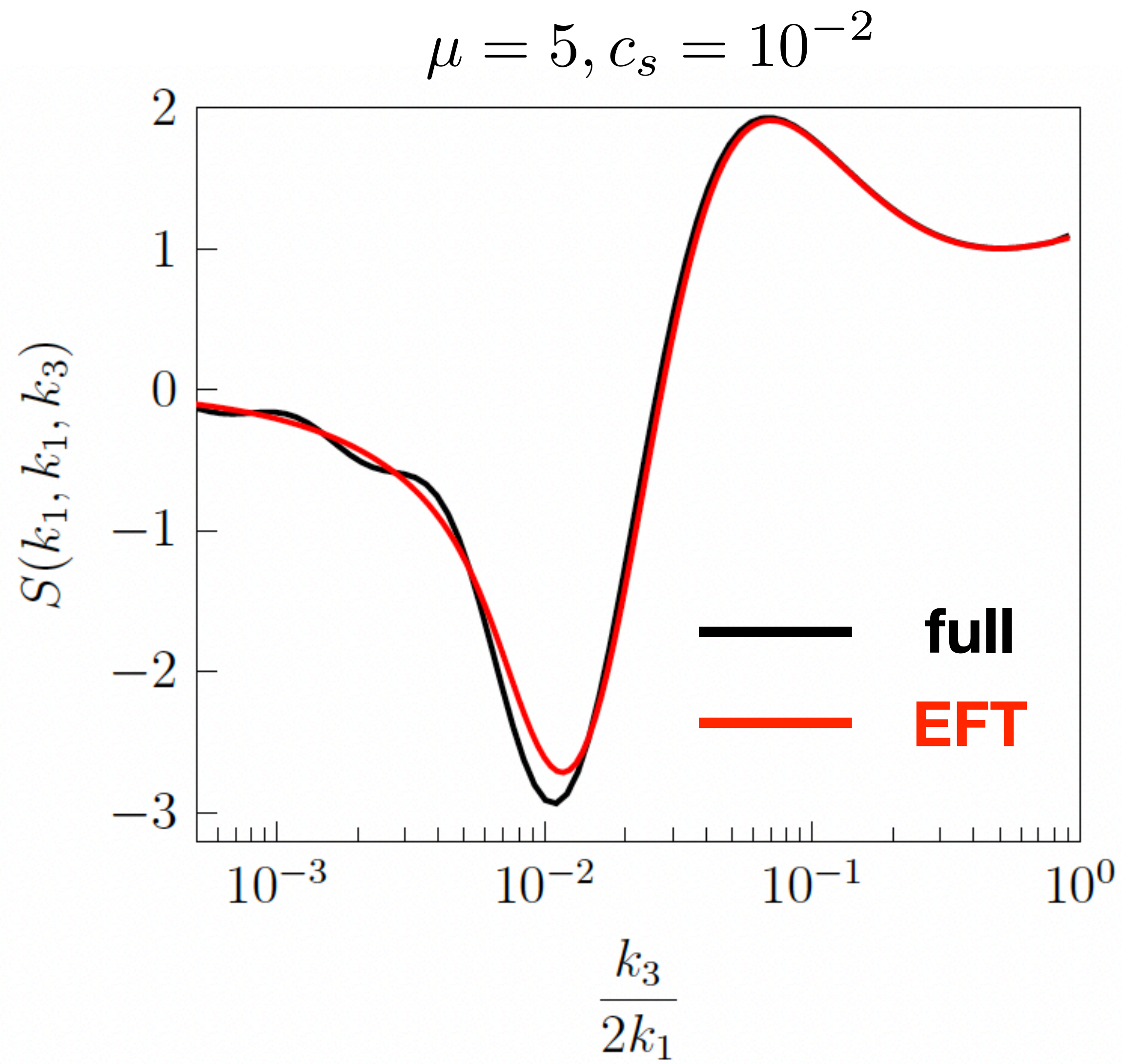
For a low sound speed, the heavy supersonic field instantaneously responds to the dynamics of the curvature perturbation

The heavy field can be integrated out, but in a non-standard manner (the field is relativistic at sound horizon crossing), yielding a (spatially) non-local single-field EFT

$$S_{\pi,\mathrm{induced}} = \int d\eta\, d^3\mathbf{x}\, a^2(\eta) \left(\frac{\rho^2}{2} \pi_c' \frac{1}{m^2 - 2H^2 - H^2\eta^2\nabla^2} \pi_c' + \frac{\rho}{a(\eta)\Lambda_1} \pi_c'^2 \frac{1}{m^2 - 2H^2 - H^2\eta^2\nabla^2} \pi_c' + \frac{\rho c_s^2}{a(\eta)\Lambda_2} (\partial_i \pi_c)^2 \frac{1}{m^2 - 2H^2 - H^2\eta^2\nabla^2} \pi_c' \right)$$

Non-local single-field EFT

Simple analytical formulae: one-parameter family of shapes, depending on order parameter $\alpha = c_s(\mu^2 + 1 + 4)^{1/2} \approx c_s m/H$



✔ Boil down to standard equilateral shapes for large alpha

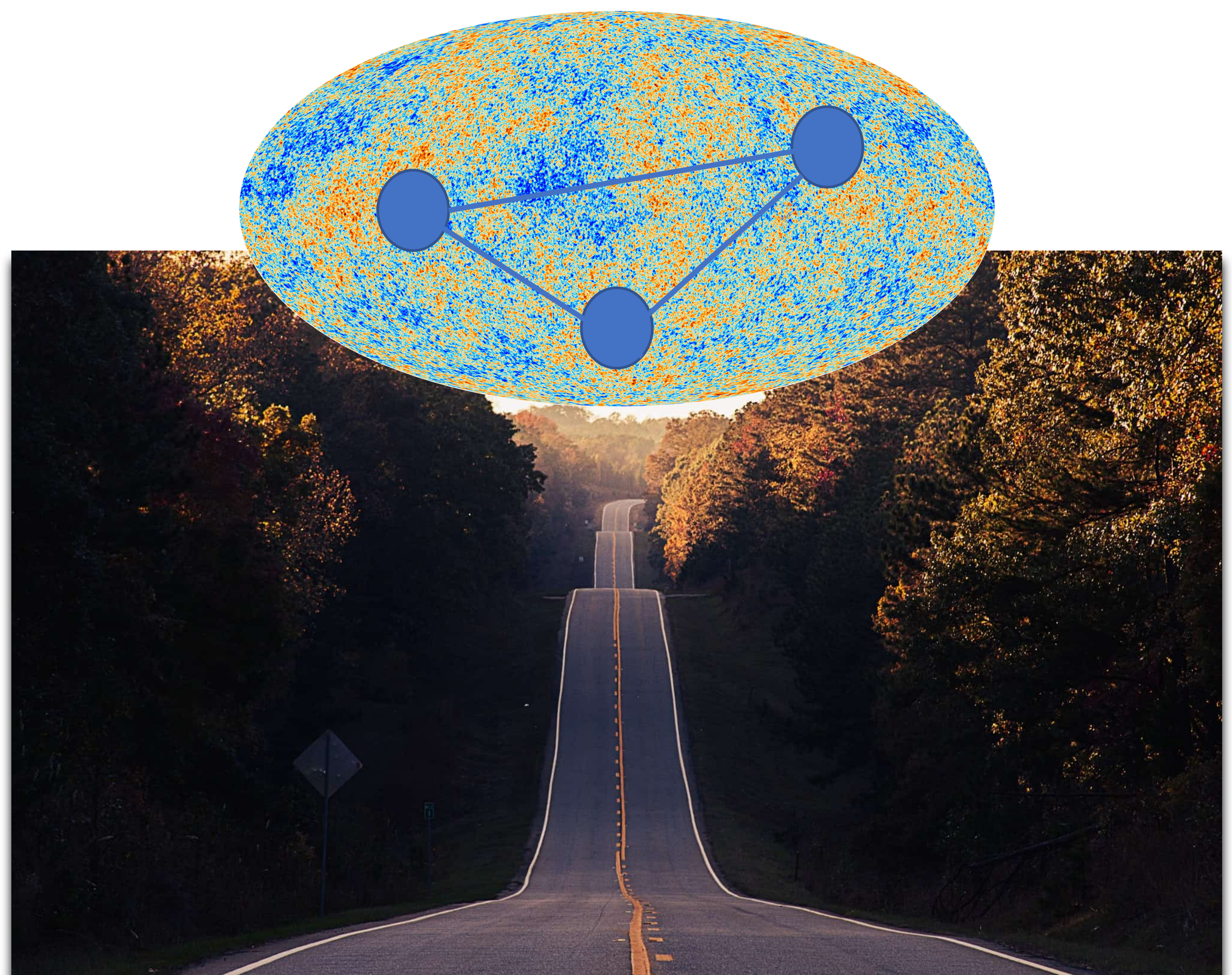
✔ Reproduce resonances of low speed collider for small alpha

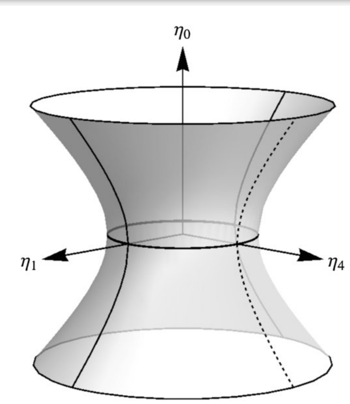
✘ Misses particle production effects

✘ Breaks down for masses too close to 3H/2

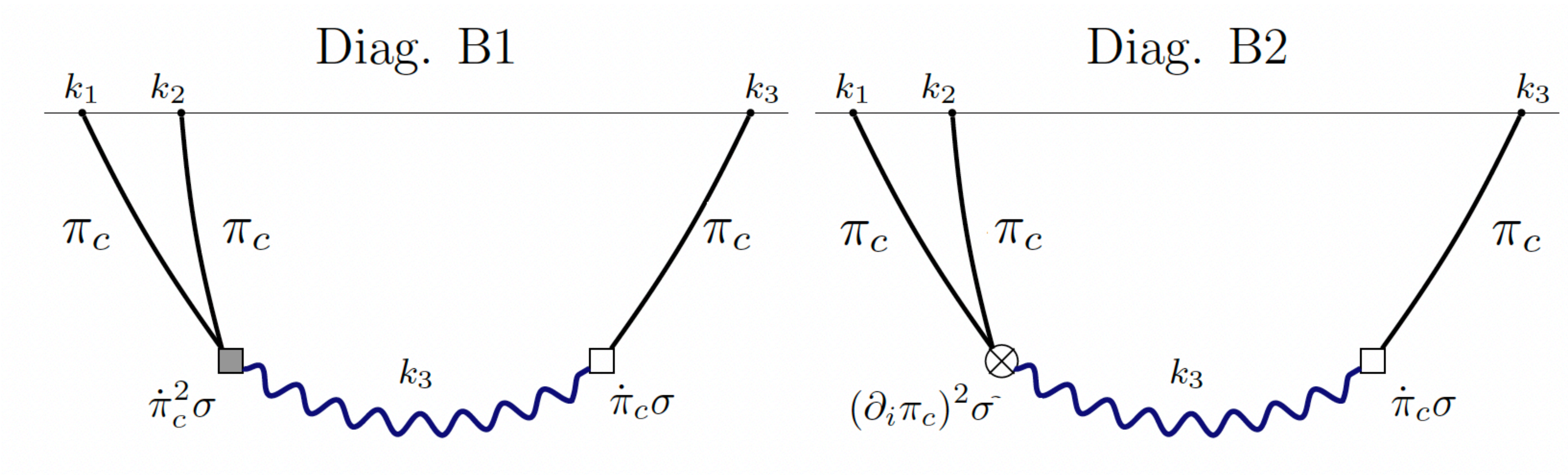
II Cosmological bootstrap

From a de Sitter seed four-point to inflationary correlators





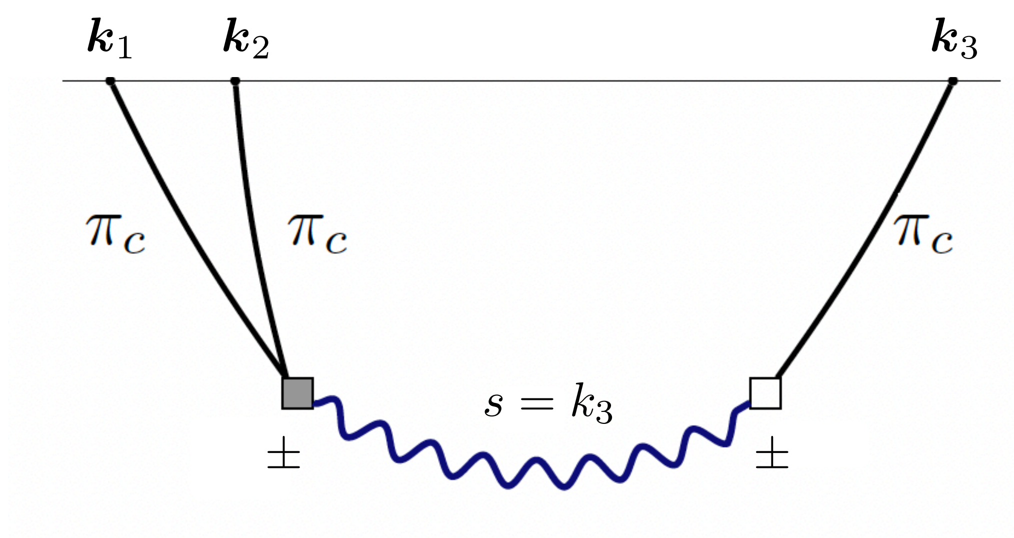
Diagrams of interest



with same method: 2 pt, 4 pt, higher order derivatives (see paper)

See 2205.00013 for another method, for bispectrum only

The In-in Computation is difficult



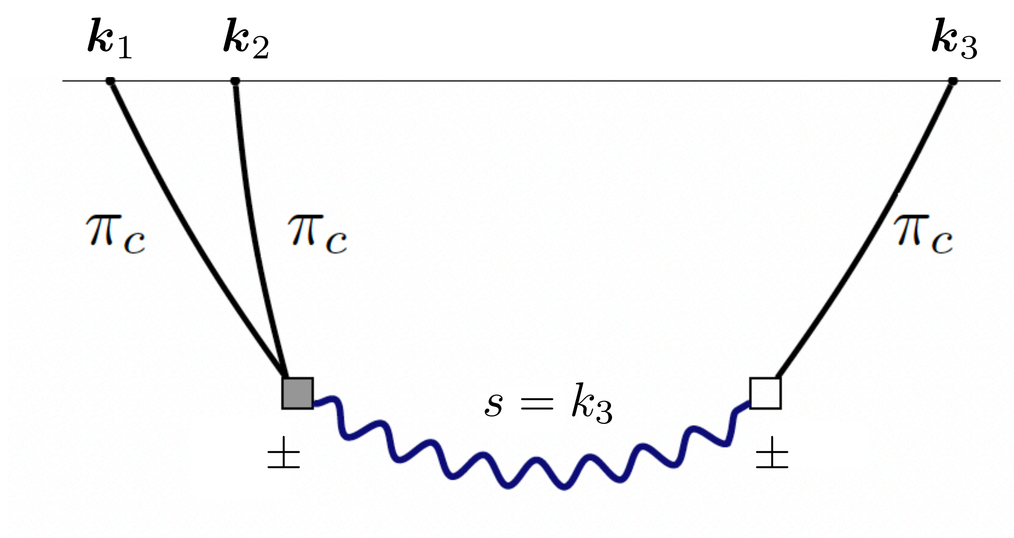
$$\langle \pi_c^3 \rangle = \sum_{\pm \text{ at each vertex}} \int \prod_{i=1}^{V} d\eta_i \ \text{vertex}_i(\boldsymbol{k}_i, \eta_i)$$

$$\prod_{j=1}^{n} \pi_c^{\mp}(k_j, \eta_i) \pi_c^{\pm}(k_j, \eta_0) \times \prod_{l=1}^{L} G_{\pm\pm}(s_l, \eta_{1l}, \eta_{2l})$$

bulk-to-boundary

bulk-to-bulk

The In-in Computation is difficult



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mode functions

$$\pi_c^{\pm}(k,\eta) = \frac{iH}{\sqrt{2c_s^3 k^3}}(1 \pm i c_s k\eta)\exp(\mp i c_s k \eta)$$

$$\sigma_+(k,\eta) = \frac{\sqrt{\pi}H}{2}\exp(-\pi\mu/2)\ \exp(i\pi/4)(-\eta)^{3/2}H_{i\mu}^{(1)}(-k\eta)$$

bulk-to-bulk propagators

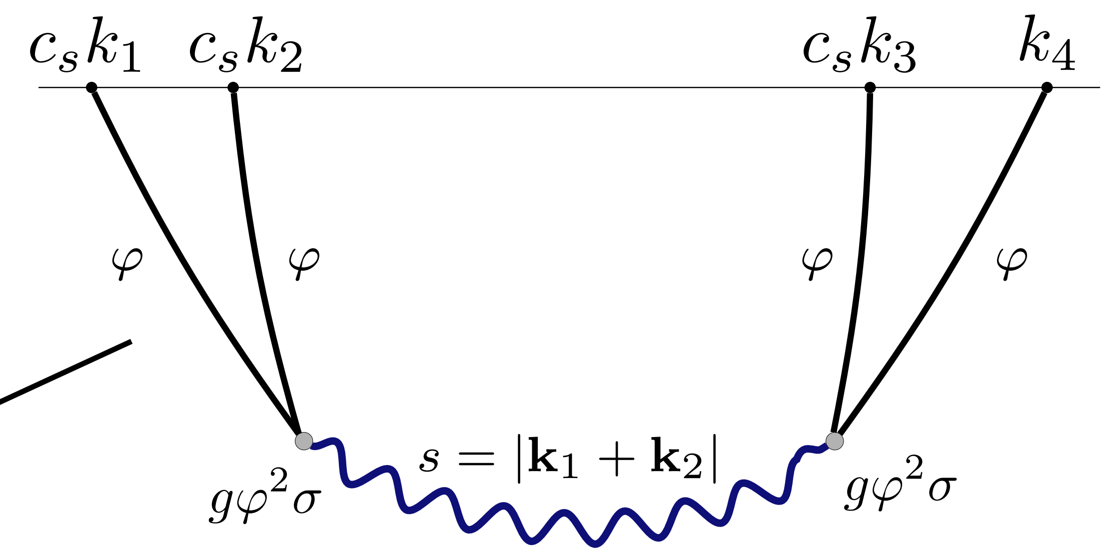
$$G_{++}(s,\eta,\eta') = \sigma_+(s,\eta)\sigma_-(s,\eta')\theta(\eta-\eta') + \eta \leftrightarrow \eta'$$

$$G_{+-}(s,\eta,\eta') = \sigma_+(s,\eta)\sigma_-(s,\eta')$$

Bispectra from a dS four-point

All our diagrams (2-,3-,4-pt) can be related to a **de Sitter-invariant seed four-point function** of a conformally coupled field, e.g.:

$$B(k_1, k_2, k_3) = W(k_i, \partial_{k_i}) \lim_{k_4 \to 0}$$



$$\varphi_\pm(k, \eta) = -\frac{H}{\sqrt{2k}}\eta \exp(\mp ik\eta)$$

(relativistic dispersion relation)

Boost breaking manifests itself both

- in the weight-shifting operators (*boost breaking vertices*)

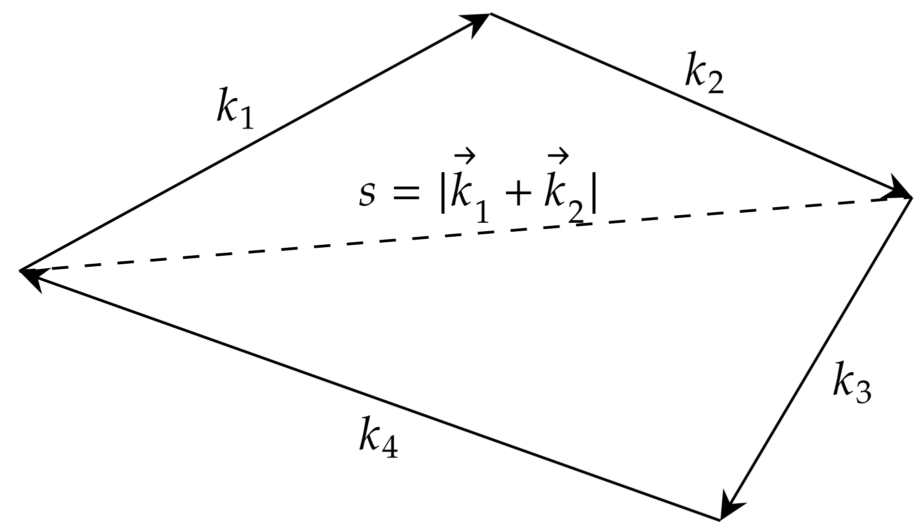
- and also in the argument of the four-point function (*different speeds of propagation*) $k_i\ (i = 1, 2, 3) \to c_s k_i$

Bispectra from a dS four-point

The seed correlator $\hat{F}(u,v)$ has been found in 1811.00024

$$u = \frac{s}{k_1 + k_2} \leq 1$$

$$v = \frac{s}{k_3 + k_4} \leq 1$$



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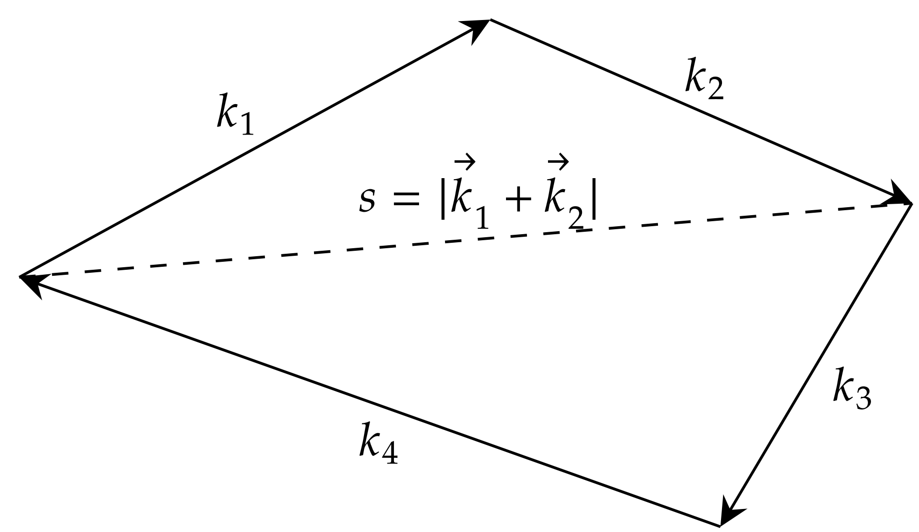


… but we need its analytical continuation beyond the kinematically allowed region

$$k_i(i=1,2,3) \to c_s k_i,\ k_4 \to 0,\ s = |\mathbf{k}_3 + \mathbf{k}_4| \to k_3$$

$$u \to \frac{k_3}{c_s(k_1+k_2)}\,, v \to \frac{1}{c_s}$$

Building blocks of the seed correlator

$$\hat{F} = \hat{F}_{++} + \hat{F}_{--} + \hat{F}_{+-} + \hat{F}_{-+}$$

Difficult

Truly nested integrals

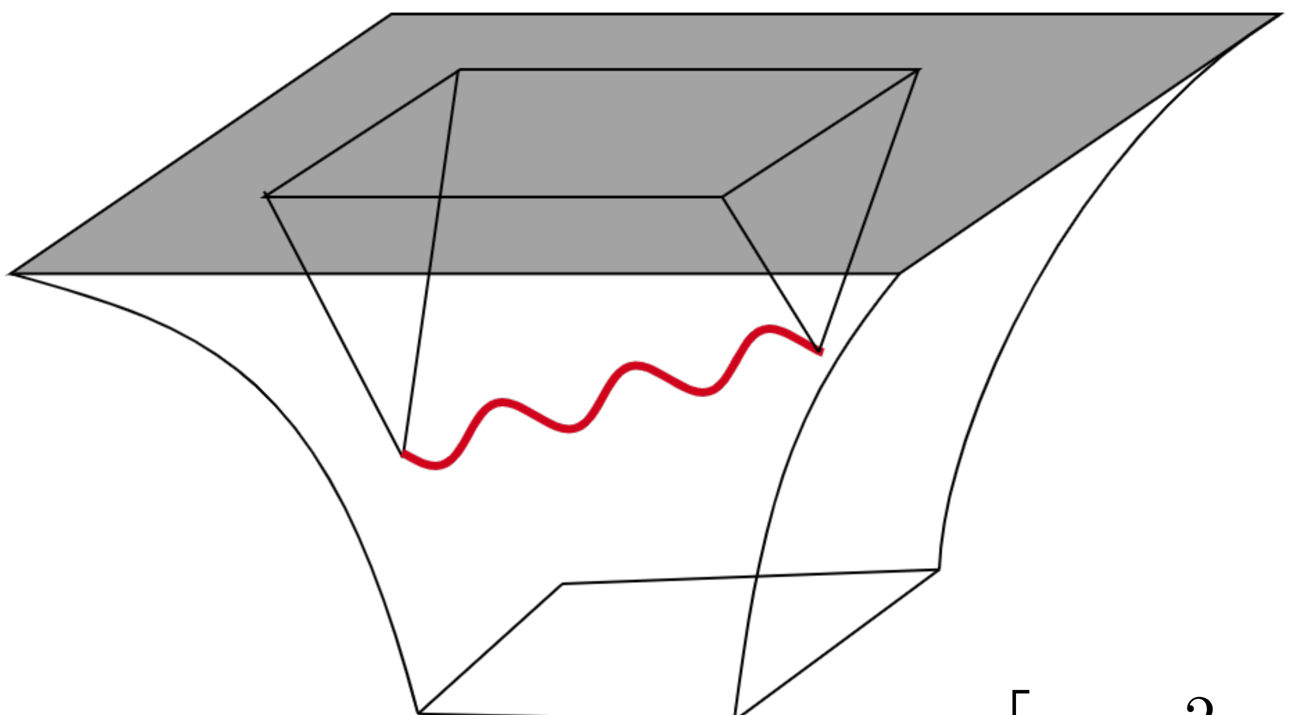
Easiest part

Factorised time integrals

$$F_{\pm\pm}(k_1,\ldots,k_4;s) = -\frac{g^2}{2H^2}\int_{-\infty(1\mp i\epsilon)}^{\eta_0}\frac{d\eta}{\eta^2}\int_{-\infty(1\mp i\epsilon)}^{\eta_0}\frac{d\eta'}{\eta'^2}e^{\pm i(k_1+k_2)\eta}\,e^{\pm i(k_3+k_4)\eta'}$$
$$\times\, G_{\pm\pm}(s,\eta,\eta')$$

$$G_{++}(s,\eta,\eta') = \sigma_+(s,\eta)\sigma_-(s,\eta')\theta(\eta-\eta') + \eta \leftrightarrow \eta'$$

Bootstrap. I Locality



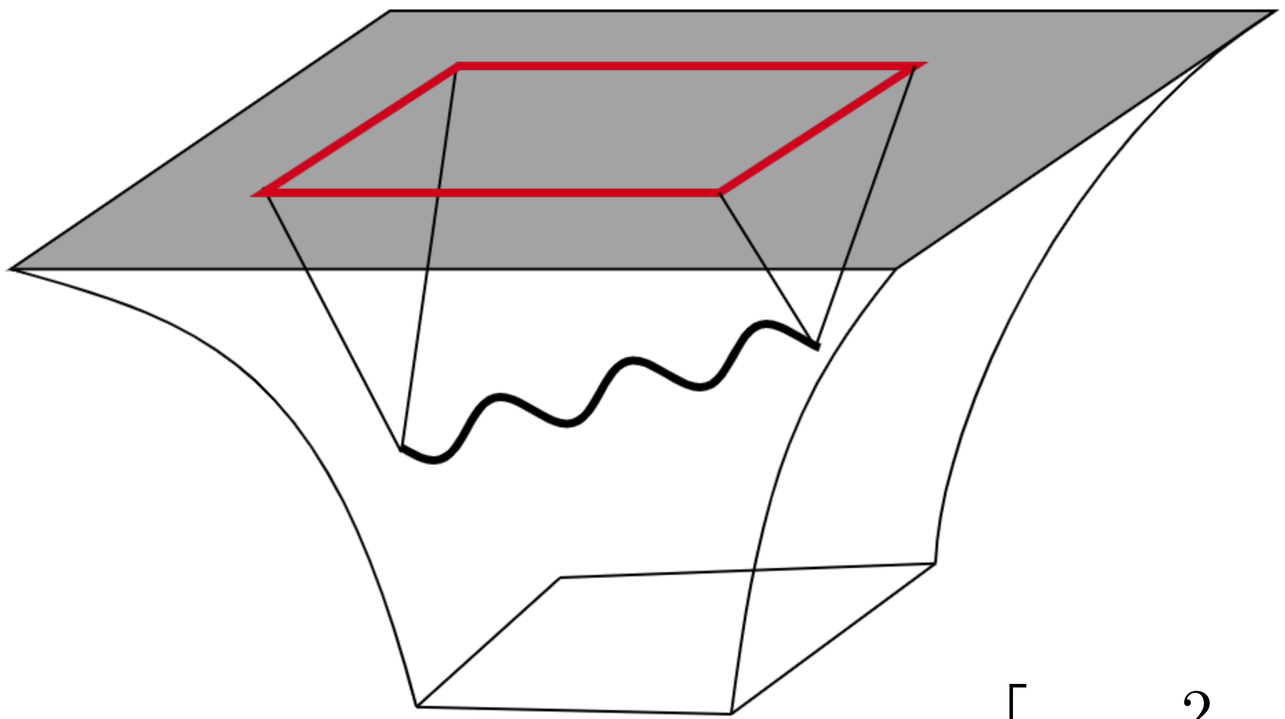
$$\left[\partial_\eta^2 - \frac{2}{\eta}\partial_\eta + k^2 + \frac{m^2}{\eta^2 H^2}\right] G_{\pm\pm}(s,\eta,\eta') = (\eta' H)^2 \delta(\eta - \eta')$$

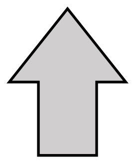
Bulk local differential equation

Bootstrap. I Locality

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \left(\mu^2 + \frac{1}{4}\right)\right]\hat{F}_{\pm\pm}(u,v) = g^2 \frac{u\,v}{2(u+v)}$$

Boundary differential equation





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$$\hat{F}_{++}(u,v) = \sum_{m,n}\left(a_{m,n} + b_{m,n}\log(u)\right)\frac{1}{u^m}\left(\frac{u}{v}\right)^n + \sum_{\pm\pm}\beta_{\pm\pm}f_\pm(u)f_\pm(v)\,, \qquad 1 < |u| < |v|$$

Suitable particular solution "from 'infinity"

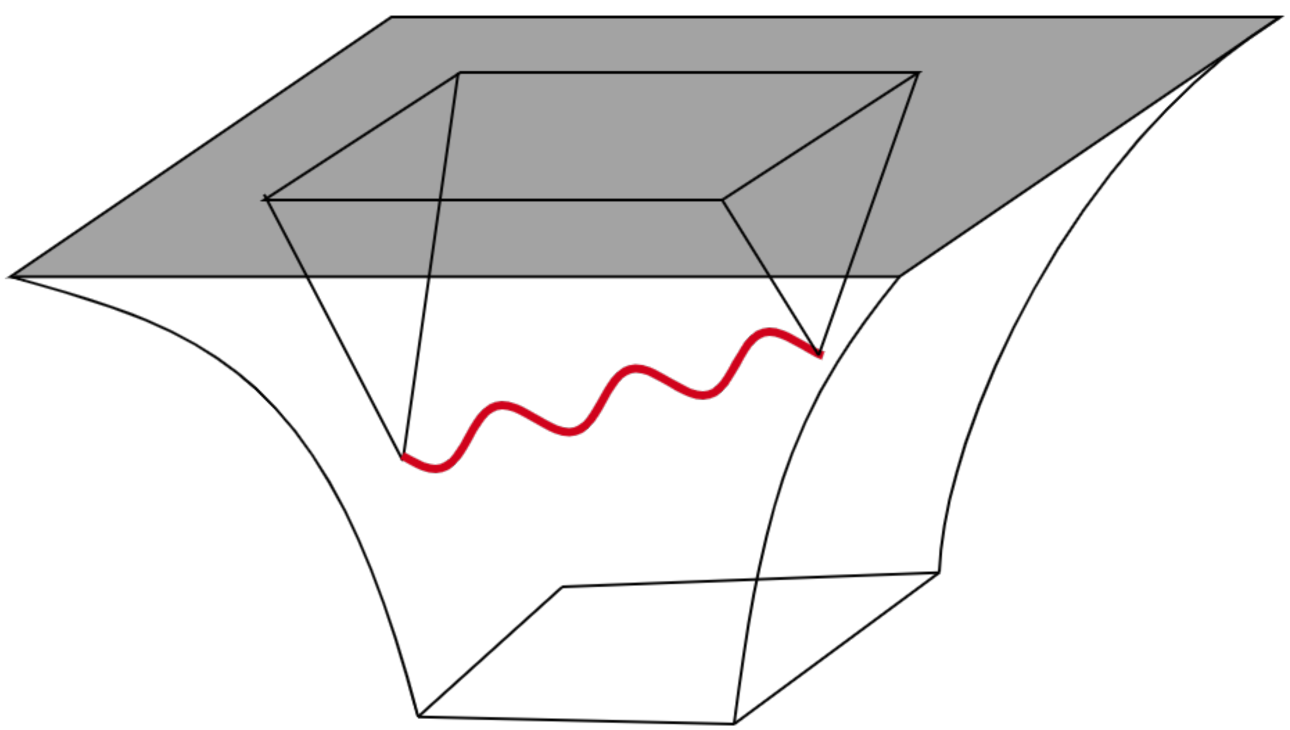
Homogeneous solution with four free parameters to determine

Series coefficients and partial resummation

$$f_+(u) = {}_2F_1\left(\frac{1}{4} - \frac{i\mu}{2}, \frac{1}{4} + \frac{i\mu}{2}; \frac{1}{2}; \frac{1}{u^2}\right)$$

$$f_-(u) = \frac{2}{u} \times {}_2F_1\left(\frac{3}{4} - \frac{i\mu}{2}, \frac{3}{4} + \frac{i\mu}{2}; \frac{3}{2}; \frac{1}{u^2}\right)$$

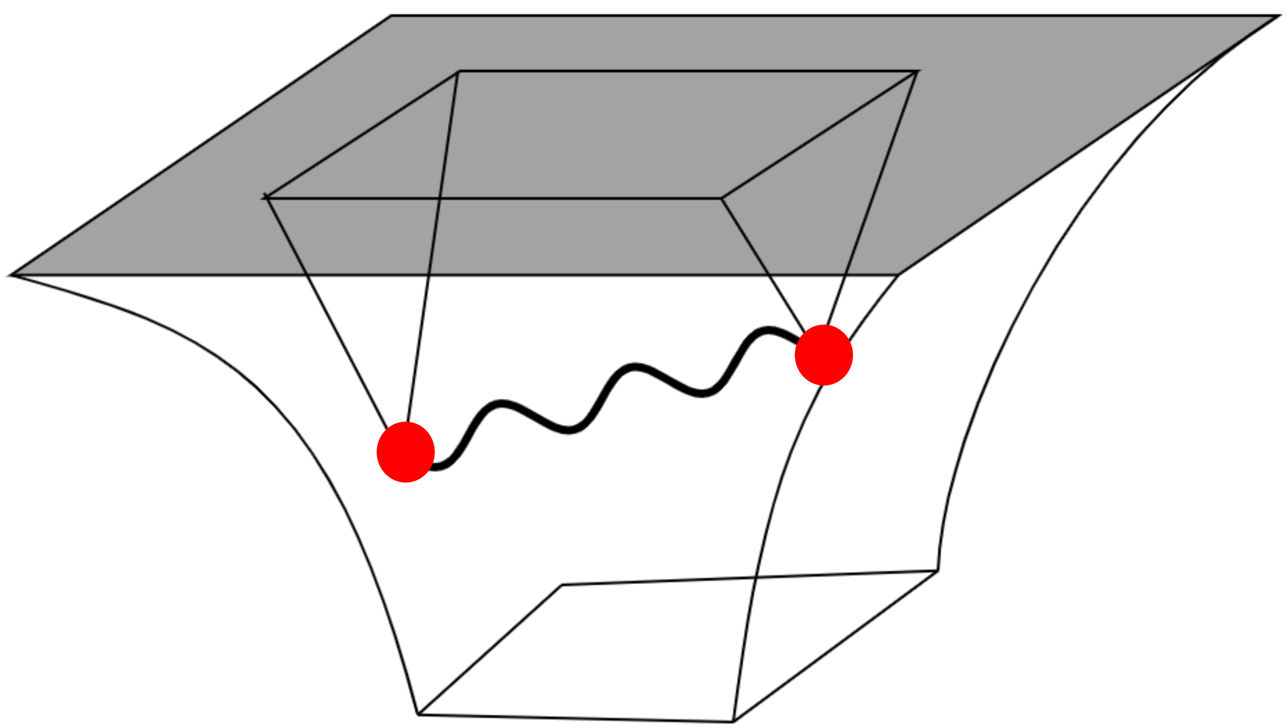
Bootstrap. 2 Unitarity



Cutting rule for the bulk to bulk propagator (energy positivity)

$$G^*_{++}(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

Bootstrap. 2 Unitarity



Reality of the couplings $g* = g$

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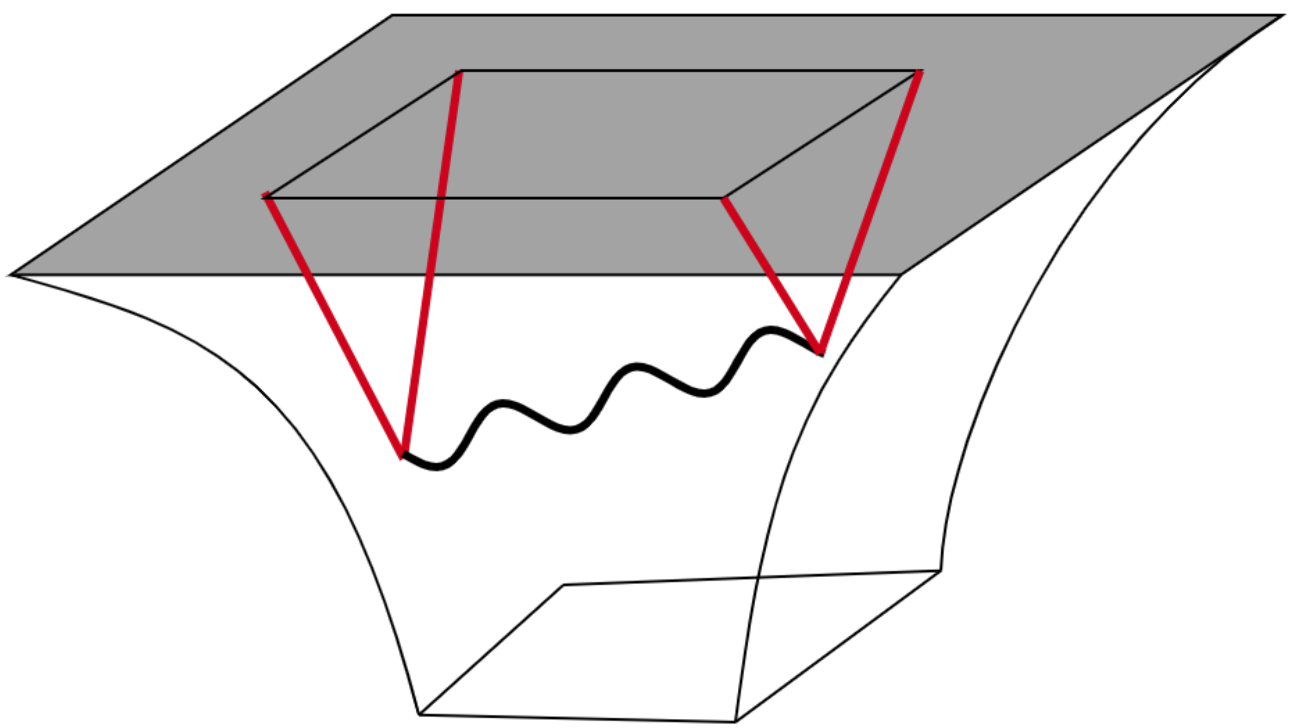

Hermitian analyticity of the bulk to boundary propagator

$$\varphi_+^*(k,\eta) = \varphi_+(-k,\eta)$$

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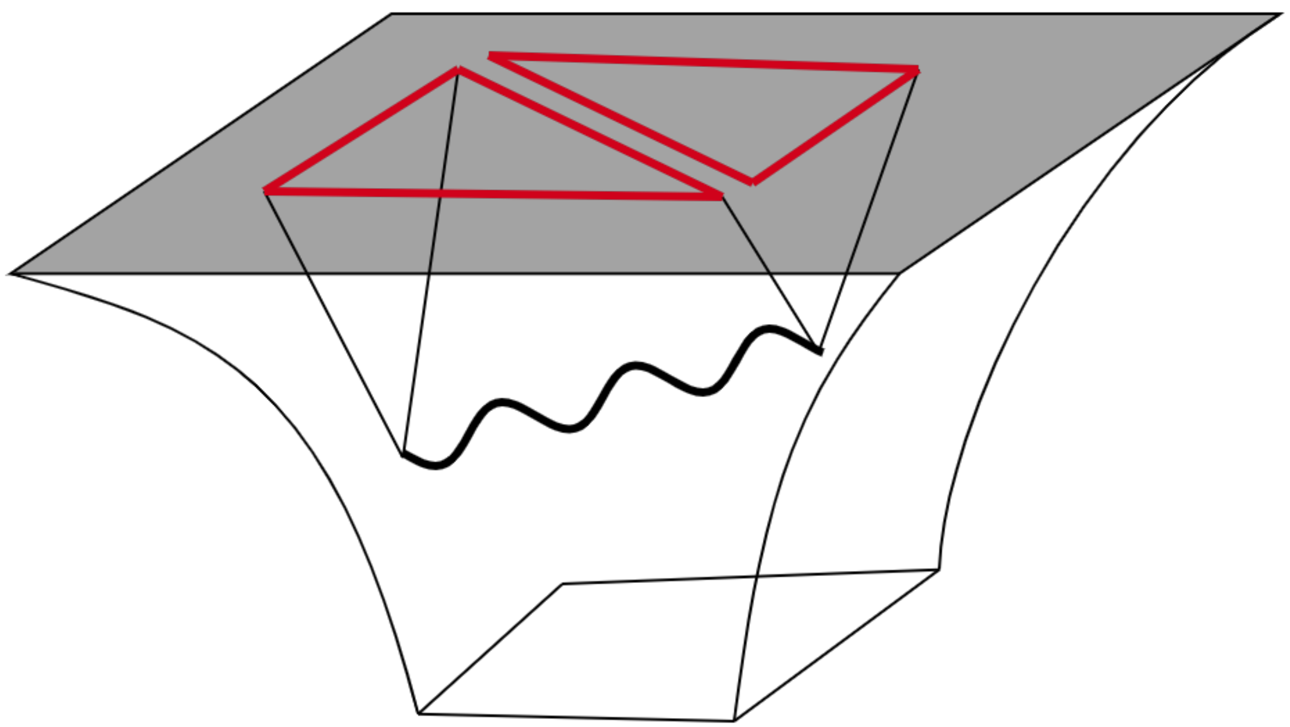
$$G_{++}^*(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

Bootstrap. 2 Unitarity

Cosmological Cutting Rule

$$\hat{F}_{++}(u,v) + \hat{F}^*_{++}(-u^*,-v^*) = -\frac{1}{2}\,\hat{f}_3(u)\hat{f}_3^*(-v^*) - \frac{1}{2}\hat{f}_3(v)\hat{f}_3^*(-u^*)$$



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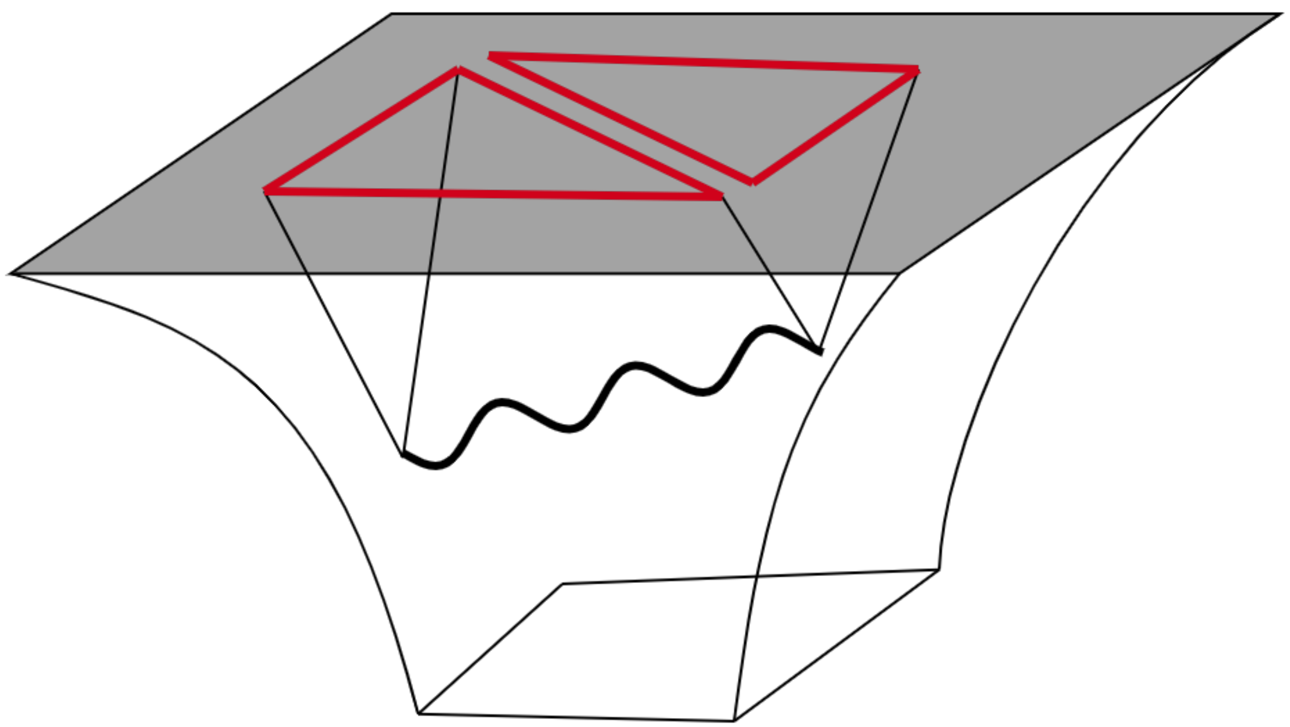
$$\hat{F}_{++}(u,v) + \hat{F}^*_{++}(-u^*,-v^*) = -\frac{1}{2}\,\hat{f}_3(u)\hat{f}_3^*(-v^*) - \frac{1}{2}\hat{f}_3(v)\hat{f}_3^*(-u^*)$$



Hermitian analyticity of the bulk to boundary propagator

$$\varphi_+^*(k,\eta) = \varphi_+(-k,\eta)$$

Reality of the couplings $g* = g$

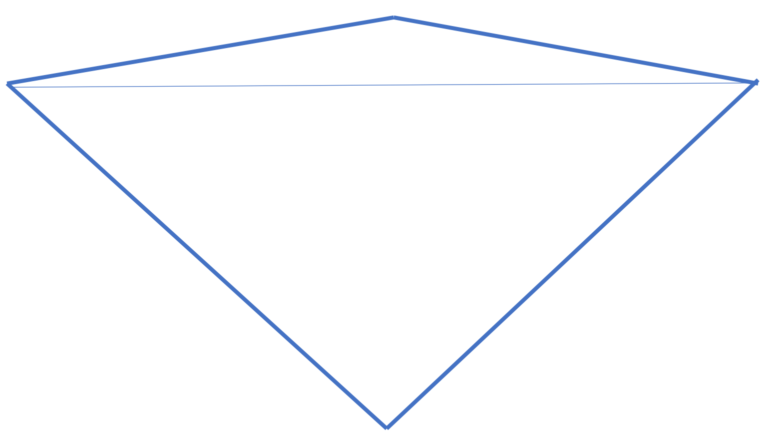
with $$\hat{f}_3(u) = \frac{ig}{2\sqrt{2\pi}}\left(|\Gamma\left(1/4 + i\mu/2\right)|^2 f_+(u) - |\Gamma\left(3/4 + i\mu/2\right)|^2 f_-(u)\right)$$

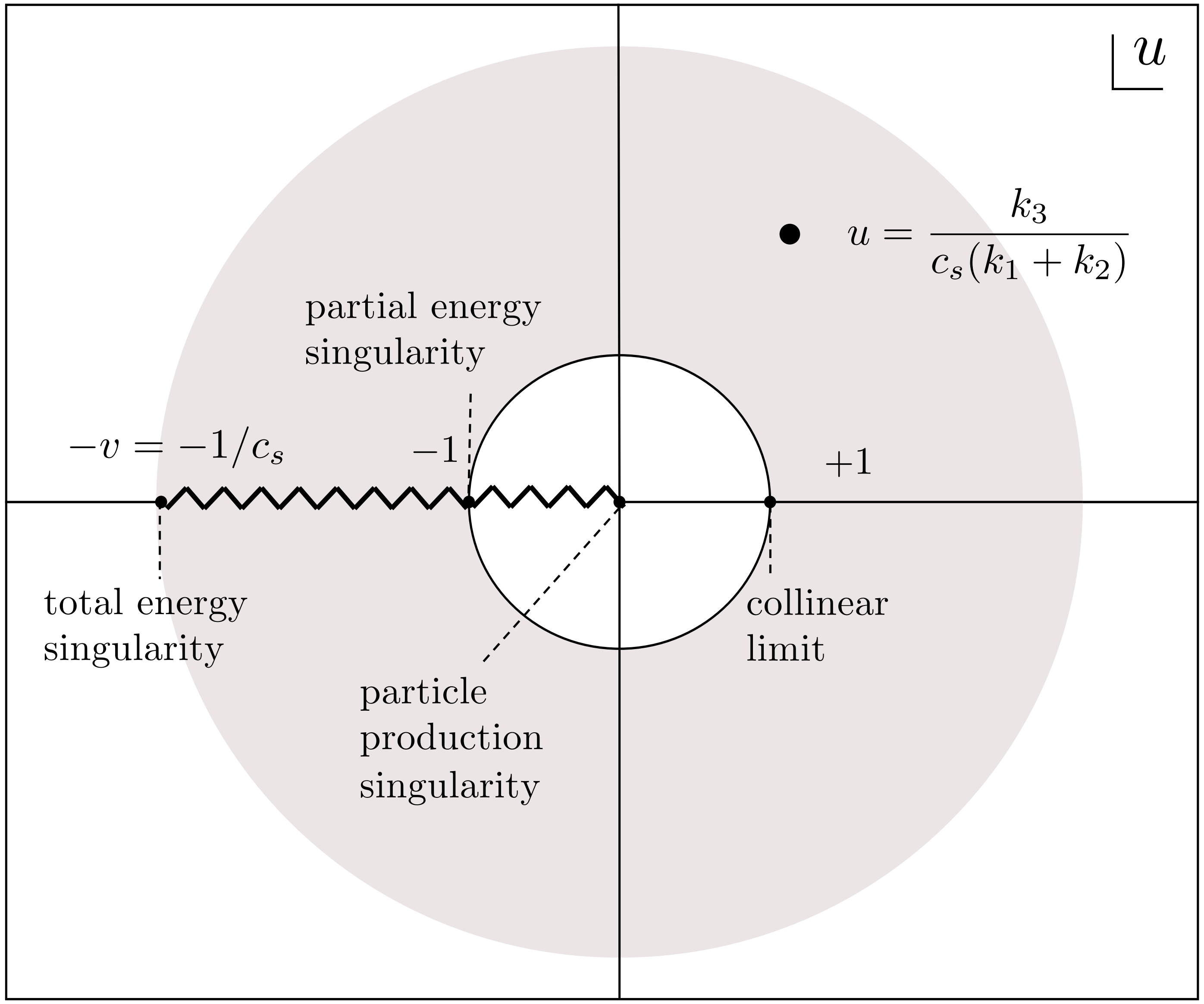
Bootstrap. 3 Analyticity

For Bunch-Davies initial conditions, no singularity can arise in the physical domain of the correlator

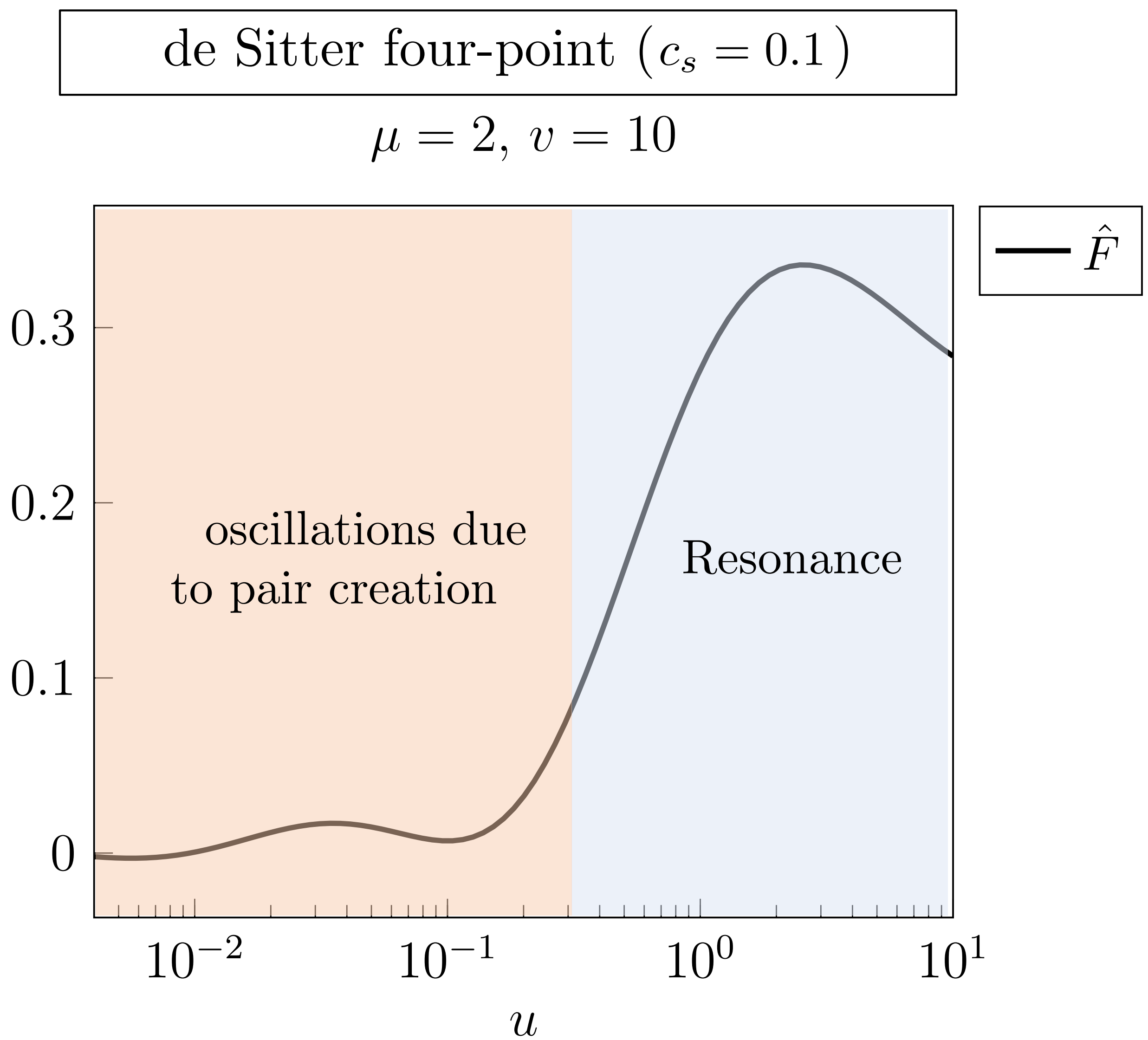
Requiring absence of singularity in the collinear (folded) limit

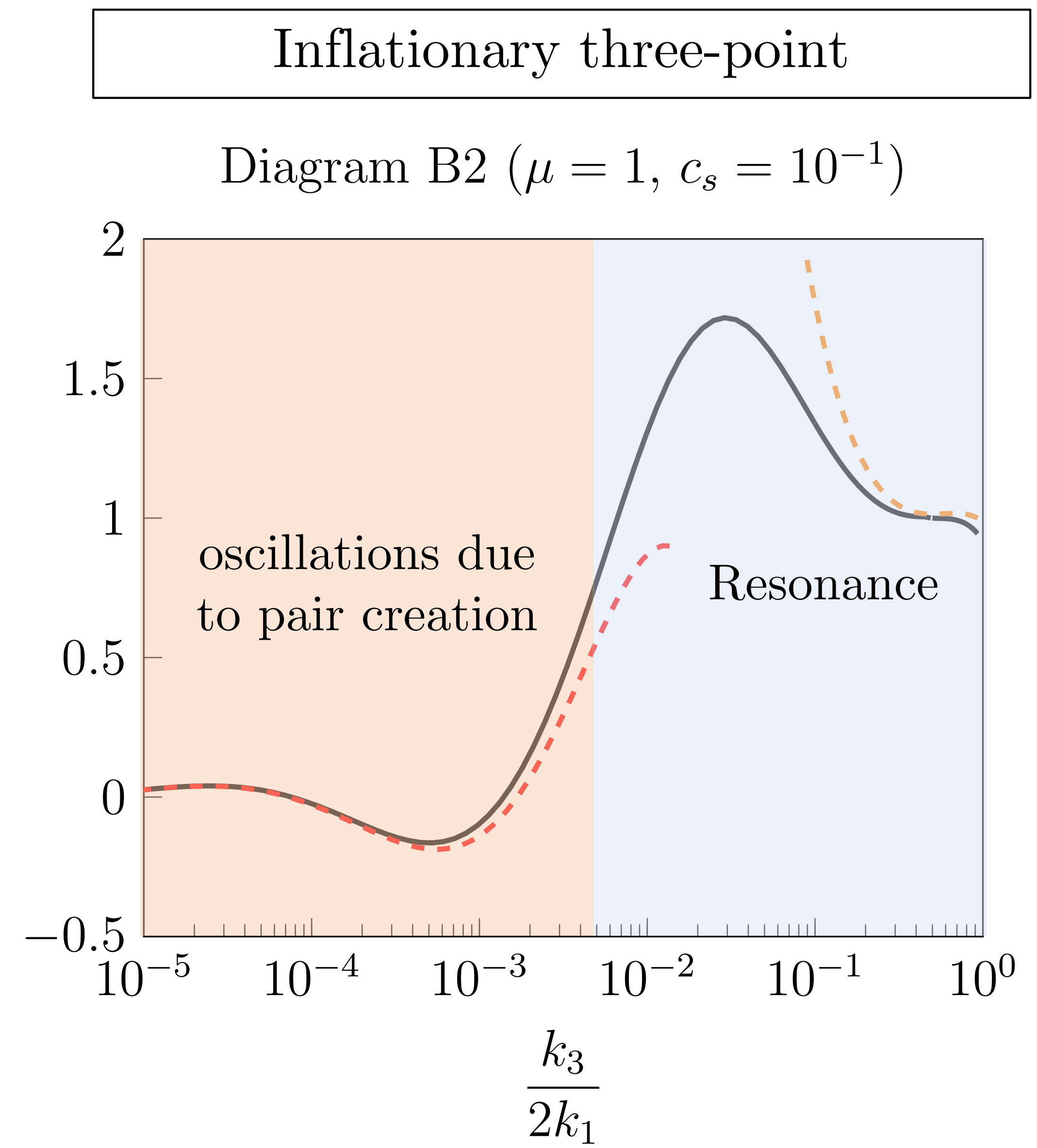
$$u \to 1$$





- Low speed collider $m < H/c_s$





Concluding remarks

Cosmological bootstrap: new methods and new insight into cosmological correlators

We extended the reach of cosmological collider + bootstrap beyond de Sitter invariant setups

Identification of new signatures of heavy fields: low speed collider with resonances in squeezed limit

Non-local EFT gives useful insight but breaks down for masses too close to Hubble, by contrast to exact bootstrap results