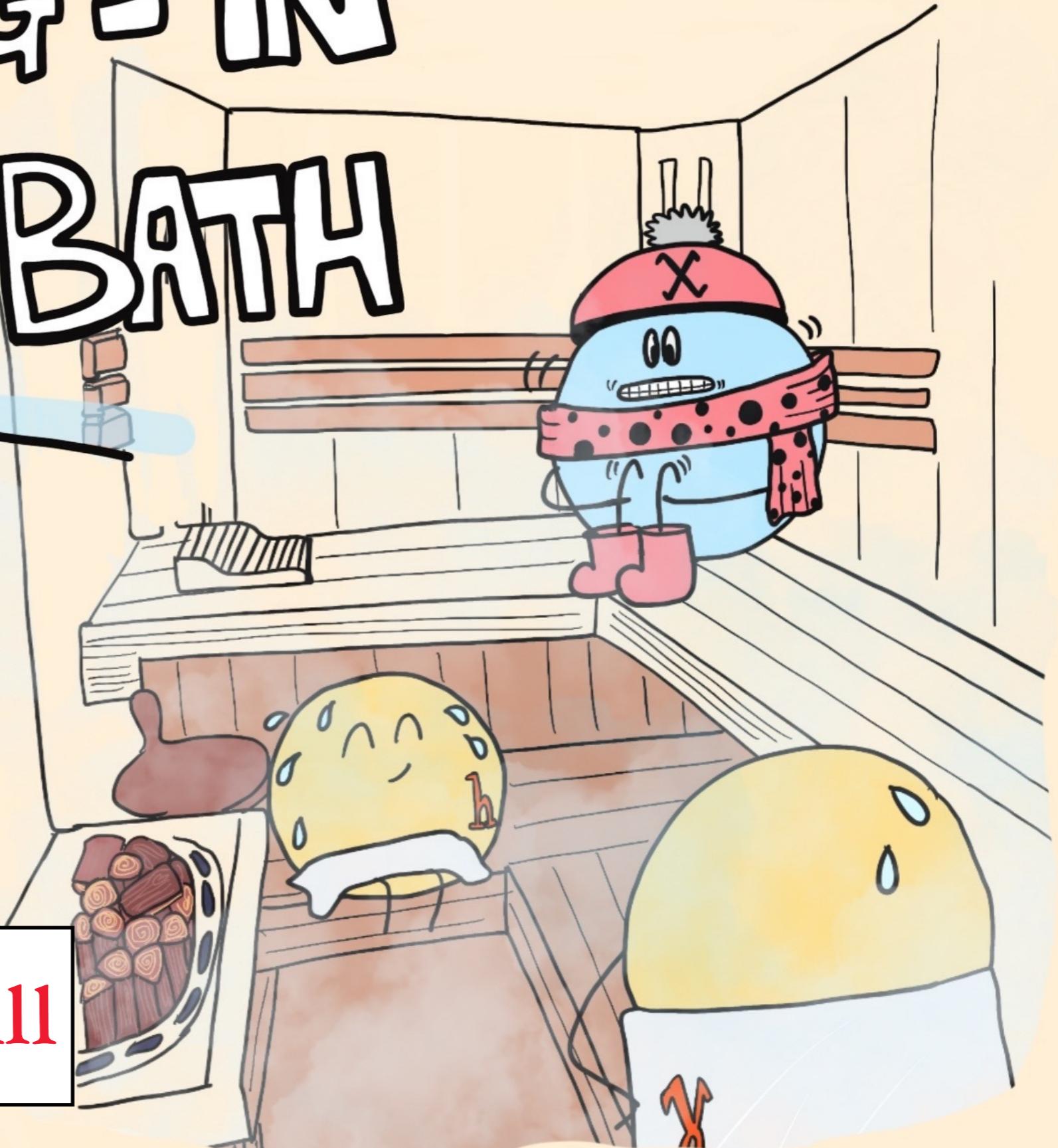


FREEZING - in A HOT BATH

SANIYA HEEBA

JHEP (110) 2022

[w/ T. Bringmann,
F. Kahlhoefer
K. Vangnes]

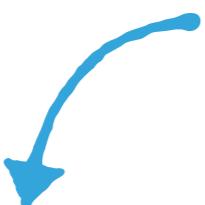


SOLVE A BOLTZMANN EQUATION

$$\dot{n}_\chi + 3Hn_\chi = C[f_\chi]$$

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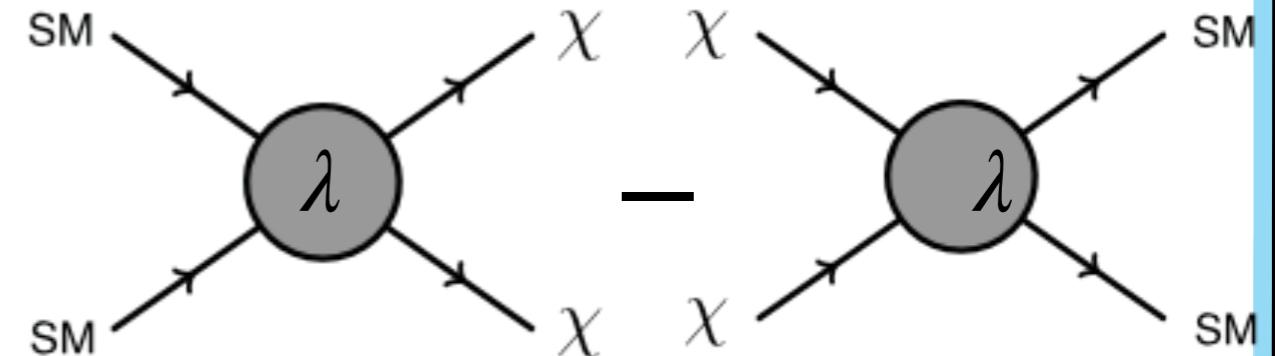
$$n_\chi = Y_\chi s$$


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Collision term

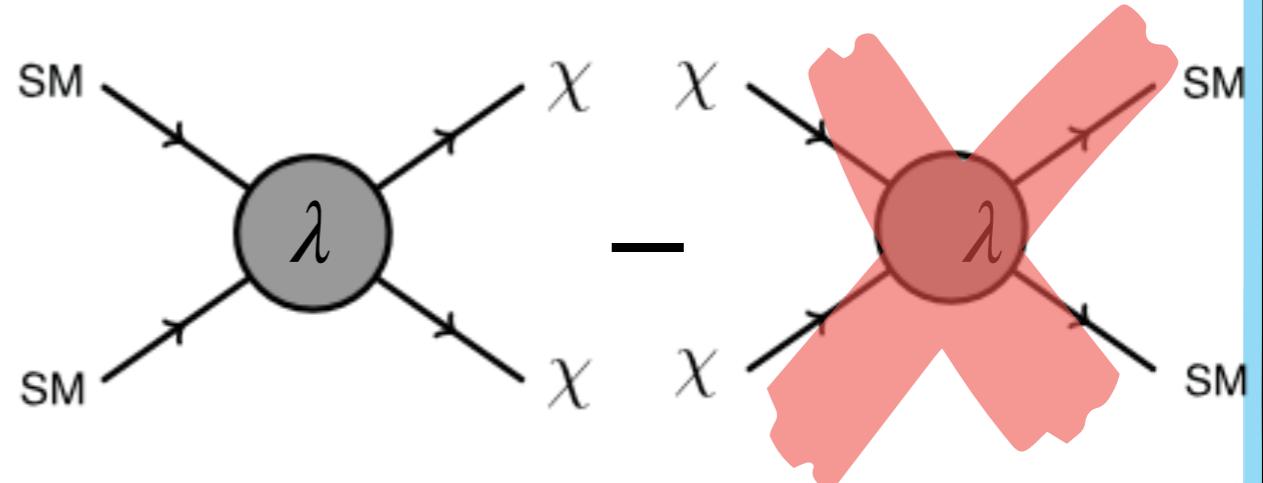


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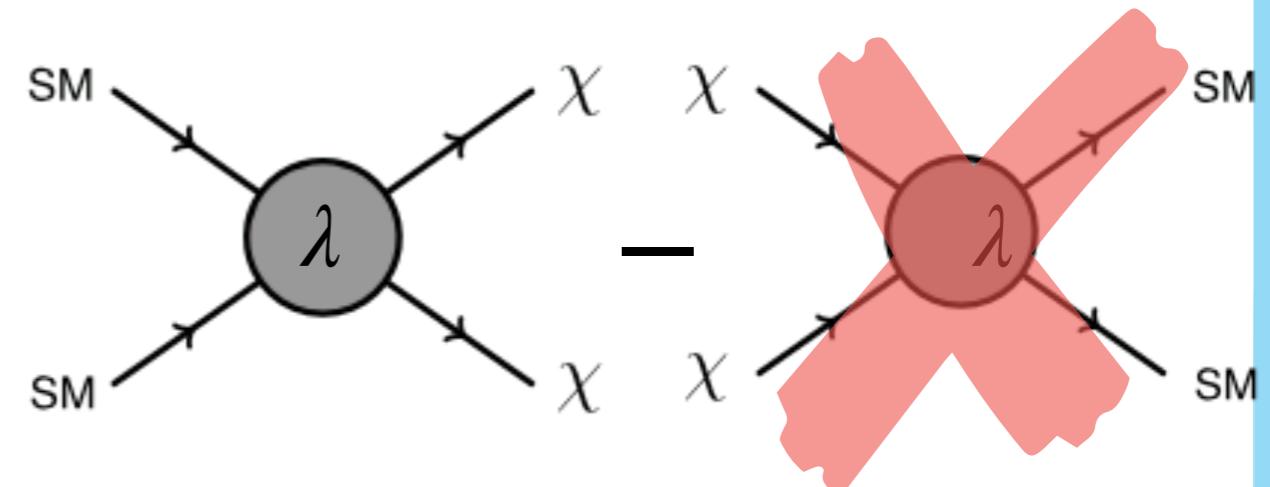
no back-reaction for freeze-in
since DM density is negligible

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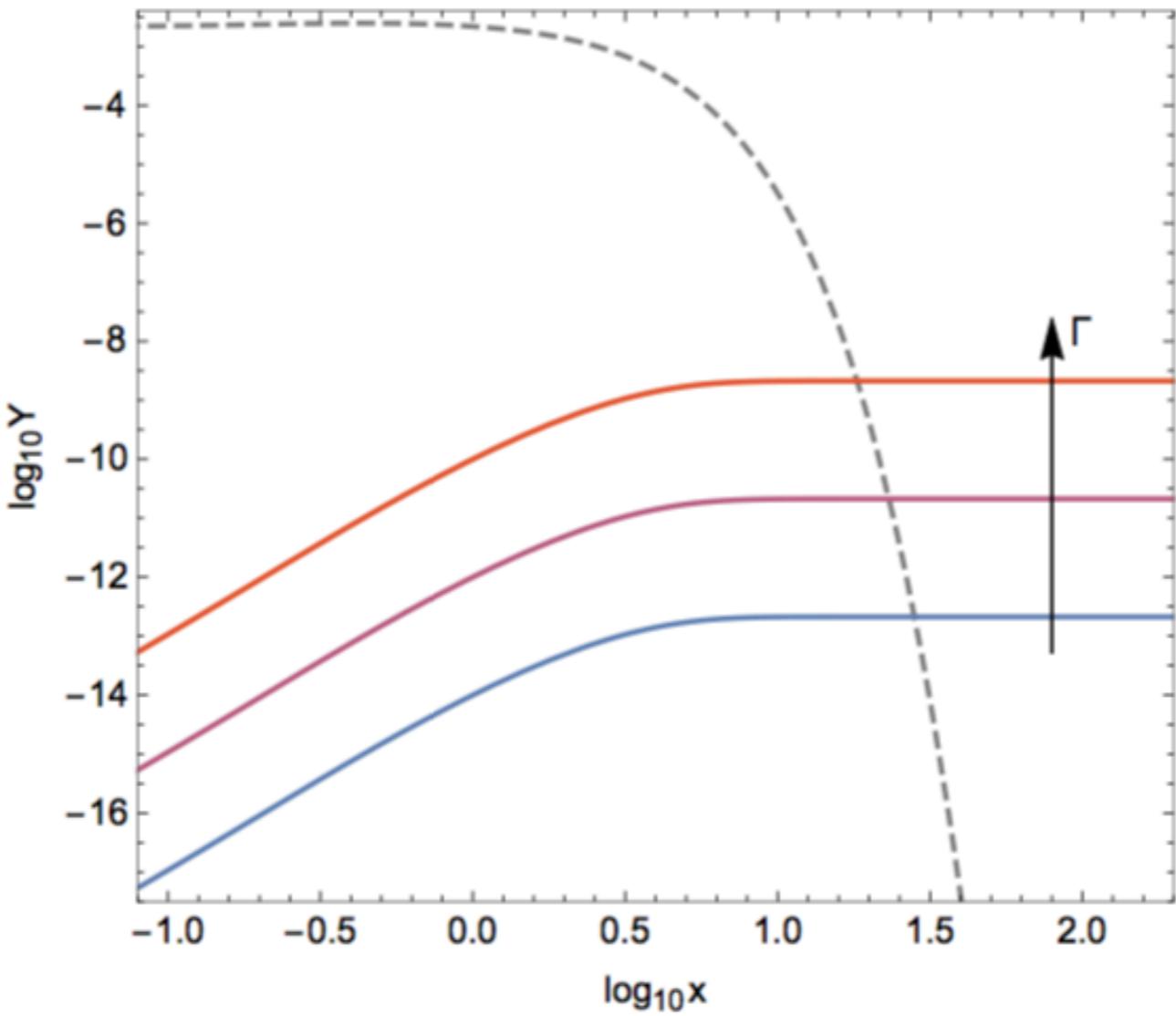
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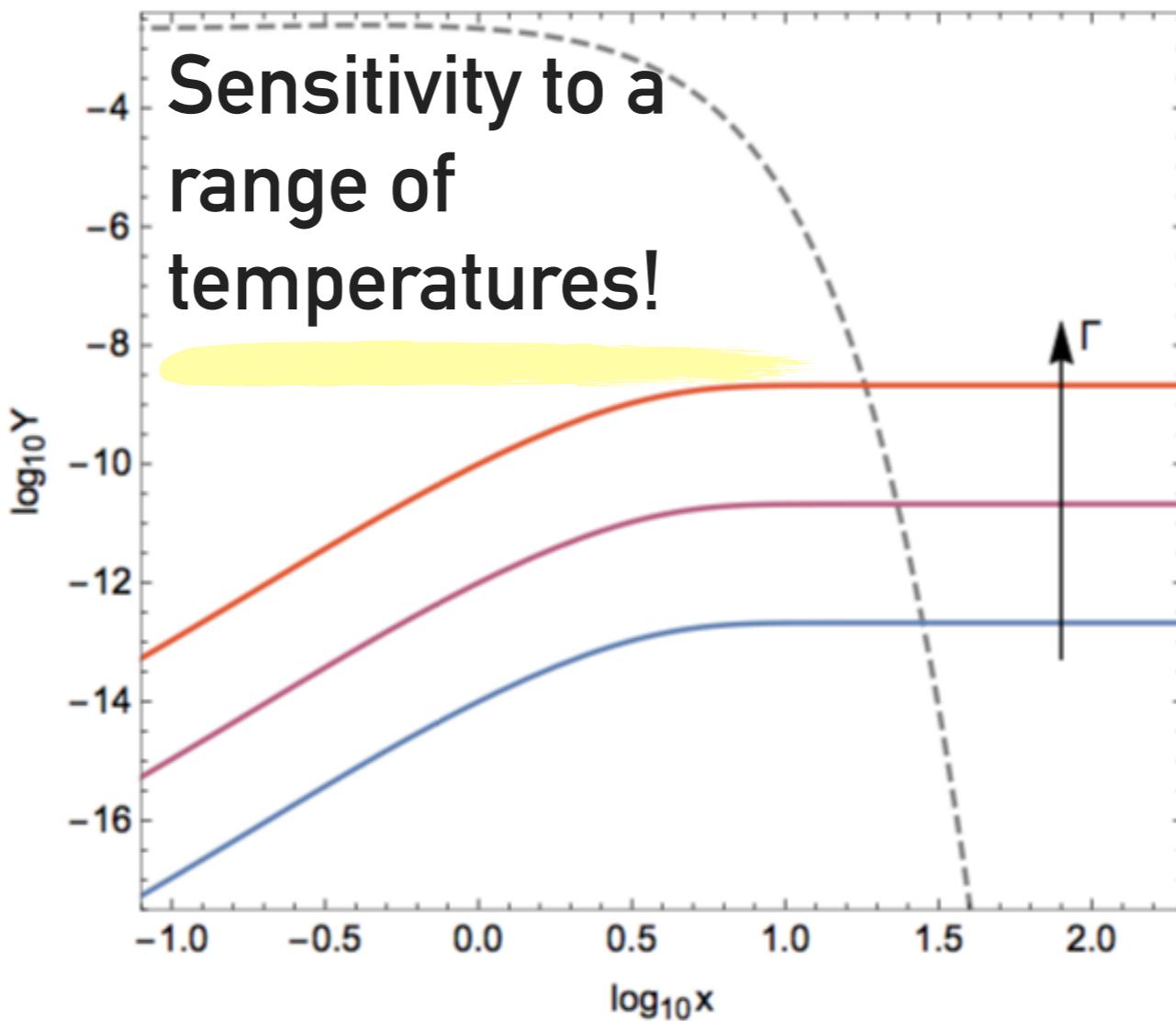


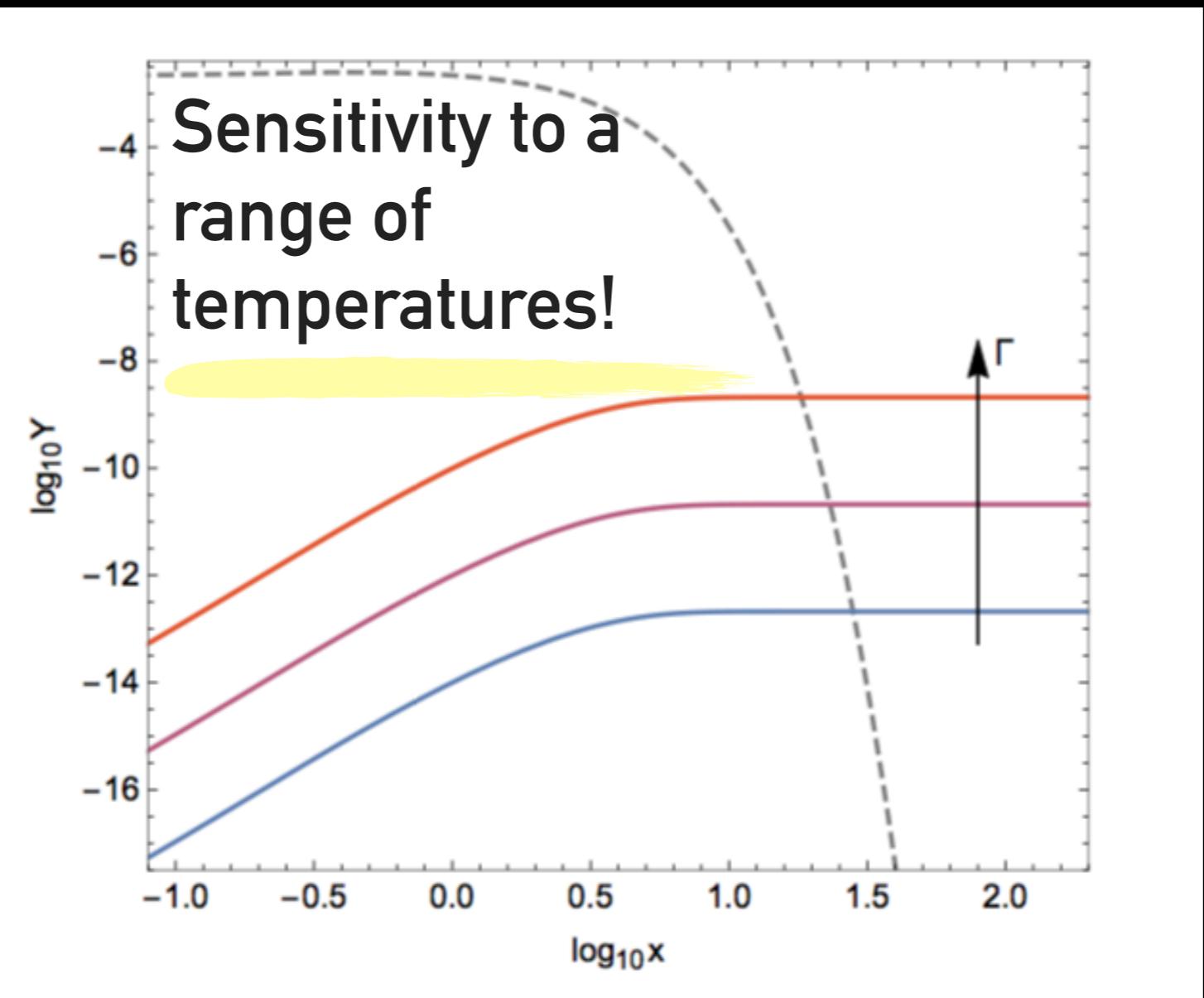
no back-reaction for freeze-in
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$$\Rightarrow \frac{dY_\chi}{dx} = \frac{s}{Hx} \langle \sigma v \rangle_{\text{SMSM} \rightarrow \chi\chi} Y_{\text{SM, eq.}}^2$$

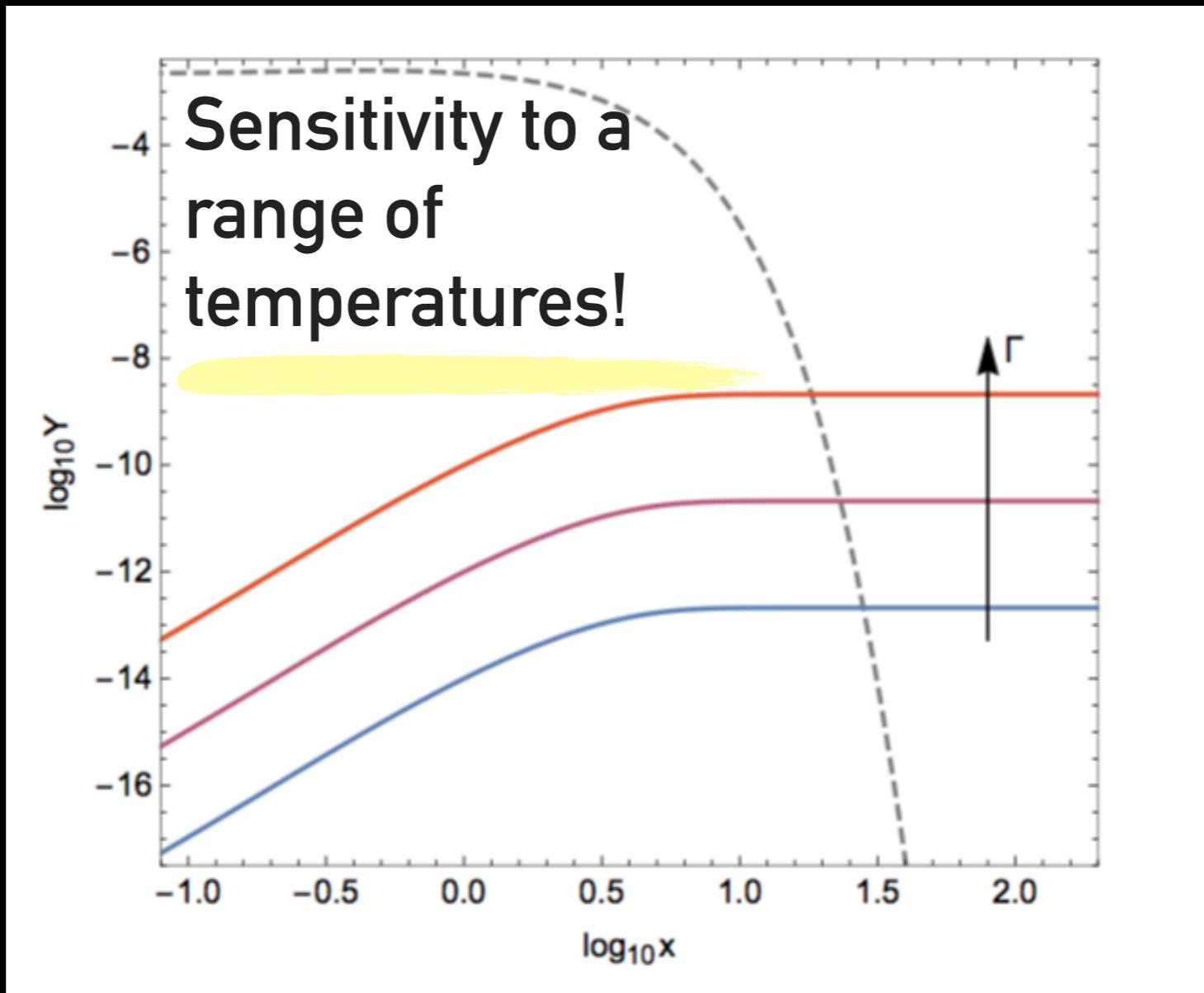


**Sensitivity to a
range of
temperatures!**



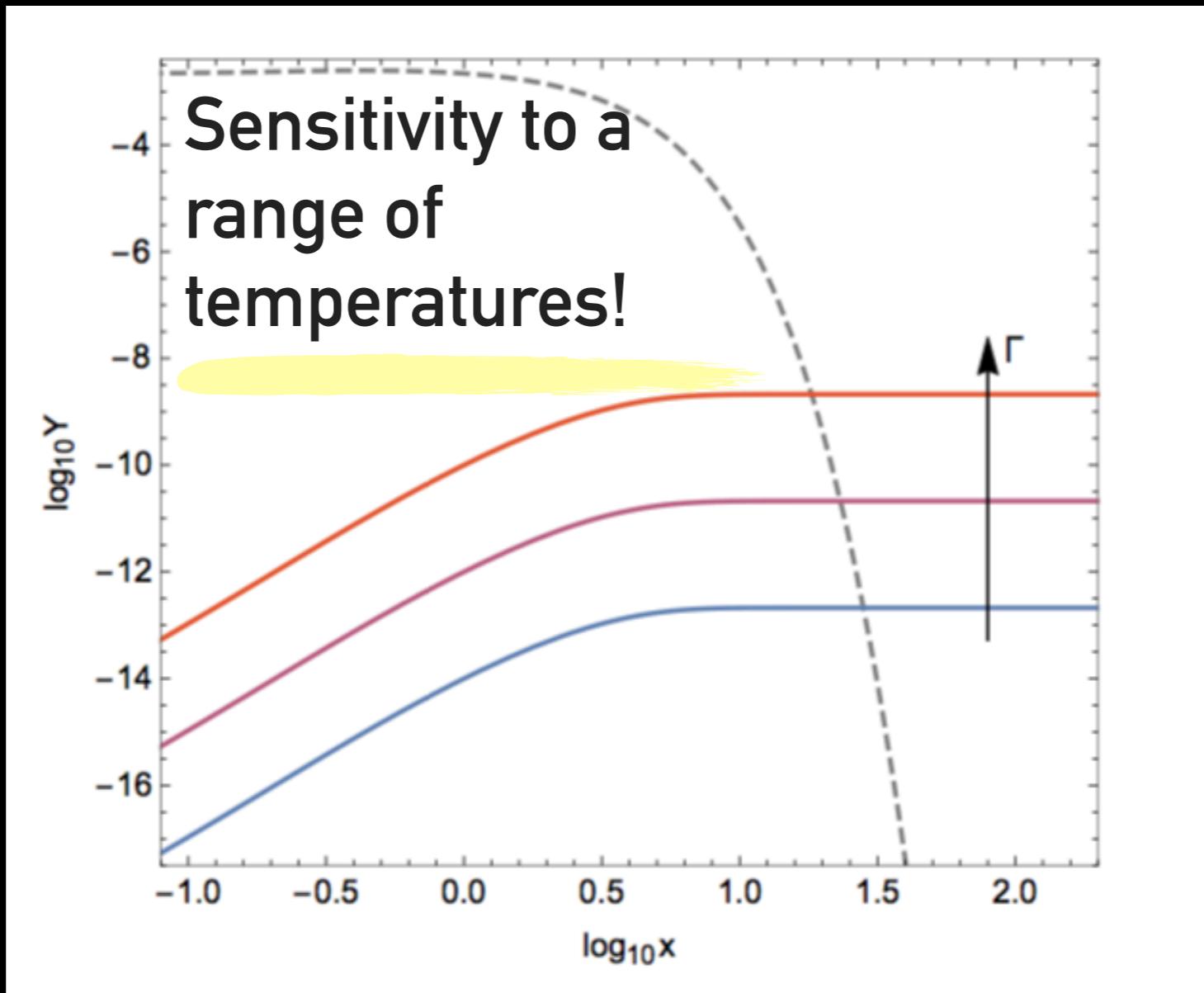


Need to account for:



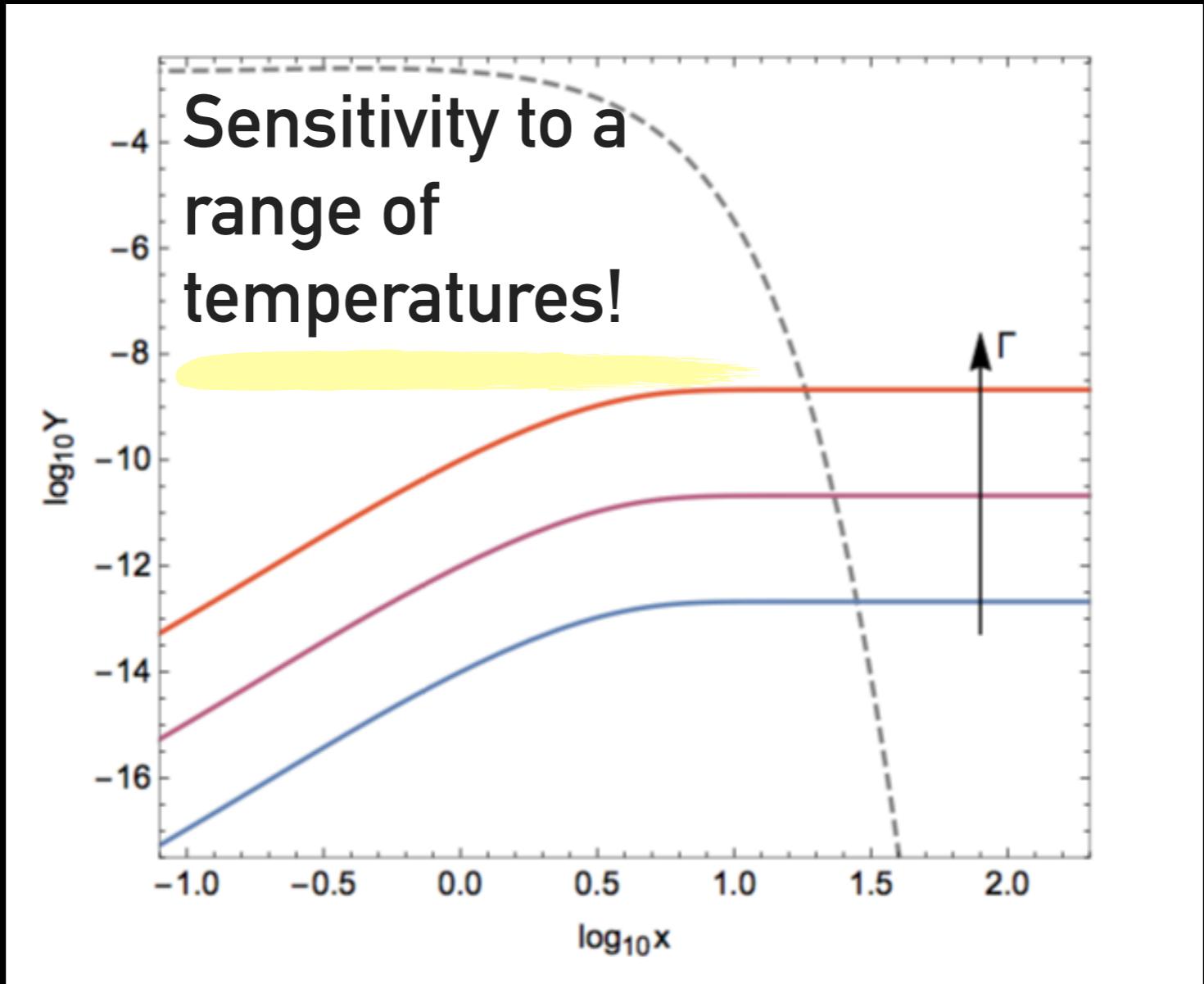
Need to account for:

- spin statistics of relativistic quantum gases: result in a frame dependence



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- spin statistics of relativistic quantum gases: result in a frame dependence
- particle widths/masses in-medium



Need to account for:

- spin statistics of relativistic quantum gases: result in a frame dependence
- particle widths/masses in-medium
- relevant degrees of freedom (phase transitions...)

I.

**REFORMULATE THE FIMP BOLTZMANN EQ TO
CONSISTENTLY ACCOUNT FOR THESE
EFFECTS**

I.

REFORMULATE THE FIMP BOLTZMANN EQ TO
CONSISTENTLY ACCOUNT FOR THESE
EFFECTS

II.

APPLY TO A SIMPLE MODEL TO STUDY THEIR
RELEVANCE

GENERIC COLLISION TERM FOR FREEZE-IN

$$C[f_\chi] = \frac{1}{N_\psi} \int \frac{d^3 p}{(2\pi)^3 2E} \int \frac{d^3 \tilde{p}}{(2\pi)^3 2\tilde{E}} \int \frac{d^3 k}{(2\pi)^3 2\omega} \int \frac{d^3 \tilde{k}}{(2\pi)^3 2\tilde{\omega}} (2\pi)^4 \delta^{(4)}(\tilde{p} + p - \tilde{k} - k) \\ \times \left[\left| \mathcal{M} \right|_{\psi\psi \rightarrow \chi\chi}^2 f_\psi(\omega) f_\psi(\tilde{\omega}) \right]$$

GENERIC COLLISION TERM FOR FREEZE-IN

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symmetry factor

$$\times \left[\left| \mathcal{M} \right|_{\psi\psi \rightarrow \chi\chi}^2 f_\psi(\omega) f_\psi(\tilde{\omega}) \right]$$

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symmetry factor



Lorentz Invariant
Phase Space



Energy-momentum
conservation

$$\times \left[\left| \mathcal{M} \right|^2_{\psi\psi \rightarrow \chi\chi} f_\psi(\omega) f_\psi(\tilde{\omega}) \right]$$

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 $\psi \equiv \text{SM}$

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symmetry factor



**Lorentz Invariant
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**Energy
conservation**

**Fiducial Maxwell-Boltzmann
DM phase-space density**

BOLTZMANN EQ. FOR FREEZE-IN:

$$\frac{dY_\chi}{dx} = \frac{\langle \sigma v \rangle_{\chi\chi \rightarrow \psi\psi}}{xsH} \left(n_\chi^{\text{MB}} \right)^2$$

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$$\langle \sigma v \rangle_{\chi\chi \rightarrow \psi\psi} = \frac{8x^2}{K_2^2(x)} \int_1^\infty d\tilde{s} \tilde{s}(\tilde{s}-1) \int_1^\infty d\gamma \sqrt{\gamma^2 - 1} e^{-2\sqrt{\tilde{s}}x\gamma} \sigma_{\chi\chi \rightarrow \psi\psi}(s, \gamma)$$

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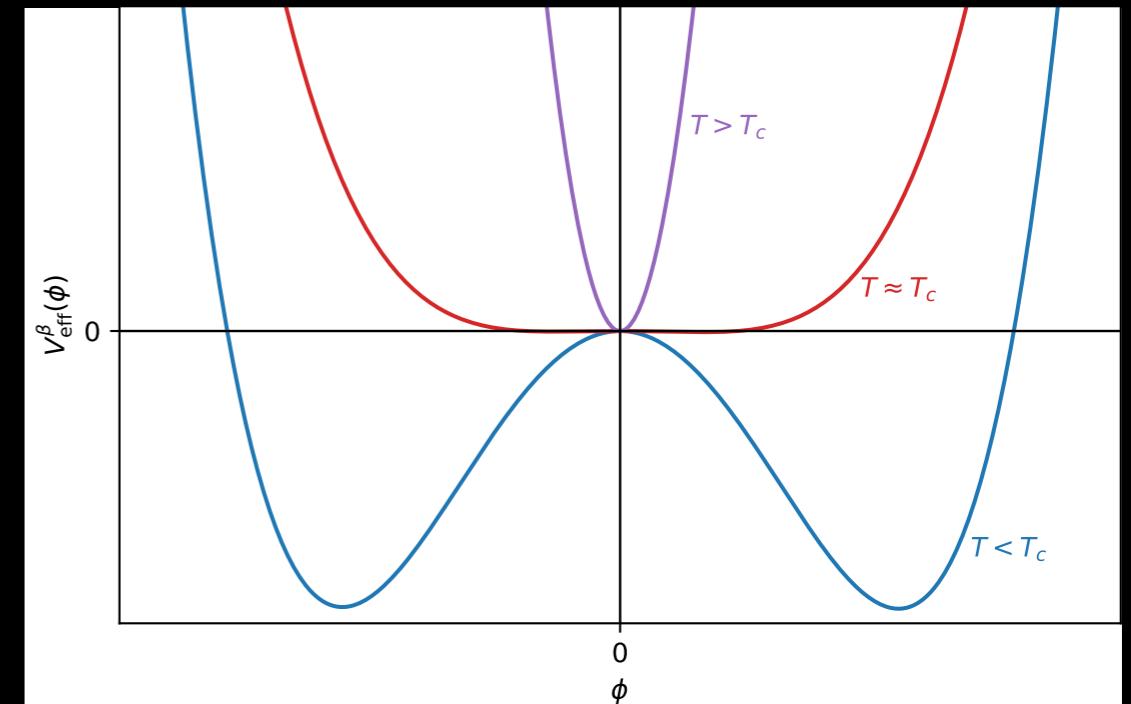
In-medium cross-section boosted to the CMS frame

II. APPLICATION TO A MODEL

$$L \supset \frac{\lambda_{hs}}{2} |H|^2 S^2$$

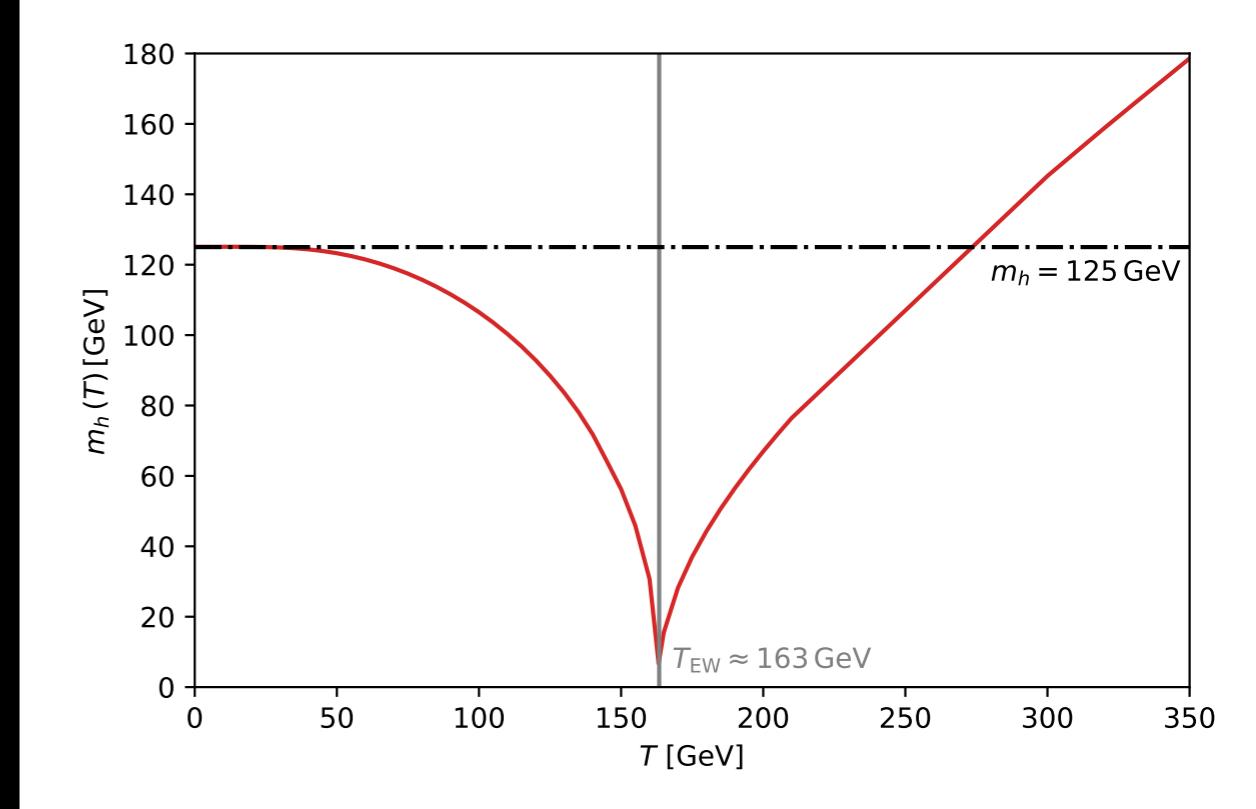
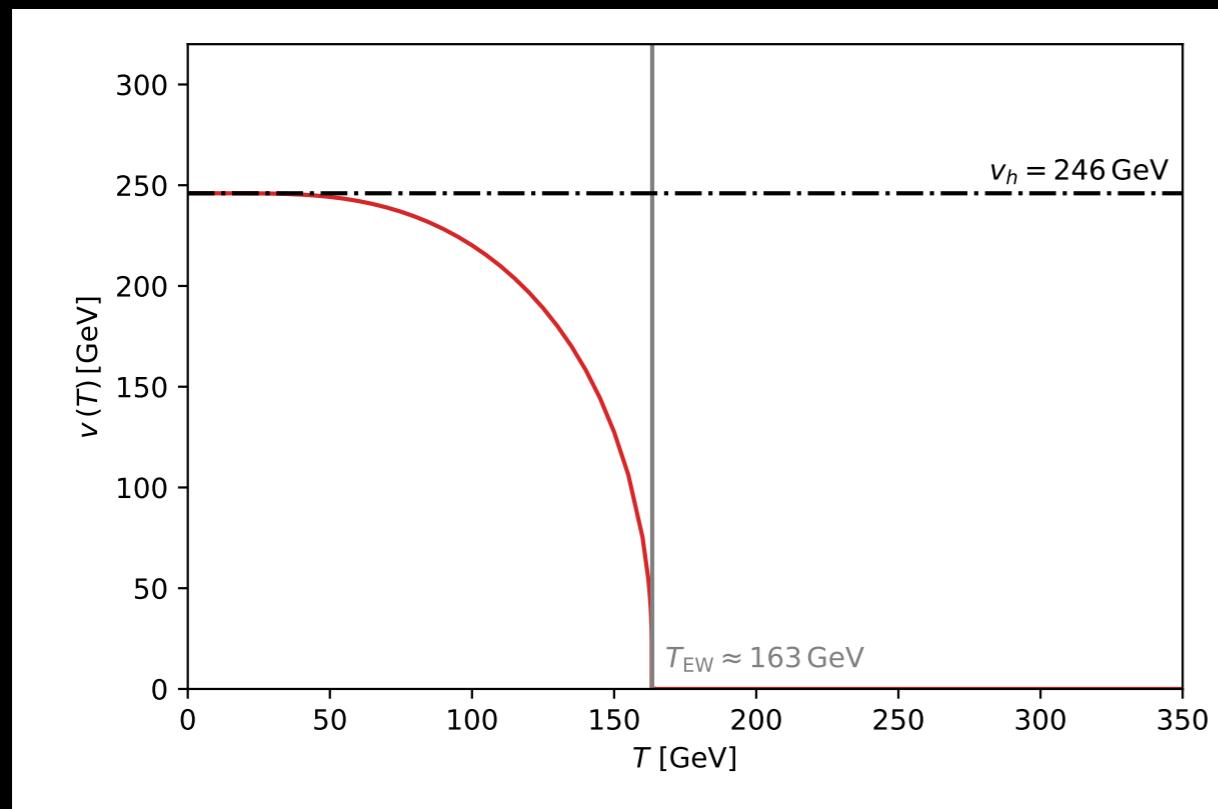
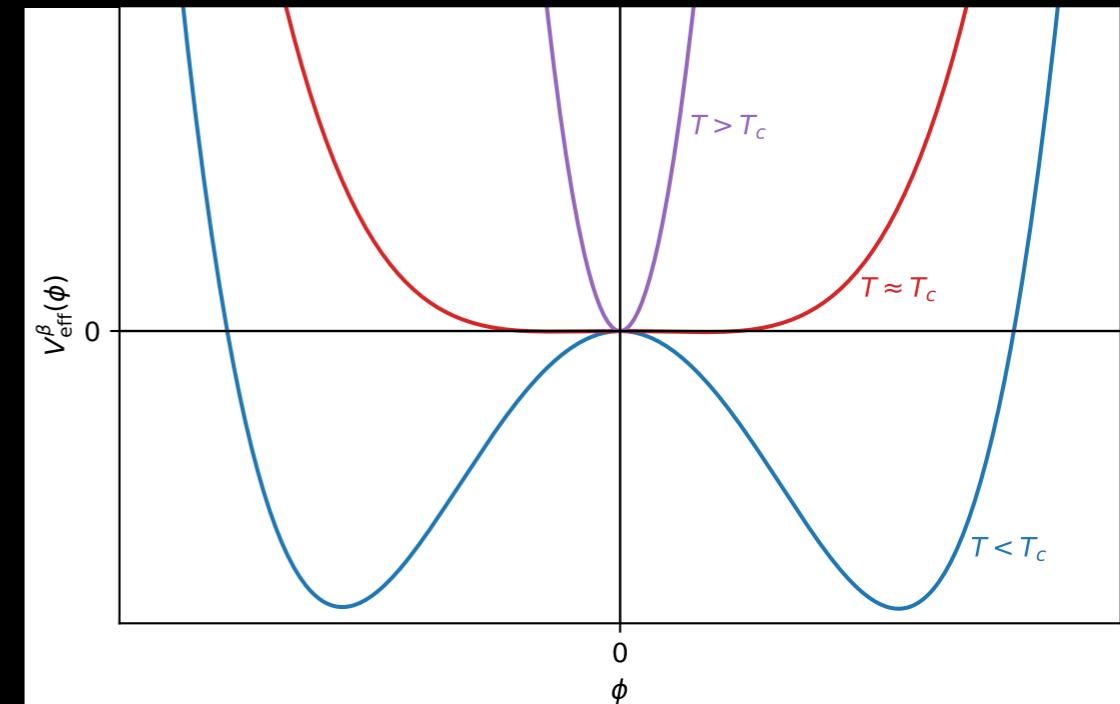


ELECTROWEAK PHASE TRANSITION:



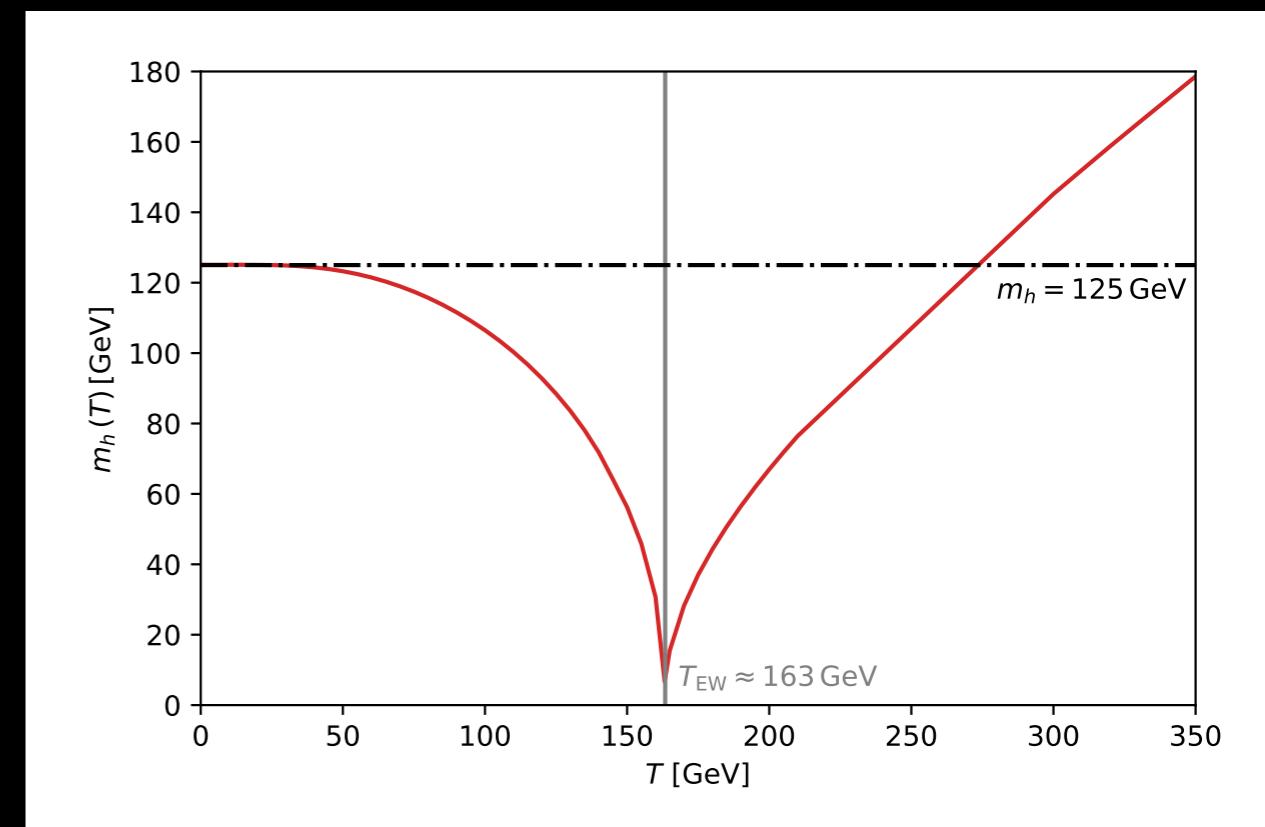
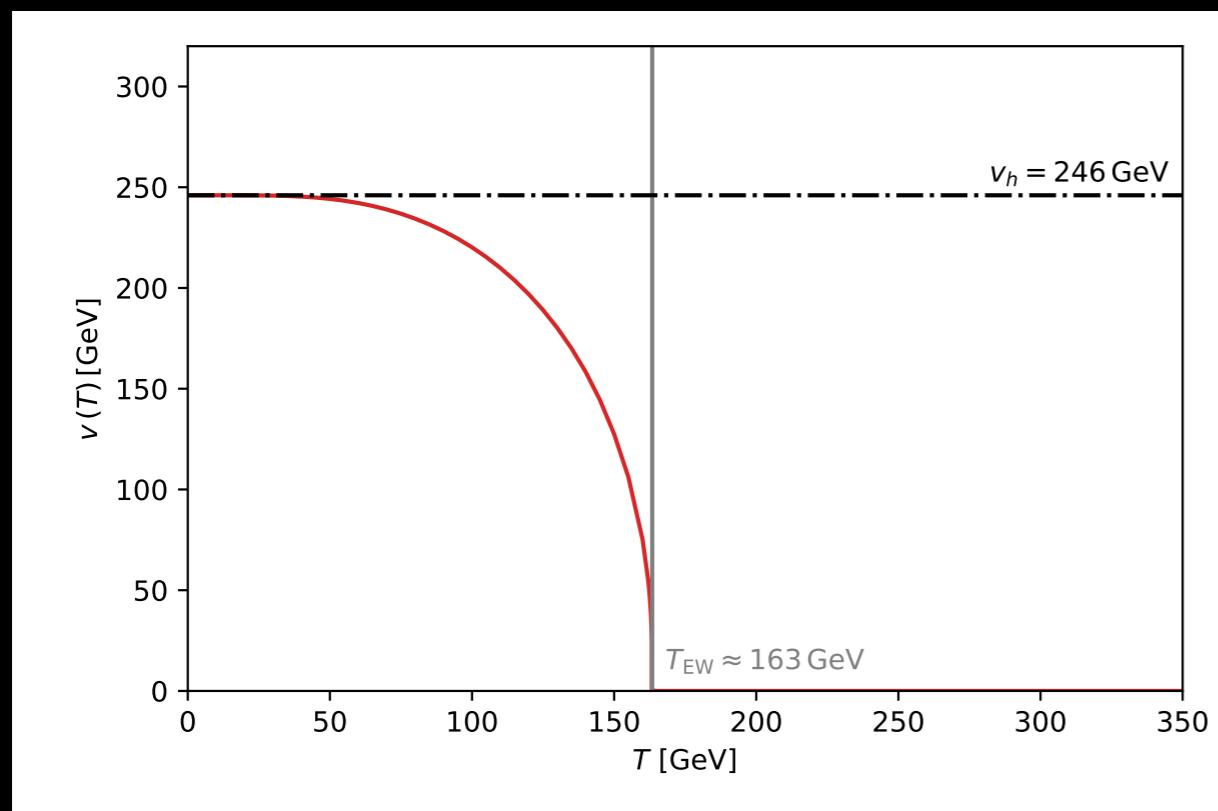
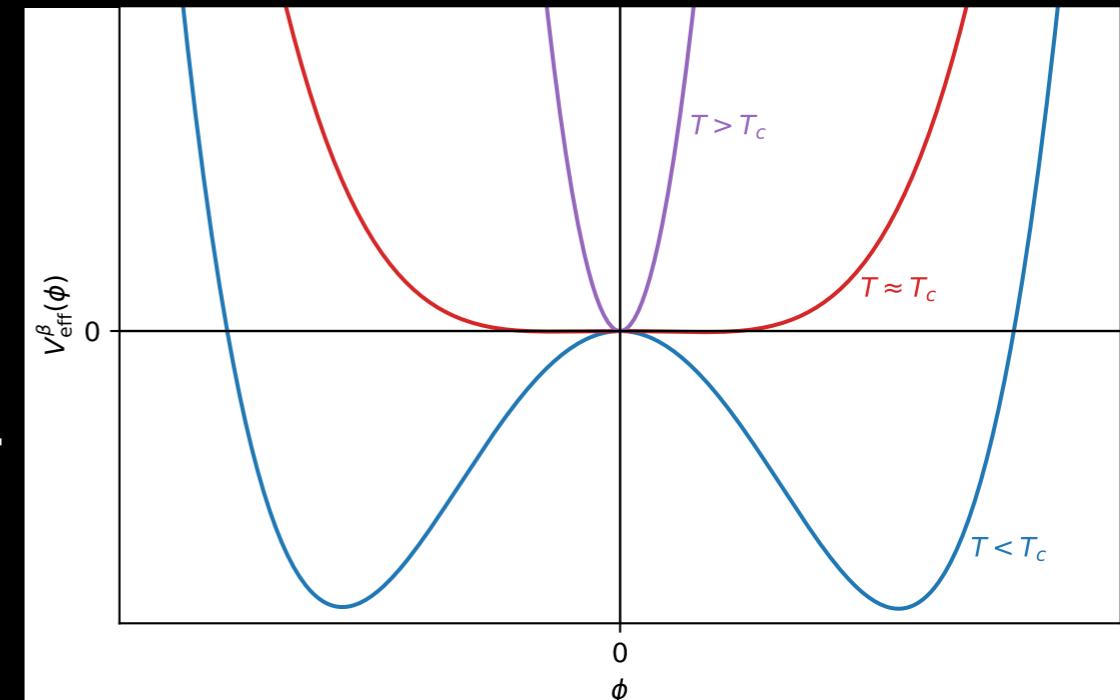
ELECTROWEAK PHASE TRANSITION:

- The **higgs mass** and **vev** are **temperature dependent**

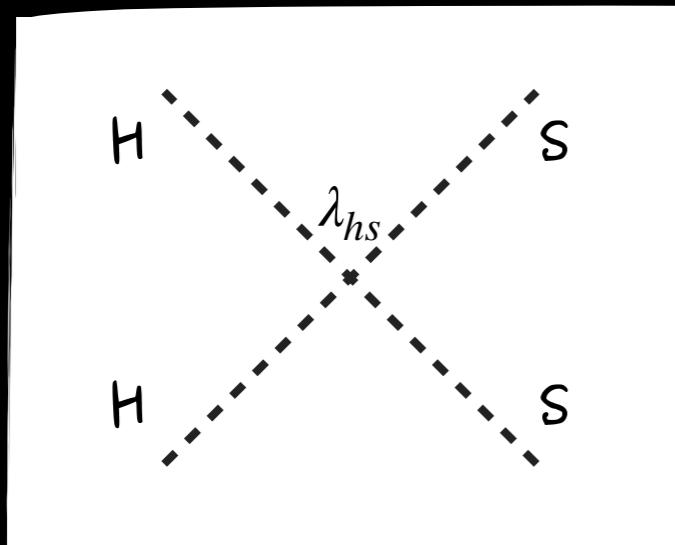


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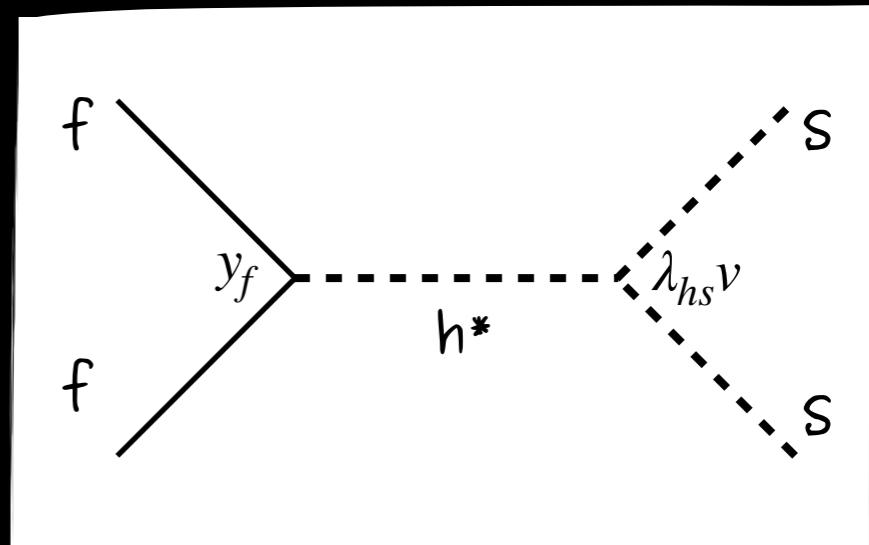
- ▶ The **higgs mass** and **vev** are **temperature dependent**
- ▶ Coupling structure of the Lagrangian changes before/after PT



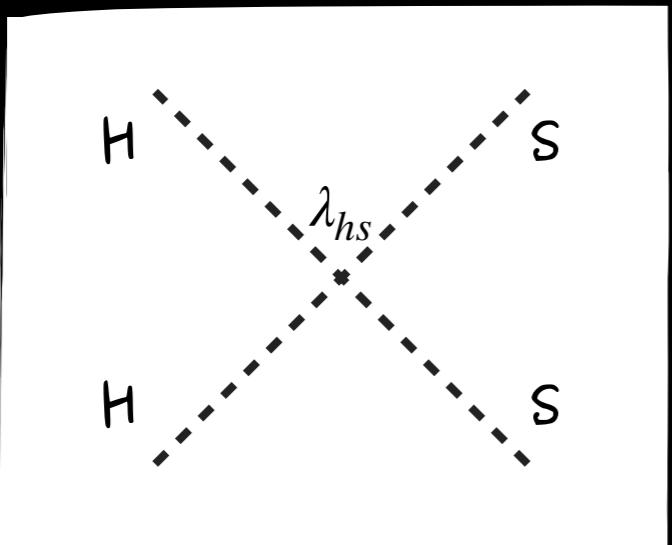
BEFORE EWPT



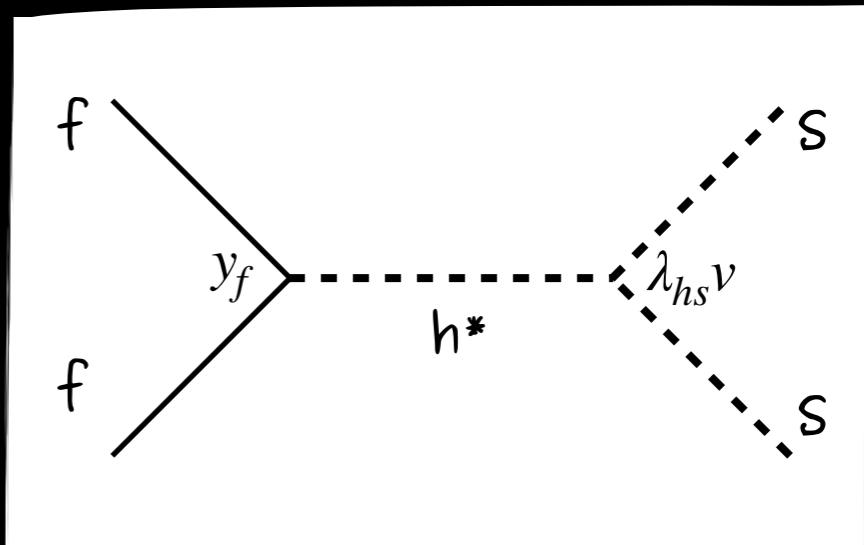
AFTER EWPT



BEFORE EWPT



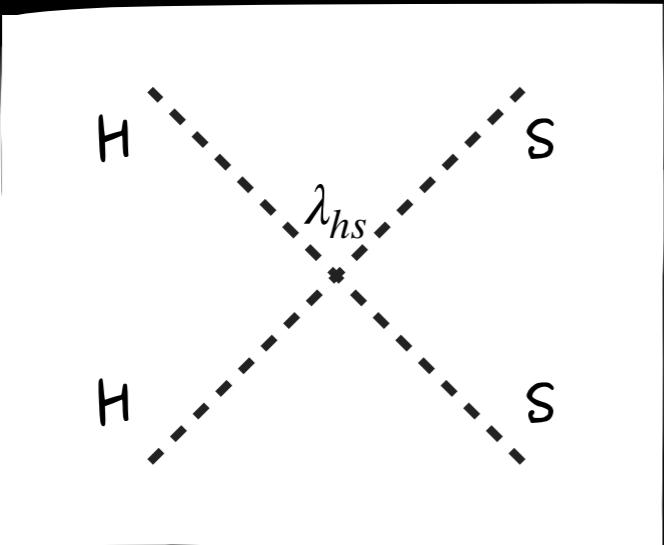
AFTER EWPT



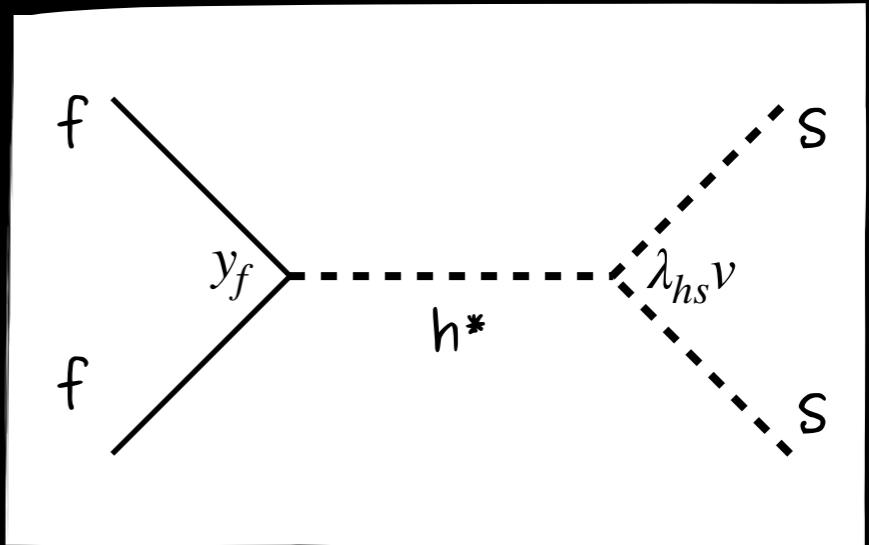
If we write this is in terms of scalar annihilation:

$$\sigma v_{ss \rightarrow h^* \rightarrow ff} \sim \frac{\lambda_{hs}^2 \nu_h}{\sqrt{s}} \frac{\Gamma_{h^*}(\sqrt{s})}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2}$$

BEFORE EWPT



AFTER EWPT

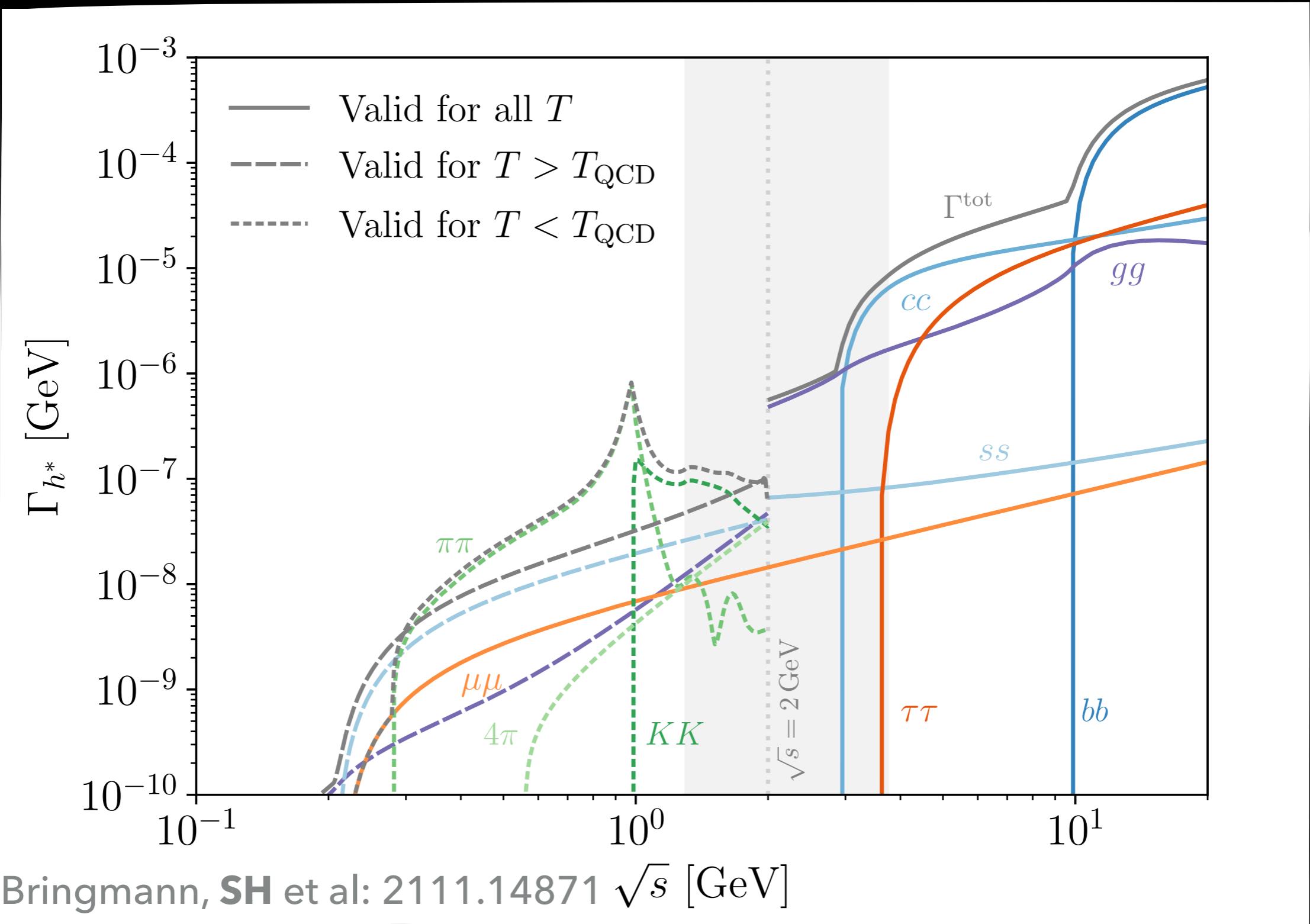


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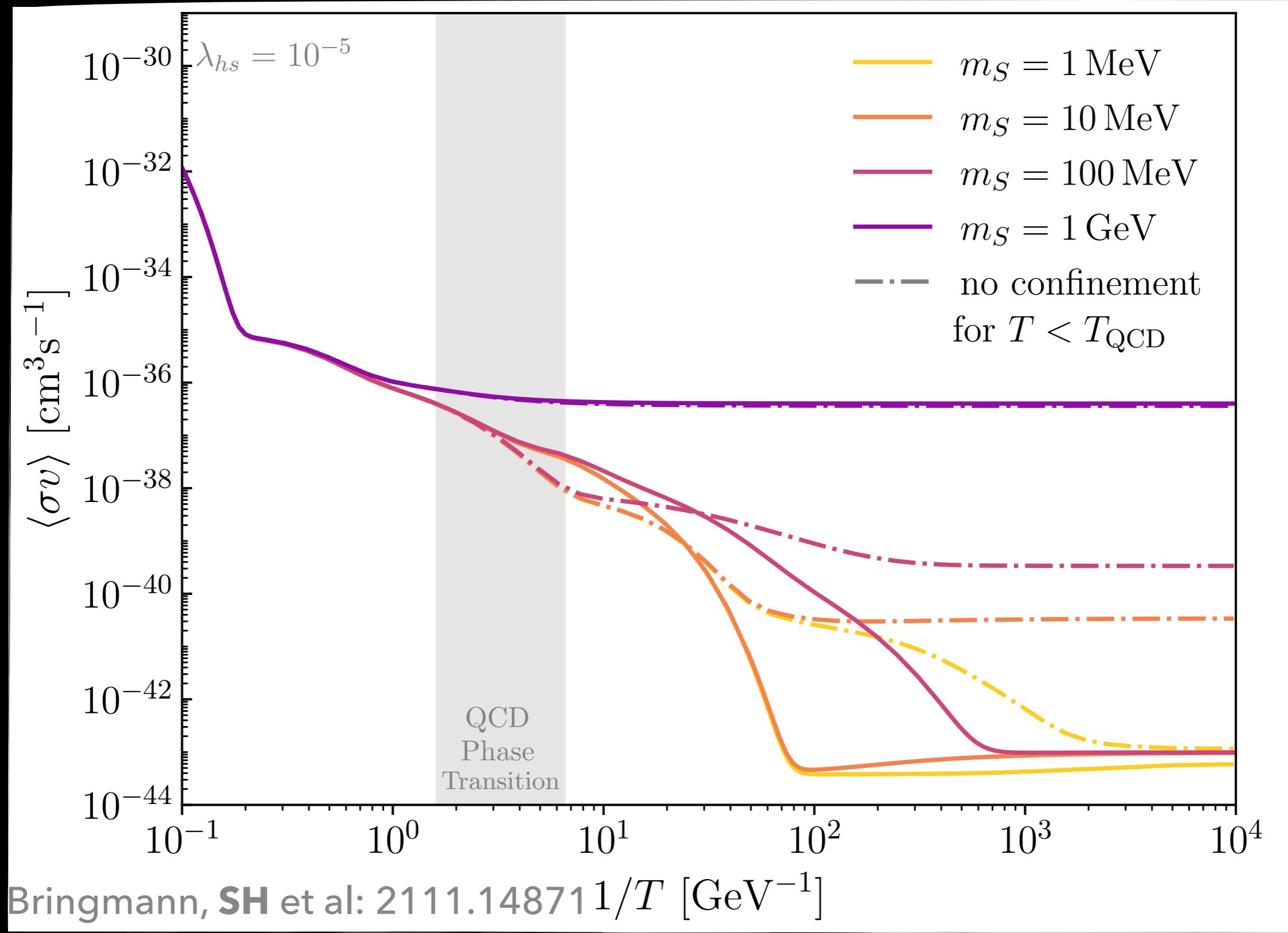
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generically include NLO
EW corrections!

QCD PHASE TRANSITION:



CROSS-SECTIONS:



UNTIL NOW:

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1. **FREEZE-IN** IN TERMS OF DM ANNIHILATION CROSS SECTIONS INCLUDING **IN-MEDIUM** AND **THERMAL EFFECTS**

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1. **FREEZE-IN** IN TERMS OF DM ANNIHILATION CROSS SECTIONS INCLUDING **IN-MEDIUM** AND **THERMAL EFFECTS**
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UNTIL NOW:

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2. INCLUDE NLO CORRECTIONS GENERICALLY
3. CONSISTENTLY ACCOUNT FOR THE RELEVANT DEGREES OF FREEDOM

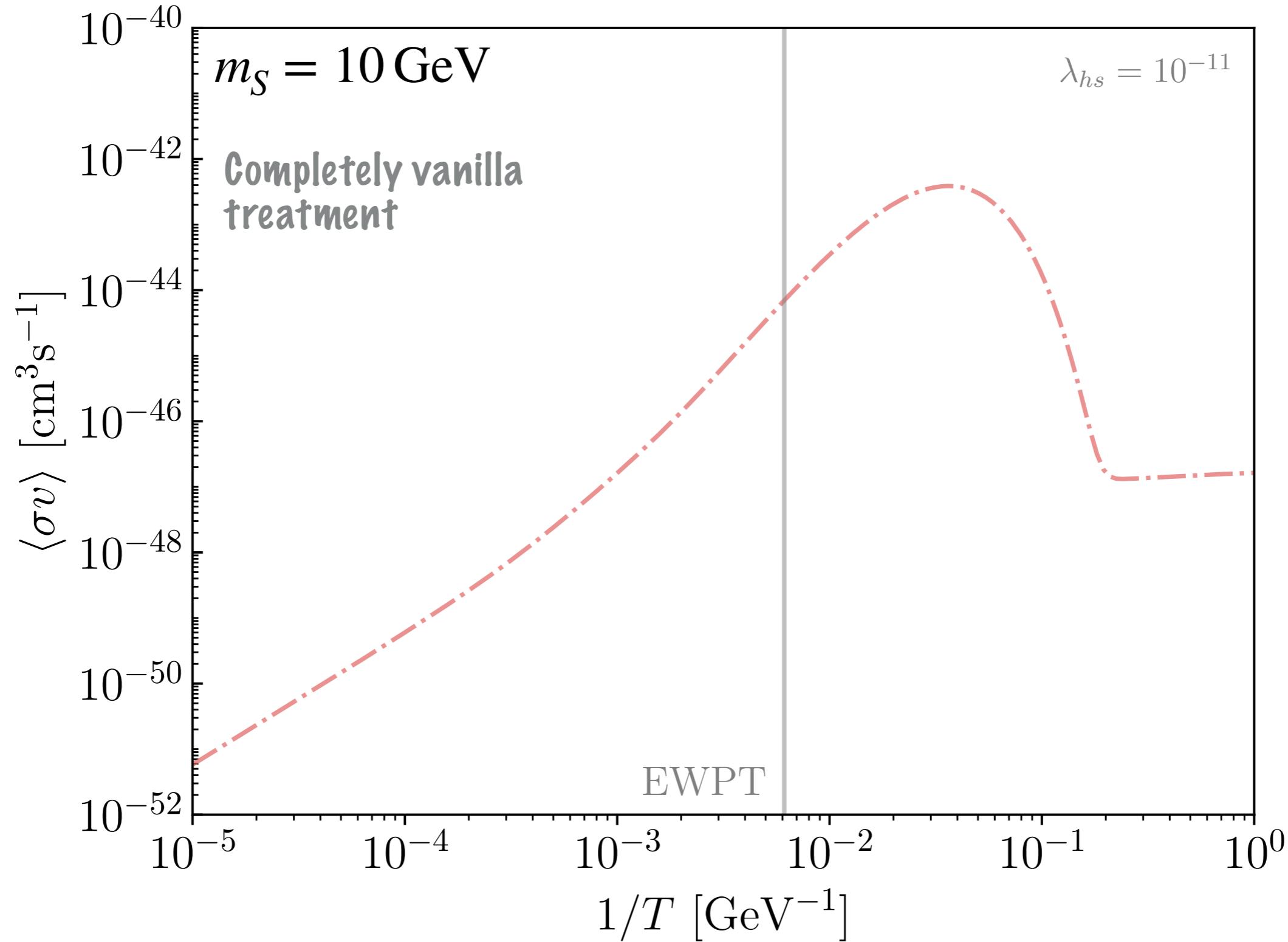
I.

IR FREEZE-IN

$(T_{\text{RH}} \gg T_{\text{EW}})$

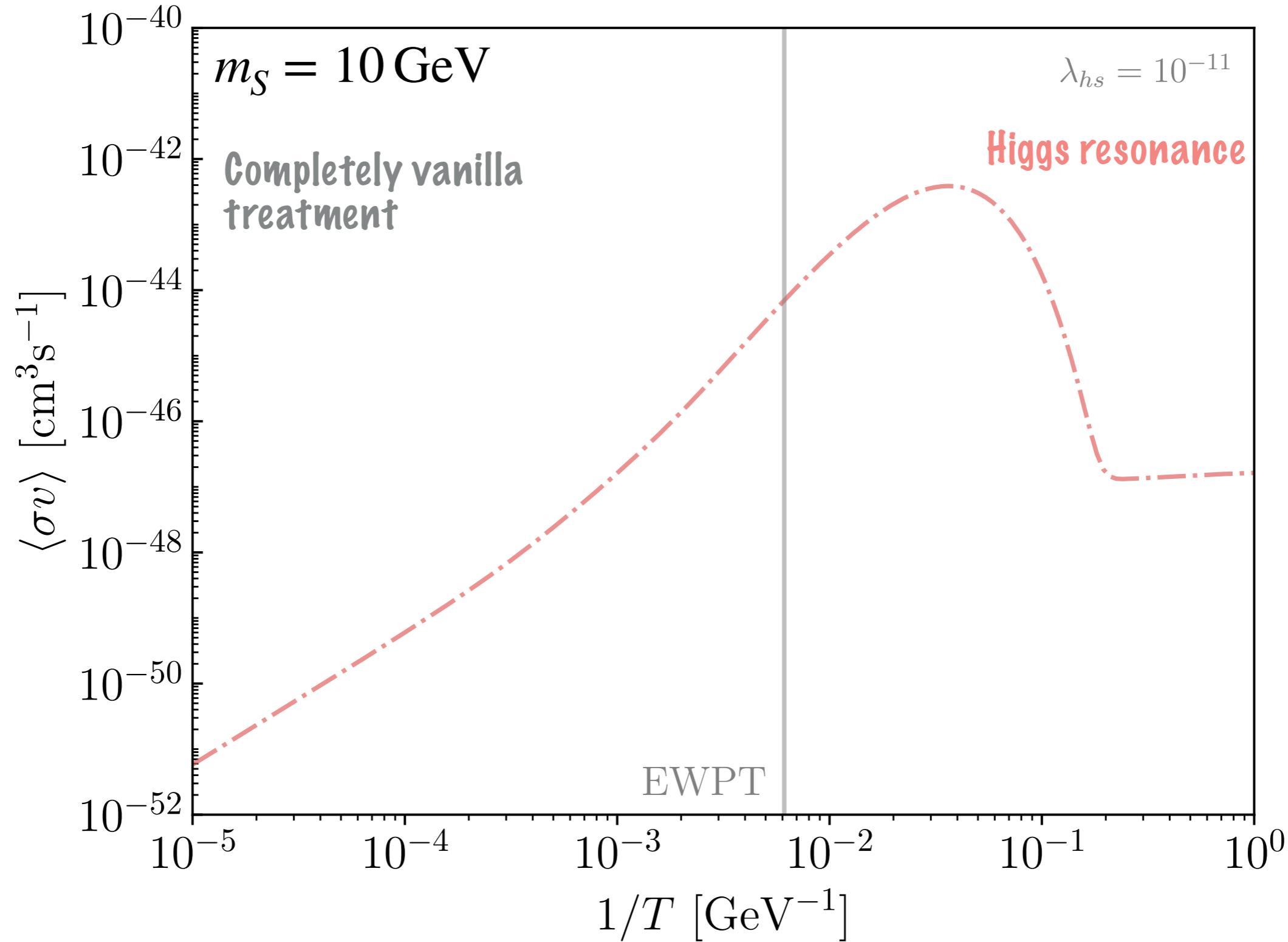
CROSS-SECTIONS:

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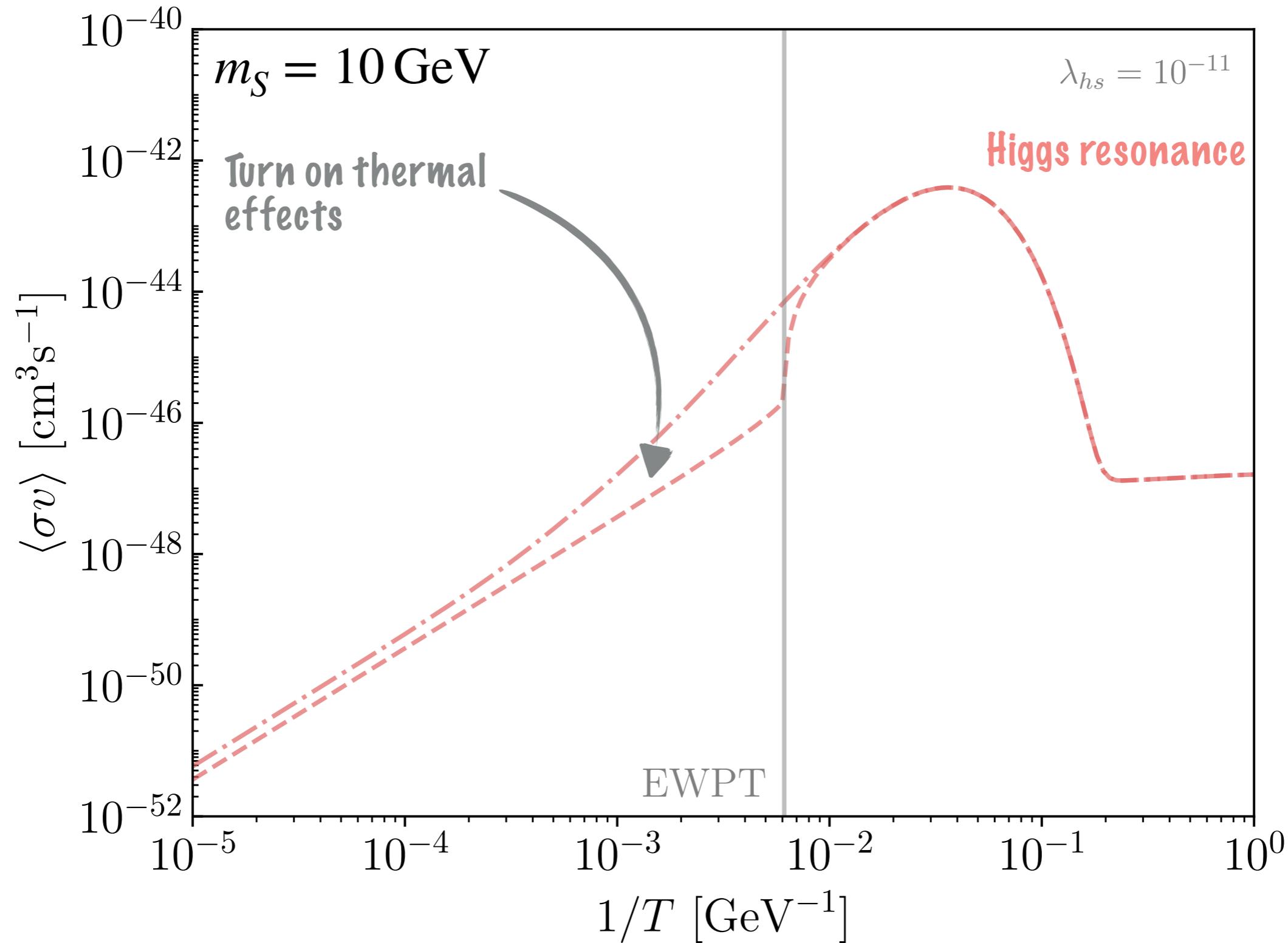
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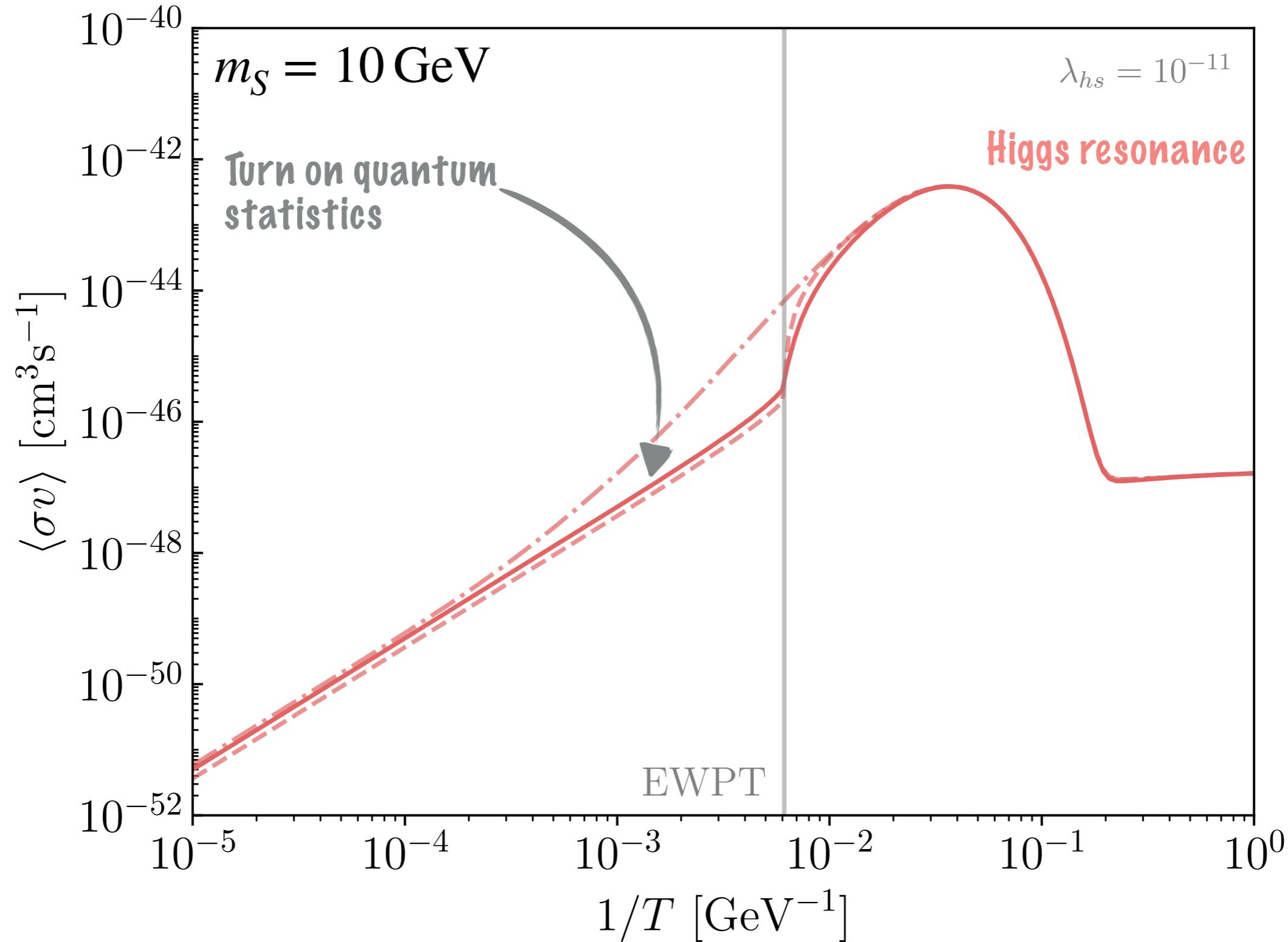
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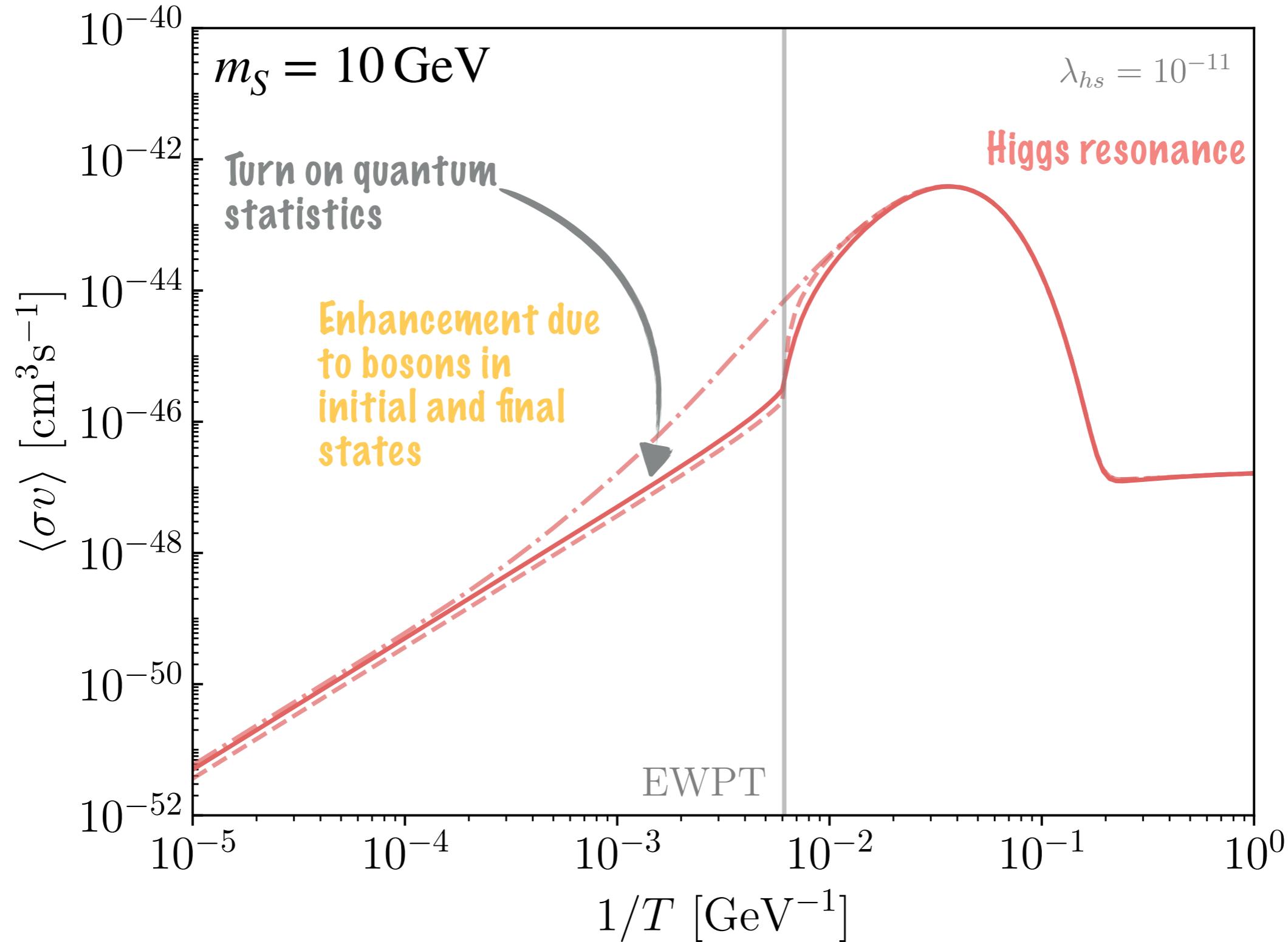
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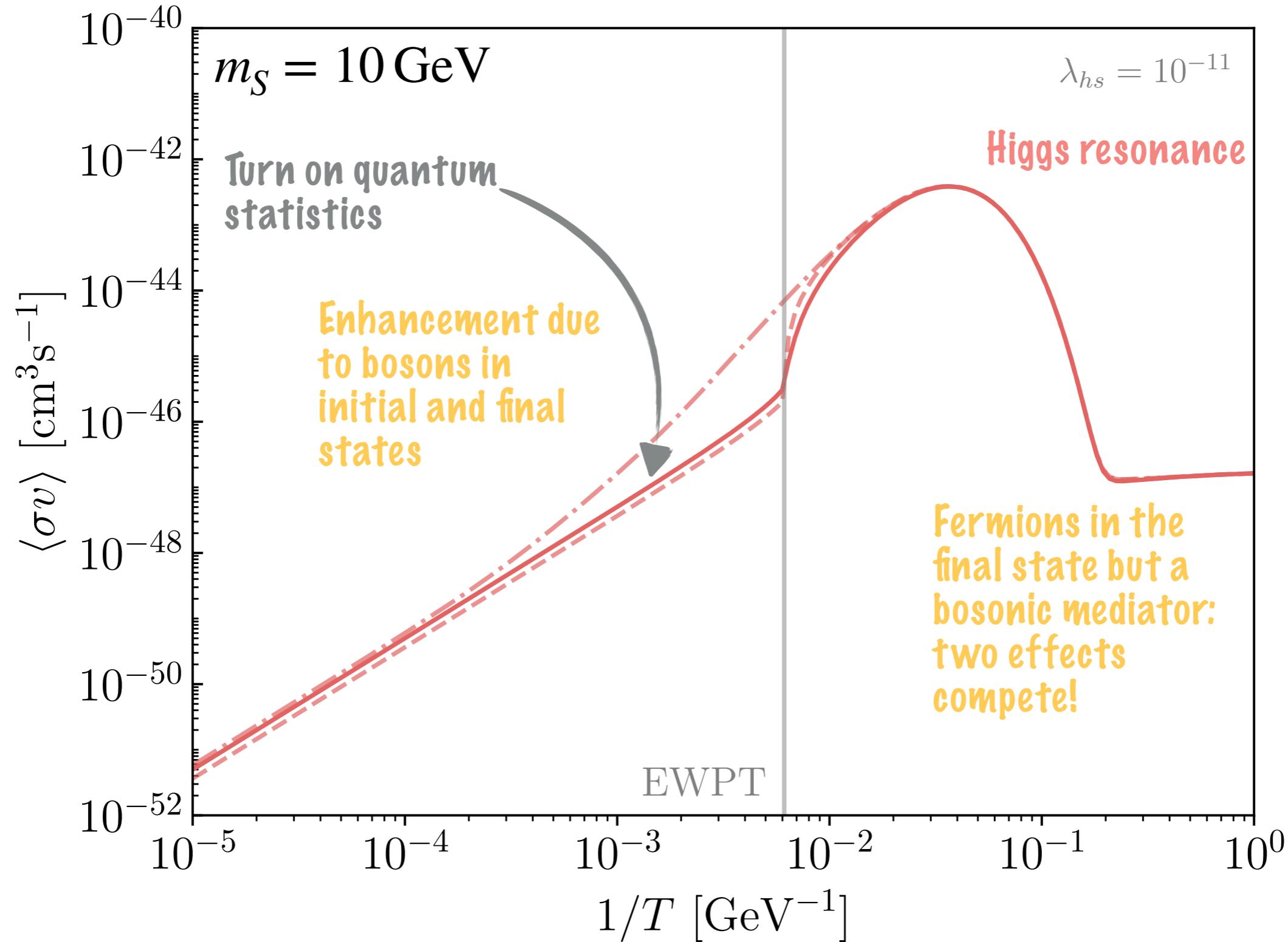
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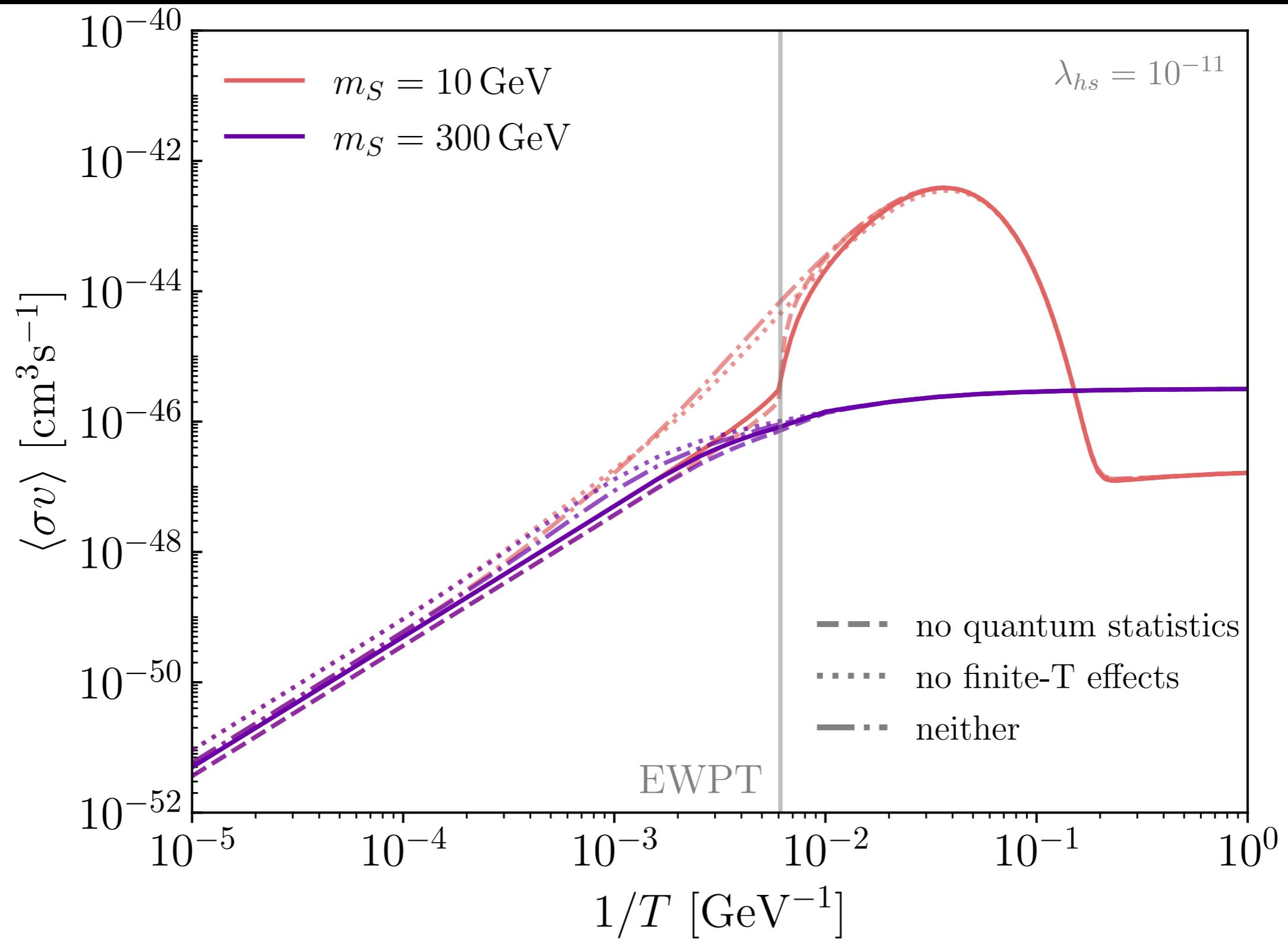
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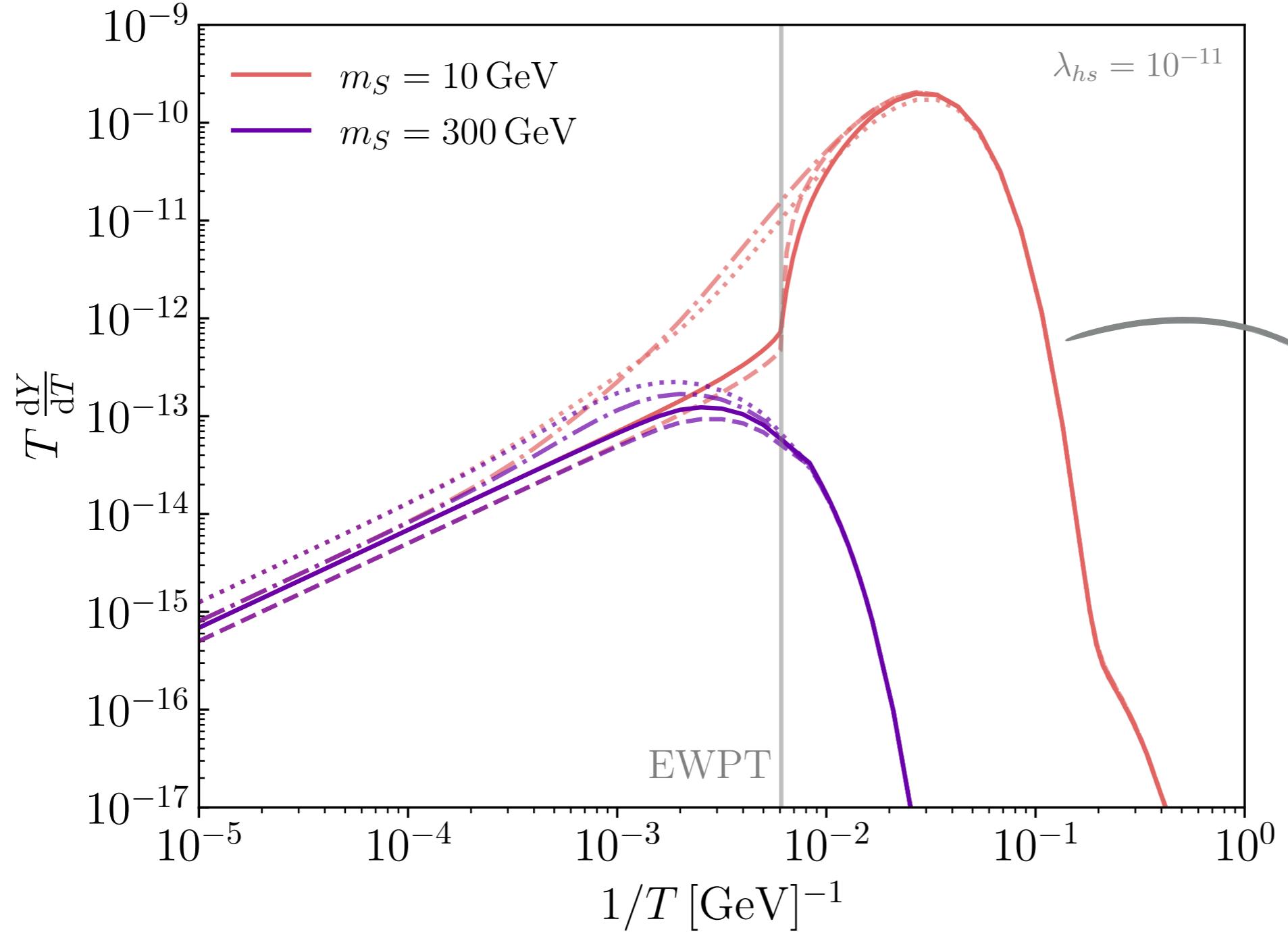
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YIELD:

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ABUNDANCE: WHAT WE EXPECT

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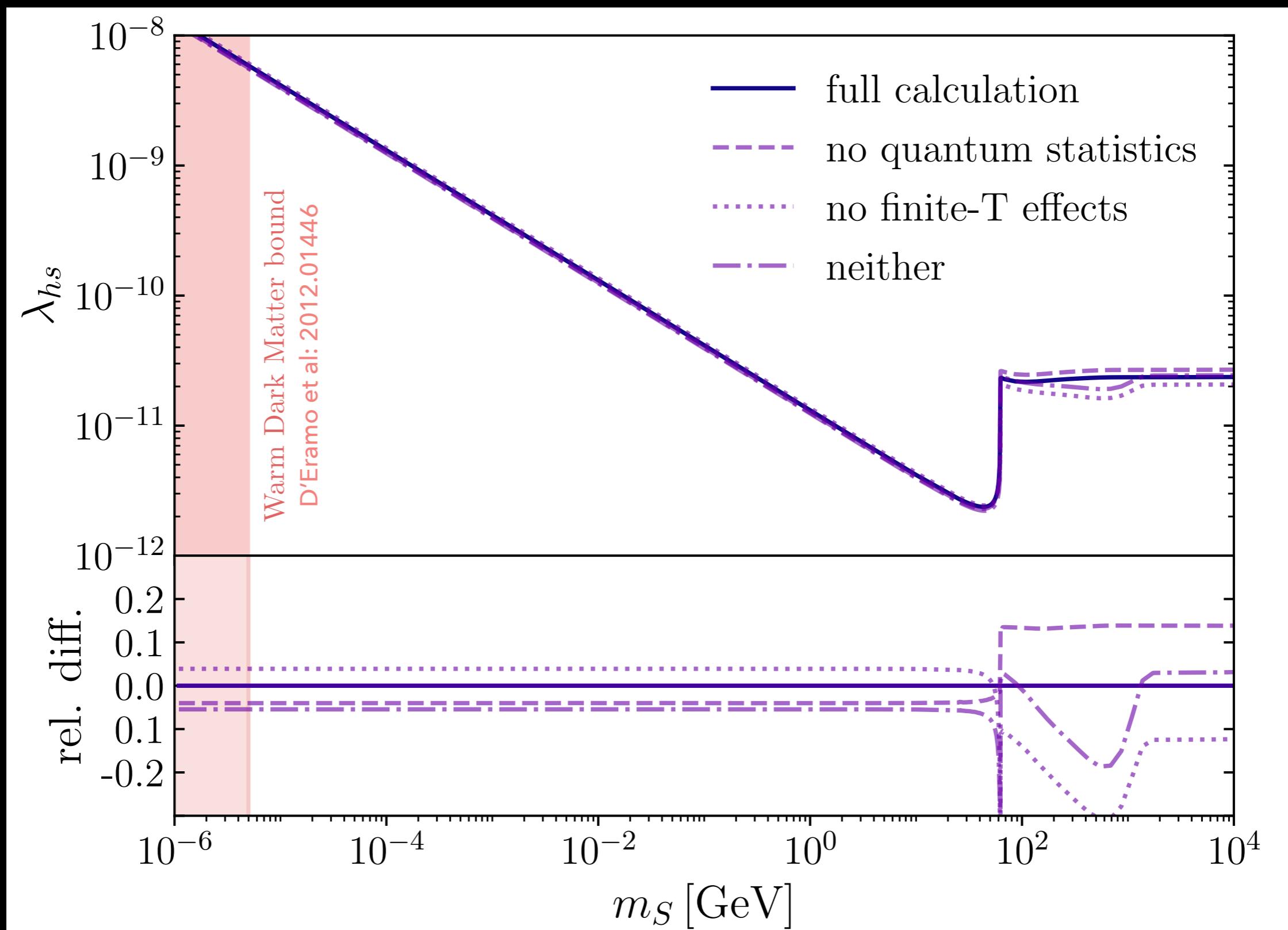
ABUNDANCE: WHAT WE EXPECT

- ▶ For $m_s < m_h/2$, production around the higgs mass:
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 - ▶ In-medium effects compete

ABUNDANCE: WHAT WE EXPECT

- ▶ For $m_s < m_h/2$, production around the higgs mass:
 - ▶ very small temperature effects
 - ▶ In-medium effects compete
- ▶ For $m_S > m_h/2$, production close to or before the electroweak phase transition, sizeable effects

λ_{hs} FOR $\Omega_S h^2 = 0.12$:



I.

UV FREEZE-IN

$(T_{\text{RH}} \ll T_{\text{EW}})$

BACKGROUND:

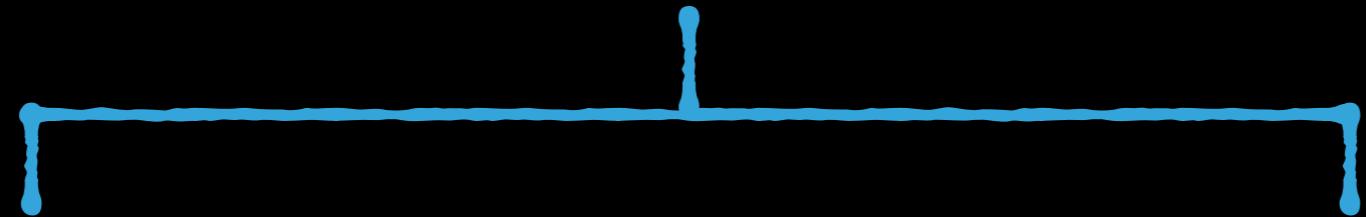
Lower bound on the reheating temperature ~ 5 MeV

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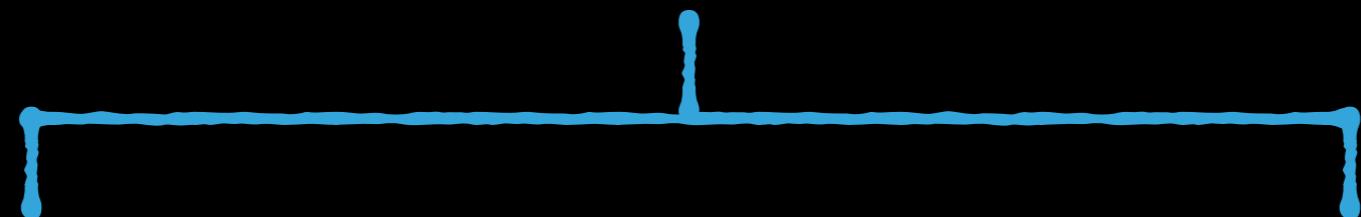
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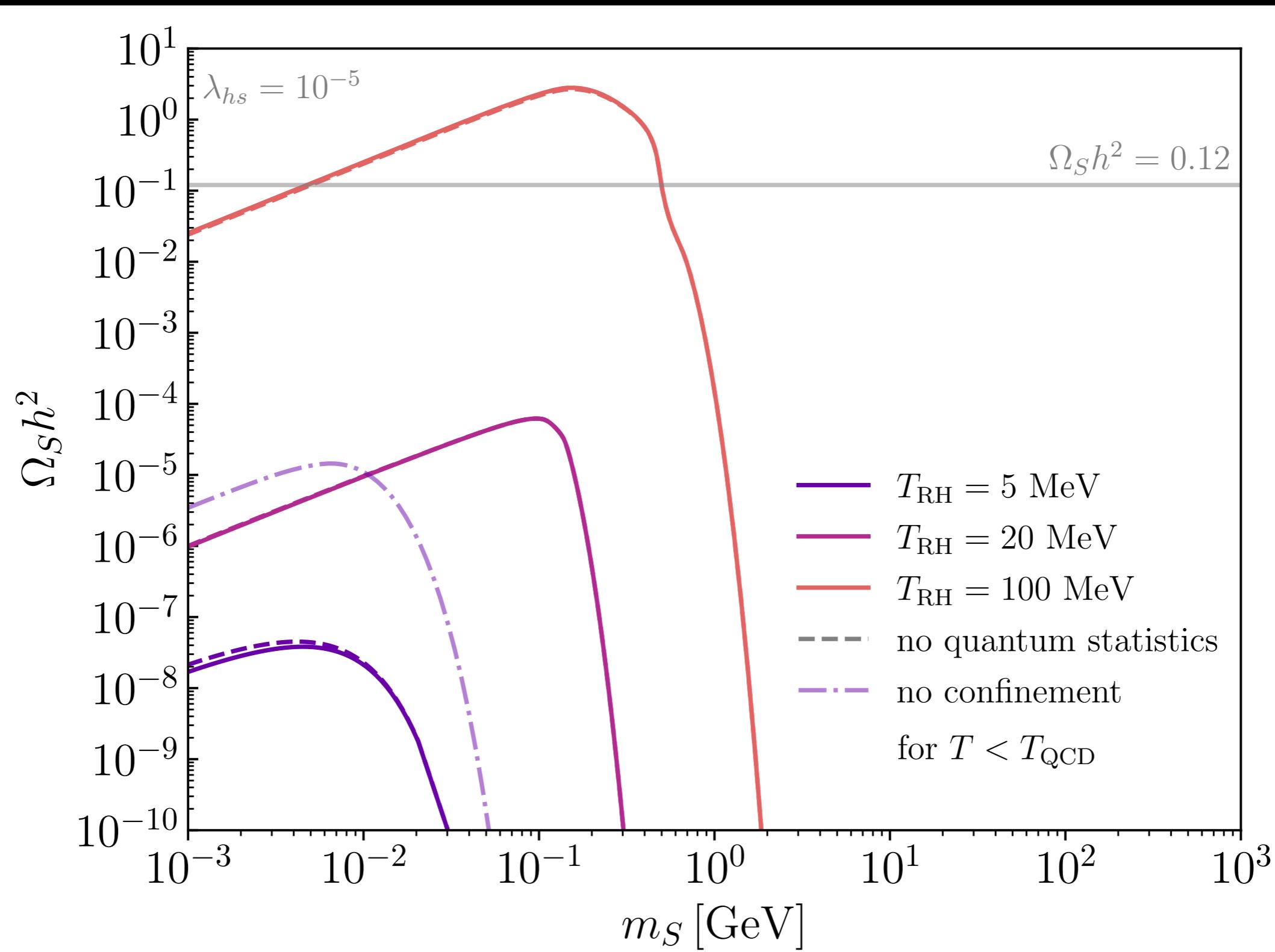
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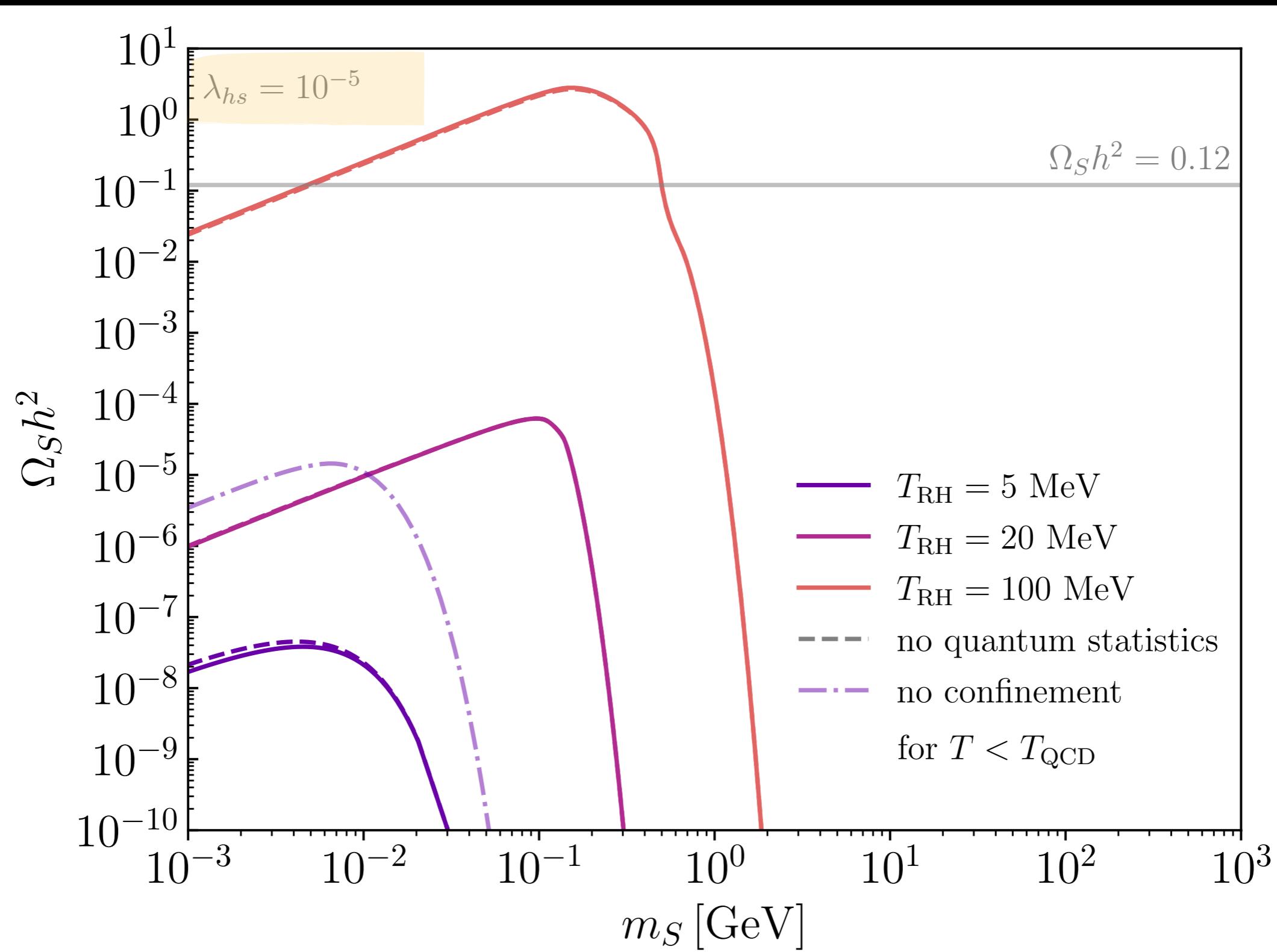
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QCD phase transition is relevant!

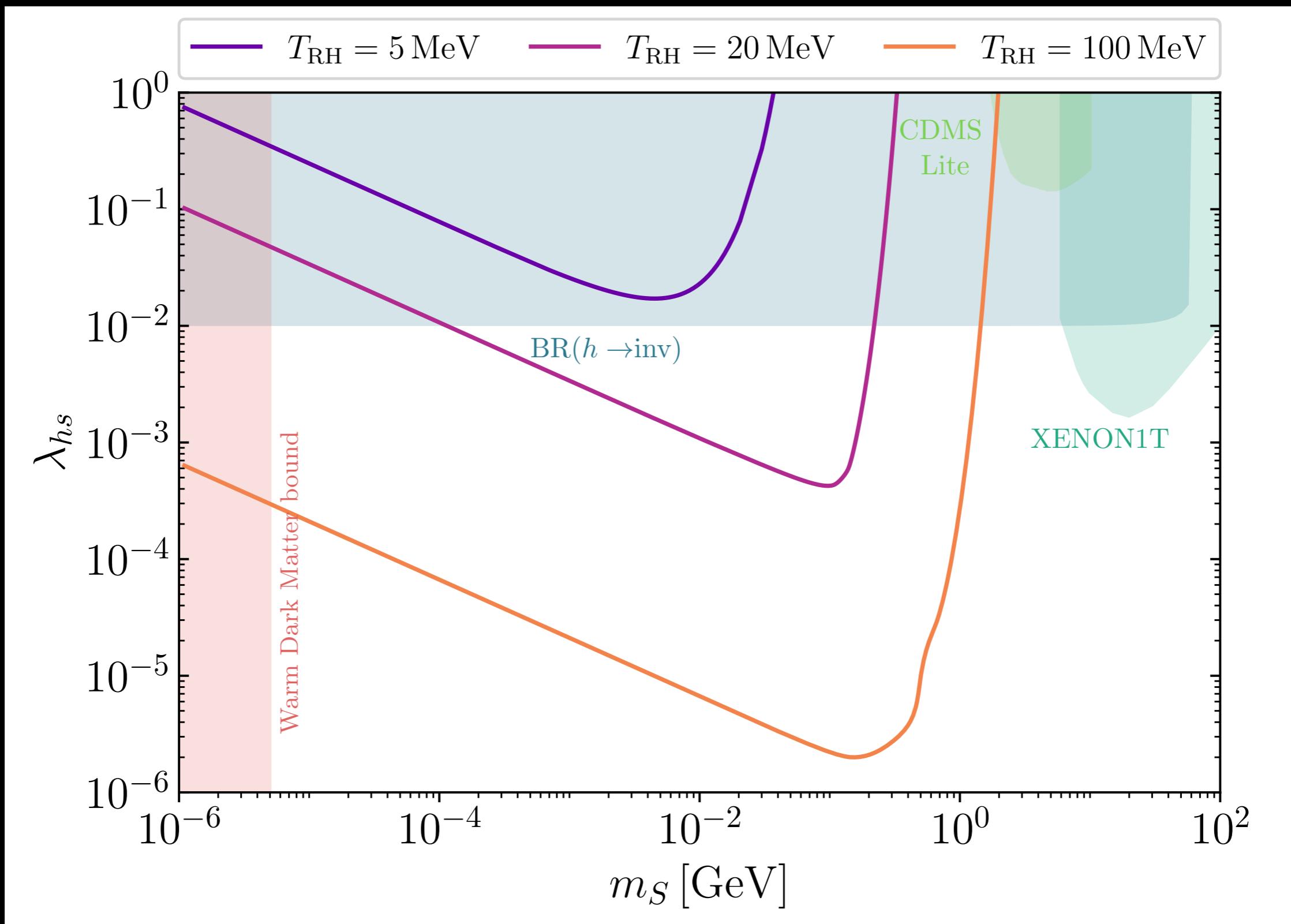
ABUNDANCE:



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λ_{hs} FOR $\Omega_S h^2 = 0.12$:



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FREEZE-IN MODELS ARE EXPERIMENTALLY TESTABLE