Instabilities appearing in dark energy models

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arXiv:2205.01055, arXiv:2204.13098, arXiv: 2107.14215

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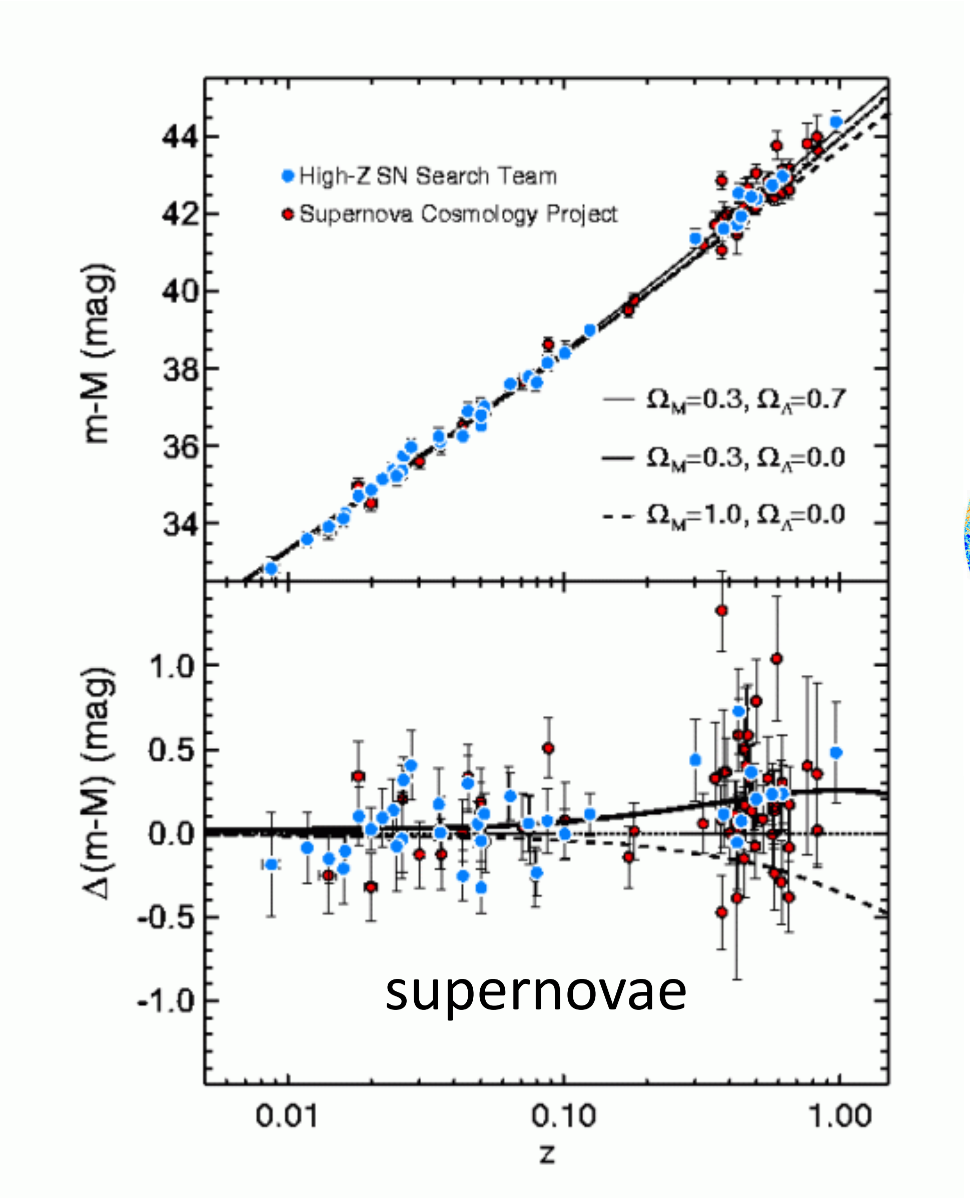
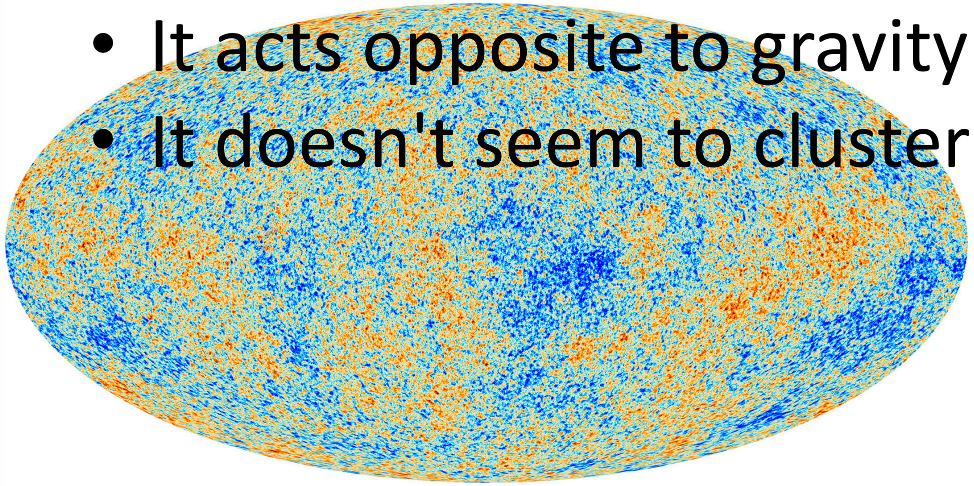
Accelerating expansion of the Universe:



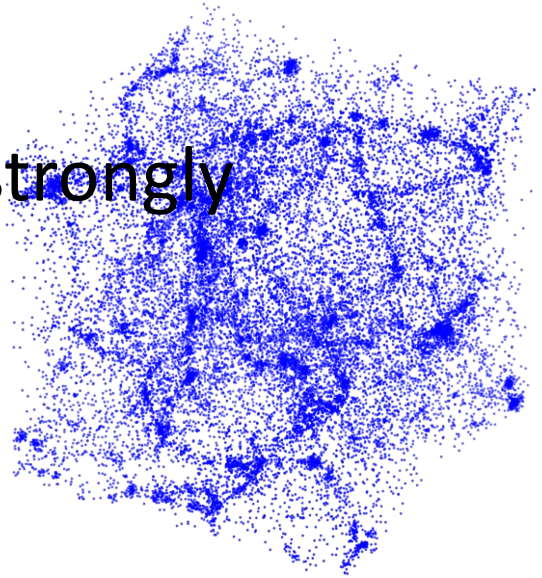
supernovae

$$\Omega_{\rm DE} \sim 0.7$$

- It acts opposite to gravity
- It doesn't seem to cluster strongly



The CMB



Large scale structure

The reason behind the cosmic acceleration:

- Possibilities:
  - Cosmological constant /Dark energy /beyond general relativity

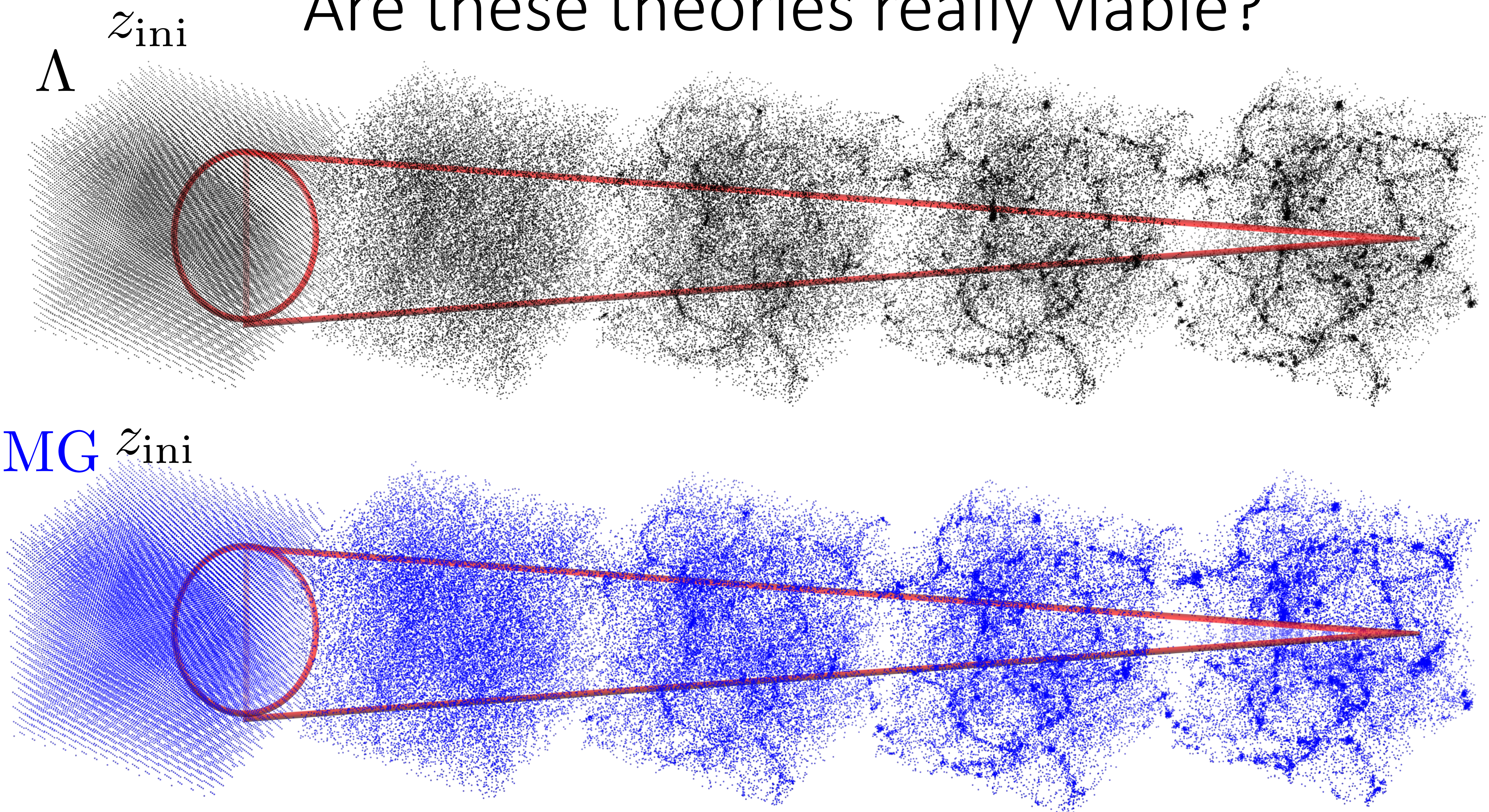
Modified gravity $$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$ Dark energy

- Effective field theory of dark energy:
  - Convenient way to describe DE and MG: a general action for class of theories
  - Connecting to observations minimally

- DE/MG theories are studied well at the level of the background

  and linear

Are these theories really viable?



Importance of non-linear modelling:

$$S_{\mathrm{DE}} = \int d^4x\sqrt{-g}P(X,\phi)$$

$$\updownarrow$$

$$w, c_s^2$$

- Non-linear evolution of the k-essence leads to an instability

  - The instability:

    - In the cosmological context (using k-evolution N-body code)
      (Hassani, Adamek, Kunz, Shi, Wittwer; arXiv: 2107.14215 & arXiv: 2204.13098)

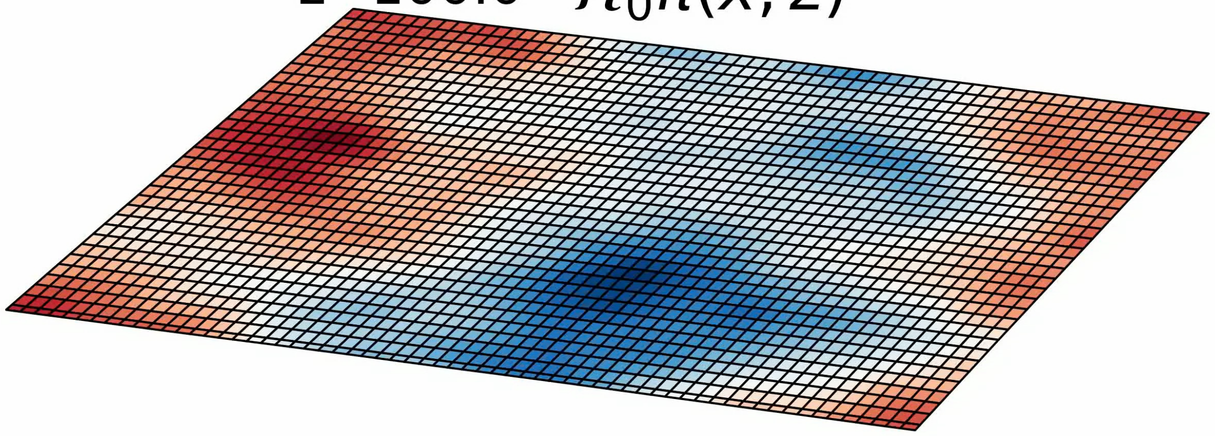
    - In 1+1 dimensions
      (Eckmann, Hassani, Zaag; arXiv: 2205.01055)

Non-linear scalar field equation: $\pi = \frac{\delta\phi}{\bar{\phi}}$

$$\partial_\tau^2\pi + \mathcal{H}(1-3w)\partial_\tau\pi + \Big(\partial_\tau\mathcal{H} - 3w\mathcal{H}^2 + 3c_s^2(\mathcal{H}^2 - \partial_\tau\mathcal{H})\Big)\pi$$
$$- \partial_\tau\Psi + 3\mathcal{H}(w - c_s^2)\Psi - 3c_s^2\partial_\tau\Phi - c_s^2\nabla^2\pi$$
$$= \mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi)$$

$$\mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi) = -\frac{\mathcal{H}}{2}\big(5c_s^2 + 3w - 2\big)\boxed{(\vec{\nabla}\pi)^2}$$
$$+ 2(1-c_s^2)\vec{\nabla}\pi\cdot\vec{\nabla}\partial_\tau\pi - \Big[(c_s^2-1)\big(\partial_\tau\pi + \mathcal{H}\pi - \Psi\big)$$
$$+ c_s^2(\Phi - \Psi) + 3\mathcal{H}c_s^2(1+w)\pi\Big]\nabla^2\pi + (2c_s^2-1)\vec{\nabla}\Psi\cdot\vec{\nabla}\pi$$
$$- c_s^2\vec{\nabla}\Phi\cdot\vec{\nabla}\pi + \frac{3(c_s^2-1)}{2}\partial_i\Big(\partial_i\pi(\vec{\nabla}\pi)^2\Big)\,.$$

$c_s^2 < 10^{-4.7}$ $\mathcal{H}_0\pi$



No instability for large c_s^2

Non-linear scalar field equation:

$$\partial_\tau^2\pi + \mathcal{H}(1-3w)\partial_\tau\pi + \Big(\partial_\tau\mathcal{H} - 3w\mathcal{H}^2 + 3c_s^2(\mathcal{H}^2 - \partial_\tau\mathcal{H})\Big)\pi$$
$$-\partial_\tau\Psi + 3\mathcal{H}(w - c_s^2)\Psi - 3c_s^2\partial_\tau\Phi - c_s^2\nabla^2\pi$$
$$= \mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi)$$

3+1 D

Spherical symmetry

$$\frac{d^2\pi}{d\tau^2} = +c_s^2\frac{\partial^2\pi}{\partial r^2} - \alpha\Big(\frac{d\pi}{dr}\Big)^2$$

1+1 D

$$c_s^2 \to 0 \qquad \frac{d^2\pi}{d\tau^2} = -\alpha\Big(\frac{d\pi}{dr}\Big)^2$$

This PDE is unstable. The curvature of the scalar field around minima blows up in finite time.

$$\frac{d^2\pi}{d\tau^2} = -\alpha\left(\frac{d\pi}{dr}\right)^2$$

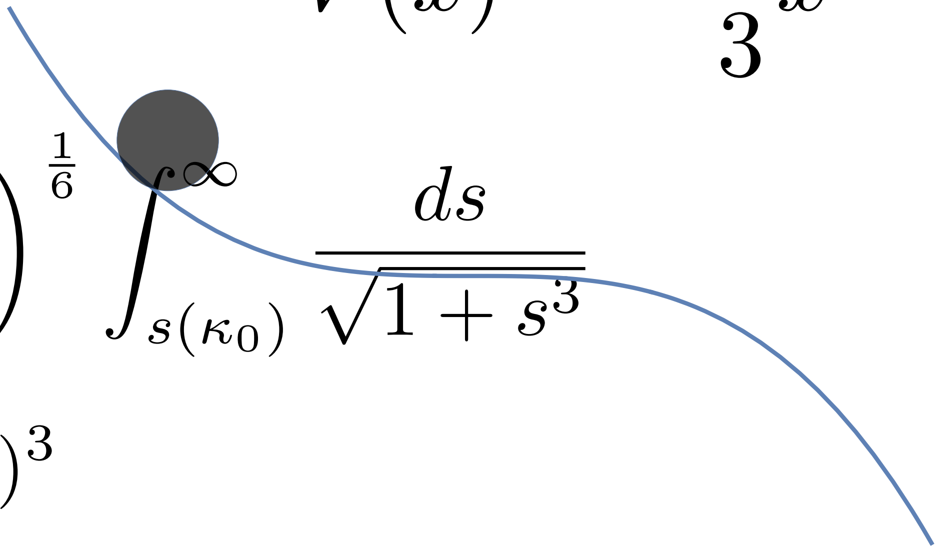
One special solution $$\pi_s(\tau, r) = \frac{\kappa(\tau)}{2} r^2$$

$$\partial_\tau^2 \kappa(\tau) = \alpha\left[\kappa(\tau)\right]^2$$

We can think of this ODE as Newton's second law

$$F(x) = x^2$$

$$V(x) = -\frac{4}{3}x^3$$



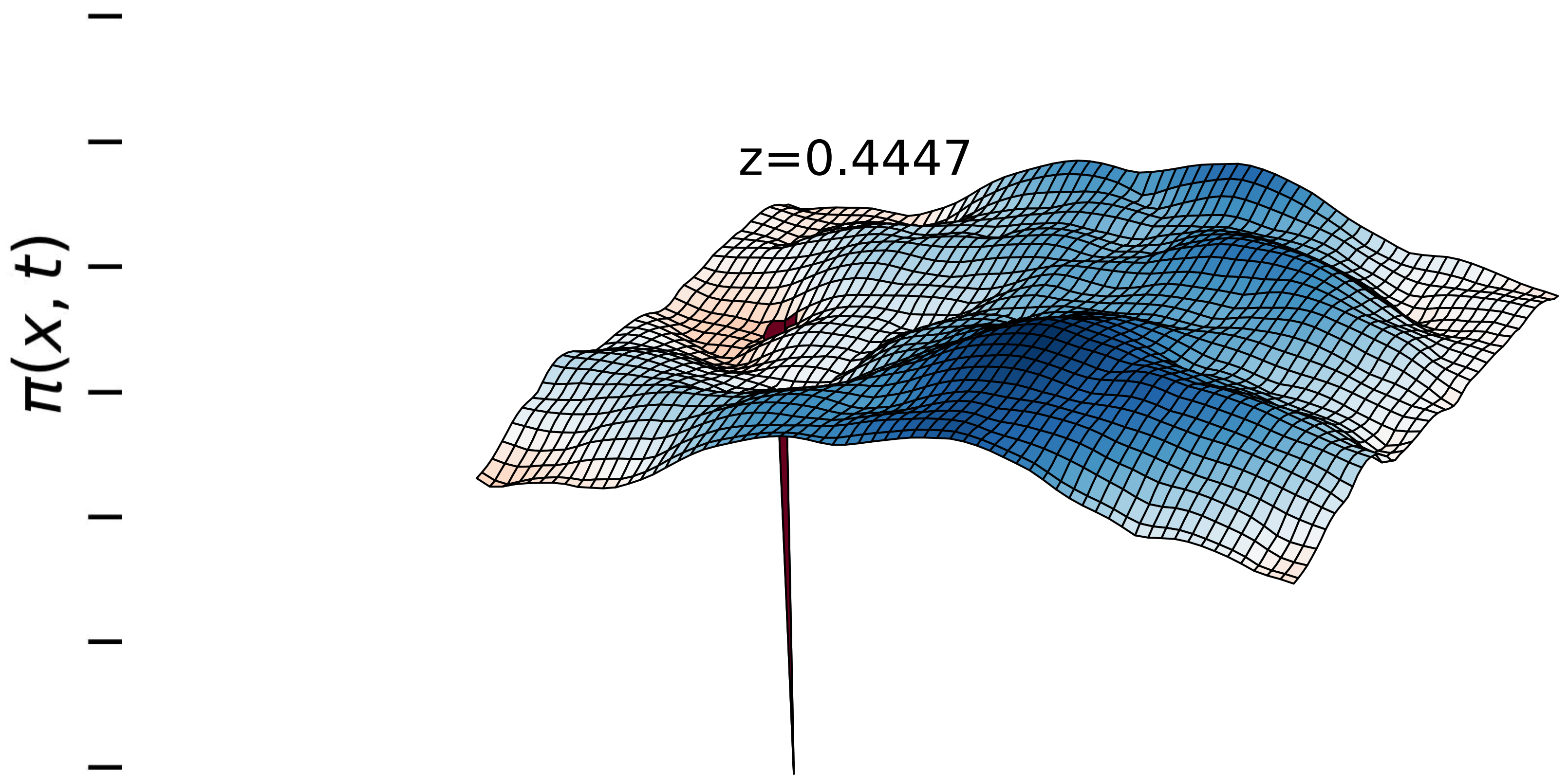
$$\kappa(\tau) = \frac{3}{2(\tau - \tau_b)^2}$$

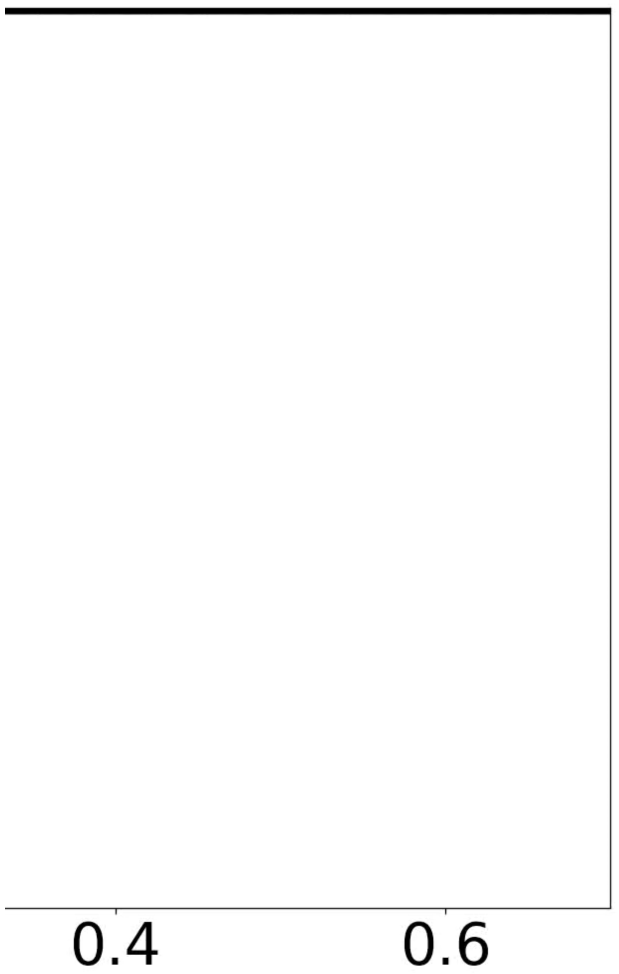
$$\tau_b = \left(\frac{3}{8\alpha}\right)^{\frac{1}{3}} \left(\frac{1}{C}\right)^{\frac{1}{6}} \int_{s(\kappa_0)}^{\infty} \frac{ds}{\sqrt{1+s^3}}$$

$$C = \kappa'(0)^2 - \frac{8}{3}\kappa(0)^3$$

Hassani, Shi, Adamek, Kunz, Shi, Wittwer; arXiv: 2107.14215

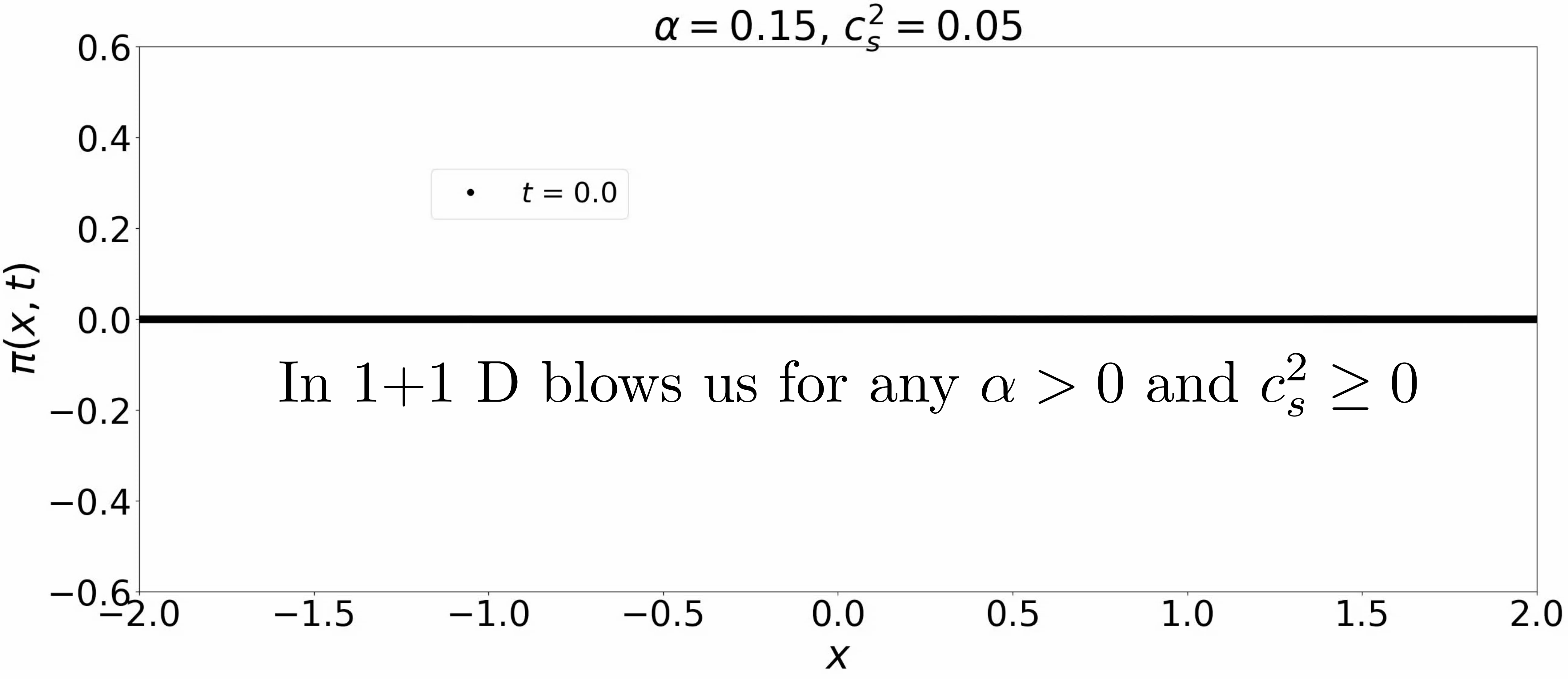
V-type blowup $\pi(0,x)=0, \partial_\tau \pi(0,x)=\Phi(x)$





(Hassani, Adamek, Kunz, Shi, Wittwer; arXiv: 2107.14215 & arXiv: 2204.13098)

M-type blowup



In 1+1 D blows us for any $\alpha > 0$ and $c_s^2 \geq 0$

$$\frac{d^2u}{d\tau^2} = +c_s^2\frac{\partial^2 u}{\partial x^2} - \alpha\left(\frac{du}{dx}\right)^2$$

(Eckmann, Hassani, Zaag; arXiv: 2205.01055)

Take home messages:

- Non-linearities can be **VERY** important
- Cosmological N-body simulations bound : $c_s^2 \geq 10^{-4.7}$
- From mathematical point of view any c_s^2 leads to an instability!
- Can k-essence be a cosmic acceleration candidate?

Thank you ☺