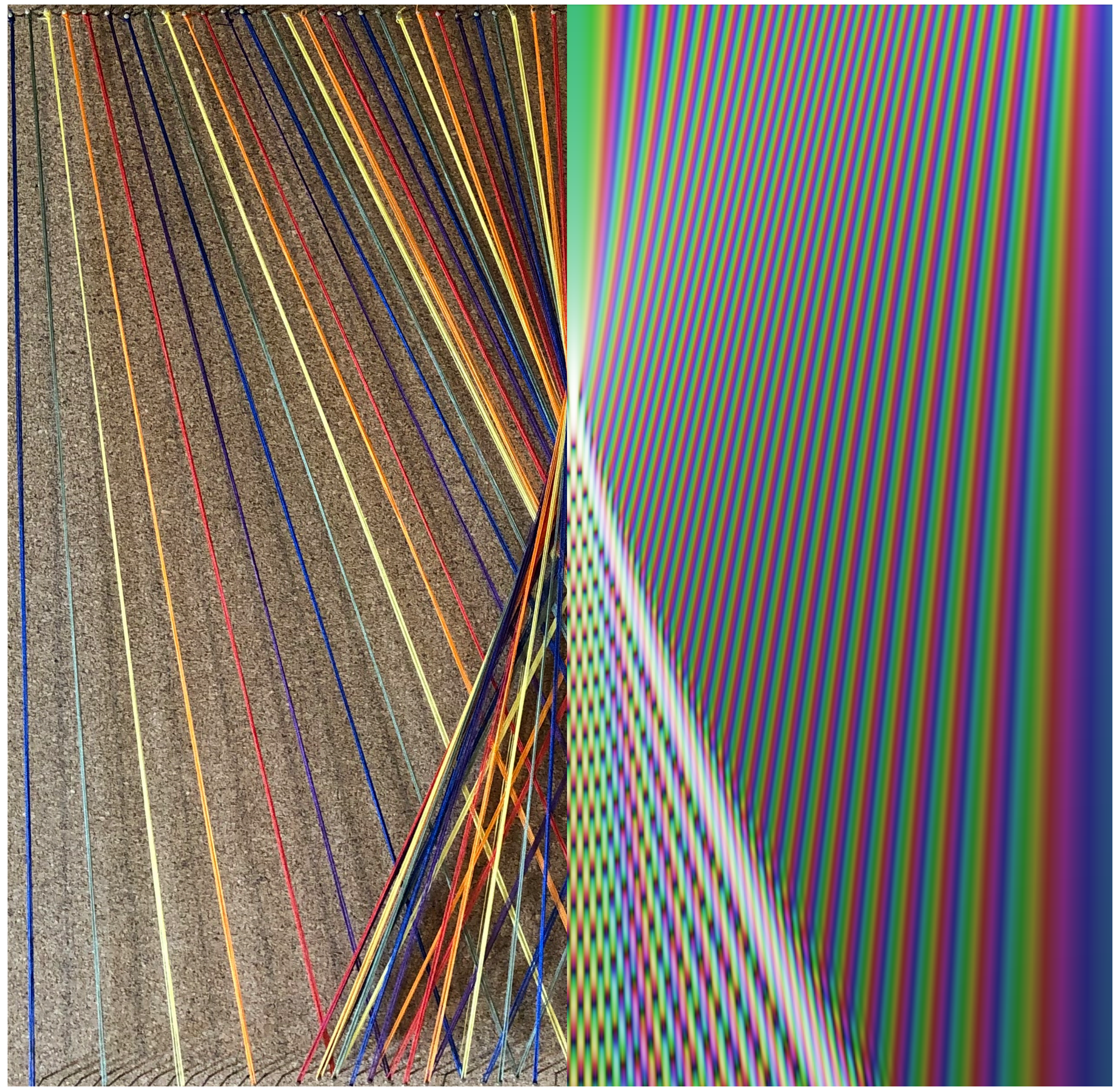
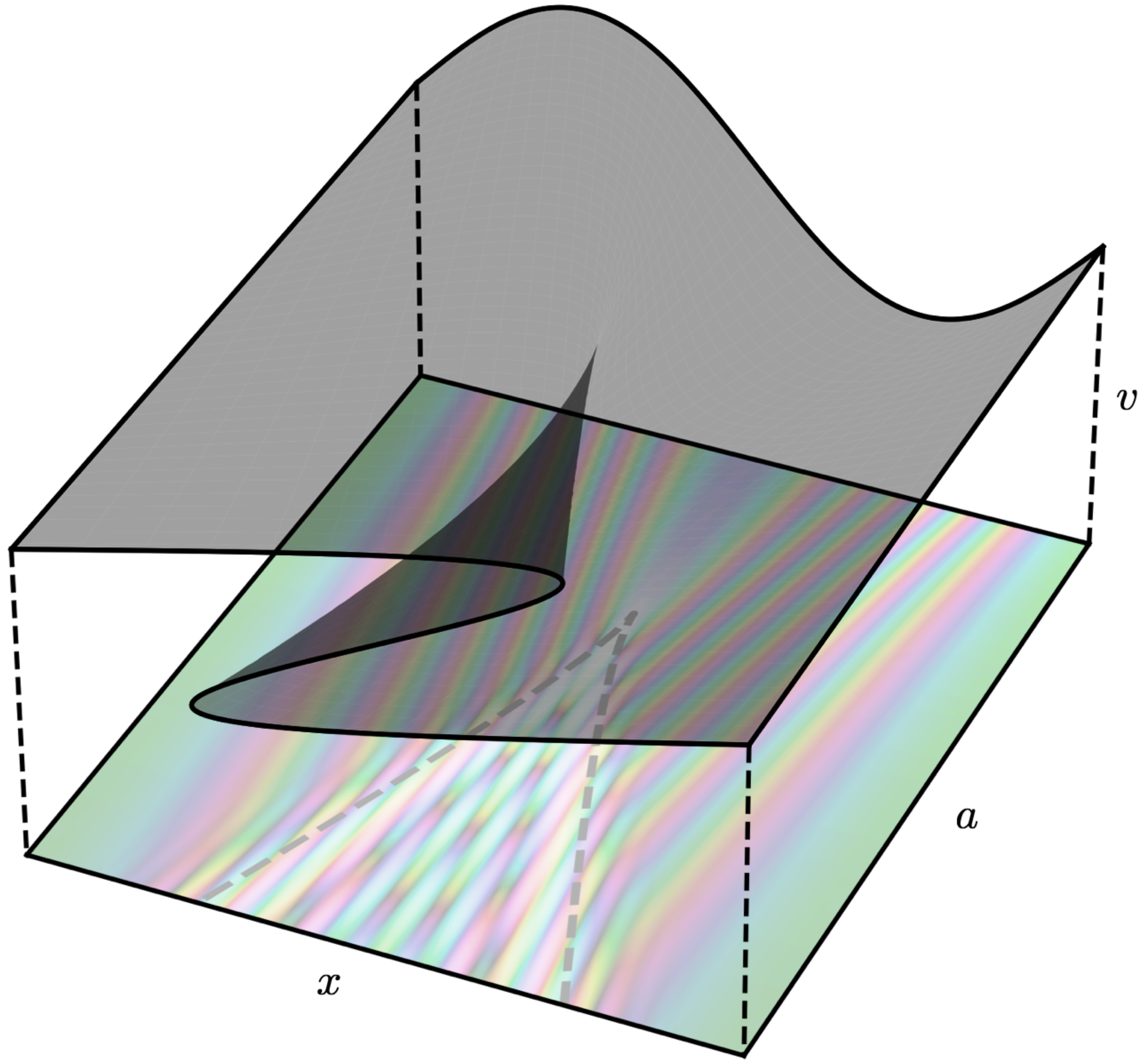
Making (dark matter) waves:
Untangling wave interference for multi-streaming dark matter





Alex Gough

with Cora Uhlemann

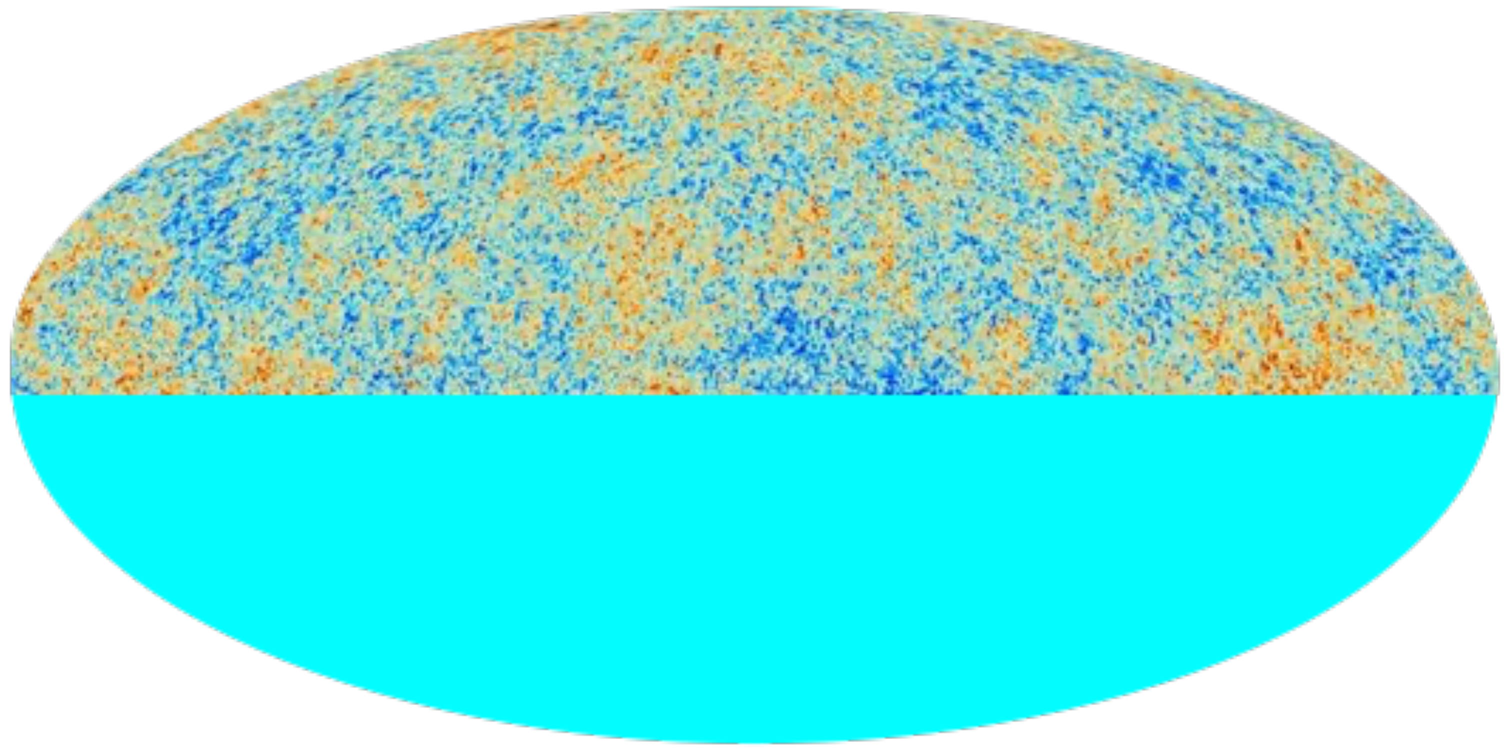
Newcastle University

arXiv: 2206.11918

Cosmology from Home 4—15 July 2022

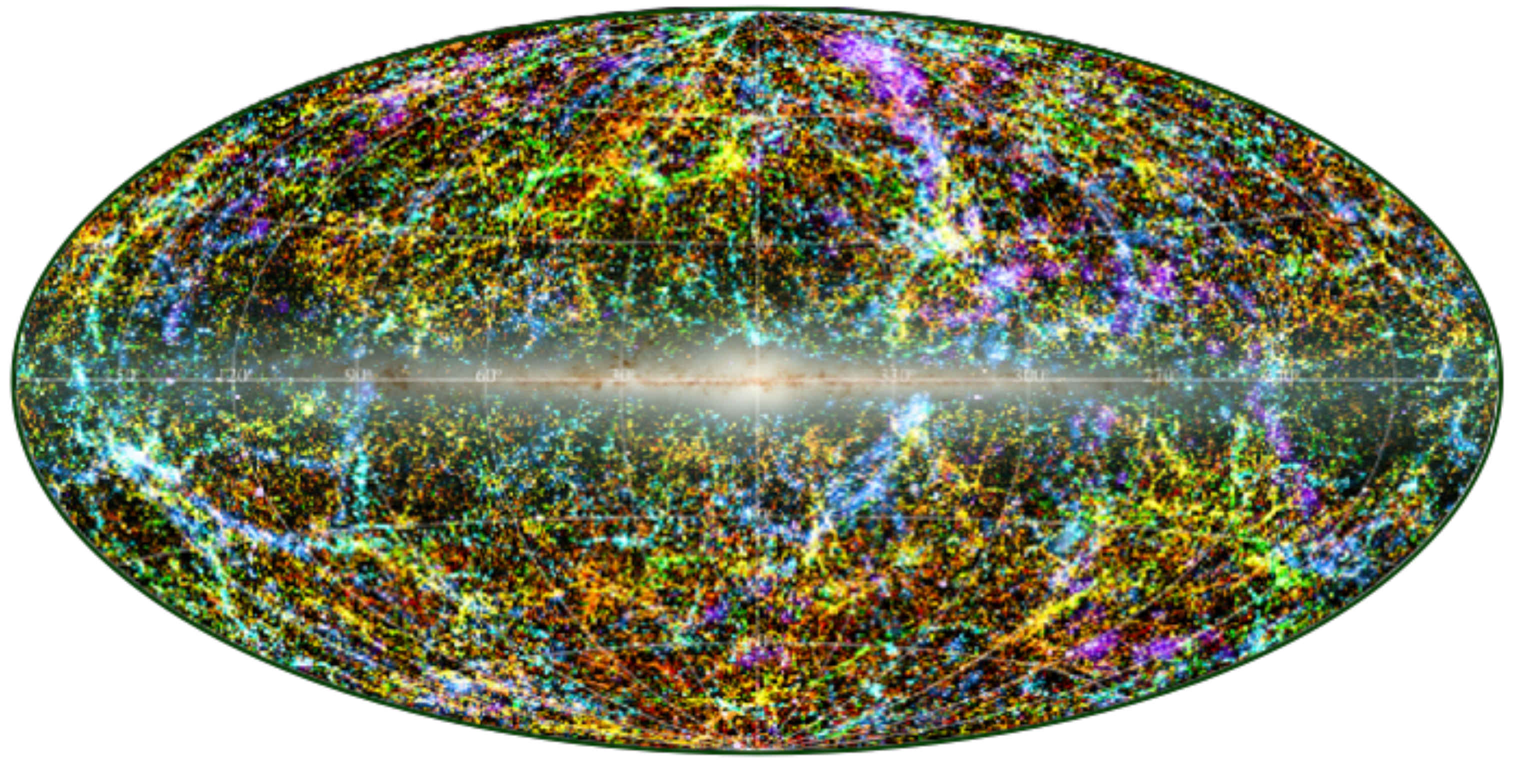
Big questions

Afterglow
of the early universe



nearly uniform

Cosmic web
of galaxies



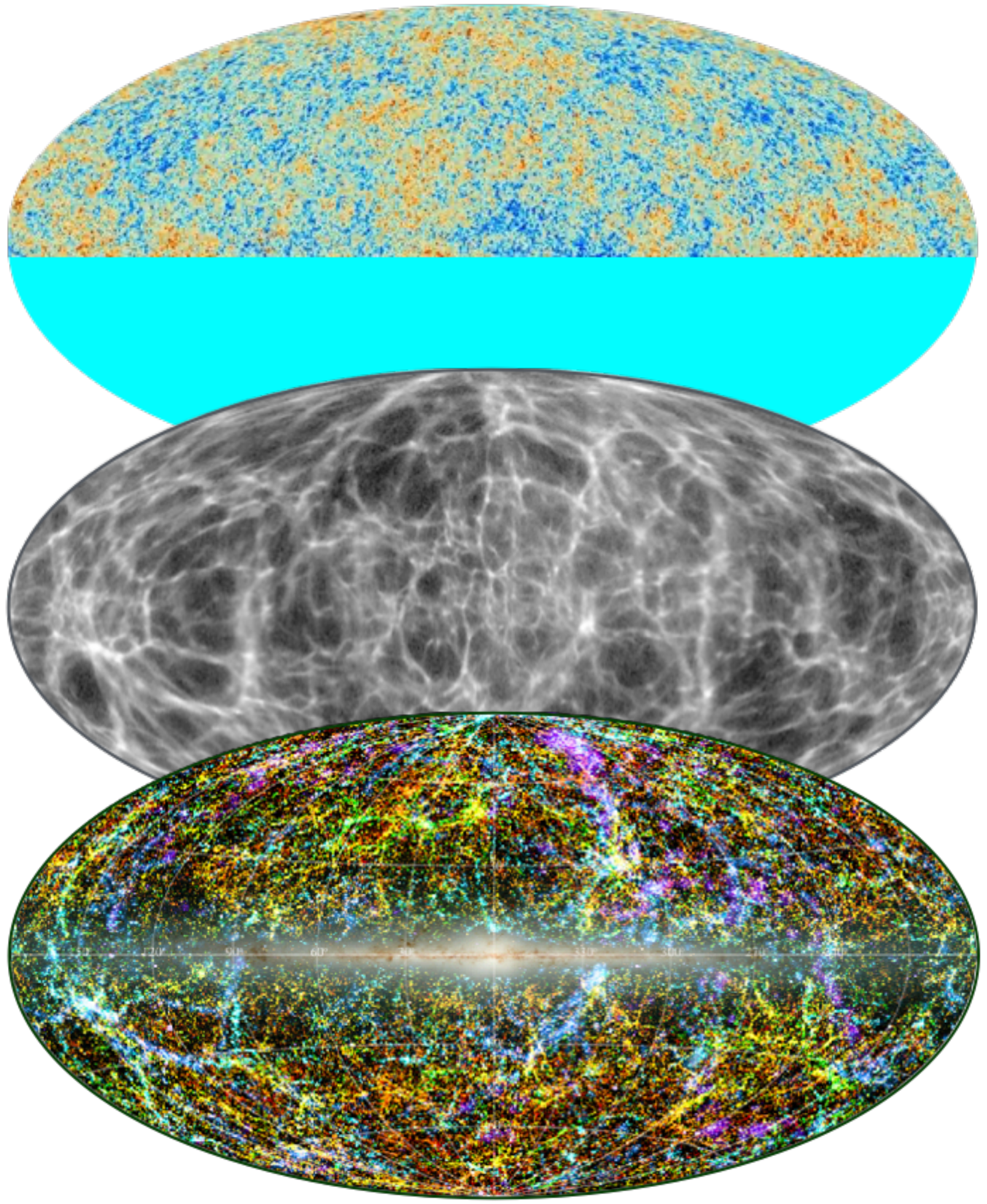
rich structure

Big questions

Afterglow
of the early universe

Skeleton
of dark matter

Cosmic web
of galaxies



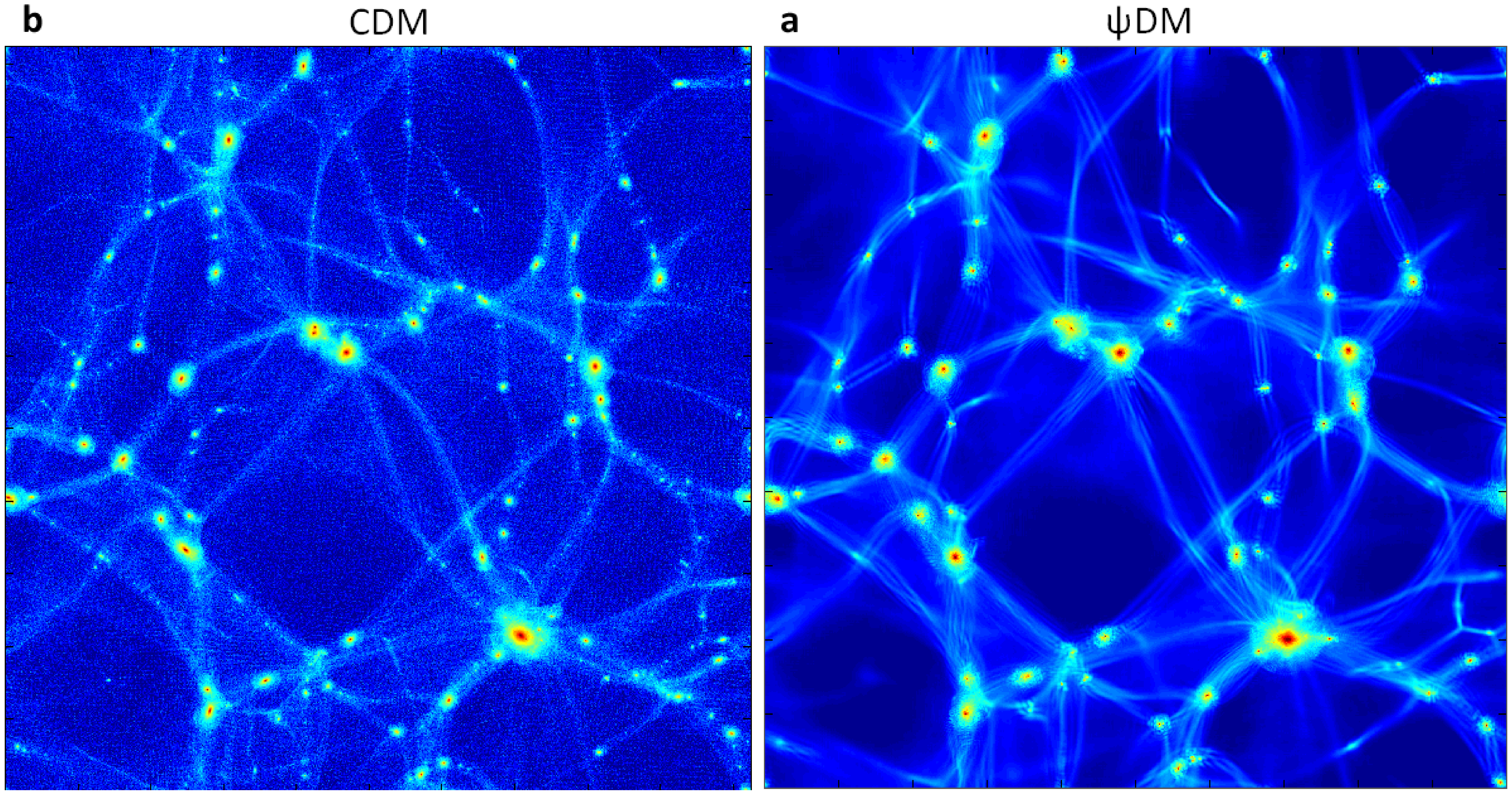
nearly uniform

rich structure

Wave dark matter

What do we see?

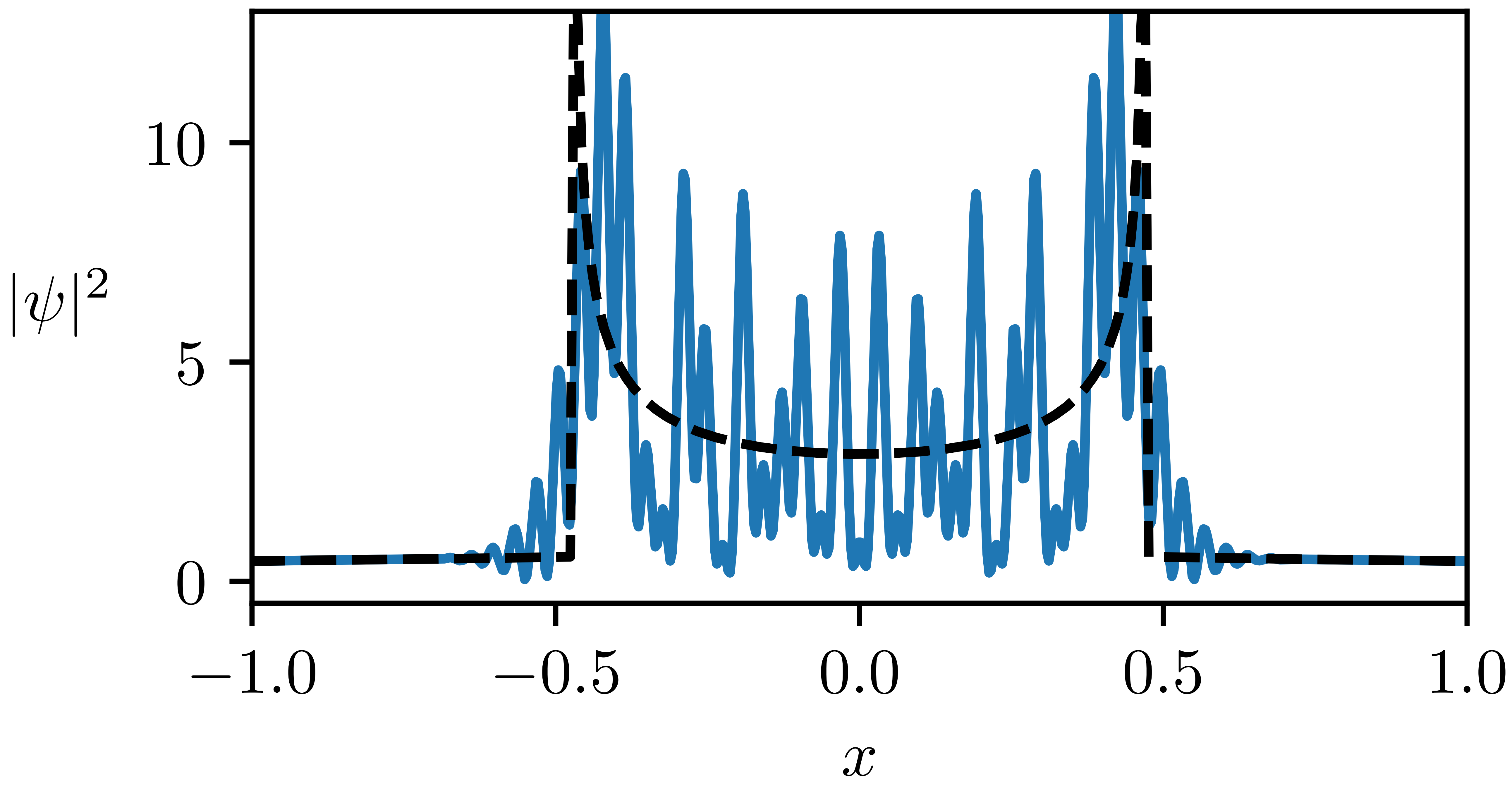
- Same large scale network as CDM
- Wave interference “decorates” the cosmic web

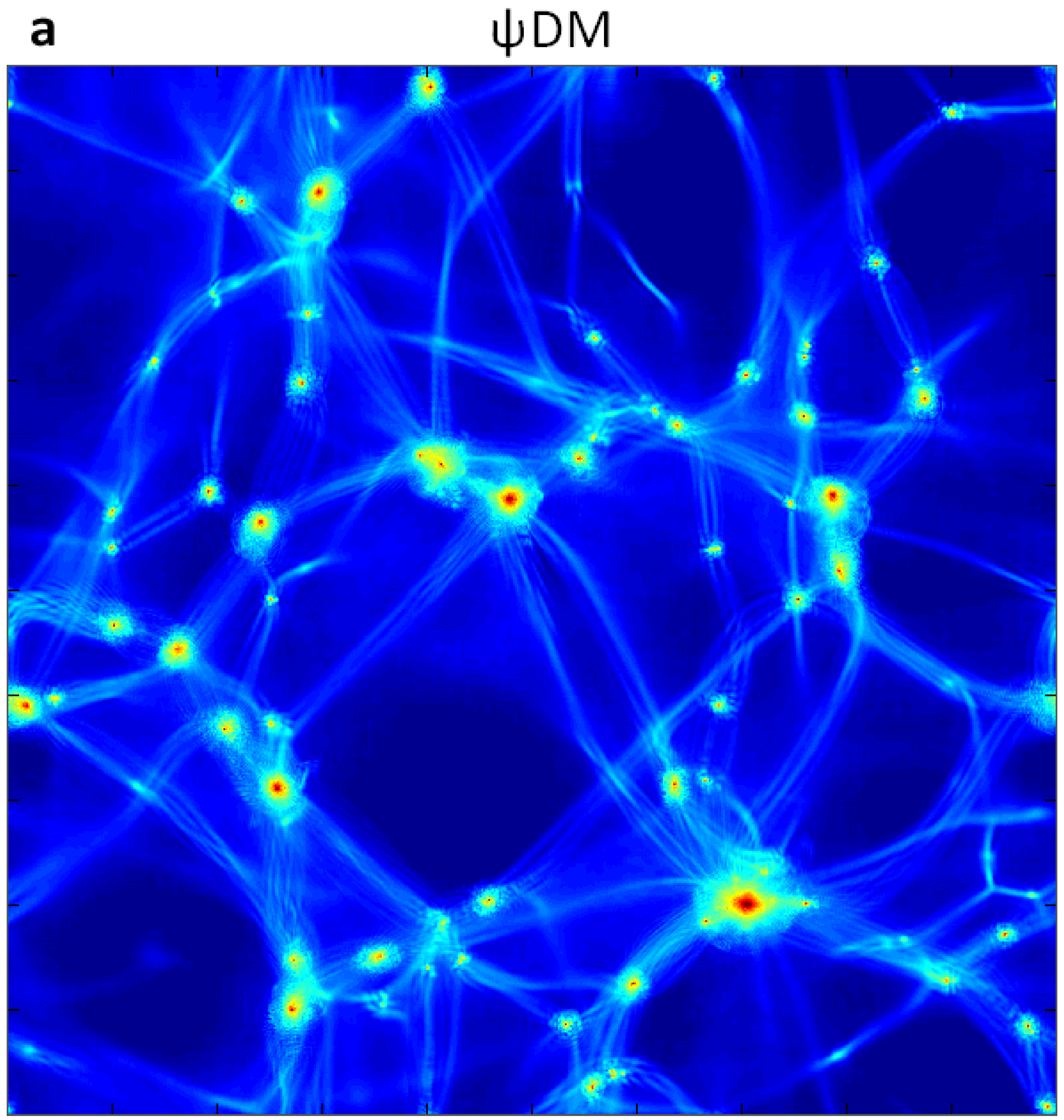


Schive ++ Nature Phys. Lett, `15

astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Wave dark matter





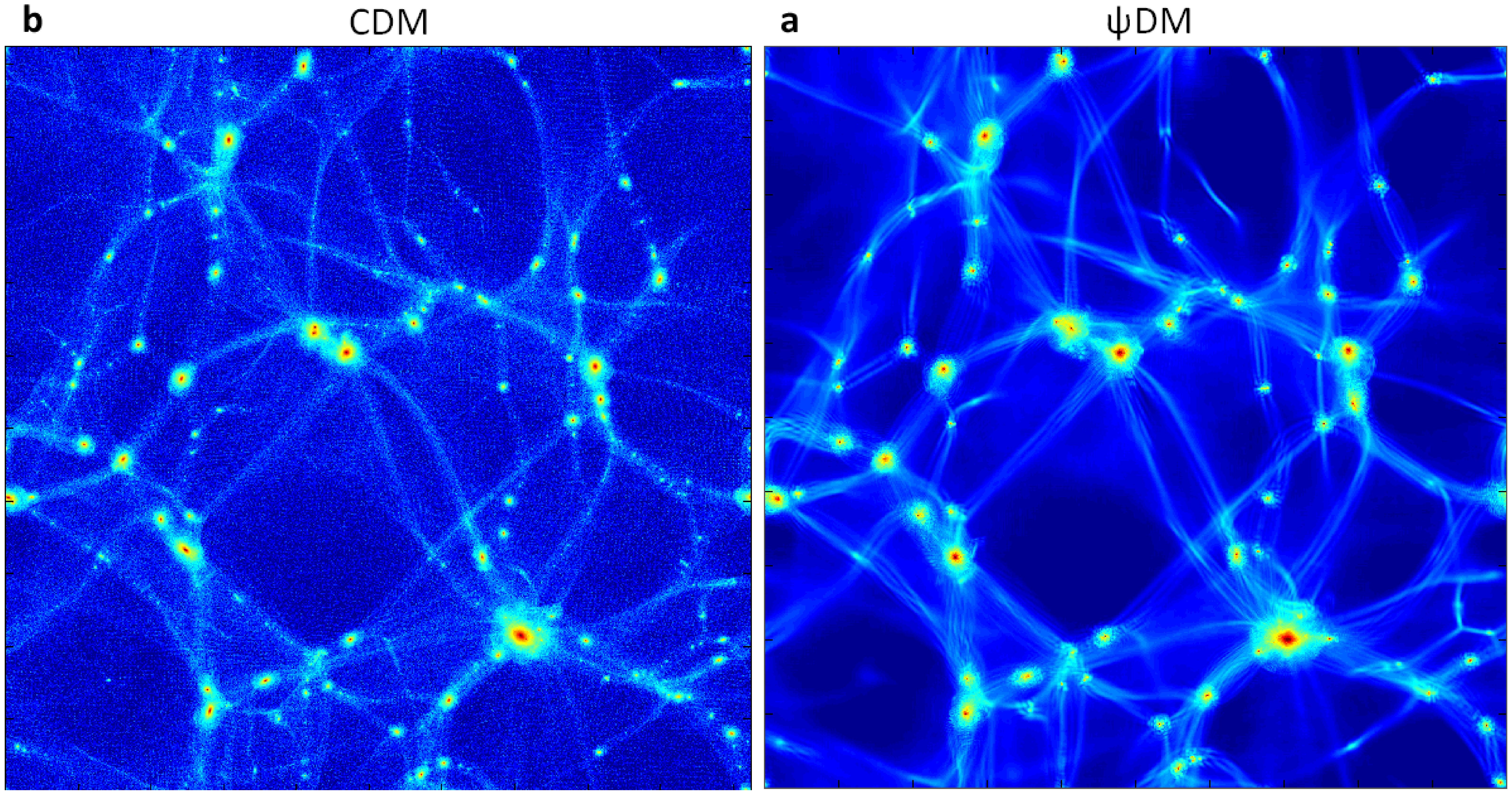
Schive ++ Nature Phys. Lett, `15
astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Wave dark matter

Why do we care?

- True wavelike dark matter (axions etc)

- Learn about CDM via analytics on wave dark matter



Schive ++ Nature Phys. Lett, `15

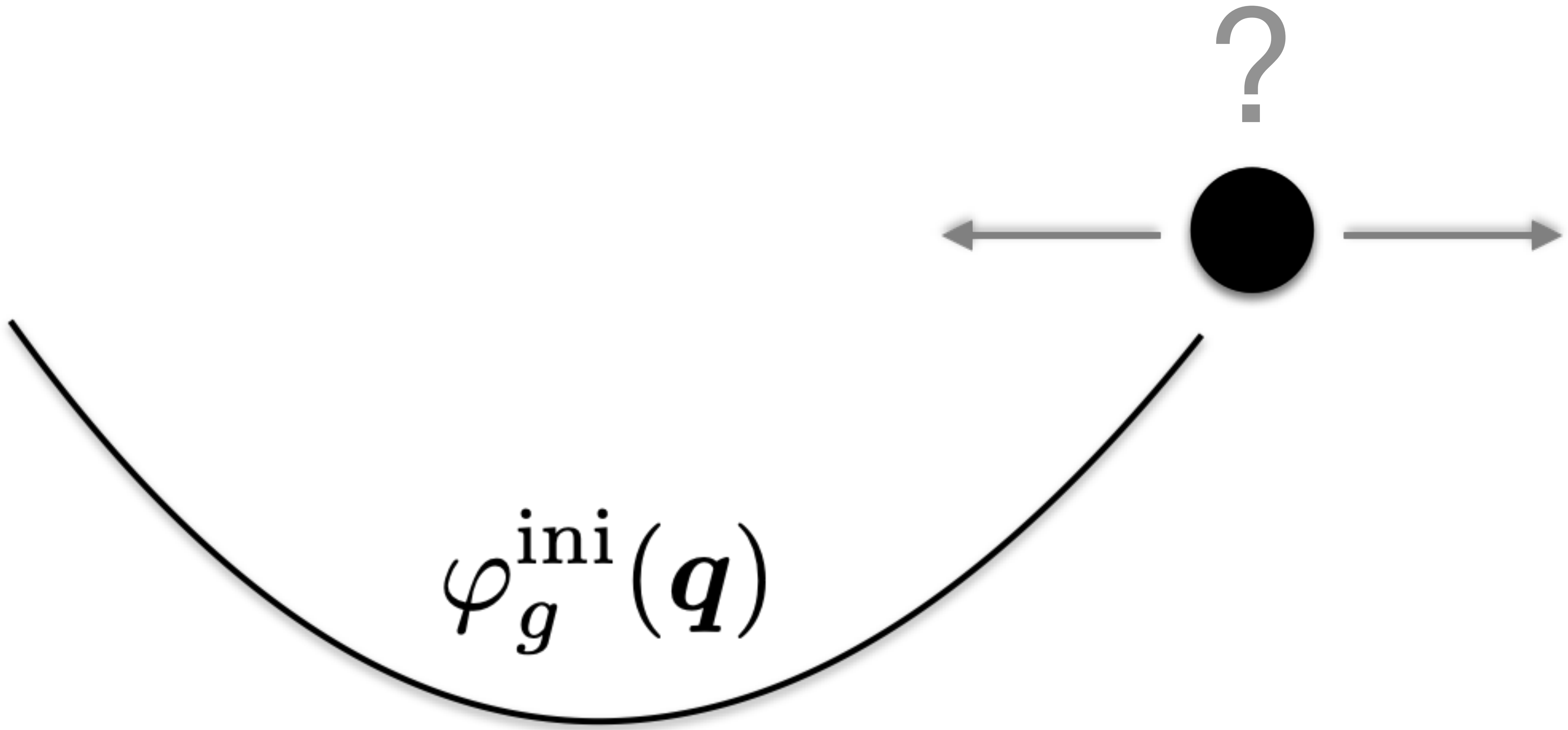
astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Classical dynamics

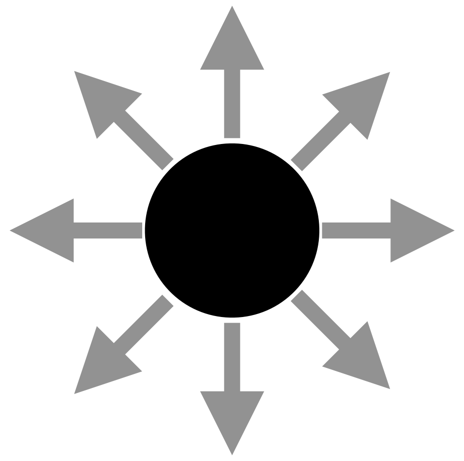
Approximate: shoot particles following initial potential

$$\boldsymbol{v}(\boldsymbol{q}, a) = -\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}(\boldsymbol{q})$$

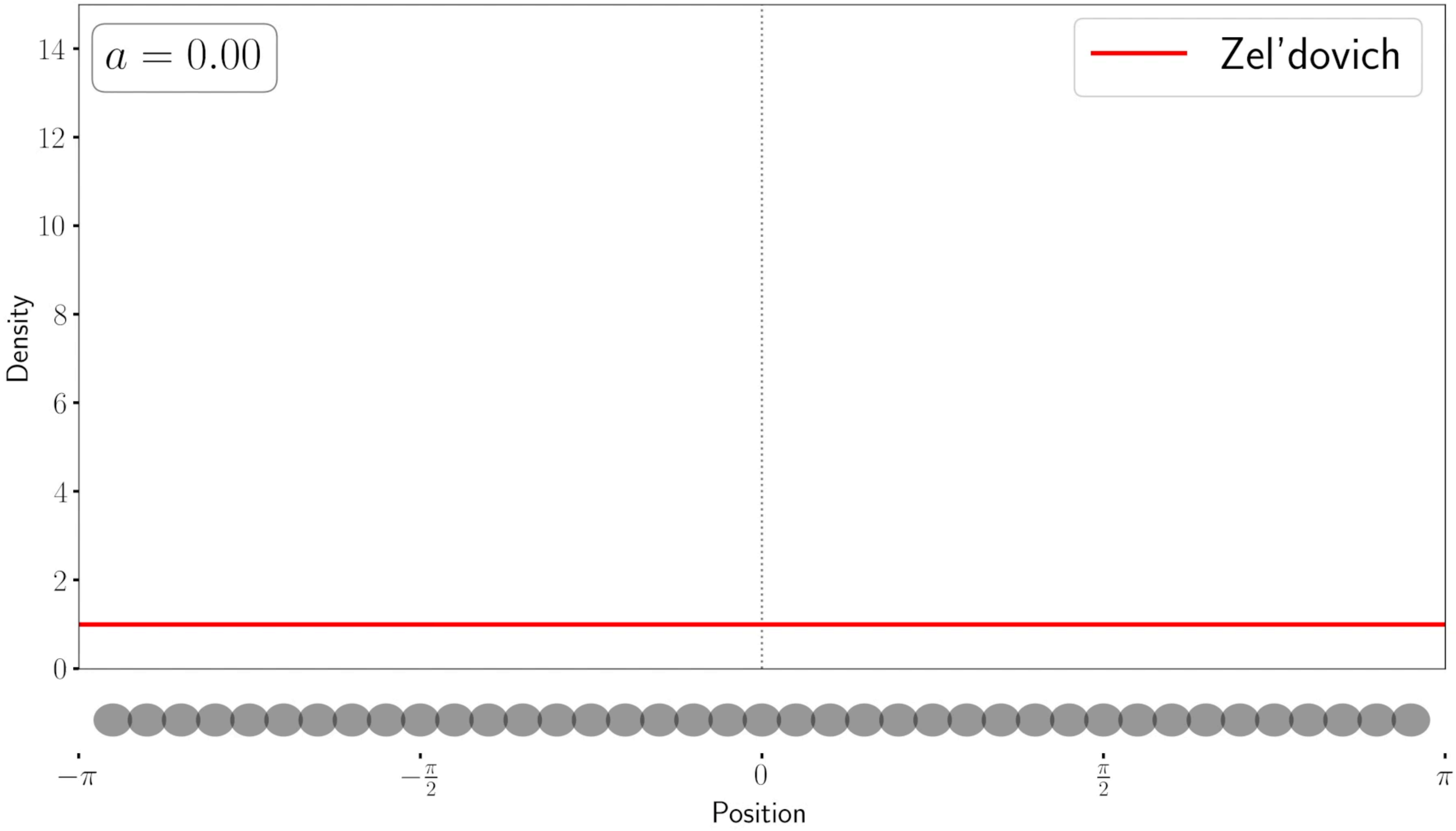
$$\boldsymbol{x}(\boldsymbol{q}, a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}(\boldsymbol{q})$$ Zel'dovich approximation*

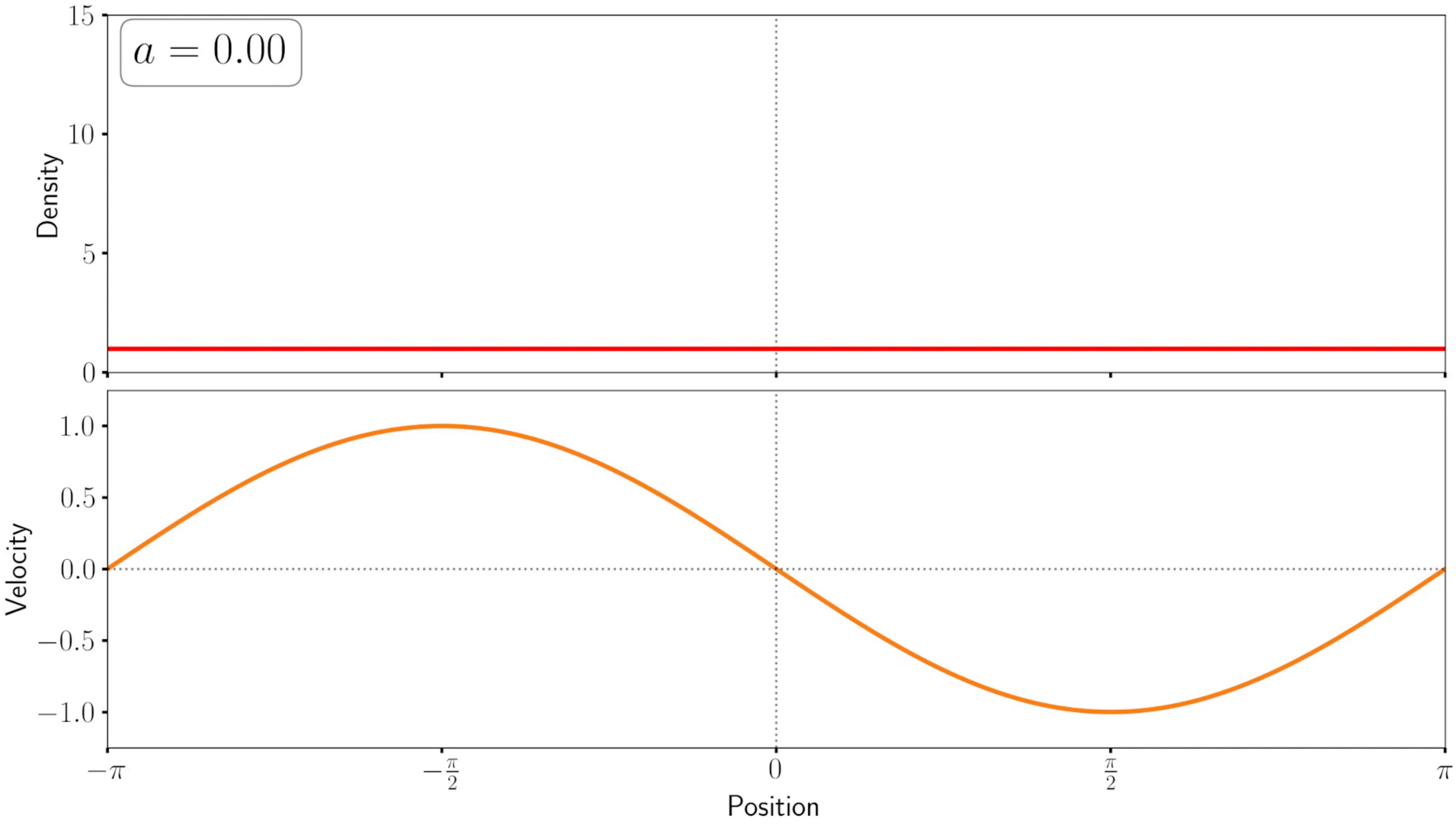


*(Lagrangian) perturbation theory:
ZA + tidal effects

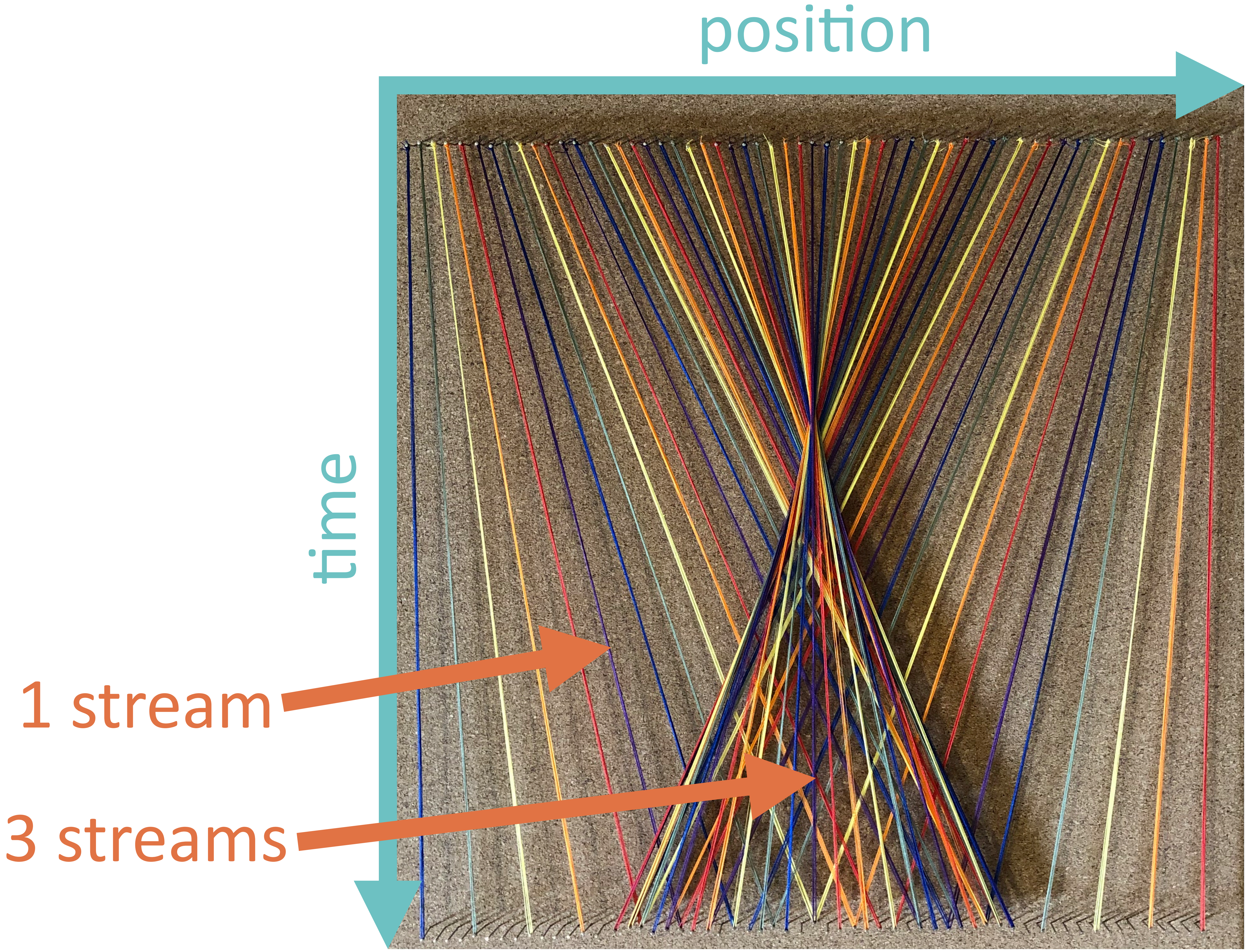


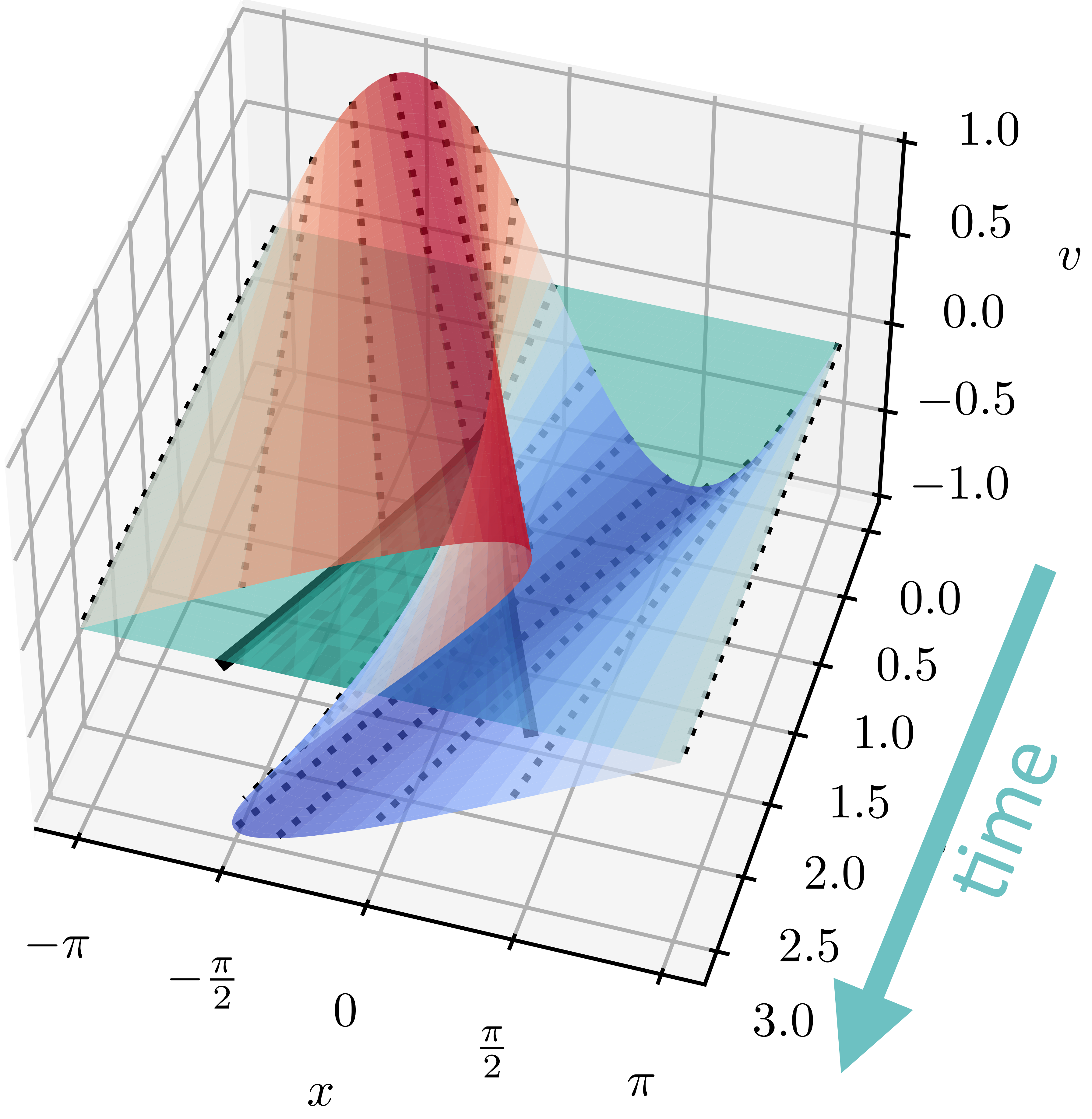
Zel'dovich A&A 1970


$a = 0.00$
Zel'dovich
Density
0
2
4
6
8
10
12
14
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position


$a = 0.00$
Density
15
10
5
0
Velocity
1.0
0.5
0.0
−0.5
−1.0
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position

Multi-streaming





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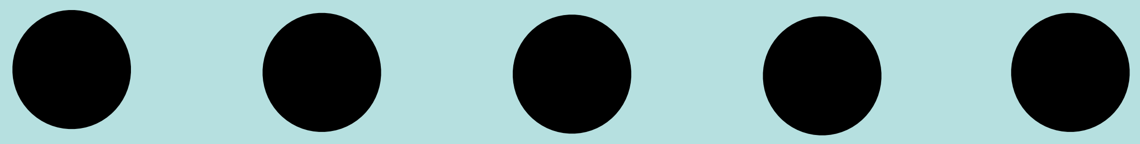
Simple models

Cold Dark Matter

Particles

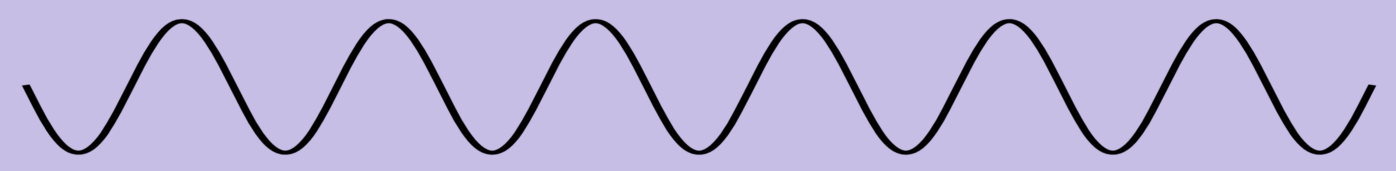


Zel'dovich approximation

$$\boldsymbol{x} = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}$$

Wave Dark Matter

Waves



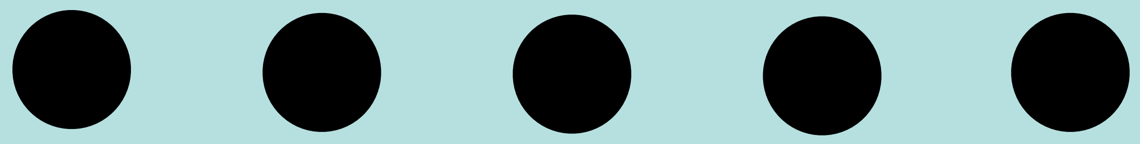
?

Widrow & Kaiser APJ 1993
Coles 2002

Simple models

Cold Dark Matter

Particles



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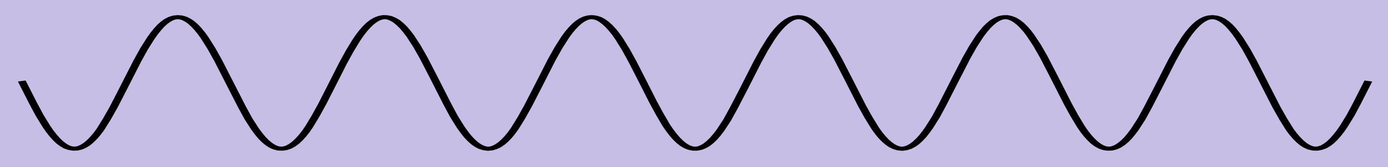
Wave Dark Matter

Waves



Free Schrödinger

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi$$

Widrow & Kaiser APJ 1993
Coles 2002

The simple wave model

How to build the analogous system for the simple example?

Observables

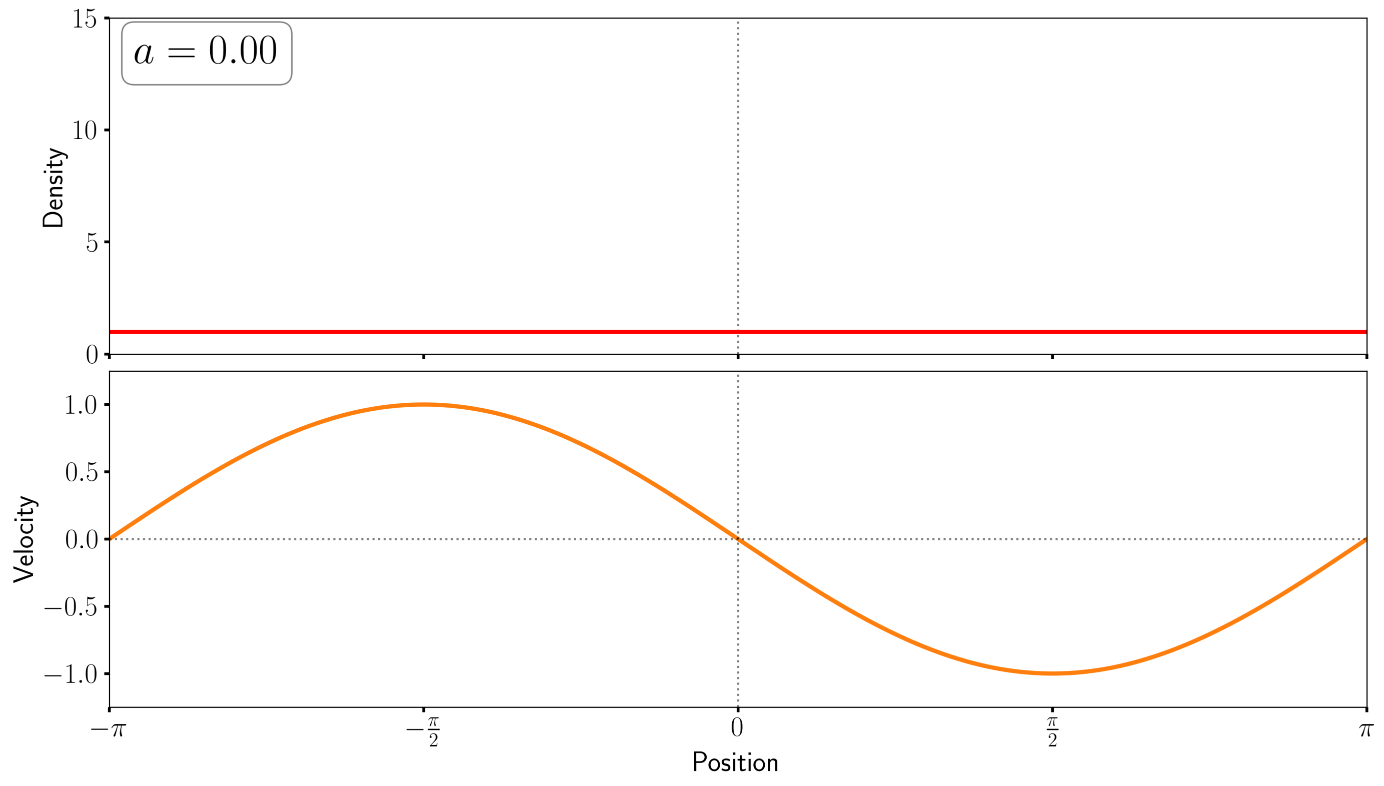
$$\psi = \sqrt{\rho}\exp\left(\frac{i}{\hbar}\phi_v\right)$$

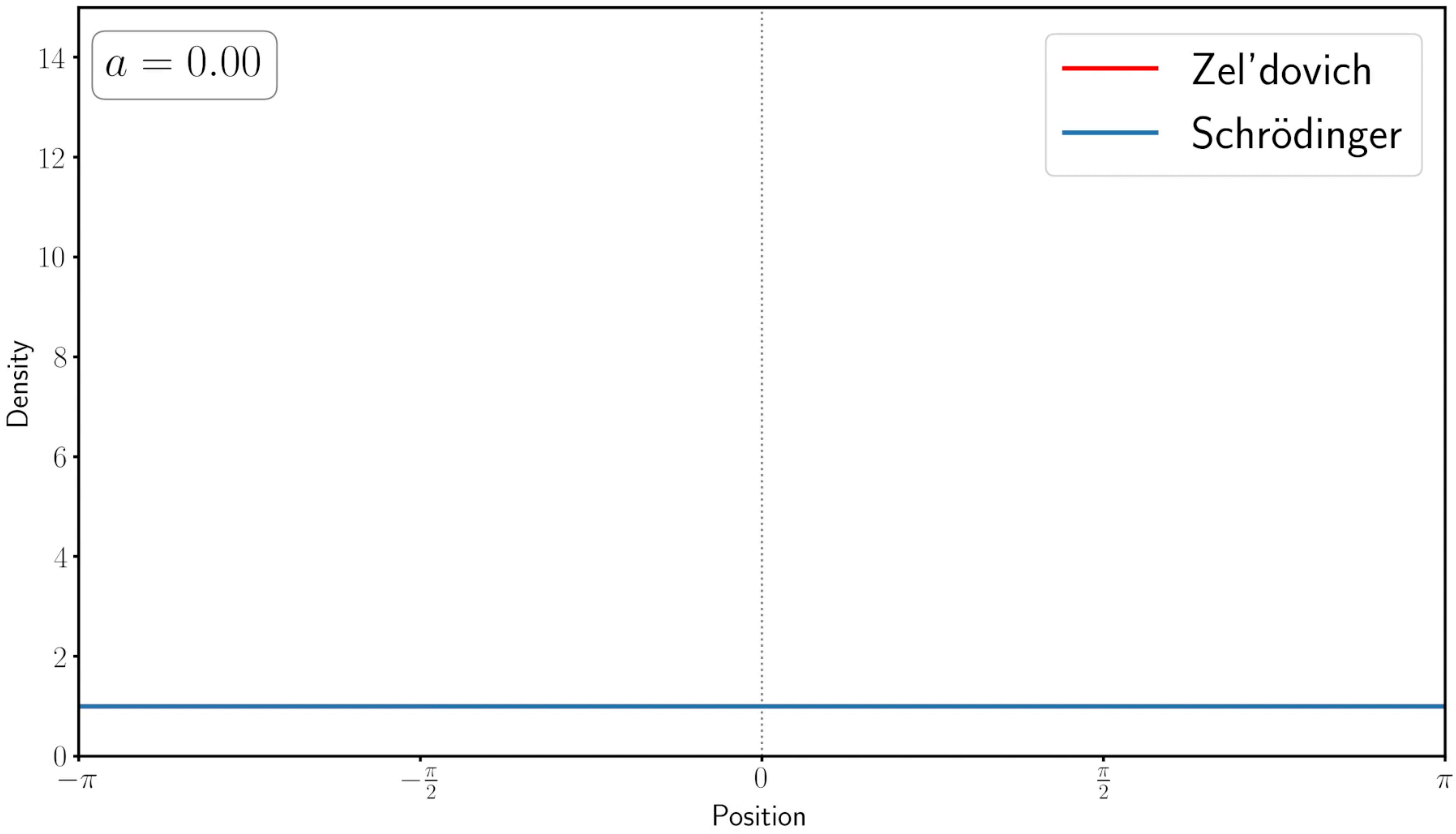
Density

Velocity $\boldsymbol{v} = \nabla\phi_v$

Zel'dovich initial conditions (uniform density, sinusoid velocity)

$$\psi^{(\mathrm{ini})}(q) = \exp\left(\frac{i}{\hbar}\cos(q)\right)$$





$a = 0.00$
Zel'dovich
Schrödinger
Density
0
2
4
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8
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12
14
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position

Free wave evolution

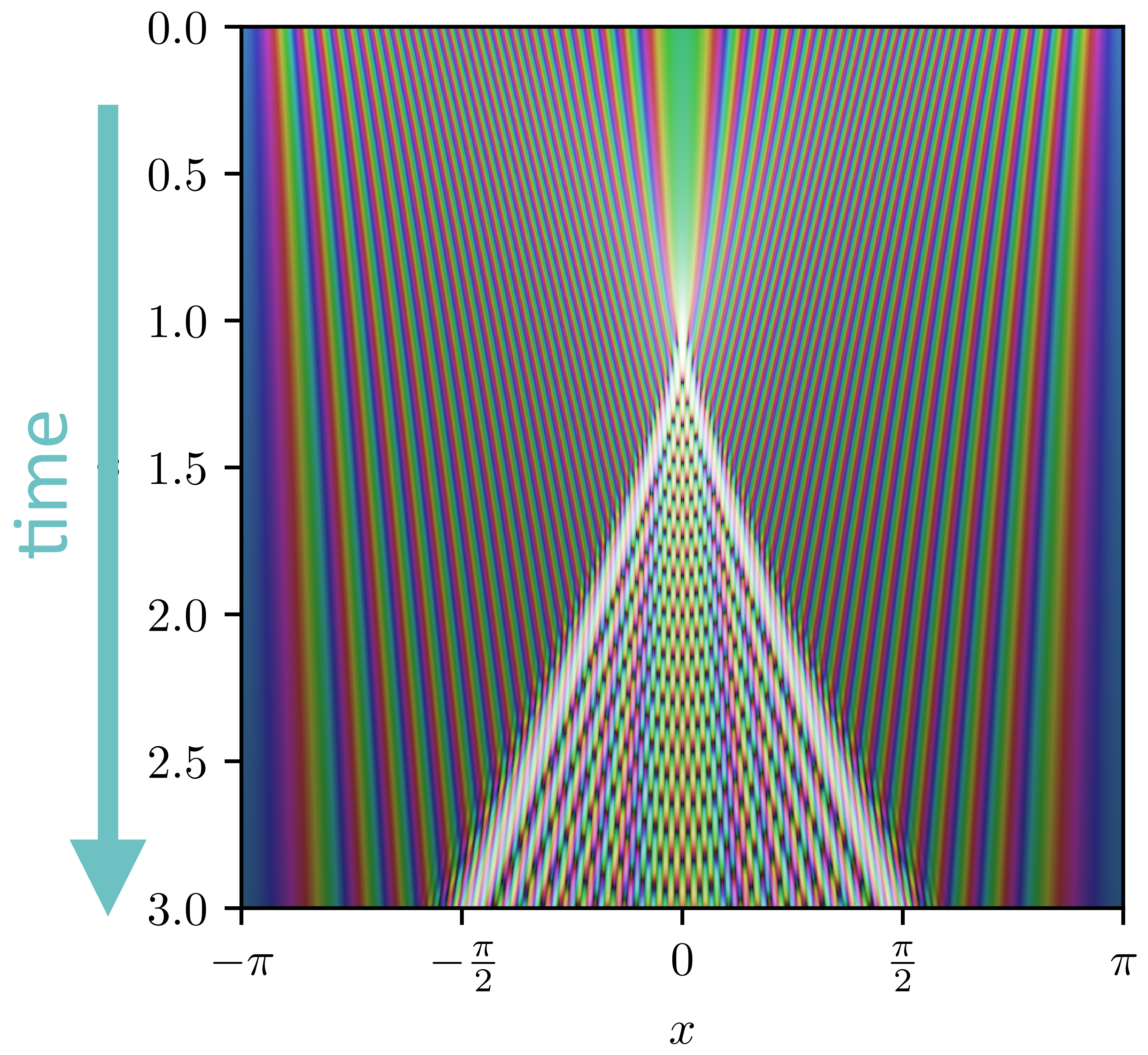
Amplitude: brightness

Phase: colour

Features

- Regularised caustic
- Interference

→ Multi-streaming



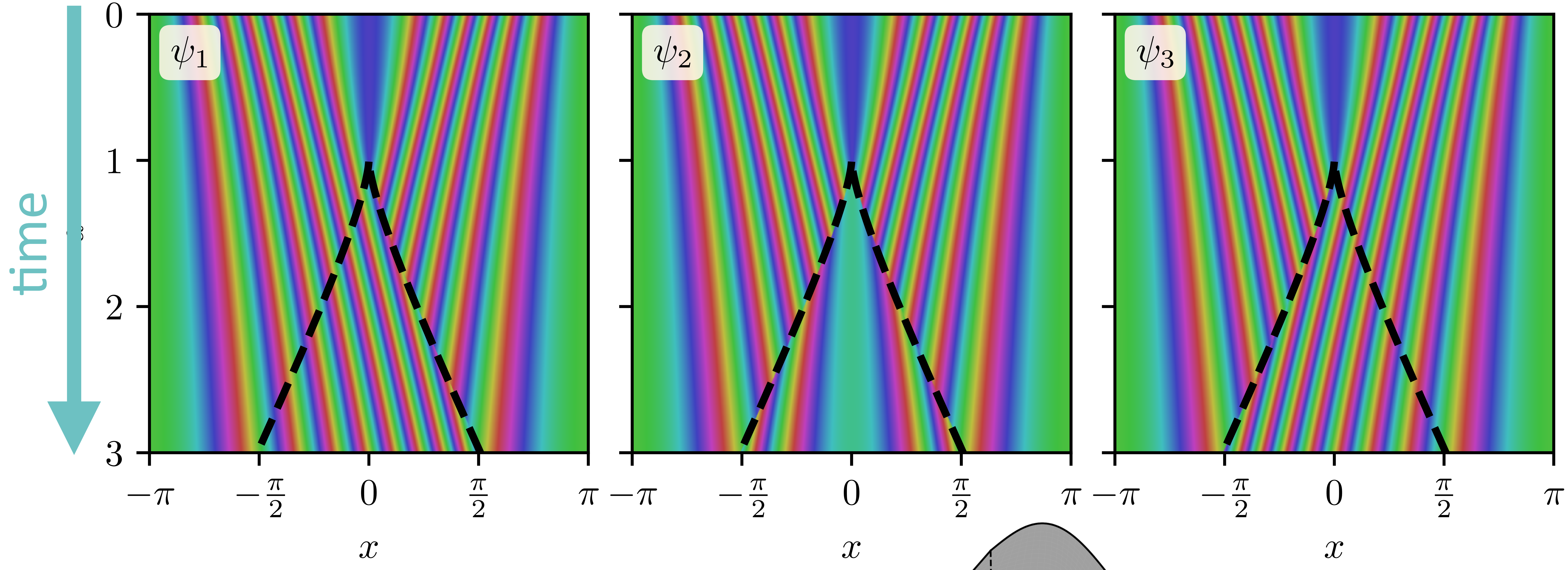
How to unweave ψ?

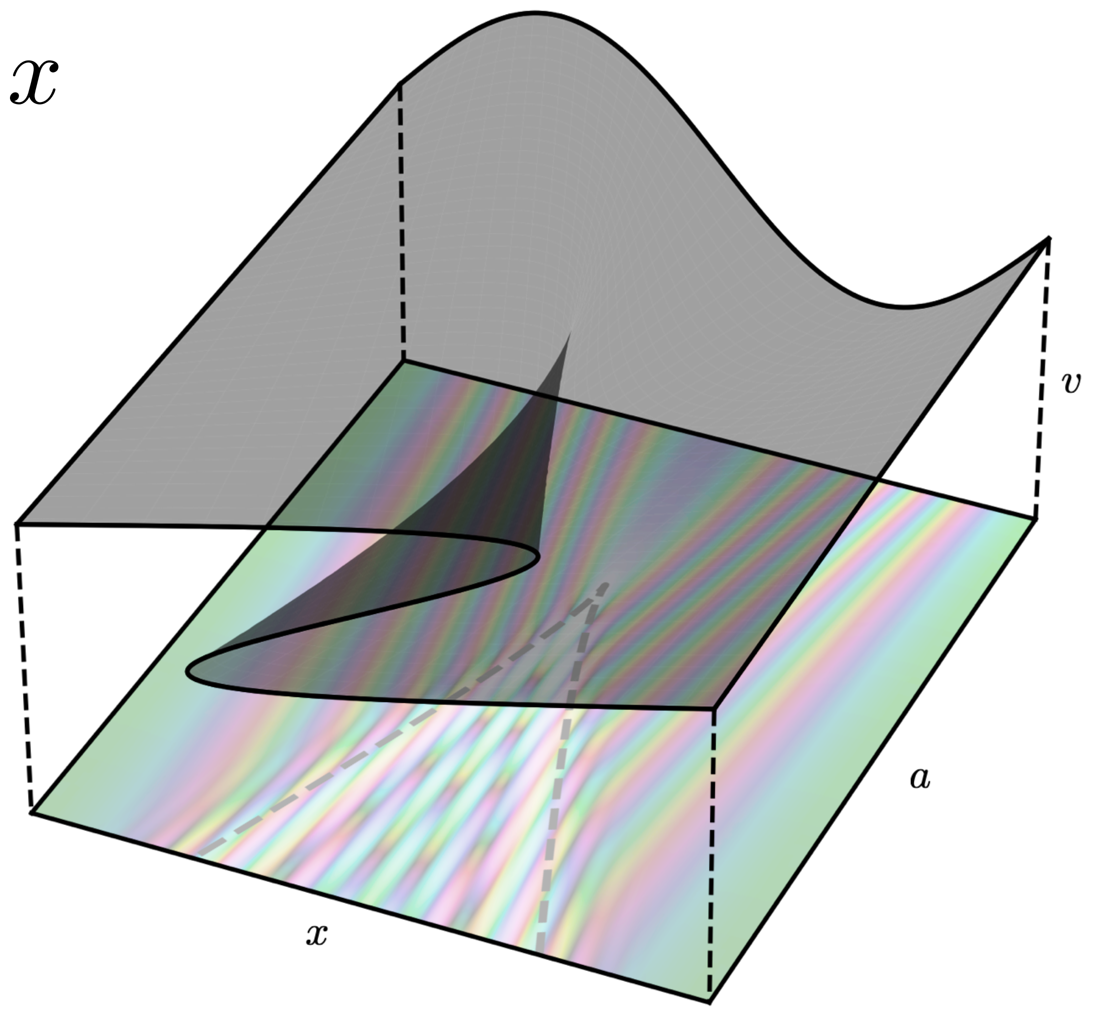
How is info stored in interference?

Uhlemann++ PRD 2019

Gough and Uhlemann 2022

Unweaving





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Unweaving (now with maths)

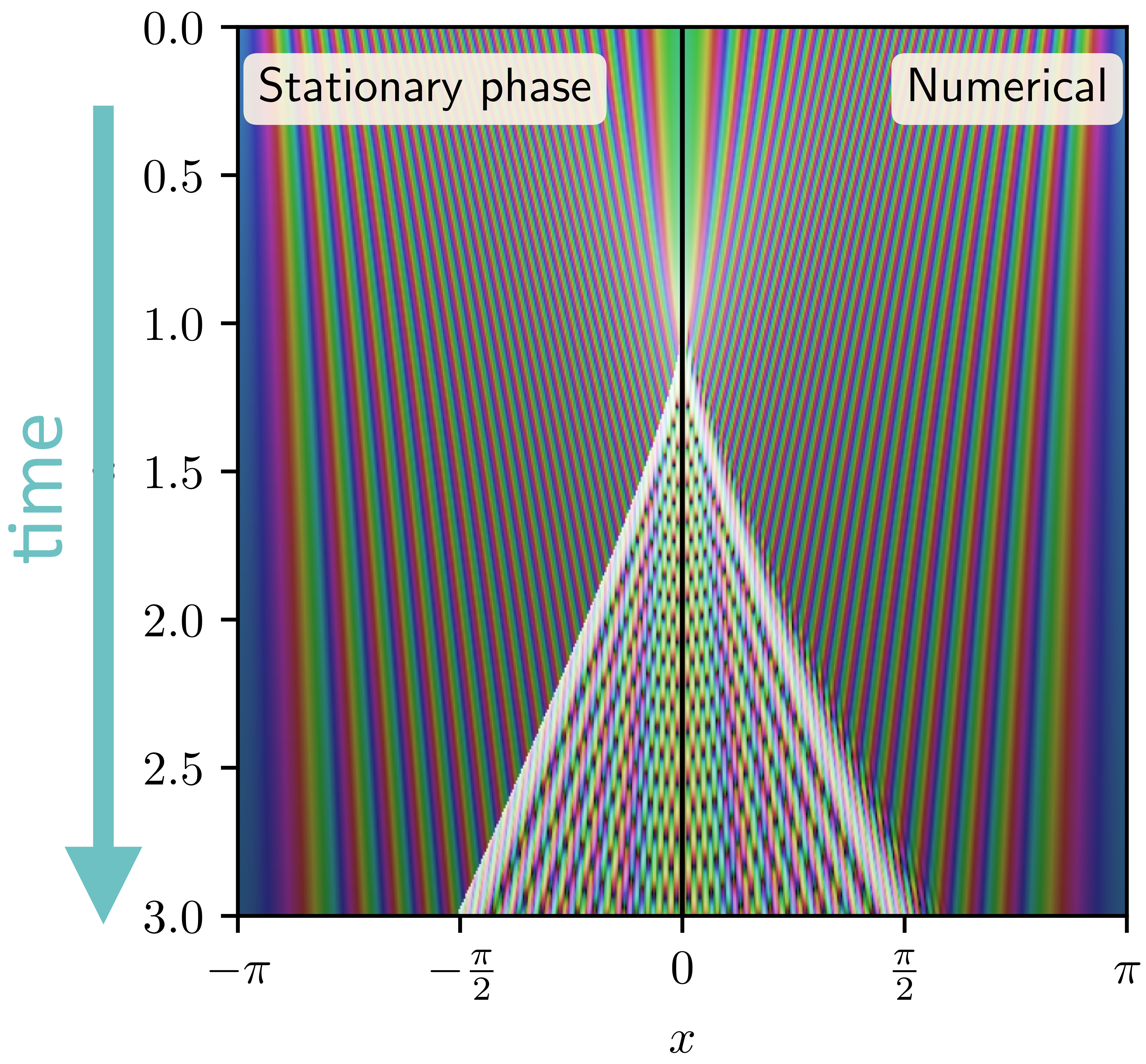
Based on the propagator

$$\psi(x,a) \sim \int \mathrm{d}q\, \underbrace{K_0(q;x,a)\psi^{(\mathrm{ini})}(q)}_{\exp\left[\frac{i}{\hbar}\zeta(q;x,a)\right]}$$

- $\zeta(q;x,a)$ contains the *action* and the *initial conditions*
- $\hbar$ small $\rightarrow$ integrand oscillatory

Stationary Phase Approximation

q where $\zeta'(q) = 0$ dominate integral



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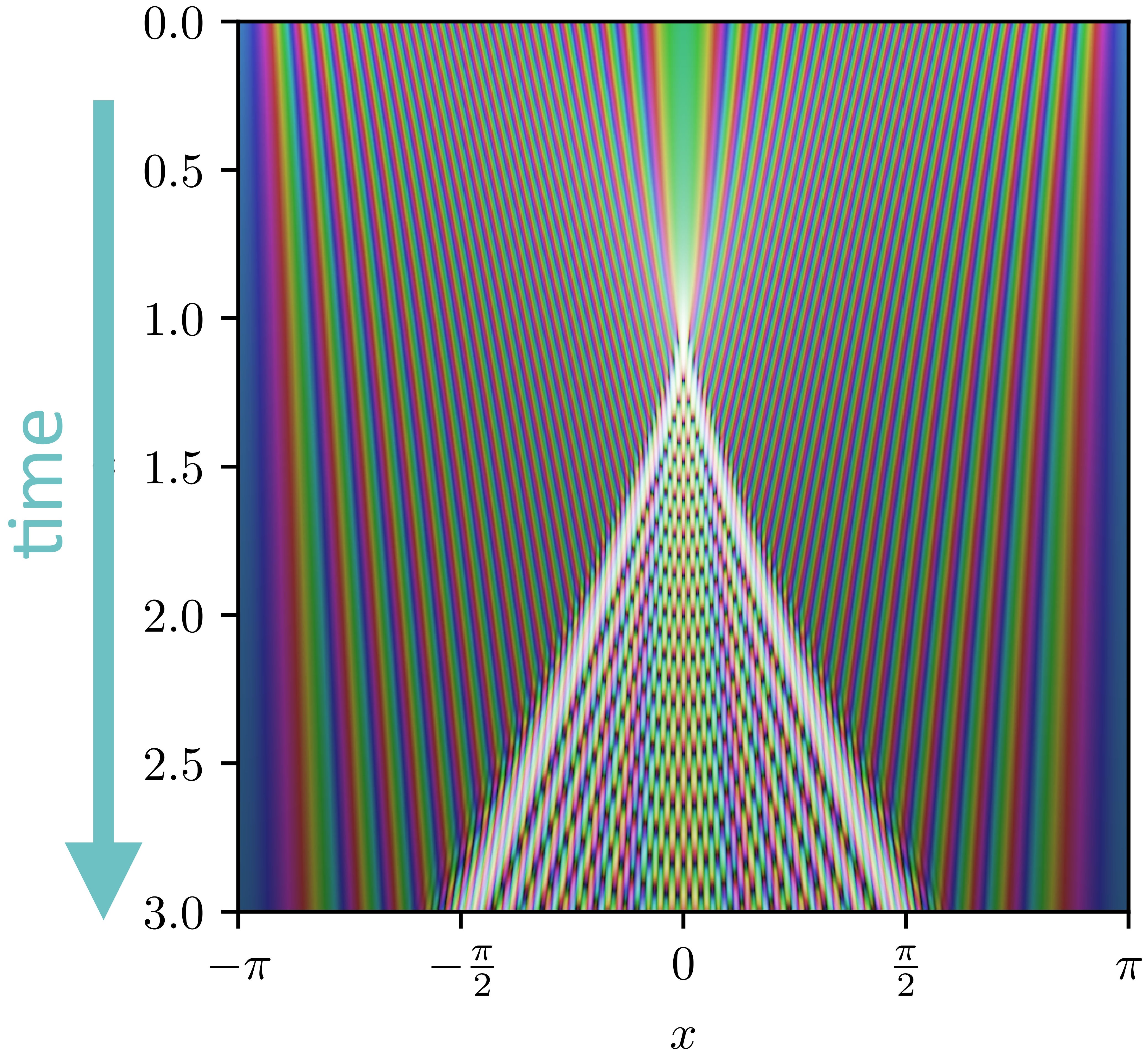
Non-potential velocity

From $\mathbf{v} = \nabla\phi_v$, it appears ψ can only encode potential velocities (like a perfect fluid)

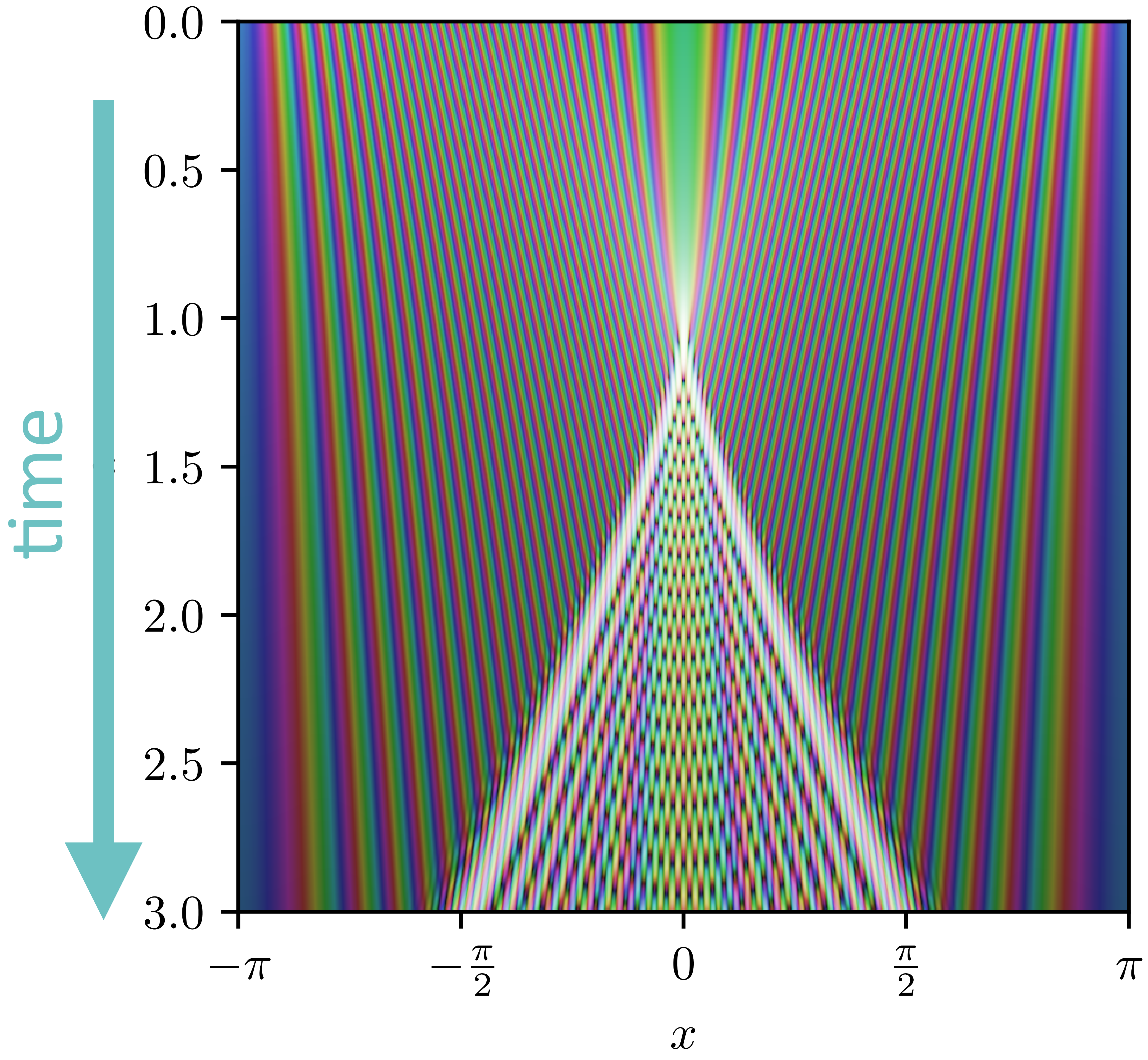
→ Multi-stream averaging means velocity cannot be potential in the classical case



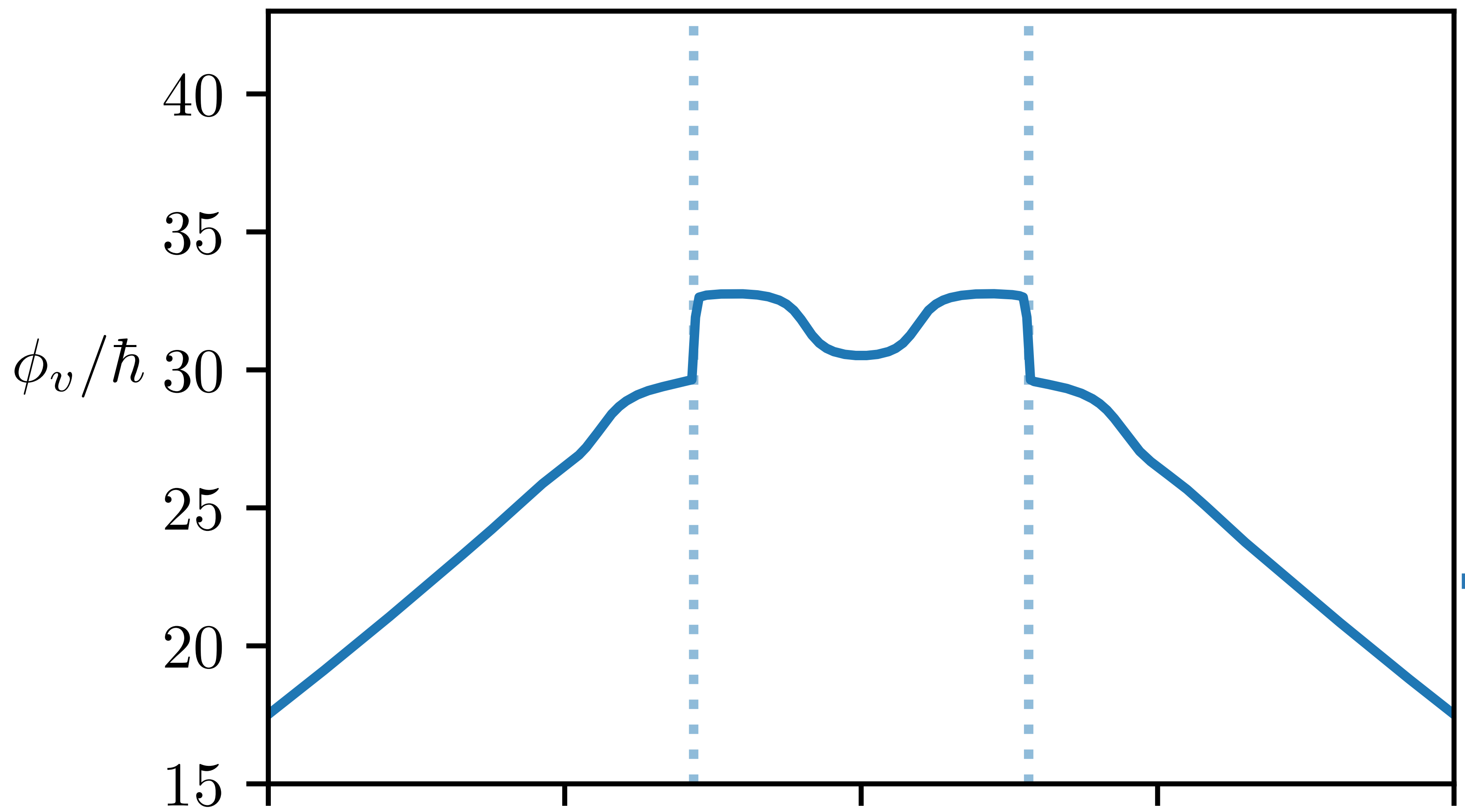
Non-potential velocity

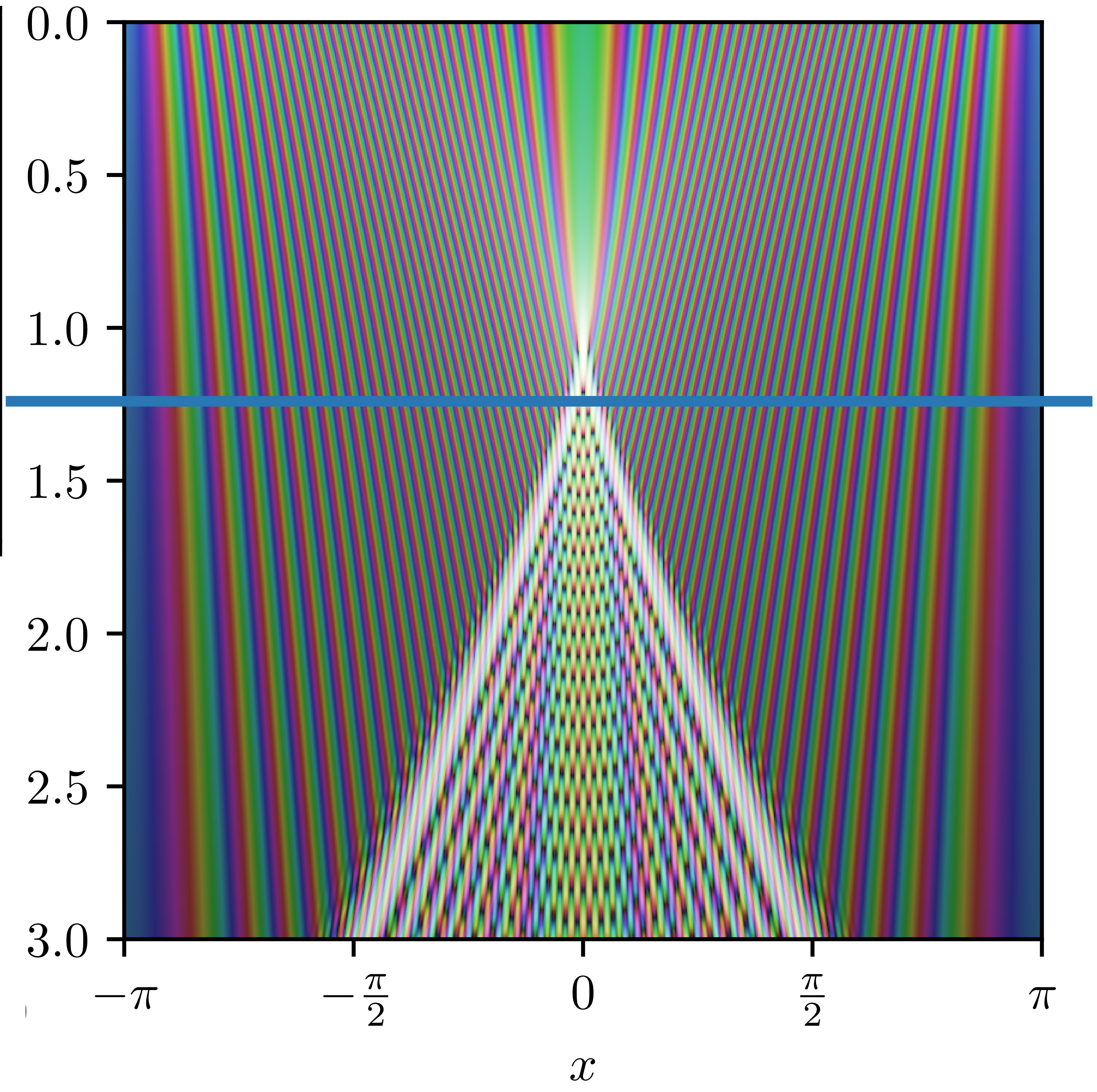
From $\mathbf{v} = \nabla \phi_v$, ppears ψ can only encode potenti locities (like a perfect fluid)

- Multi-stream averaging means velocity cannot be potential in the classical case
- If ϕ_v is discontinuous, then $\mathbf{v}$ can be non-potential
- Where $\rho = 0$, the phase is undefined, could jump there?



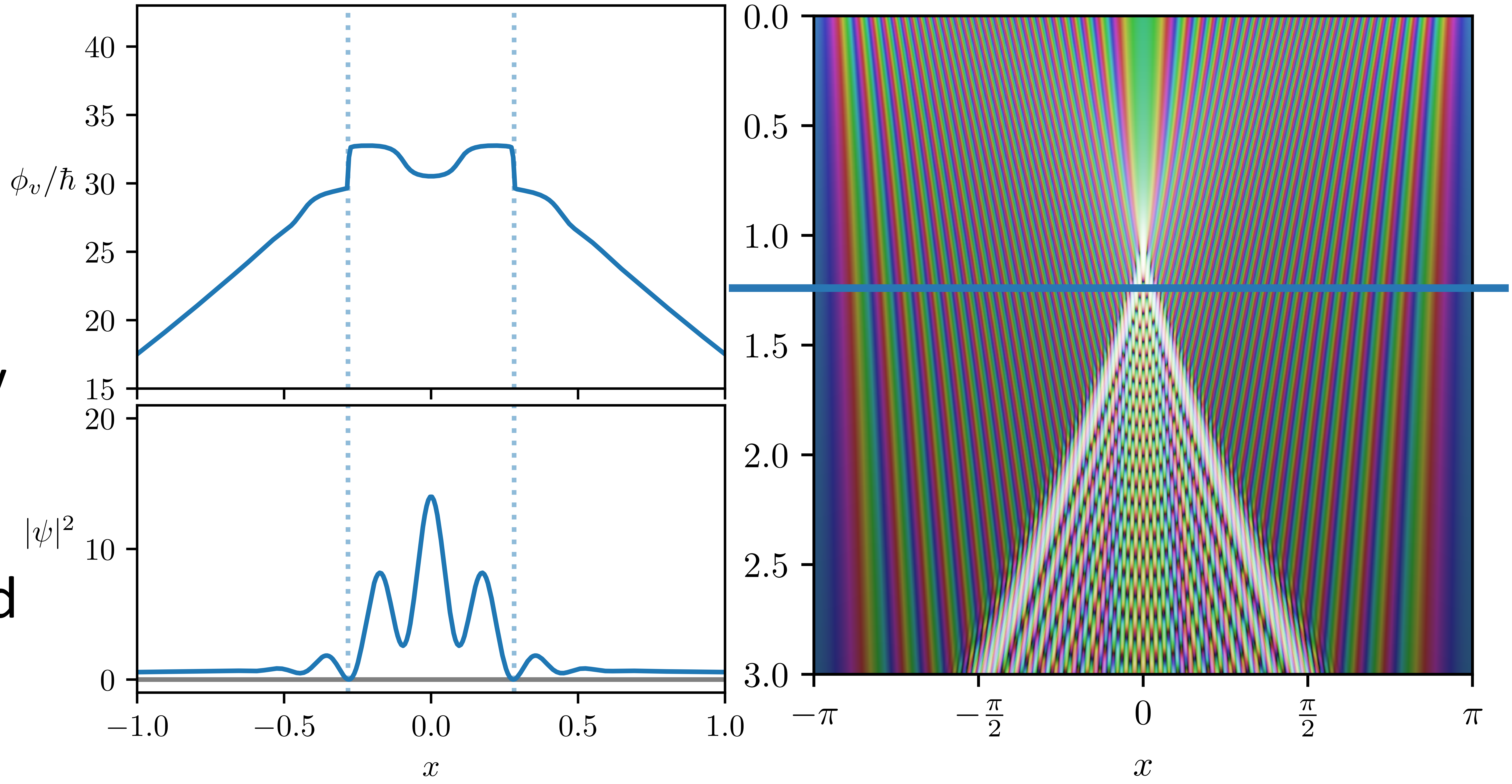
Non-potential velocity





Non-potential velocity

- Phase jumps correspond to zeros in the density
- ψ encodes information beyond a perfect fluid!



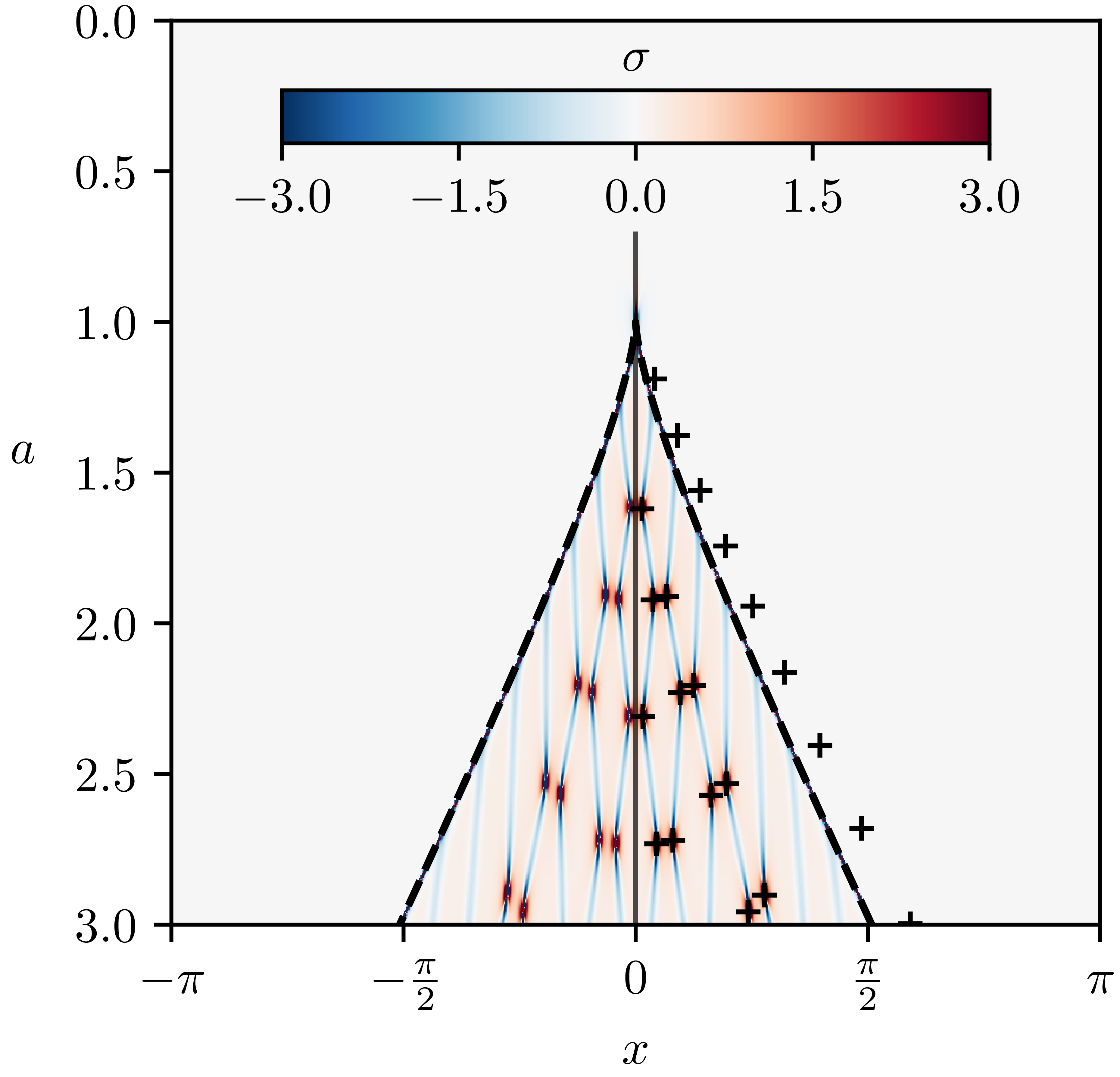
Non-potential velocity

- Velocity dispersion (+ other cumulants)
- Vorticity in 2 or 3 dimensions

$$\psi \approx \psi_{\mathrm{avg}} \times \psi_{\mathrm{hidden}}$$

Fluid part

Oscillatory



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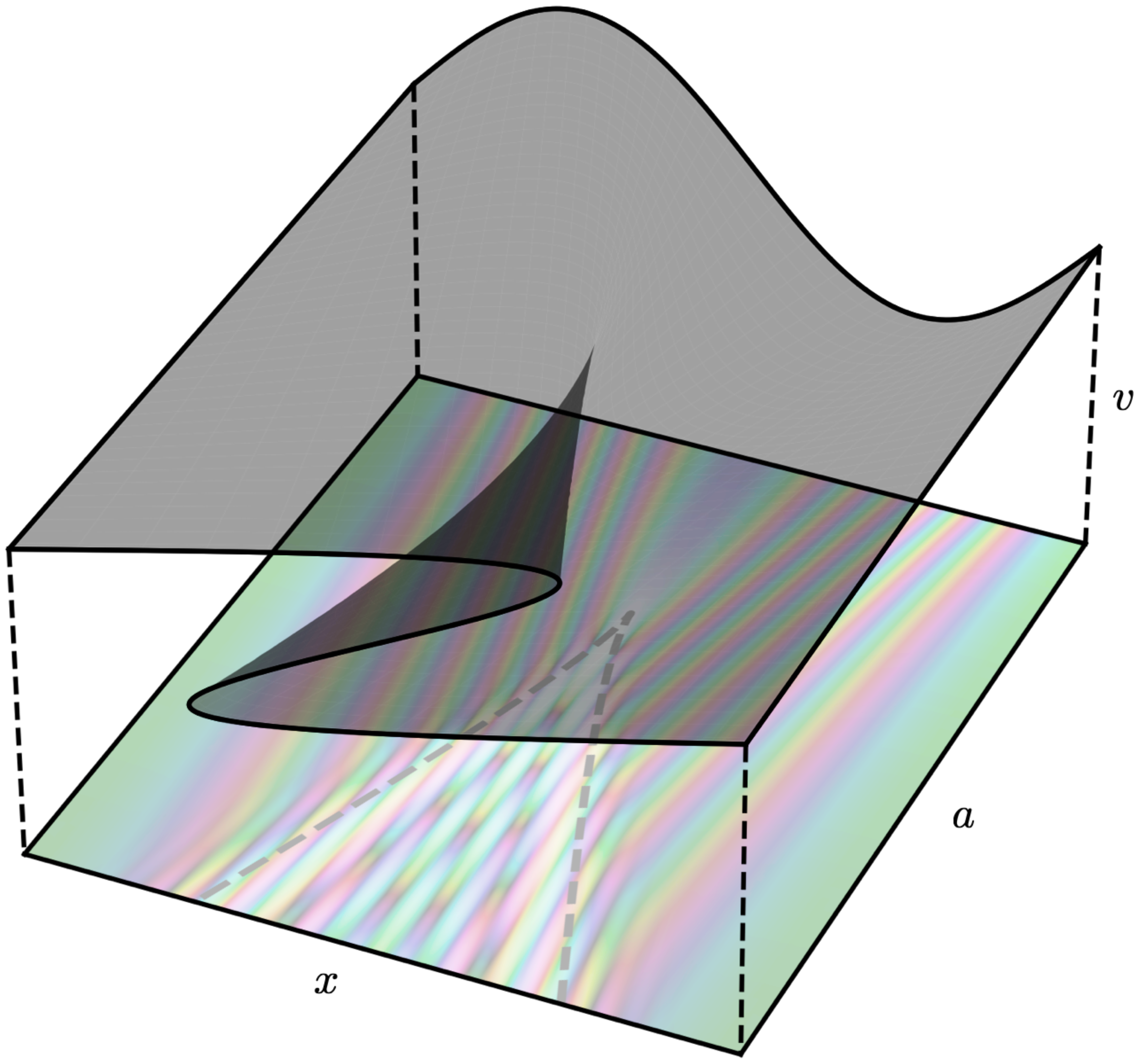
Success of SPA

SPA allows ψ to be decomposed into classical-like wavefunctions

- Separation can be done without constructing phase space for ψ
- Multi-streaming $\sim$ interference
- Beyond perfect fluid $\sim$ phase jumps and density zeros



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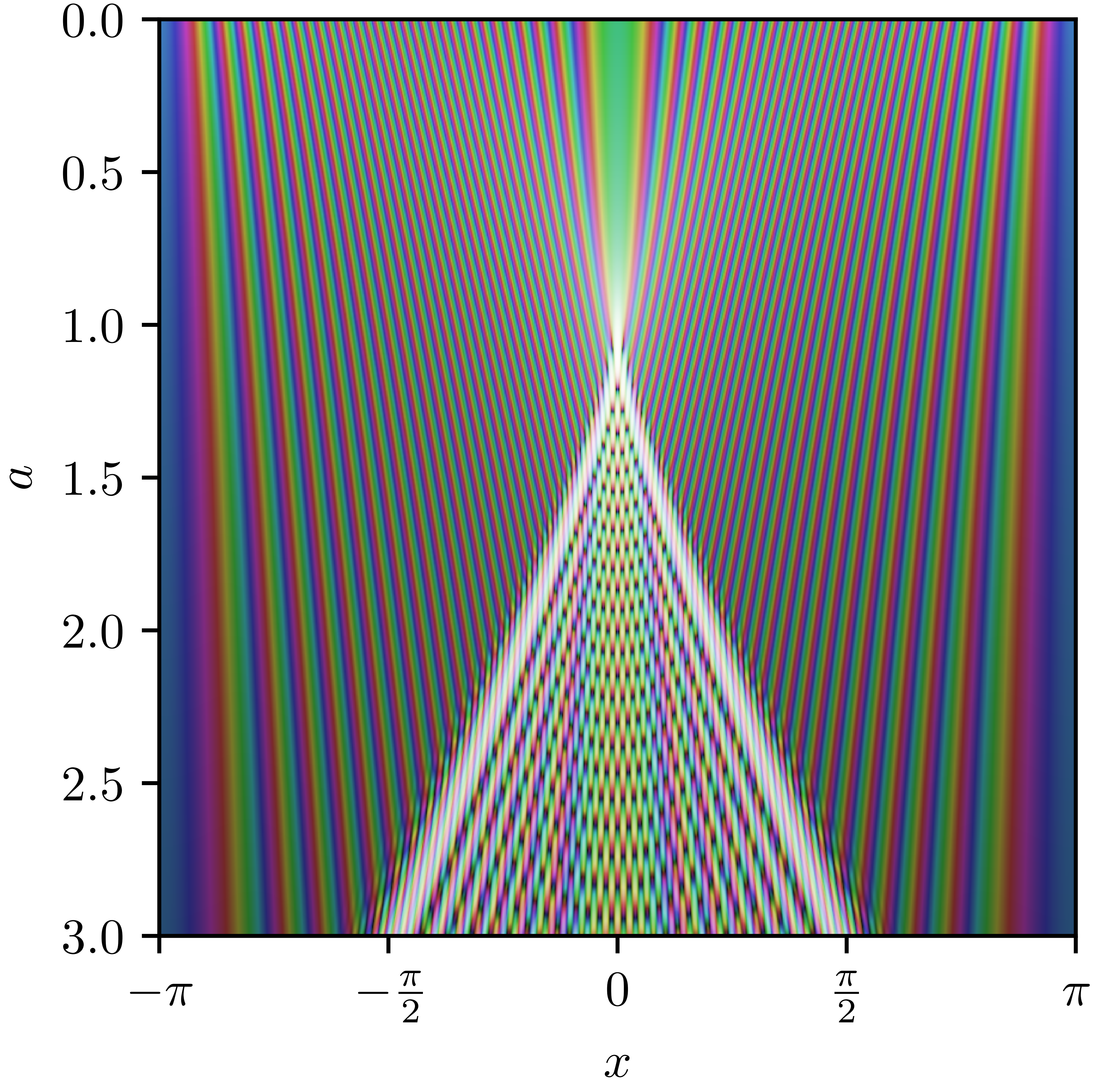
Caustic features

Dark matter



Optics

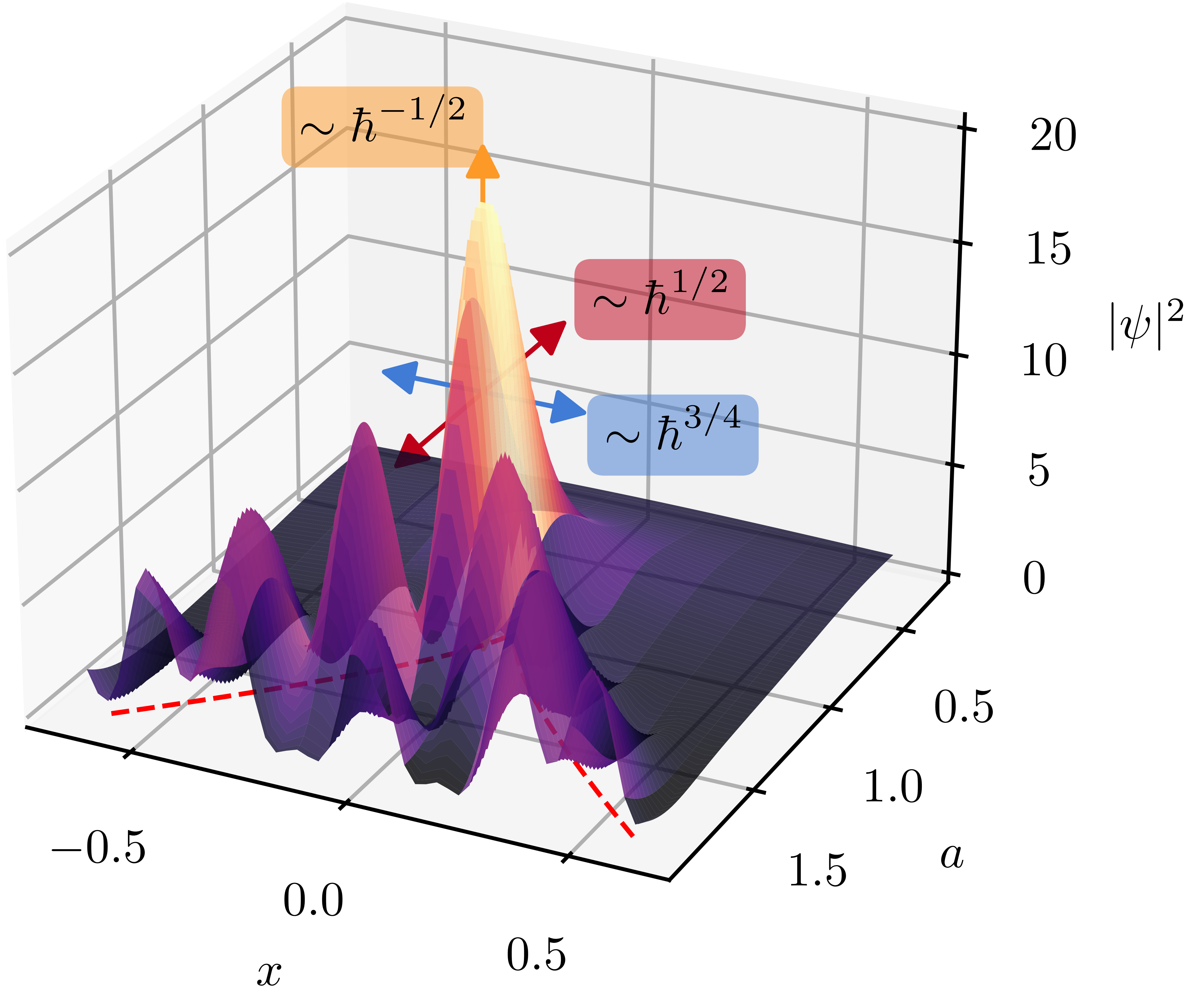
Universal properties

Stable caustics fully classified into just a few types

→ standard forms of ζ

Near a particular caustic some features have universal behaviour

- maximum amplitude
- fringe spacing



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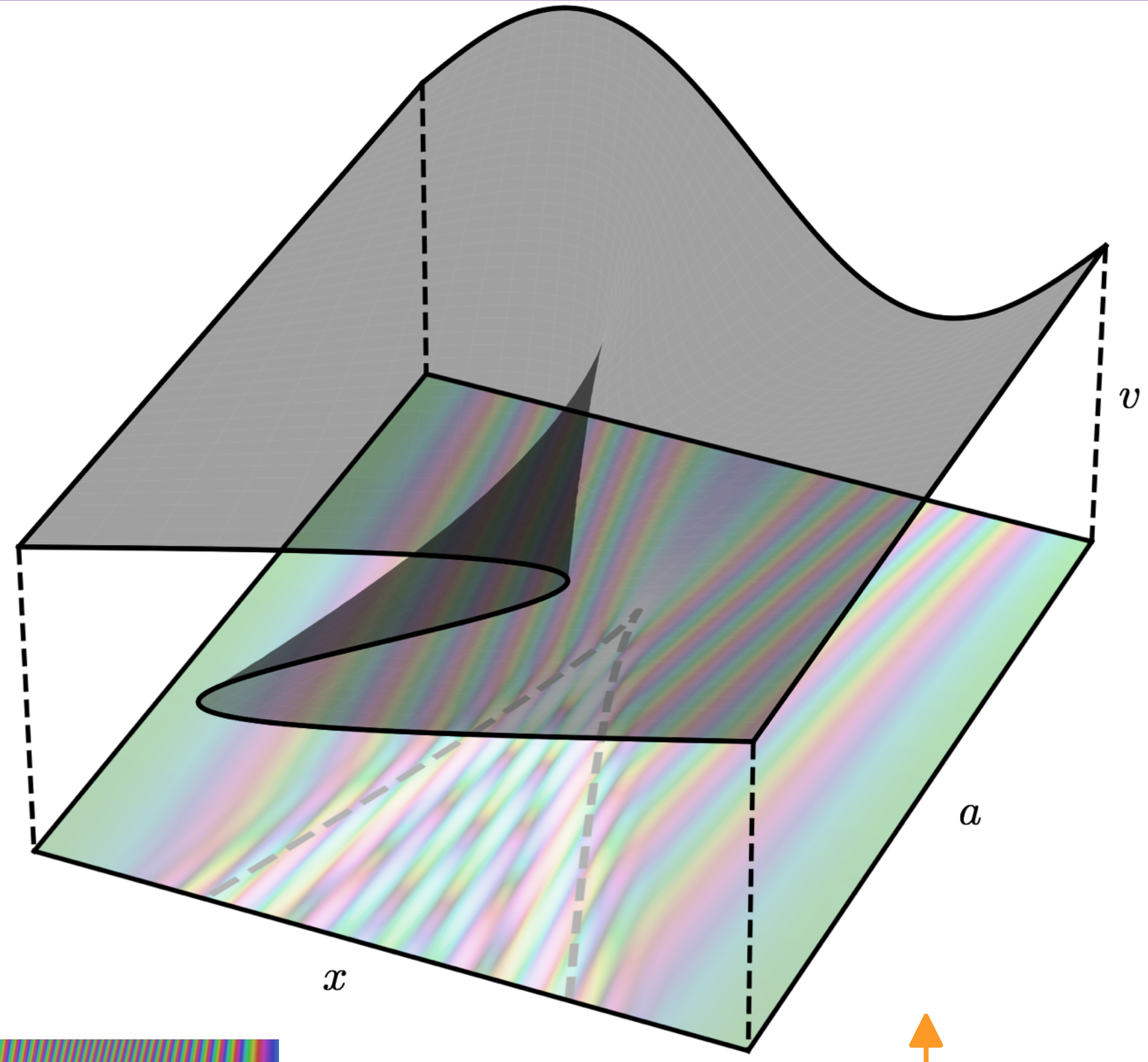
Takeaways

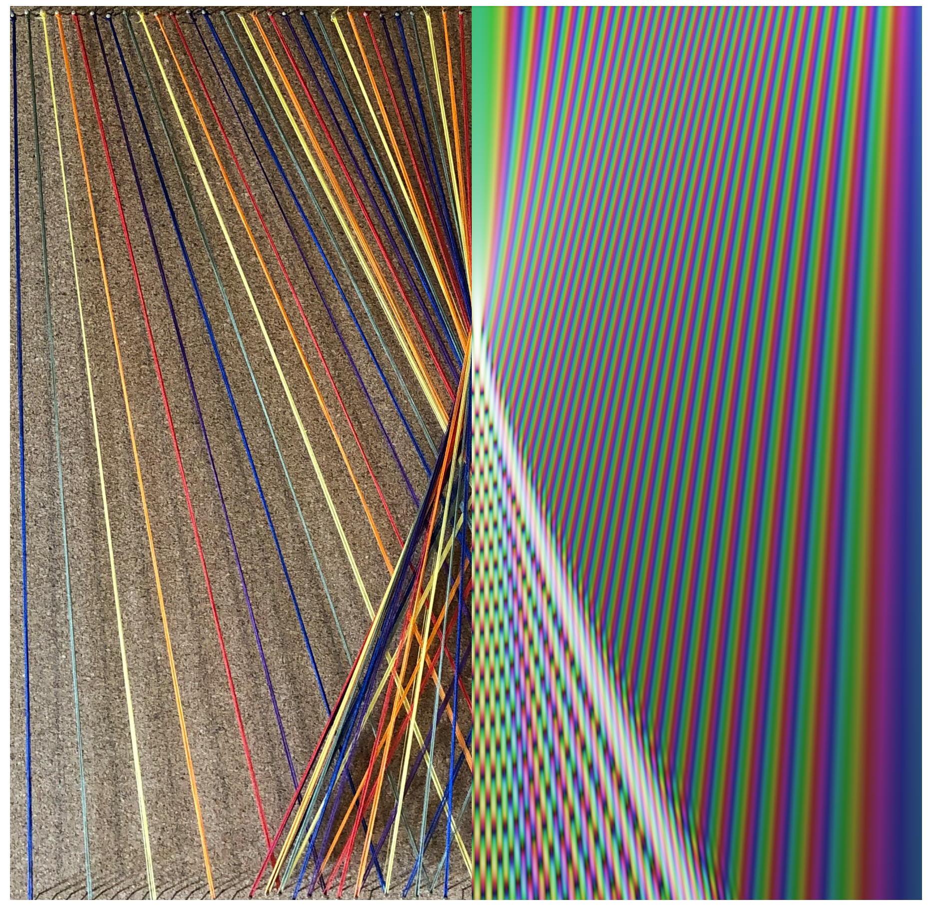
Wave DM presents rich phenomenology, decorating the cosmic web

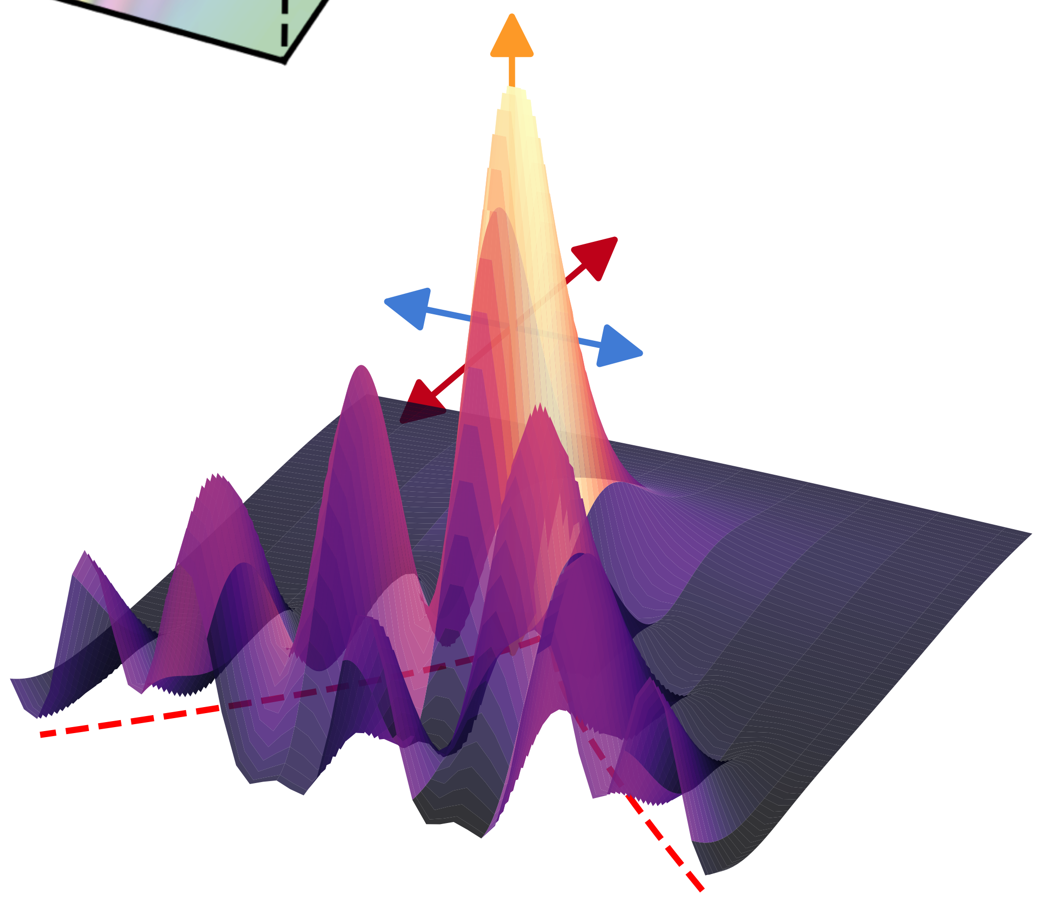
- interference $\sim$ multi-streaming
- phase jumps $\sim$ non-potential velocity



Wave models of CDM efficiently capture information beyond fluid models

- prospects for analytic modelling and complementing numerics





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