Cosmological Bootstrap Primer

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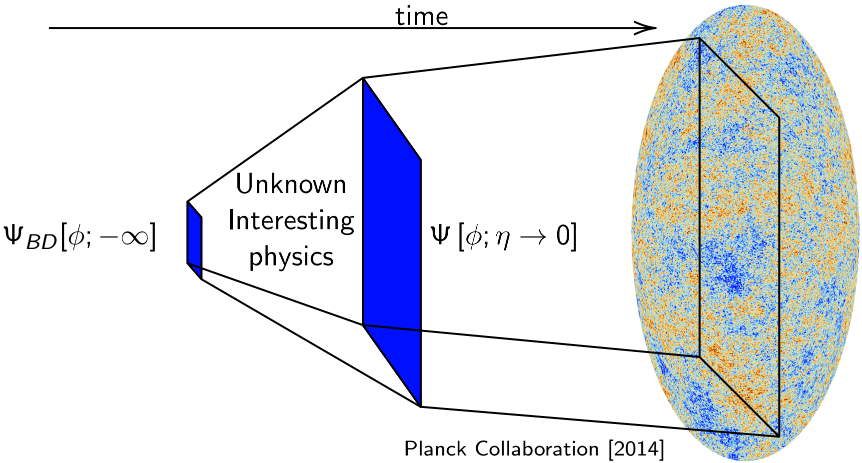
Contents

Motivation

- Cosmological surveys measure correlators between fields
- These depend on the quantum fluctuations in the early universe
- Our goal is thus to understand QFT in de Sitter



The wavefunction of the universe

- The wavefunction of the universe is computed from the action

$$\Psi[\bar{\phi};\eta_0] = \int_{\text{vacuum}}^{\bar{\phi}} \mathcal{D}\phi\mathcal{D}\pi\, e^{i\int d^4x[\phi'\pi - \mathcal{H}(\phi,\pi)]}$$

- We express it in terms of Fourier coefficients which we call the wavefunction coefficients

$$\Psi[\bar{\phi}] = \exp\left[-\sum_{n=2}^{\infty}\frac{1}{n!}\int_{\mathbf{k}^n}\psi_n(2\pi)^3\delta^3\left(\sum \mathbf{k}_a\right)\bar{\phi}(\mathbf{k}_1)\ldots\bar{\phi}(\mathbf{k}_n)\right]$$

- These coefficients are simply related to the more typical correlators

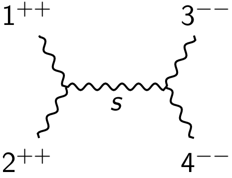
$$P(k) = \frac{1}{2\,\mathrm{Re}\,\psi_2(k)}, \qquad B_3 = -\frac{2\,\mathrm{Re}\,\psi_3}{\prod_{a=1}^{3} 2\,\mathrm{Re}\,\psi_2(k_a)},$$

$$B_4 = -\frac{2}{\prod_{a=1}^{4} 2\,\mathrm{Re}\,\psi_2(k_a)}\left[\mathrm{Re}\,\psi_4 - \frac{\mathrm{Re}\,\psi_3\,\mathrm{Re}\,\psi_3}{\mathrm{Re}\,\psi_2(s)} - t - u\right].$$

Model Based Calculations

Fundamental Principles → Propose Model → Compute Observables

Speed of Light
Lorentz Invariance
Equivalence Principle
→ General Relativity →



Issues:

- Only works when we have a specific model that we want to check
- Over complicates the problem
- Impact of the fundamental principles is obscured

Bootstrapping

Locality
Lorentz Invariance
Dimensional Analysis

$$\frac{[12]^4\langle 34\rangle^4}{stu}$$

Benefits:

- Helps when we don't have a single theory we want to test
- Clarifies link between Fundamental Principles and observables
- Gives insight into non-perturbative results and positivity bounds
- Simplifies calculations

Fundamental Principles

The most important principles in the Cosmological Bootstrap are:

- Flat Space Limit
  - $\lim_{k_T \to 0} \psi_n \propto \frac{A_n}{k_T^p}$

Maldecena and Pimental [2011], Raju [2012]

- de Sitter isometries

Maldacena and Pimental [2011], Arkani-Hamed et al. [2018], Baumann et al. [2019]

- Locality
  - $\lim_{k_1 \to 0} \partial_{k_1} \psi_n(k_a) = 0$

Jazayeri et al. [2021]

- Scale Invariance
  - $[\psi_n] = [k^3]$
- Unitarity

HG, Jazayeri, Pajer [2020], HG, Jazayeri, Lee and Pajer [2021]

Implications of Unitarity

- We assume a unitary time evolution operator

$$U^\dagger U = 1$$

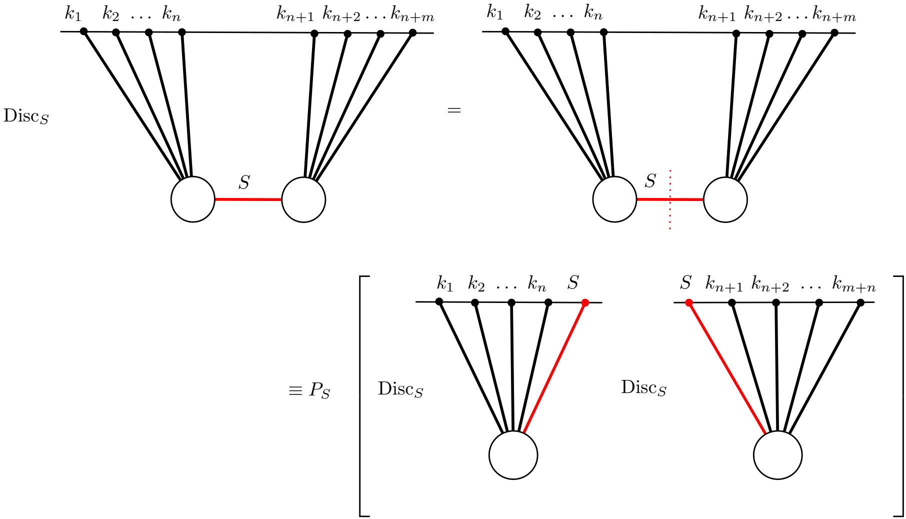
- In perturbation theory this can be written as

$$\delta U + \delta U^\dagger = -\delta U \delta U^\dagger$$

- Sandwiching this between two states and inserting the identity gives

$$\langle in| \delta U |in\rangle + \langle in| \delta U^\dagger |in\rangle = -\int \sum_f \langle in| \delta U |f\rangle \langle f| \delta U^\dagger |in\rangle$$

Implications of Unitarity



Conclusions

- We have made a lot of progress towards understanding the perturbative dS wavefunction
- Locality, Unitarity and Scale Invariance pose strong constraints on observables (for more details see Gordon's talk)
- They allow us to directly boostrap all tree level bispectra without reference to a Lagrangian, as will be discussed in Ayngaran and Dong Gang's talks