Study of Quintessence and Phantom scalar field models through a general parametrization of the Hubble parameter

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Cosmology from Home 2022

Based on "Quintessence or Phantom: Study of scalar field dark energy models through a general parametrization of the Hubble parameter."
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State of the Art

- Our universe is not only expanding but it is also accelerating!!
- ΛCDM model has been constrained with unprecedented accuracy.
- With the improvement in our ability to constrain the cosmological parameters, a few statistically significant tensions has emerged.
- It seems that the late time cosmological data and early time cosmological data are in tension.
- We need to extent our imagination beyond standard ΛCDM.

Hubble Tension

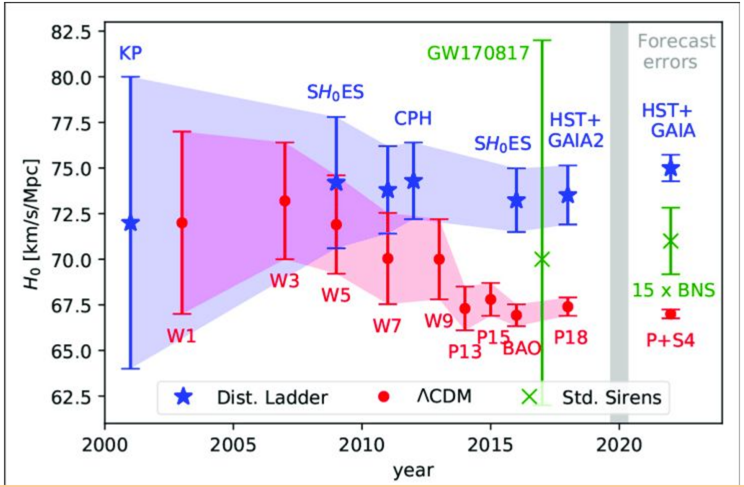


Fig 1(a)

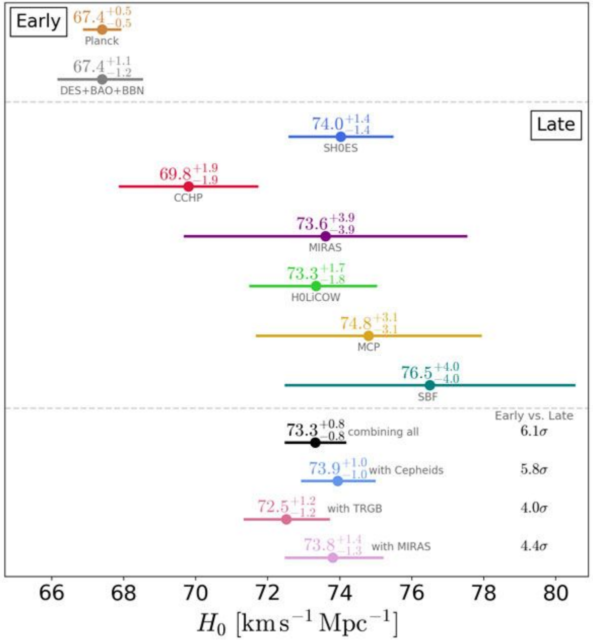
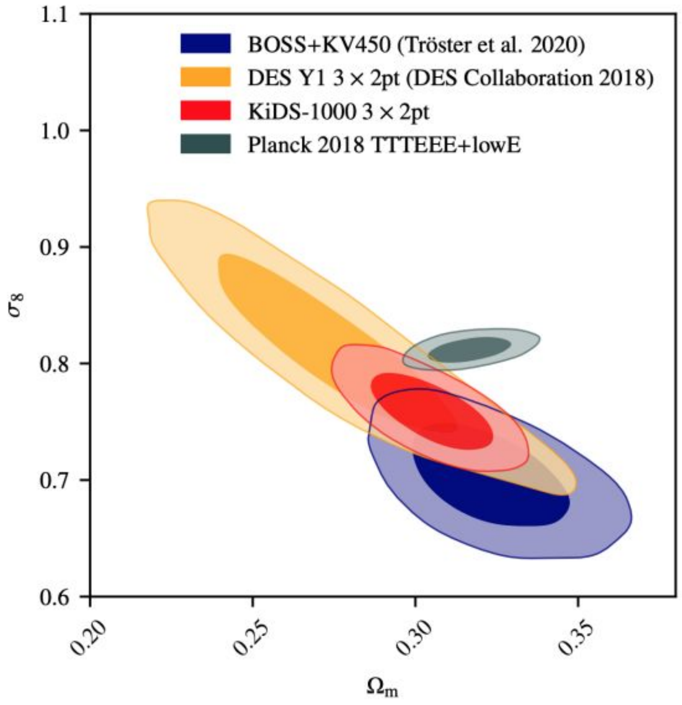


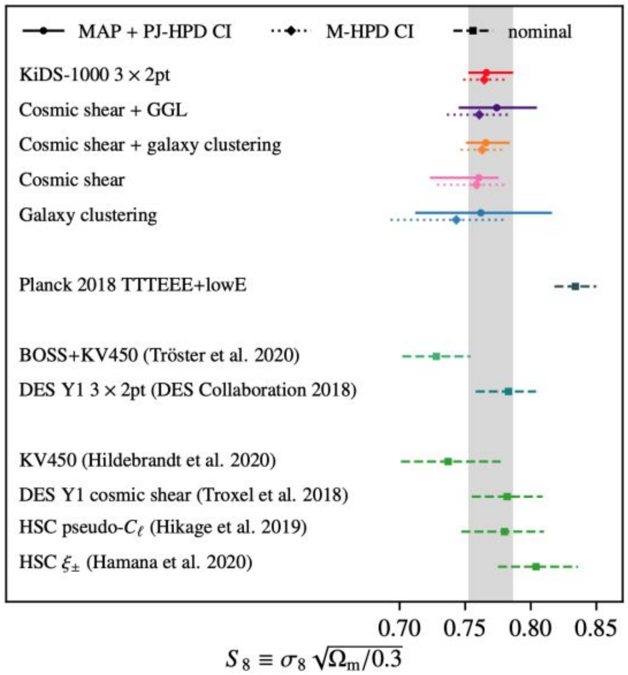
Fig 1(b)

CMB Planck data together with BAO, BBN, and DES have constraint the Hubble parameter to be H0 ~ (67.0 – 68.5)km/s/Mpc. On the other hand, cosmic distance ladder and time delay measurement like those reported by SH0ES and H0LiCOW collaborations have reported H0 = (74.03 ± 1.42) km/s/Mpc and H0 = (73.3 +1.7 −1.8) km/s/Mpc respectively by observing the local Universe.

arXiv: 2008.11284

σ_8 Tension





Apart from the Hubble tension, another tension between the Planck data with the weak lensing and the redshift surveys has been reported.

Fig 2

arXiv:2008.11285

Scalar Field as Dark energy

- The ΛCDM model happens to be most consistent with the observations but it suffers from problems arising from both theoretical and observational aspects.
- From the theoretical side it has to deal with the cosmological constant problem, coincidence problem and the fine tuning problem.
- From the observational side it is unable to explain the tension between the early time (Planck, BAO) and late time observations (SH0ES).
- There could be new physics involved and we should think beyond ΛCDM model.
- Scalar fields models are considered as one of the best alternatives to the cosmological constant.

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \mathcal{L}_\phi \right] + S_M$$

$$\mathcal{L}_\phi = -\epsilon \frac{1}{2} g^{\mu v} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

Scalar Field Dynamics

For a spatially flat, homogeneous and isotropic universe filled with matter and nonminimally coupled scalar field components

$$3H^2 = \rho_m + \rho_\phi = \rho_m + \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi)$$

$$2\dot{H} + 3H^2 = -p_\phi = -\frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi)$$

$$\epsilon\ddot{\phi} + 3\epsilon H\dot{\phi} + \frac{dV}{d\phi} = 0$$

ϵ → Switch parameter

$$\epsilon = \begin{cases} +1, & \text{Quintessence} \\ -1, & \text{Phantom} \end{cases}$$

ρ_m → Matter energy density

$\rho_\phi = \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi)$ → Scalar field energy density

$p_\phi = \frac{1}{2}\epsilon\dot{\phi}^2 - V(\phi)$ → Pressure component

Governing equations

By simple rearrangement of the field equations we can rewrite the derivative of the Hubble parameter and the potential of the scalar field

$$2\dot{H} = -\frac{\rho_{m0}}{a^3} - \epsilon\dot{\phi}^2$$

$$V(\phi) = \dot{H} + 3H^2 - \frac{\rho_{m0}}{2a^3}$$

The time derivatives are then converted to the derivatives w.r.t the scale factor as

$$\dot{H} = \frac{1}{2}a\frac{d}{da}(H^2)$$

$$\dot{\phi} = aH\left(\frac{d\phi}{da}\right)$$

$$a\,\frac{d(H^2)}{da} + \frac{\rho_{m0}}{a^3} = -\,\epsilon\,\dot{\phi}^2$$

$$\frac{d\phi}{dz} = \left[\frac{2E\frac{dE}{dz} - 3\Omega_{m0}(1+z)^2}{\epsilon E^2(1+z)}\right]^{\frac{1}{2}}$$

$$\frac{V(z)}{3H_0^2} = -\frac{(1+z)}{3}E\frac{dE}{dz} + E^2 - \frac{1}{2}\Omega_{m0}(1+z)^3$$

$$E(z) = \frac{H(z)}{H_0} \Rightarrow \text{Normalized Hubble parameter}$$

$$\Omega_{m0} = \frac{\rho_{m0}}{3\,H_0^2} \Rightarrow \text{Present value of the matter density parameter}$$

arXiv: 2201.09306

Cosmological parameters

It is also possible to write down relevant cosmological parameters in a similar fashion in terms of $E(z)$

$$\Omega_m(z) = \frac{\rho_m}{3H^2} = \frac{\Omega_{m0}(1+z)^3}{E^2},$$

$$\Omega_\phi(z) = 1 - \Omega_m(z) = 1 - \frac{\Omega_{m0}(1+z)^3}{E^2},$$

$$w_\phi(z) = \frac{-1 - \frac{2\dot{H}}{3H^2}}{\Omega_\phi} = \frac{\frac{2}{3}(1+z)E\frac{dE}{dz} - E^2}{E^2 - \Omega_{m0}(1+z)^3},$$

$$q(z) = -1 - \frac{\dot{H}}{H^2} = \frac{(1+z)}{E}\frac{dE}{dz} - 1.$$

- Apparently all these expressions are independent of the switch parameter ϵ.
- However the dependence on the switch parameter comes through the evolution of the $E(z)$

.

General Condition for Phantom Barrier crossing

$$\left(\frac{d\phi}{dz}\right)^2 = \left[\frac{2E\frac{dE}{dz} - 3\Omega_{m0}(1+z)^2}{\epsilon E^2(1+z)}\right]$$

$$w_\phi(z) = -1 + \frac{1+z}{3\Omega_\phi E^2}\left(2E\frac{dE}{dz} - 3\Omega_{mo}(1+z)^2\right)$$

$2E\frac{dE}{dz} - 3\Omega_{m0}(1+z)^2 > 0$	Quintessence
$2E\frac{dE}{dz} - 3\Omega_{m0}(1+z)^2 < 0$	Phantom

Since here all parameters are expressed in terms of $E(z)$, we can consider a functional form of $E(z)$ for further analysis (instead of choosing $V(\phi)$)

We propose a general condition for the Phantom barrier crossing at $z = z_\lambda$.

$$2E\frac{dE}{dz} - 3\Omega_{m0}(1+z_\lambda)^2 = 0$$

Any dark energy model for which

$$2E\frac{dE}{dz} - 3\Omega_{m0}(1+z_\lambda)^2 > 0 \quad \text{for} \quad z > z_\lambda$$

$$2E\frac{dE}{dz} - 3\Omega_{m0}(1+z_\lambda)^2 < 0 \quad \text{for} \quad z < z_\lambda$$

will undergo a phantom barrier crossing.

A Toy Model

We have considered the following parameterization:

$$E(z) = \left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right]^{\frac{1}{2}}$$

$$H^2(z) = H_0^2\left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right]$$

$$\Omega_m(z) = \frac{\Omega_{m0}(1+z)^3}{\left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right]}$$

$$\Omega_\phi(z) = 1 - \frac{\Omega_{m0}(1+z)^3}{\left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right]}$$

$$w_\phi(z) = \frac{p\left(b - 2bz + \frac{2z}{c} - \frac{z^2}{c} + \frac{3z^2}{d}\right) - 3}{3\left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right) - \Omega_{m0}(1+z)^3\right]}$$

$$q(z) = \frac{p\left(b - bz + \frac{2z}{c} + \frac{3z^2}{d} + \frac{z^3}{d}\right) - 2}{2\left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right]}$$

$$\phi(z) = \int\left[\frac{p\left(b + \frac{2z}{c} + \frac{3z^2}{d}\right) - 3\Omega_{m0}(1+z)^2}{\epsilon(1+z)\left[1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right]}\right]^{\frac{1}{2}} dz$$

$$\frac{V(z)}{3H_0^2} = -\frac{p(1+z)\left(b + \frac{2z}{c} + \frac{3z^2}{d}\right)}{6} + \left[\left(1 + pz\left(b + \frac{z}{c} + \frac{z^2}{d}\right)\right)\right] - \frac{1}{2}\Omega_{m0}(1+z)^3$$

Consideration of a parametrization of E(z) or H(z) is already studied. But this approach helps to study the quintessence and phantom model in a single setup.

Equation of state parameter for the toy Model

- The EOS of the scalar field can be written in a more compact form as

$$w_\phi = \frac{3w_0\left(\frac{a}{a_0}\right)^3 + 2w_1\left(\frac{a}{a_0}\right)^2 + w_3\left(\frac{a}{a_0}\right)}{3\left[w_2 - w_0\left(\frac{a}{a_0}\right)^3 - w_1\left(\frac{a}{a_0}\right)^2 - w_3\left(\frac{a}{a_0}\right)\right]},$$

- The new parameters have been written in terms of the old parameters as

$$w_0 = [p(bcd - d + c) - cd], \quad w_1 = p[2d - bcd - 3c]$$
$$w_3 = p(3c - d) \text{ and } w_2 = pc - \Omega_{m0}cd$$

$$w_{\phi 0} = -1 + \delta(w_0, w_1, w_2, w_3)$$

$$\delta(w_0, w_1, w_2, w_3) = \frac{w_1 - 3w_2 + 2w_3}{3(w_0 + w_1 + w_3 - w_2)}$$

Can be thought of as an deviation from the ΛCDM case. Depending on the choice of parameters it could be either phantom or quintessence.

Cosmological parameters in terms of (w_0, w_1, w_2, w_3)

$$E^2(z) = \frac{f_3}{f_2}$$

$$q(z) = \frac{1}{2f_3}\left((f_1 - w_2)(1+z)^3 + w_1(\Omega_{m0}-1)(1+z) + 2w_0(\Omega_{m0}-1)\right)$$

$$\Omega_m(z) = \frac{\Omega_{m0} f_2 (1+z)^3}{f_3}$$

$$\Omega_\phi = 1 - \frac{\Omega_{m0} f_2 (1+z)^3}{f_3}$$

$$\frac{V(z)}{3H_0^2} = \frac{1}{6f_2}[-(1+z)(3(f_1-w_2)(1+z)^2 - 2w_3(\Omega_{m0}-1)(1+z) - w_1(\Omega_{m0}-1)) + 6(f_1-w_2)(1+z)^3 - 6w_3(\Omega_{m0}-1)(1+z)^2 - 6w_1(\Omega_{m0}-1)(1+z) - 6w_0(\Omega_{m0}-1)] - \frac{1}{2}\Omega_{m0}(1+z)^3$$

$$f_1 = (\Omega_{m0}(w_0 + w_1 + w_3))$$
$$f_2 = (w_0 + w_1 + w_3 - w_2)$$
$$f_3 = (f_1 - w_2)(1+z)^3 - w_3(\Omega_{m0}-1)(1+z)^2 - w_1(\Omega_{m0}-1)(1+z) - w_0(\Omega_{m0}-1)$$

Comparison between numerical and analytical solutions

- We have amended a version of the publicly available CLASS code.
- The dynamics of the scalar field has been implemented in the CLASS code as an fluid with the EOS derived from our parametrization of the E(z).

$$w_\phi = \frac{3w_0\left(\frac{a}{a_0}\right)^3 + 2w_1\left(\frac{a}{a_0}\right)^2 + w_3\left(\frac{a}{a_0}\right)}{3\left[w_2 - w_0\left(\frac{a}{a_0}\right)^3 - w_1\left(\frac{a}{a_0}\right)^2 - w_3\left(\frac{a}{a_0}\right)\right]},$$

- A comparison between the analytical solutions and the numerical solutions are shown below. The percentage difference is less than 1%.
- The CLASS code has used the integral of the EoS of the dark energy over redshift whereas for obtaining the analytical solutions the KG equations has been used.

Comparison between numerical and analytical solutions

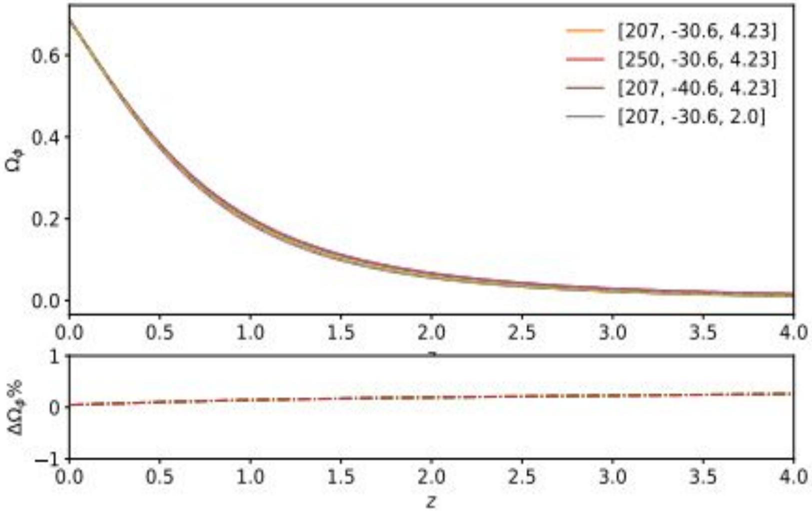


Fig 3(a)

Comparison of the dark energy density between the numerical and analytical solutions. Bottom panel shows percentage difference.

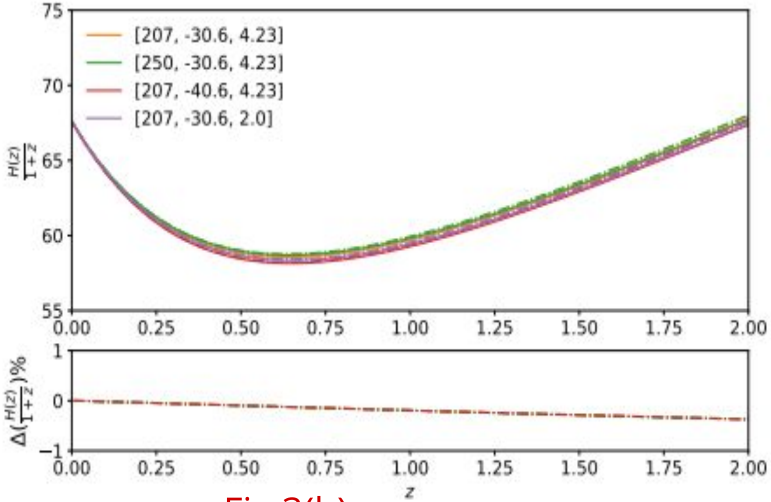


Fig 3(b)

Comparison of the expansion rate H(z)/(1+z) of the universe between the numerical and analytical solutions. Bottom panel shows percentage difference.

Numerical Investigation and Observational Constraint

We have considered following data sets:

- Pantheon
- BAO (BOSS DR12, 6dFGS, eBOSS DR14, WiggelZ)
- SDSS LRG DR7, SDSS LRG DR4
- A SH0ES Prior together with compressed Planck Likelihood

Parameters	Priors
$100\,\omega_b$	[1.9, 2.5]
ω_{cdm}	[0.095, 0.145]
H_0	[60, 80] $km\,s^{-1}Mpc^{-1}$
w_0	[0,300]
w_1	[-50,50]
w_3	[-50,50]

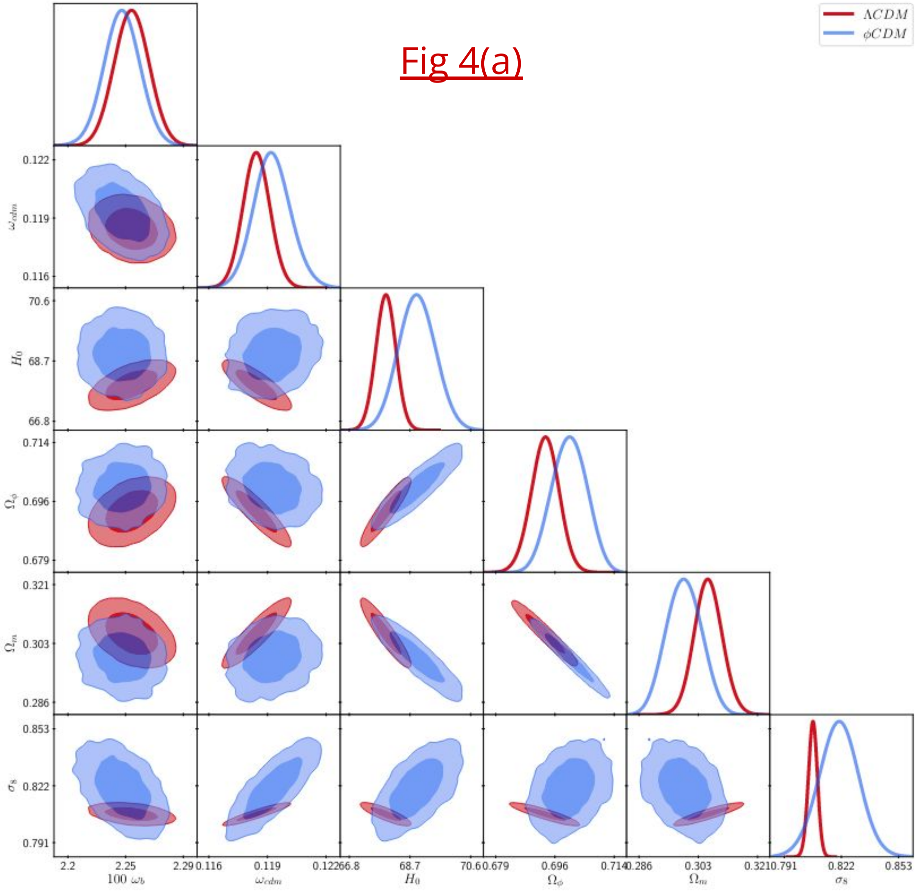
For the stability and minimization of the shooting faliure of the CLASS code we have considered $w_2 = 0$.

$$w_2 = c(p - \Omega_{m0}\, d)$$

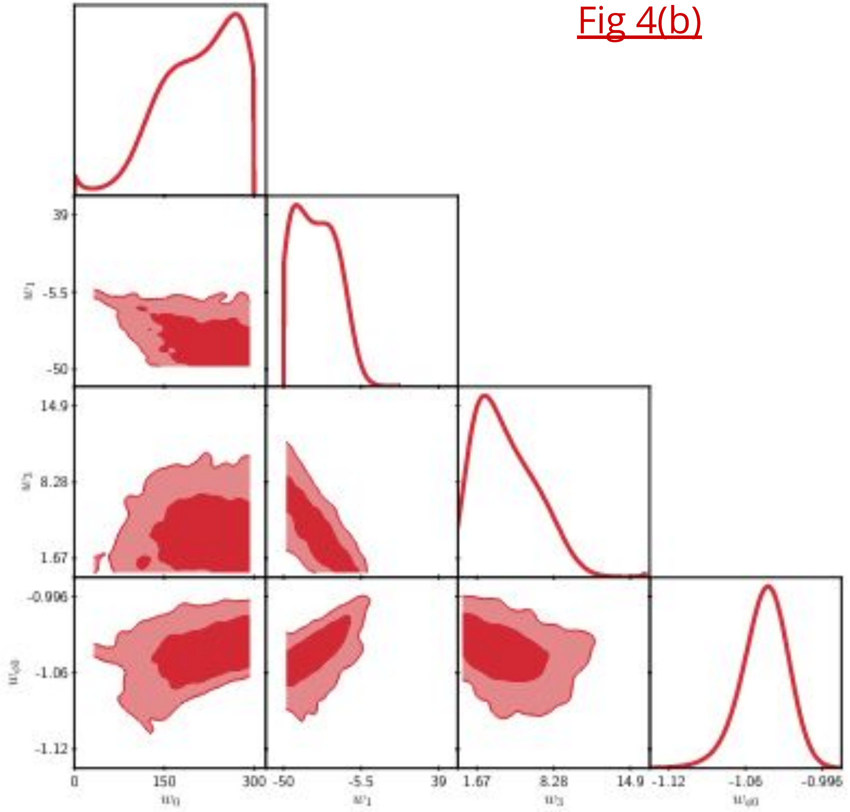
There will be always room for suitable choices for parameters which can lead to a viable cosmological model

Parameter	ΛCDM	ϕCDM
$100\omega_b$	$2.25^{+0.013}_{-0.0129}$	$2.24^{+0.016}_{-0.0128}$
ω_{cdm}	$0.118^{+0.0069}_{-0.000705}$	$0.119^{+0.0096}_{-0.0098}$
H_0	$68^{+0.326}_{-0.319}$	$68.9^{+0.573}_{-0.602}$
Ω_{DE}	$0.694^{+0.00426}_{-0.00406}$	$0.701^{+0.60478}_{-0.0016}$
Ω_m	$0.306^{+0.00406}_{-0.00426}$	$0.299^{+0.0516}_{-0.00478}$
σ_8	$0.807^{+0.00255}_{-0.00254}$	$0.821^{+0.011}_{-0.010}$
w_{DE}	-1	$-1.04^{+0.0204}_{-0.0166}$
w_0	–	> 96.97
w_1	–	< -10.03
w_3	–	$4.25^{+1.65}_{-3.46}$
$\Delta\chi^2_{\rm min}$	0	-5
$\ln B_{\phi\lambda}$	0	$+2.005$

The bayesian evidence suggests that the phantom model is moderately preferred over the LCDM



Triangular plot of 2D and 1D posterior distribution of different cosmological parameters.



Triangular plot of 2D and 1D posterior distribution of the model parameters.

Comment on Hubble Tension

- Although there is a slight increment in the best fit value of the Hubble parameter but it is far from solving the Hubble Tension.

- Similar results has been obtained in
  - *J. A. Vazquez, D. Tamayo, A. A. Sen, and I. Quiros, Phys. Rev. D 103, 043506 (2021), 2009.01904.*
  - *F. X. Linares Cedeno, N. Roy, and L. A. Urena Lopez, Phys. Rev. D 104, 123502 (2021), 2105.07103*

- This is in agreement with the recent results in

  Dinda, Bikash R. "Cosmic expansion parametrization: Implication for curvature and H_0 tension." arXiv preprint arXiv:2106.02963 (2021).

  where it has been shown that CMB+BAO+SN data put stronger constraints on H0 and on other background cosmological parameters and that the addition of H0 prior from SH0ES (or from similar other local distance observations) can not significantly pull the H0 value towards the corresponding SH0ES value.

Results

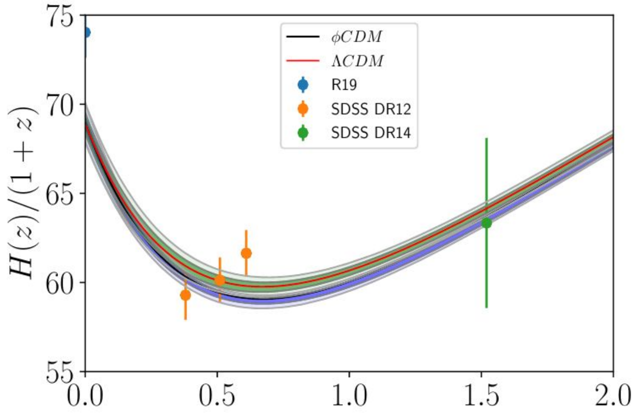


Fig 5(a): Posterior probability H(z)/(1+z)

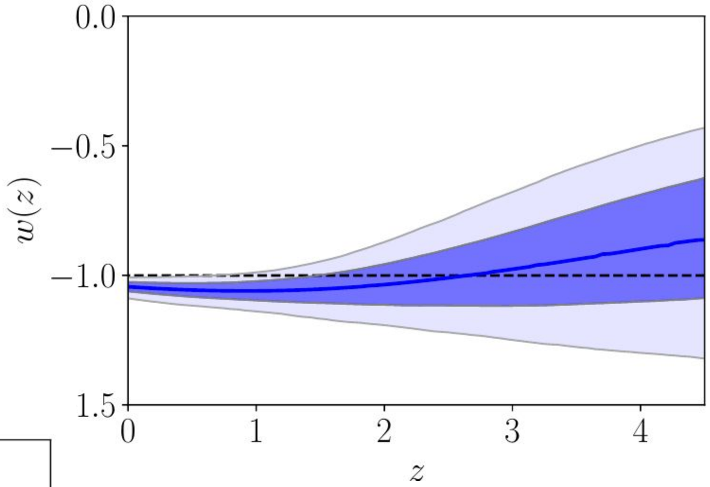


Fig 5(b): Posterior probability of DE EoS

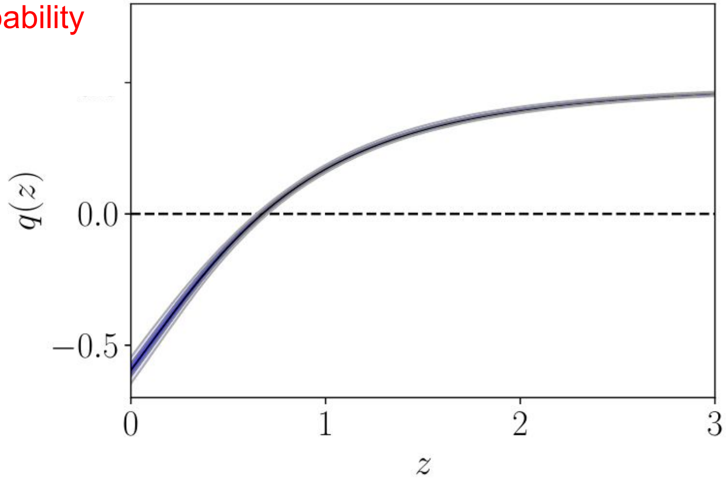


Fig 5(c): Posterior probability of the deceleration parameter

Numerical evolution of the system

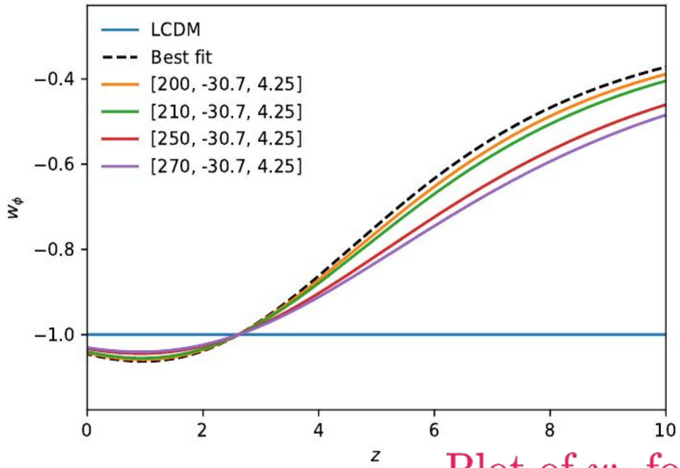


Fig 6(a)

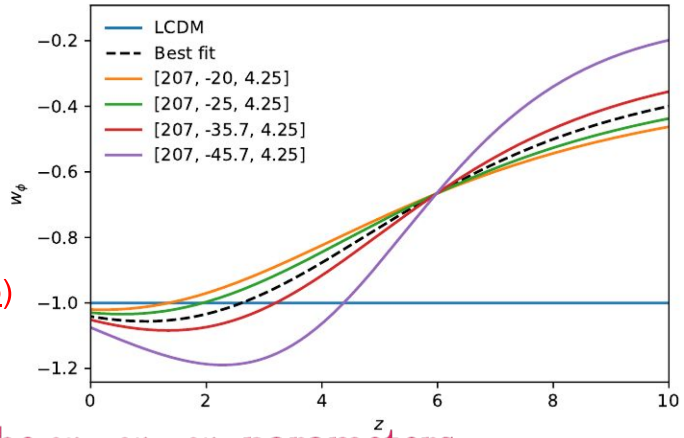


Fig 6(b)

Plot of w_ϕ for different values of the w_0, w_1, w_3 parameters.

Best fit values

$[w_0, w_1, w_3 : 207, -30.7, 4.25]$

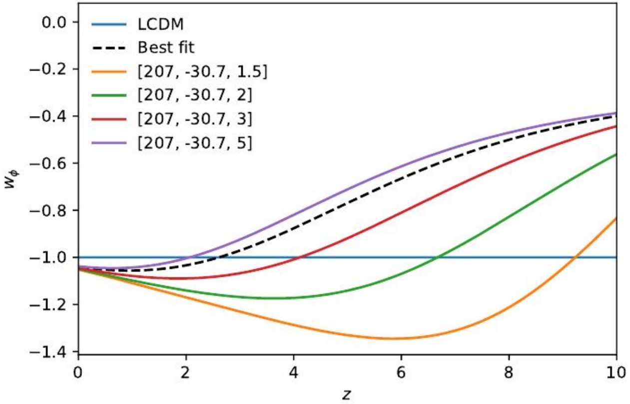


Fig 6(c)

We have varied one parameter while keeping other two parameters fixed at the best fit value.

Numerical evolution of the system

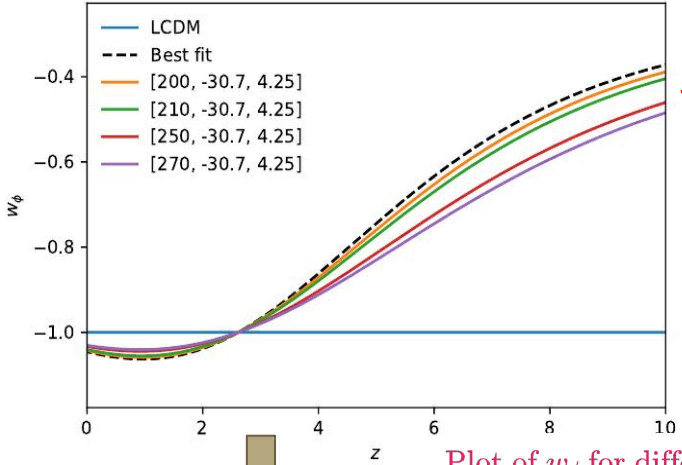


Fig 7(a)

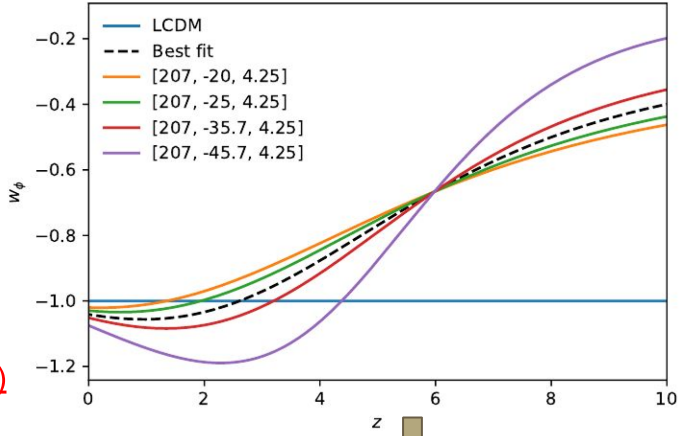


Fig 7(b)

Plot of w_ϕ for different values of the w_0, w_1, w_3 parameters.

The effect on the EoS is not significant for the w_0 at late times and also the the redshift at which phantom barrier happens.

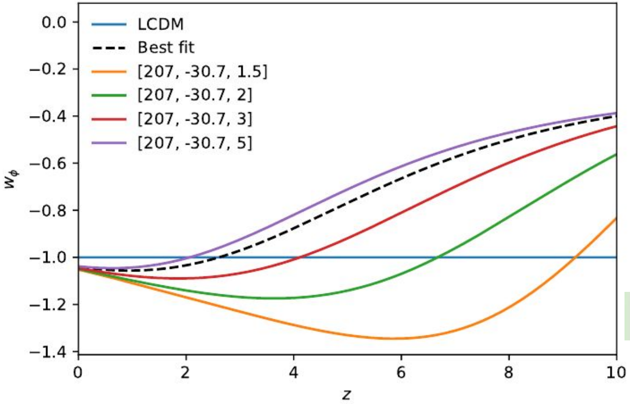


Fig 7(c)

The evolution of the EoS is quite sensitive to the w_1 and w_3 parameters.

w_3	–	$4.25^{+1.65}_{-3.46}$

Results

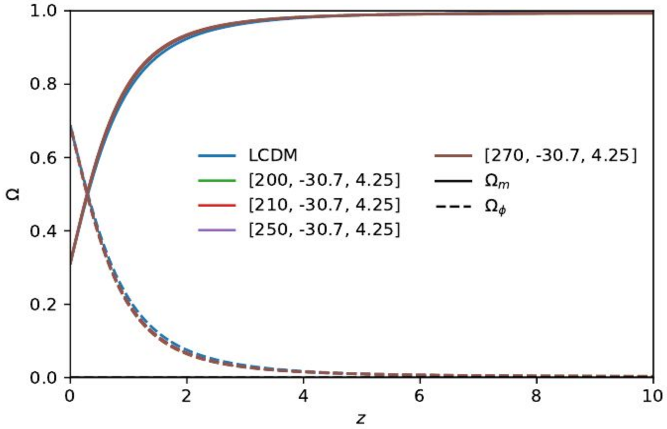


Fig 8(a)

Plot of dark energy density parameter

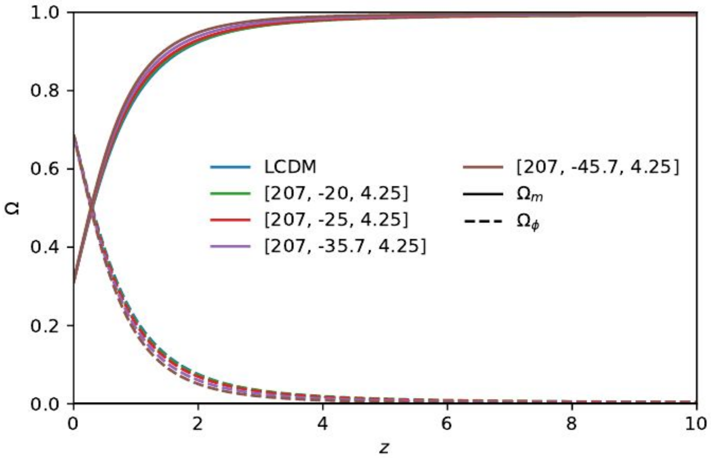


Fig 8(b)

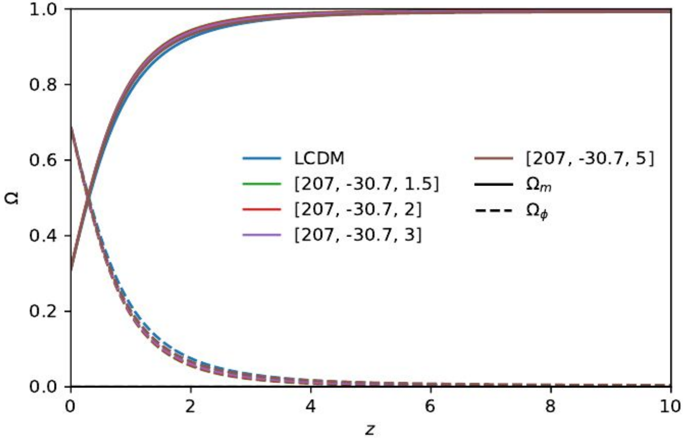


Fig 8(c)

Results

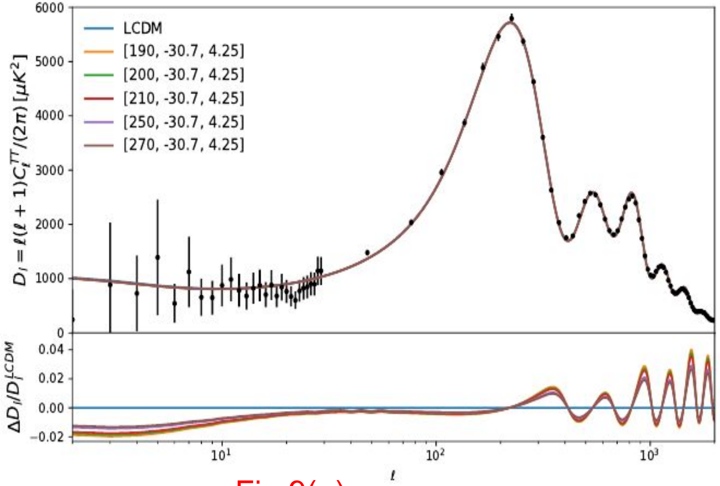


Fig 9(a)

Plot of the CMB anisotropies for the same set of parameters.

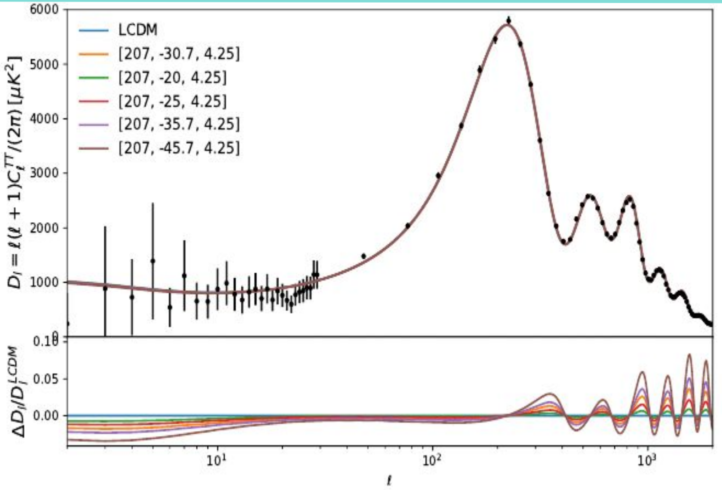


Fig 9(b)

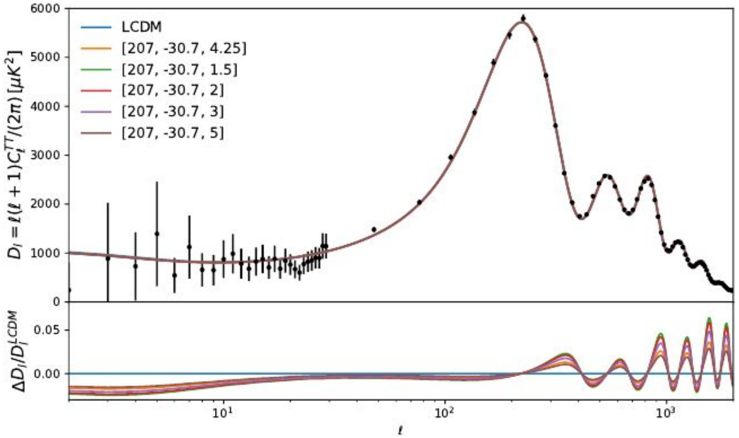


Fig 9(c)

Plot of MPS for the same set of parameters

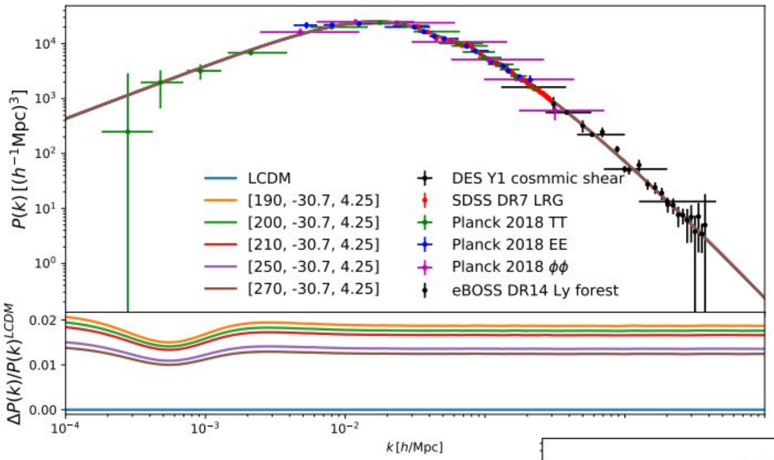


Fig 10(a)

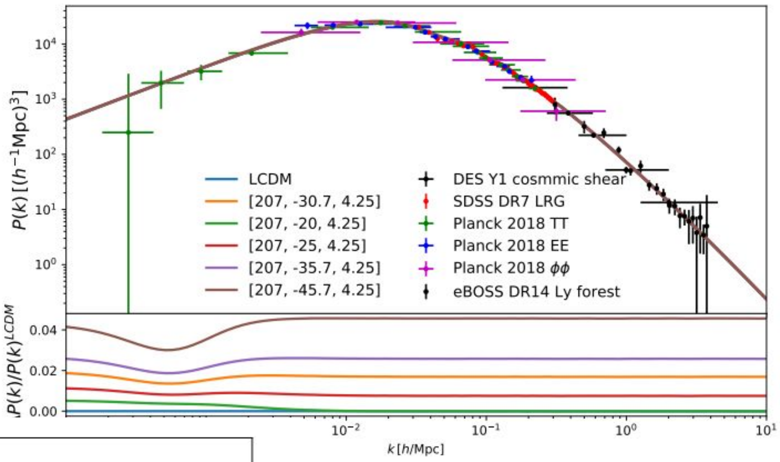


Fig 10(b)

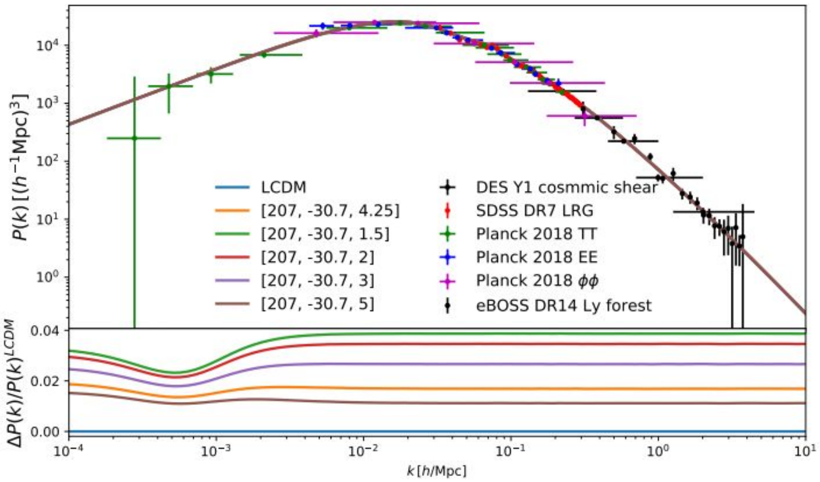


Fig 10(c)

Conclusions

- In this work we have revisited the dynamics of the scalar field dark energy models and proposed a general scheme which can include both the quintessence and the phantom scalar field models.
- Using our method it is possible to express all the cosmological parameters in terms of the normalized Hubble parameter (E), present value of the matter energy density and redshift z.
- A general condition for the phantom barrier crossing has been proposed. This general condition can help us to check if a dark energy model will have phantom barrier crossing.
- A parameterization of the H(z) has been considered. For this parametrization a phantom barrier crossing has been observed but it can not alleviate the Hubble tension.
- There is slight deviation in the $p(k)$ and D_l curve compared to the ΛCDM
- A comparison between φCDM and ΛCDM models have been carried out using the concept of Bayes Factor and the φCDM model is found to have positive preference over the ΛCDM.

Thank You