

The CMB x LSS Skew-Spectrum

Priyesh Chakraborty (Harvard University)

Based on arxiv:2202.11724 with Shu-Fan Chen and Cora Dvorkin

Outline

Background
Skew-Spectra
Fisher Analysis
Simulations Analysis

Today in Cosmology

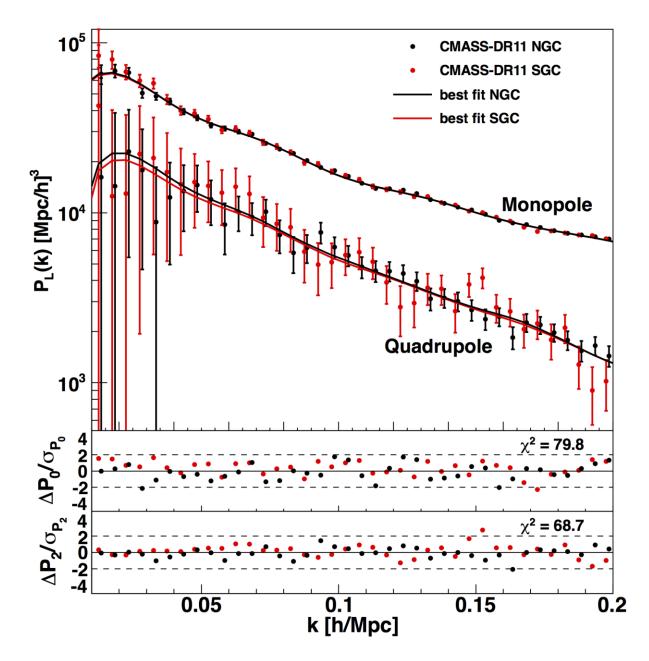
- ACDM model well constrained by CMB T&P!
- There are still potential issues:
 - H₀ tension (now 5 sigma)
 - S₈ tension (2-3 sigma)
- And new physics to constrain:
 - Nature of Dark Matter
 - Inflation

• ...

Multipole moment, ℓ 10 50 500 1000 1500 2000 2500 Temperature fluctuations [μ K 2] 6000 5000 4000 3000 2000 1000 18° 1° 0.2° 0.1° 0.07° 90° Angular scale

Large-Scale Structure

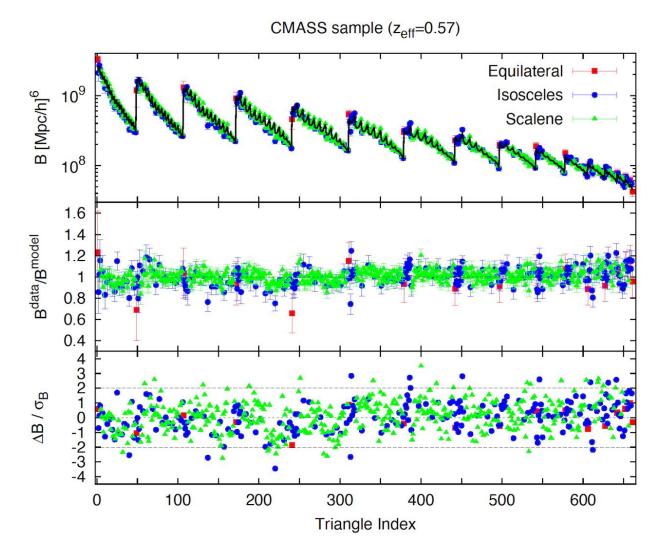
• In principle, access to orderof-magnitude more modes Better constraints! **Resolve** issues + avenue for new physics!



BOSS power spectra – Beutler et al (2014)

Large-Scale Structure

- Contrast to CMB, LSS contains lot of info in higher order correlators, e.g. bispectrum
- Recently measured and analysed for BOSS



Gil-Marin (2020)

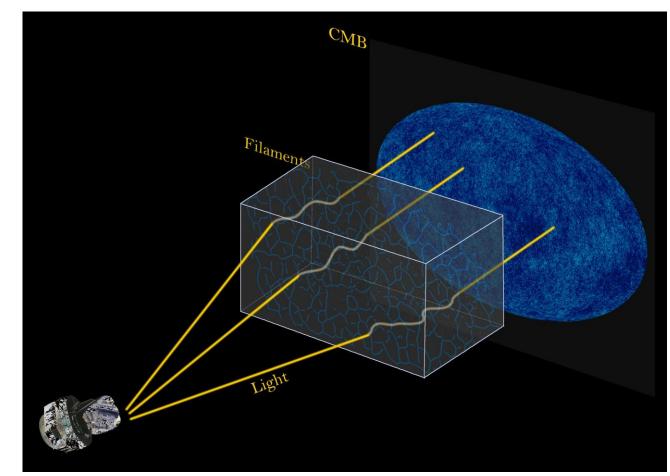
CMB Weak Lensing

 CMB photons deflected by matter from source to present

Determined by intermediate matter

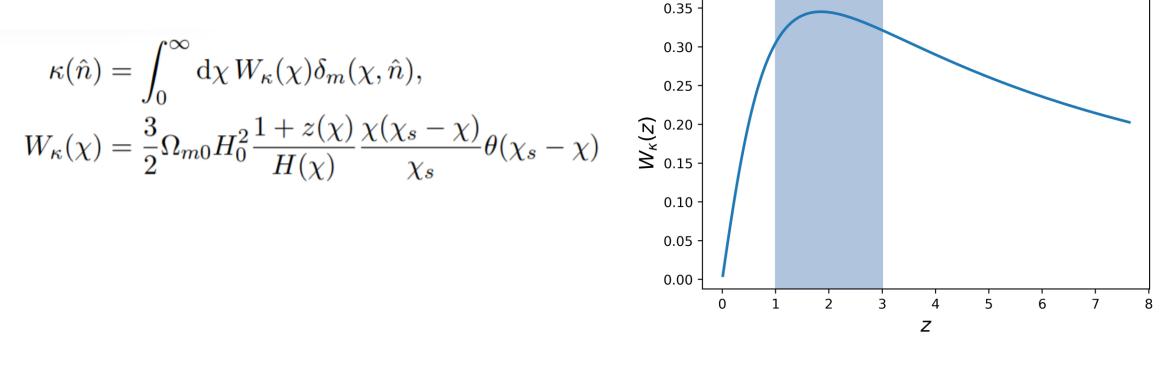
$$\vec{T}(\hat{x}) = T(\hat{x} + \hat{d})$$

$$\vec{\kappa}(\hat{n}) = \nabla \cdot \hat{d}(n)$$



CMB Weak Lensing

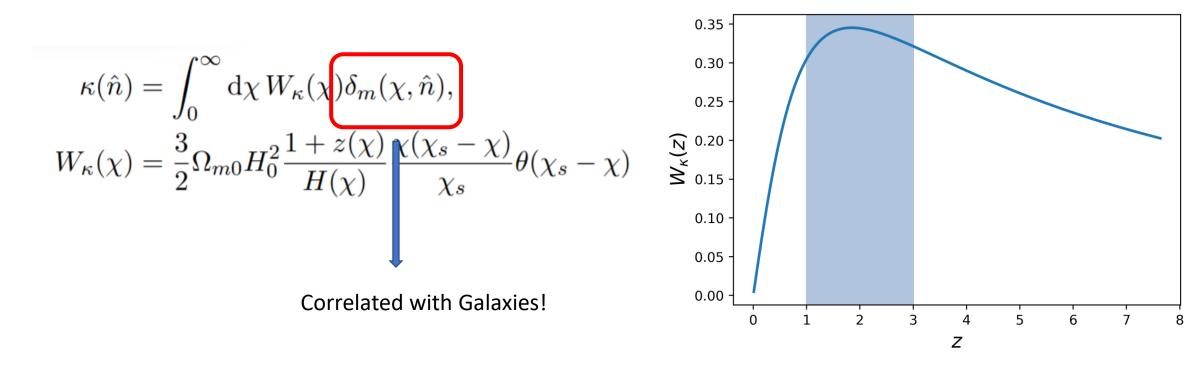
CMB photons deflected by matter from source to present



Kernel peaks z=1-3

CMB Weak Lensing

• CMB photons deflected by matter from source to present

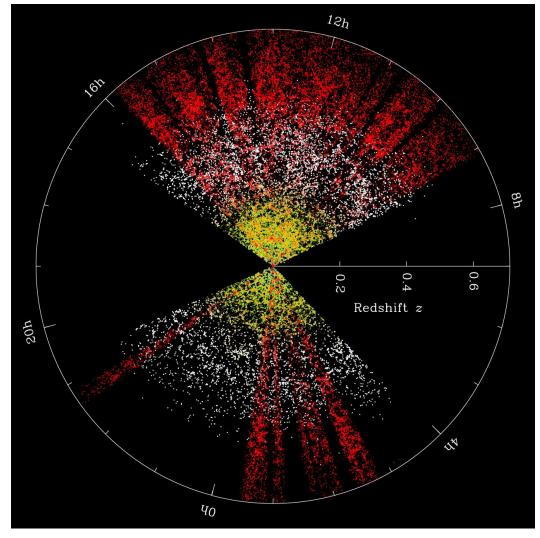


Kernel peaks z=1-3

Tomography

Observe galaxy angular positions
 + redshifts

$$\begin{split} \delta^i_g(\hat{n}) &= \int_0^\infty \mathrm{d}\chi \, W^i_g(\chi) \delta_g(\chi, \hat{n}) \,, \\ W^i_g(\chi) &= \frac{1}{\bar{n}_i} \frac{\mathrm{d}n_i}{\mathrm{d}\chi} \,, \quad \bar{n}_i \equiv \int_0^\infty \mathrm{d}\chi \, \frac{\mathrm{d}n_i}{\mathrm{d}\chi} \end{split}$$



SDSS III - BOSS

• Power spectra:
$$\langle \delta^a_{\ell m} \delta^{b*}_{\ell' m'} \rangle = \delta^{K}_{\ell \ell'} \delta^{K}_{mm'} C^{ab}_{\ell}$$

• Bispectra:
$$\langle \delta^a_{\ell_1 m_1} \delta^b_{\ell_2 m_2} \delta^c_{\ell_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b^{abc}_{\ell_1 \ell_2 \ell_3}$$

...

• Polyspectra (in principle)

• Bispectra: $\langle \delta^a_{\ell_1 m_1} \delta^b_{\ell_2 m_2} \delta^c_{\ell_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b^{abc}_{\ell_1 \ell_2 \ell_3}$

• Bispectra:
$$\langle \delta^a_{\ell_1 m_1} \delta^b_{\ell_2 m_2} \delta^c_{\ell_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b^{abc}_{\ell_1 \ell_2 \ell_3}$$

• Theory angular bispectra obtained by projecting Fourier space ones

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \xrightarrow{\int \mathrm{d}\chi} b_{\ell_1 \ell_2 \ell_3}$$

Perturbation Theory/N-body

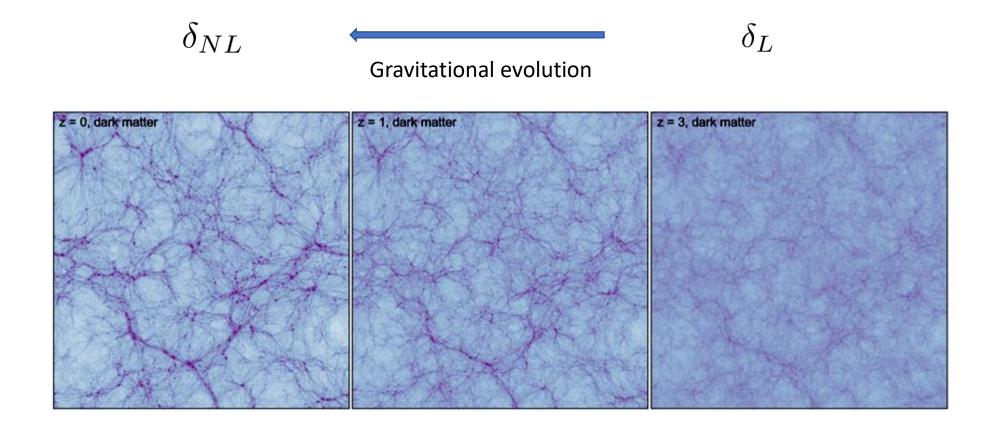
• Bispectra:
$$\langle \delta^a_{\ell_1 m_1} \delta^b_{\ell_2 m_2} \delta^c_{\ell_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b^{abc}_{\ell_1 \ell_2 \ell_3}$$

• In perturbation theory, can be written as:

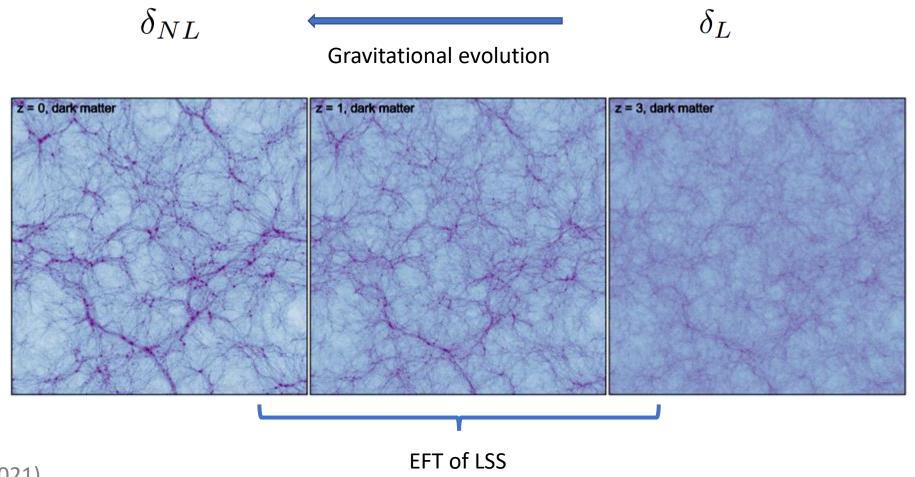
$$b^{abc}_{\ell_1\ell_2\ell_3} = \sum_{n_1n_2n_3} c^{abc}_{n_1n_2n_3} \int_0^\infty \mathrm{d}r \, r^2 \tilde{I}^{ac}_{\ell_1}(r;n_1) \tilde{I}^{bc}_{\ell_2}(r;n_2) \tilde{I}^c_{\ell_3}(r;n_3)$$

$$\tilde{I}_{\ell}^{ab}(r;n) \equiv 4\pi \int_{0}^{\infty} \mathrm{d}\chi \, W_{a}(\chi) D_{+}(\chi) \int_{0}^{\infty} \mathrm{d}k \, k^{2+n} j_{\ell}(kr) j_{\ell}(kr) P^{ab}(k) \Big]$$
Can be computed efficiently using
$$\tilde{I}_{\ell}^{a}(r;n) \equiv 4\pi \int_{0}^{\infty} \mathrm{d}\chi \, W_{a}(\chi) D_{+}^{2}(\chi) \int_{0}^{\infty} \mathrm{d}k \, k^{2+n} j_{\ell}(kr) j_{\ell}(kr)$$
Limber/FFTLog!

Chen, Lee, Dvorkin (2021)



IllustrisTNG (2021)



IllustrisTNG (2021)

 δ_{NL} Gravitational evolution δ_L

• Galaxy bispectrum:

$$B^{ggg}(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3}) = 2\left(\frac{17}{21}b_{1}^{3} + b_{1}^{2}b_{2} - \frac{2}{3}b_{1}^{2}b_{\mathcal{G}_{2}}\right)\mathbf{P}_{0}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2})P(k_{1})P(k_{2}) + 2b_{1}^{3}\frac{1}{2}\left(\frac{k_{1}}{k_{2}} + \frac{k_{2}}{k_{1}}\right)\mathbf{P}_{1}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2})P(k_{1})P(k_{2}) + 2\left(\frac{4}{21}b_{1}^{3} + \frac{2}{3}b_{1}^{2}b_{\mathcal{G}_{2}}\right)\mathbf{P}_{2}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2})P(k_{1})P(k_{2}) + 2 \text{ perms.}$$

Desjacques, Jeong, Schmidt (2018)

 δ_{NL} Gravitational evolution δ_L

• Galaxy bispectrum:

$$B^{ggg}(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3}) = 2 \left(\frac{17}{21} b_{1}^{3} + b_{1}^{2} b_{2} - \frac{2}{3} b_{1}^{2} b_{\mathcal{G}_{2}} \right) \mathbf{P}_{0}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2}) \mathbf{P}(k_{1}) P(k_{2}) + 2 b_{1}^{3} \frac{1}{2} \left(\frac{k_{1}}{k_{2}} + \frac{k_{2}}{k_{1}} \right) \mathbf{P}_{1}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2}) P(k_{1}) P(k_{2}) + 2 \left(\frac{4}{21} b_{1}^{3} + \frac{2}{3} b_{1}^{2} b_{\mathcal{G}_{2}} \right) \mathbf{P}_{2}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2}) P(k_{1}) P(k_{2}) + 2 \text{ perms.}$$

Desjacques, Jeong, Schmidt (2018)

Post-Born Corrections

Correction to Born approximation produces second order terms

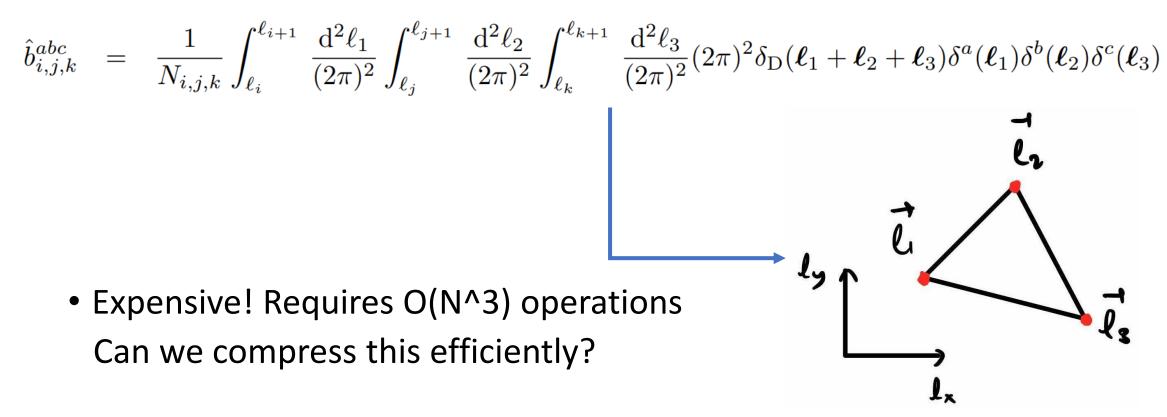
Born:
$$\kappa(\hat{n}) = \int_0^\infty d\chi W_\kappa(\chi) \delta_m(\chi, \hat{n})$$

 $\kappa^{(2)}(\boldsymbol{L}) = -4 \int d\chi W_\kappa(\chi, \chi_s) \int d\chi' W_\kappa(\chi', \chi) \int_{L'} \boldsymbol{L}' \cdot \boldsymbol{L} \boldsymbol{L}' \cdot (\boldsymbol{L} - \boldsymbol{L}') \Psi(\boldsymbol{L}', \chi) \Psi(\boldsymbol{L} - \boldsymbol{L}', \chi')$
Generate tree-level bispectra

 Subdominant for power spectra but important for bispectra even at low ell

Bispectrum Measurement

• Need to compute average over all triangles



Coulton, Liu et al (2018)

• Minimum variance (bispectrum) amplitude estimator

$$\hat{f}_{\rm NL} = \frac{\sigma^2}{(4\pi f_{\rm sky})^2} \sum_{\ell_1 \ell_2 \ell_3} g_{\ell_1 \ell_2 \ell_3}^2 \frac{b_{\ell_1 \ell_2 \ell_3}^{\rm th} \hat{b}_{\ell_1 \ell_2 \ell_3}^{\rm obs}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$
$$= \frac{\sigma^2}{(4\pi f_{\rm sky})^2} \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \sum_{\ell_3 m_3} \mathcal{G}_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} b_{\ell_1 \ell_2 \ell_3}^{\rm th} \frac{\delta_{\ell_1 m_1} \delta_{\ell_2 m_2} \delta_{\ell_3 m_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

• Minimum variance (bispectrum) amplitude estimator

$$\hat{f}_{\rm NL} = \frac{\sigma^2}{(4\pi f_{\rm sky})^2} \sum_{\ell_1 \ell_2 \ell_3} g_{\ell_1 \ell_2 \ell_3}^2 \frac{b_{\ell_1 \ell_2 \ell_3}^{\rm th} \hat{b}_{\ell_1 \ell_2 \ell_3}^{\rm obs}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

$$= \frac{\sigma^2}{(4\pi f_{\rm sky})^2} \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \sum_{\ell_3 m_3} \mathcal{G}_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} b_{\ell_1 \ell_2 \ell_3}^{\rm th} \frac{\delta_{\ell_1 m_1} \delta_{\ell_2 m_2} \delta_{\ell_3 m_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

$$\hat{f}_{\rm NL} \propto \sum_{\ell} \hat{\tilde{C}}_{\ell}$$

• Minimum variance (bispectrum) amplitude estimator

$$\hat{\tilde{C}}_{\ell} = \frac{1}{2\ell+1} \sum_{m} D_{\ell m} \frac{\delta_{\ell m}^*}{C_{\ell}}$$

$$D_{\ell m} = (-1)^m \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \mathcal{G}_{\ell_1 \ell_2 \ell}^{m_1 m_2 (-m)} b_{\ell_1 \ell_2 \ell}^{\text{th}} \frac{\delta_{\ell_1 m_1}}{C_{\ell_1}} \frac{\delta_{\ell_2 m_2}}{C_{\ell_2}}$$

Munshi, Heavens (2009)

Fourier space: Schmittfull, Baldauf, Seljak

(2015)

Moradinezhad Dizgah, Lee, Schmittfull,

Dvorkin (2020)

• With two fields 'a' and 'b':

$$\hat{\tilde{C}}_{\ell}^{ab} = \frac{1}{2\ell+1} \sum_{m} D_{\ell m}^{ab} \tilde{\delta}_{\ell m}^{b*}$$

$$D^{ab}_{\ell m} = (-1)^m \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \mathcal{G}^{m_1 m_2 (-m)}_{\ell_1 \ell_2 \ell} b^{aab, \text{th}}_{\ell_1 \ell_2 \ell} \tilde{\delta}^a_{\ell_1 m_1} \tilde{\delta}^a_{\ell_2 m_2}$$

$$\delta^a_{\ell m}/C^a_\ell \equiv \tilde{\delta}^a_{\ell m}$$

• Can we do better?

Recall: Galaxy bispectrum in redshift space:

$$B^{ggg}(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3}) = 2\left(\frac{17}{21}b_{1}^{3} + b_{1}^{2}b_{2} - \frac{2}{3}b_{1}^{2}b_{\mathcal{G}_{2}}\right)\mathbf{P}_{0}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2})P(k_{1})P(k_{2}) \\ + 2b_{1}^{3}\frac{1}{2}\left(\frac{k_{1}}{k_{2}} + \frac{k_{2}}{k_{1}}\right)\mathbf{P}_{1}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2})P(k_{1})P(k_{2}) \\ + 2\left(\frac{4}{21}b_{1}^{3} + \frac{2}{3}b_{1}^{2}b_{\mathcal{G}_{2}}\right)\mathbf{P}_{2}(\hat{\mathbf{k}}_{1} \cdot \hat{\mathbf{k}}_{2})P(k_{1})P(k_{2}) \\ + 2 \text{ perms.}$$

Schmittfull, Baldauf, Seljak (2015)

• Can we do better?

Galaxy bispectrum in harmonic space:

$$b_{\ell_1\ell_2\ell_3}^{ggg} = \left(\frac{17}{21}b_1^3 + b_1^2b_2 - \frac{2}{3}b_1^2b_{\mathcal{G}_2}\right)h_{0,\ell_1\ell_2\ell_3}^{ggg} + b_1^3h_{1,\ell_1\ell_2\ell_3}^{ggg} + \left(\frac{4}{21}b_1^3 + \frac{2}{3}b_1^2b_{\mathcal{G}_2}\right)h_{2,\ell_1\ell_2\ell_3}^{ggg}$$

$$\begin{split} h_{0,\ell_{1}\ell_{2}\ell_{3}}^{aab} &\to \mathbf{P}_{0}(\hat{k}_{1} \cdot \hat{k}_{2})P(k_{1})P(k_{2}) + 2 \text{ perms.} \\ h_{1,\ell_{1}\ell_{2}\ell_{3}}^{aab} &\to \frac{1}{2} \left(\frac{k_{1}}{k_{2}} + \frac{k_{2}}{k_{1}} \right) \mathbf{P}_{1}(\hat{k}_{1} \cdot \hat{k}_{2})P(k_{1})P(k_{2}) + 2 \text{ perms.} \\ h_{2,\ell_{1}\ell_{2}\ell_{3}}^{aab} &\to \mathbf{P}_{2}(\hat{k}_{1} \cdot \hat{k}_{2})P(k_{1})P(k_{2}) + 2 \text{ perms.} . \end{split}$$

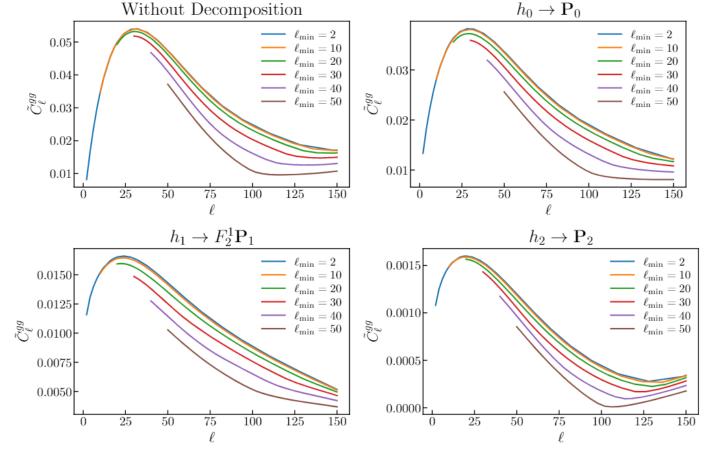
• Can we do better?

Galaxy bispectrum in harmonic space:

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$$D_{\ell m}^{ab,i} = (-1)^m \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \mathcal{G}_{\ell_1 \ell_2 \ell}^{m_1 m_2 (-m)} h_{\ell_1 \ell_2 \ell}^{aab,i} \tilde{\delta}_{\ell_1 m_1}^a \tilde{\delta}_{\ell_2 m_2}^a$$

- Can we do better?
- Yes! Use the $h_{i,\ell_1\ell_2\ell_3}^{aab}$ as kernels instead

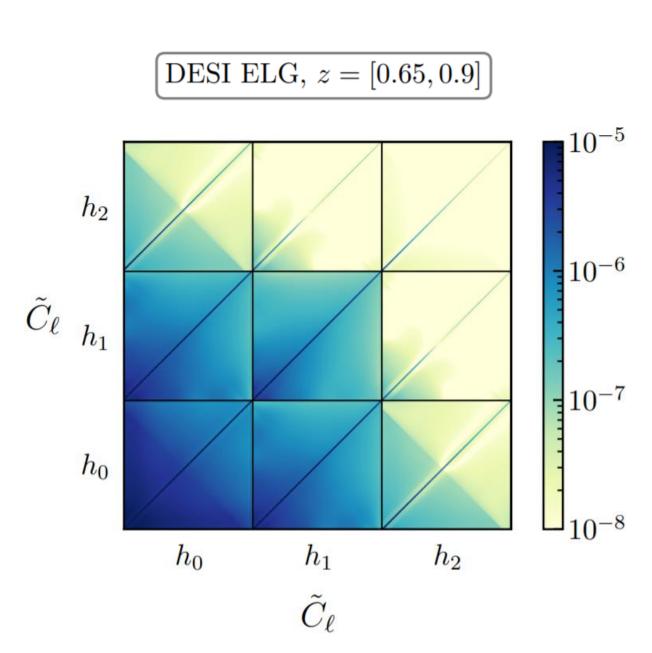


Theoretical Expression

 $\tilde{C}_{\ell}^{ab,i} = \frac{1}{2\ell+1} \sum_{\ell_1\ell_2} g_{\ell_1\ell_2\ell}^2 \frac{h_{i,\ell_1\ell_2\ell}^{aab} b_{\ell_1\ell_2\ell}^{aab,\text{obs}}}{C_{\ell_1}^a C_{\ell_2}^a C_{\ell}^b}$

Example skew-spectra for LSST redshift bin [0.8,1.]

- Covariance matrix
- Cannot ignore off-diagonals



Moradinezhad Dizgah, Lee, Schmittfull, Dvorkin (2020)

 Forecasted constraints for bias and A_{shot} for a single redshift bin of DESI-ELG

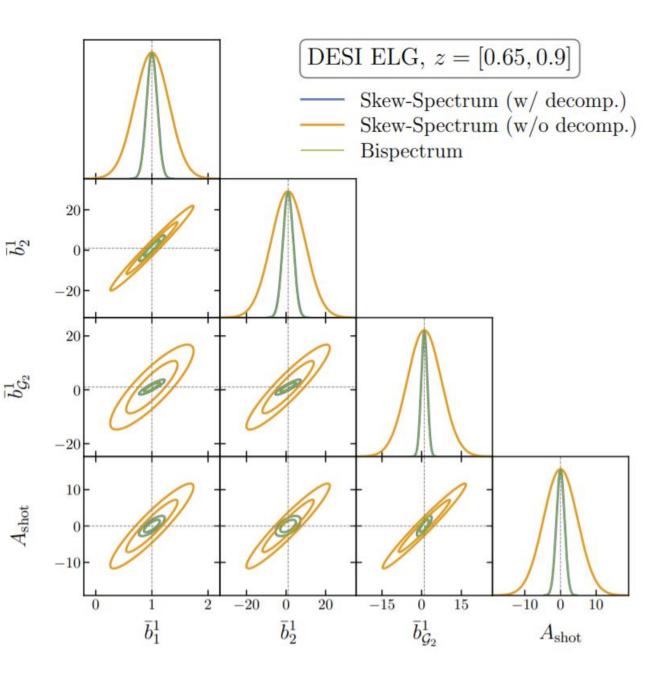
• Forecasted constraints for bias and A_{shot}^{\star} for a single redshift bin of DESI-ELG

$$P_{\rm shot}(k) = P_{\rm Poisson}(1 - A_{\rm shot})$$

$$B_{\rm shot}(k) = B_{\rm Poisson}(1 - A_{\rm shot})$$

Peebles (1980) Schmittfull, Baldauf, Seljak (2015)

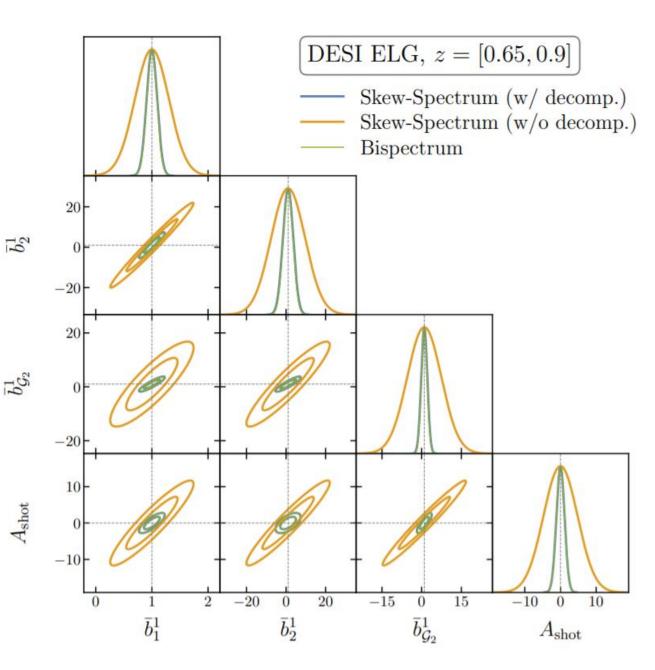
 Forecasted constraints for bias and A_{shot} for a single redshift bin of DESI-ELG



 Forecasted constraints for bias and A_{shot} for a single redshift bin of DESI-ELG

$$\tilde{C}_{\ell} = \sum_{i} \alpha_{i} \tilde{C}_{\ell}^{i}$$

Not invertible!



• Bispectrum kernels can be factorized:

$$D_{\ell m}^{ab,i} = (-1)^m \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \mathcal{G}_{\ell_1 \ell_2 \ell}^{m_1 m_2 (-m)} h_{\ell_1 \ell_2 \ell}^{aab,i} \tilde{\delta}_{\ell_1 m_1}^a \tilde{\delta}_{\ell_2 m_2}^a$$

$$h_{\ell_1\ell_2\ell_3}^{abc} = \sum_{n_1n_2n_3} c_{n_1n_2n_3}^{abc} \int_0^\infty \mathrm{d}r \, r^2 \tilde{I}_{\ell_1}^{ac}(r;n_1) \tilde{I}_{\ell_2}^{bc}(r;n_2) \tilde{I}_{\ell_3}^c(r;n_3) \tilde{I}_{\ell_3}^{bc}(r;n_3) \tilde{I}_{\ell_3}^{bc}(r;$$

• Bispectrum kernels can be factorized:

$$\begin{split} D^{ab}_{\ell m} &= (-1)^m \sum_{n_1 n_2 n_3} c^{aab}_{n_1 n_2 n_3} \\ &\times \int_0^\infty \mathrm{d} r \, r^2 \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \mathcal{G}^{m_1 m_2 (-m)}_{\ell_1 \ell_2 \ell} (\tilde{I}^{ab}_{\ell_1}(r, n_1) \tilde{\delta}^a_{\ell_1 m_1}) (\tilde{I}^{ab}_{\ell_2}(r, n_2) \tilde{\delta}^a_{\ell_2 m_2}) \tilde{I}^b_{\ell}(r, n_3) \\ & \Delta^{ab}_{\ell m}(r, n) \end{split}$$

• Bispectrum kernels can be factorized:

$$D_{\ell m}^{ab} = (-1)^m \sum_{n_1 n_2 n_3} c_{n_1 n_2 n_3}^{aab} \int_0^\infty \mathrm{d}r \, r^2 D_{\ell m}^{ab}(r; n_1, n_2) \tilde{I}_{\ell}^b(r, n_3)$$

• Reduce to trivial convolution
(modulo an integral)
$$D_{\ell m}^{ab}(r; n_1, n_2) = \int \mathrm{d}^2 \hat{n} \, Y_{\ell m}^*(\hat{n}) \Delta^{ab}(\hat{n}; r, n_1) \Delta^{ab}(\hat{n}; r, n_2)$$

Skew-Spectrum Estimation Algorithm

- 1. Obtain fields $\delta^a(\hat{n})$ and $\delta^b(\hat{n})$.
- 2. Compute the filtered harmonic coefficients, $\tilde{\delta}^a_{\ell m}$ and $\tilde{\delta}^b_{\ell m}$.
- 3. Pointwise product between $\tilde{\delta}^a_{\ell m}$ and $\tilde{I}^{ab}_{\ell}(r, n_i)$ to obtain $\Delta^{ab}_{\ell m}(r, n_i)$.
- 4. Return to position space to obtain $\Delta^{ab}(\hat{n}; r, n_i)$.
- 5. Compute harmonic transform of $\Delta^{ab}(\hat{n}; r, n_1) \Delta^{ab}(\hat{n}; r, n_2)$.
- 6. Product with $\tilde{I}_{\ell}^{b}(r, n_{3})$ and integrate over r to obtain the quadratic field.
- 7. Cross-correlate with $\tilde{\delta}^b_{\ell m}$ to obtain $\hat{\tilde{C}}^{ab}_{\ell}$.

Requires O(N log N) time!

Fisher Analysis (DESI ELG x Planck Lensing)

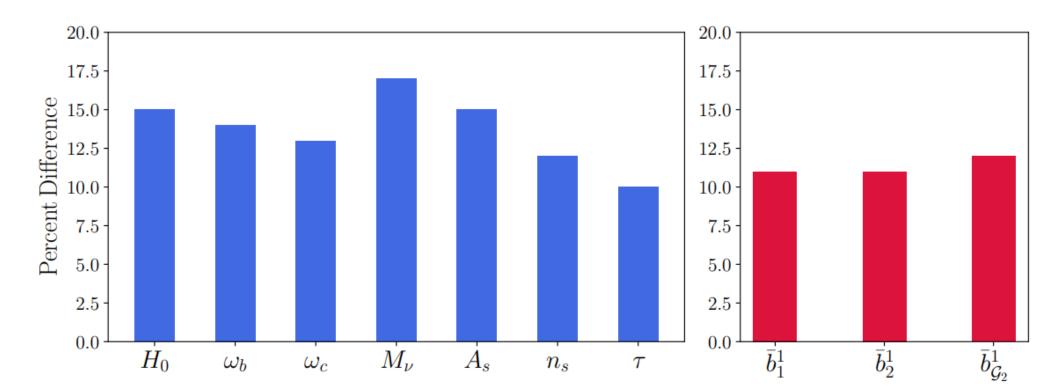
Assume Gaussian likelihood

• Vary all parameters $\lambda = \{H_0, \omega_b, \omega_c, A_s, n_s, \tau, M_\nu\} \cup \bigcup_{i=1}^{N_{\text{bin}}} \{\bar{b}_1^i, \bar{b}_2^i, \bar{b}_{\mathcal{G}_2}^i\} \cup \{A_{\text{shot}}\}$

- Fix $k_{\rm max} \sim 0.1 \ h {
 m Mpc^{-1}}$ to remain in tree-level regime
- Regardless, incorporate non-linearities using fitted PT kernel $F_2^{\text{fit}}(\mathbf{k}_1, \mathbf{k}_2)^{\star}$

Gil-Marin, Wagner, et al (2012)
 Munshi, Namikawa, et al (2020)

Results



Relative difference between marginalized errors using bispectrum vs skewspectrum

Simulations

We apply the skew-spectra to estimate biases and A_{shot} from MassiveNuS halo catalogue + lensing (ray-traced using Lenstools)

Flat-sky approximation

- Angular maps of 3.5 x 3.5 deg²
 Gridded data from simulations
- Sharp cutoff at $\ell = 1350$ (k~0.5 h/Mpc)
- Halos with minimum mass = $1.36 \times 10^{12} h^{-1} M_{\odot}$ at z=1.04

Petri (2016) Liu et al (2018)

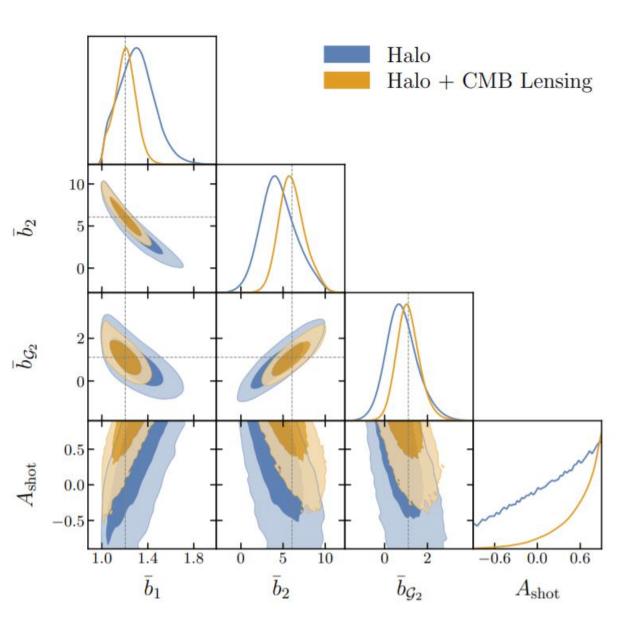
MCMC Analysis

• 68% errors for biases and 68% lower bound for A_{shot}:

Parameter	\overline{b}_1	\overline{b}_2	$ar{b}_{{\mathcal G}_2}$	A_{shot}
Uniform Prior	[1,3]	[-2,12]	[-5,5]	[-1, 1]
Halo + CMB Lensing	1.203 ± 0.091	6.068 ± 1.546	1.111 ± 0.562	≥ 0.581
Halo	1.304 ± 0.153	4.502 ± 2.104	0.822 ± 0.758	≥ 0.006

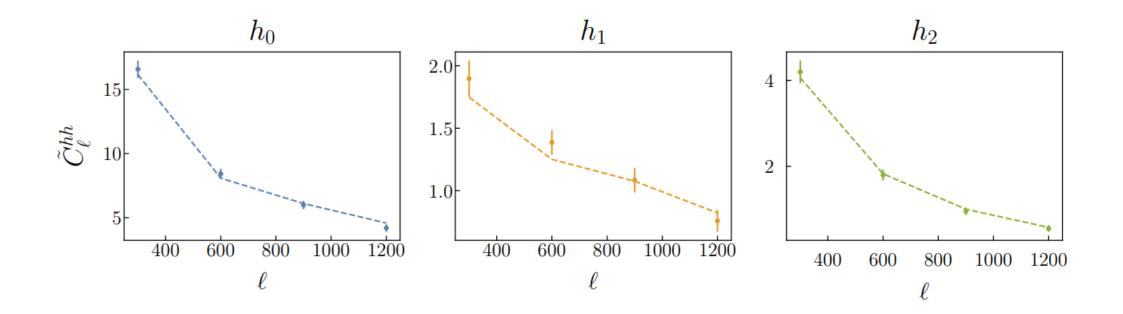
MCMC Analysis

• Posterior for halo skew-spectra and halo x lensing



MCMC Analysis

• Estimated halo-halo skew-spectra



Summary

- We compress the bispectrum to optimally capture amplitude information for CMBxLSS, finding that it also captures non-amplitude parameters as well to <17%
- We describe an algorithm to efficiently compute it from the data, requiring only O(N log N) steps
- We test our model on MassiveNuS simulations and constrain b₁ to percent level
- Future work: Analysis of BOSS data using Skew-Spectra