Supercomputers vs strong coupling in gravity

Hamiltonian analysis on silicon, and at scale

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Strong coupling in gravity

- If perturbation theory around desirable exact solution fails to capture some *inherently nonlinear* d.o.f...
- ...that d.o.f is said to be *strongly coupled* on the background, which becomes dynamically unreachable 2009.08197

Strong coupling in gravity

- In $D > 4$, fine-tuned Gauss–Bonnet admixture strongly couples the whole graviton on a maximally symmetric background 0807.2864

Strong coupling in gravity

- Hořava gravity suggested to be strongly coupled in the IR 1701.06087, 0905.2579, 0911.1299

Strong coupling in gravity

- Massive gravity has problematic vDVZ scalar, but nonlinear completions show that vDVZ becomes strongly coupled (screened) M. Fierz et al. (1939), H. van Dam et al. (1970), V. I. Zakharov (1970), D. G. Boulware et al. (1972), hep-th/0210184, A. I. Vainshtein (1972), hep-th/0106001

Strong coupling in gravity

- Same strong coupling effect blamed for Boulware–Deser ghost (final status of massive gravity contested) D. G. Boulware et al. (1972), gr-qc/0505134, 1401.4173, 1007.0443, 1011.1232

Non-Riemannian gravity (geometric formulation)

- Most general connection contains *contorsion* and *disformation*

$$\nabla_\nu V^\mu \equiv \partial_\nu V^\mu + \Gamma^\mu{}_{\lambda\nu} V^\lambda, \quad \Gamma^\mu{}_{\nu\sigma} = C^\mu_{\nu\sigma} + K^\mu{}_{\nu\sigma} + L^\mu{}_{\nu\sigma}$$

- These introduce *torsion* and *non-metricity* $Q_{\mu\nu\sigma} \equiv \nabla_\mu g_{\nu\sigma}$

$$L^\mu{}_{\nu\sigma} \equiv \frac{1}{2} Q^\mu{}_{\nu\sigma} - Q_{(\nu|}{}^\mu{}_{|\sigma)}, \quad K^\mu{}_{\nu\sigma} \equiv \frac{1}{2} T^\mu{}_{\nu\sigma} + T_{(\nu|}{}^\mu{}_{|\sigma)}$$

- Einstein–Hilbert Lagrangian of GR is really

$$L_\mathrm{G} = -\frac{1}{2\kappa} R + \frac{1}{\kappa} \lambda_\mu{}^{\rho\sigma} T^\mu{}_{\rho\sigma} + \frac{1}{\kappa} \hat{\lambda}_\mu{}^{\rho\sigma} Q^\mu{}_{\rho\sigma}$$

Torsion particle zoo (particle physics formulation)

- Metric (tetrad) and torsion (spin connection) are 40 d.o.f

$$16[h_a^{\mu}] + 24[A^{ab}{}_{\mu}] = 40$$

- Gauge fixing Poincaré symmetry leaves *up to* 20 d.o.f

$$40 - 2[\text{gauge}] \times 10[\mathbb{R}^{1,3} \rtimes \mathrm{SO}^+(1,3)] = 20$$

- These are the 2^+ *graviton* and $0^{\pm}$, $1^{\pm}$, $2^{\pm}$ *massive rotons*

$$20 = 2[2^+] + 1[0^+] + 1[0^-] + 3[1^+] + 3[1^-] + 5[2^+] + 5[2^-]$$

Hamiltonian for missing velocities

- Lagrangian $L(\psi^n, \dot{\psi}^n)$ has N d.o.f ψ^n, only $M < N$ dynamical
- Hamiltonian $H(\psi^n, \pi_n) \equiv \pi_n \dot{\psi}^n - L$ has $2N$ d.o.f ψ^n, π_n
- Non-dynamical d.o.f from missing $\dot{\psi}^m$, demands *constraints*

$$\pi_m \equiv \frac{\partial L}{\partial \dot{\psi}^m} = 0, \quad \varphi_m(\psi^n, \pi_n) \equiv \pi_m \approx 0$$

- ‘Weak’ equality $\varphi_m \approx 0$ defines physical part of phase space
- Hamiltonian made invertible for all $\dot{\psi}^n$ using multipliers u^n

$$H_{\mathrm{T}} \equiv H - u^n \varphi_n$$

The Dirac algorithm

- So we can use the total Hamiltonian $H_{\text{T}} \equiv H - u^n \varphi_n$ to study the dynamics
- But need condition that we keep $\varphi_m \approx 0$, equivalently

$$
\begin{aligned}
\dot{\varphi}_m &\equiv \int \mathrm{d}^3 x' \left\{ \varphi_m, H'_{\text{T}} \right\} \\
&\equiv \int \mathrm{d}^3 x' \Big[\left\{ \varphi_m, H' \right\} - u^{n\prime} \left\{ \varphi_m, \varphi'_n \right\} \Big] \approx 0
\end{aligned}
$$

- Depending if $\{\varphi_m, \varphi_n\} \approx 0$ this determines u^n or sets $\chi_m \approx 0$

$$
\chi_m(\psi^n, \pi_n) \equiv \{\varphi_m, H\} \approx 0, \quad u^n \approx \{\varphi_m, \varphi_n\}^{-1} \{\varphi_m, H\}
$$

Finding the d.o.f

- Repeat to find total of C *primaries* φ_n, *secondaries*, χ_n...
- Of these C_{FC} commute with everything (*first class*)
- While $C_{\text{SC}} \equiv C - C_{\text{FC}}$ do not commute (*second class*)
- Remember $L(\psi^n, \dot{\psi}^n)$ has N d.o.f ψ^n, only $M < N$ dynamical
- Dirac algorithm gives *systematic* way to find this

$$M \equiv \frac{1}{2}\left(2N - 2C_{\text{FC}} - C_{\text{SC}}\right)$$

- Poisson matrix $\{\varphi_n, \varphi_m\}$ is thus particle spectrum ‘barcode’

Yo–Nester no-go theorem

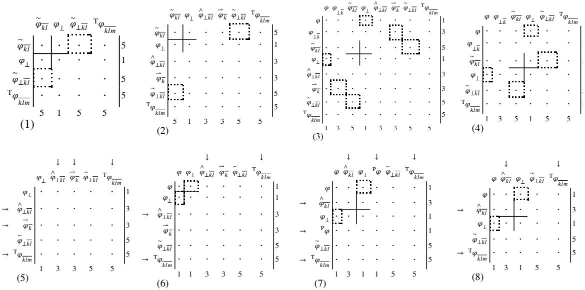
- Remember the 2^{+} *graviton* and $0^{\pm}$, $1^{\pm}$, $2^{\pm}$ *massive rotons*

$$20 = 2[2^{+}] + 1[0^{+}] + 1[0^{-}] + 3[1^{+}] + 3[1^{-}] + 5[2^{+}] + 5[2^{-}]$$

- Famous Hamiltonian analyses Hsin Chen et al. (1998), gr-qc/9902032, gr-qc/0112030
- Activating any one roton strongly couples others (possibly except 0^{+} and 0^{-})
- It has become doctrine to avoid non-Riemannian extensions beyond exact GR analogues 1903.06830, 1903.12072, Daniel Kristoffer Blixt (2021), 2111.04716, 2006.07406

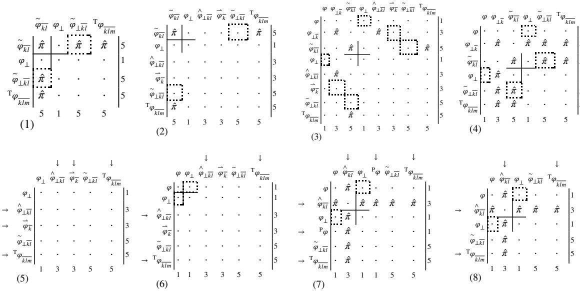
Even the ‘new IR’ is strongly coupled

- In 2101.02645 we tested novel *quadratic* $L_G = R^2 + T^2$ theories from 1812.02675, 1910.14197, 2003.02690, 2006.03581
- Linearised...

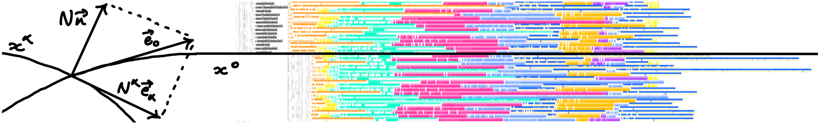


Even the 'new IR' is strongly coupled

- In 2101.02645 we tested novel *quadratic* $L_{\text{G}} = R^2 + T^2$ theories from 1812.02675, 1910.14197, 2003.02690, 2006.03581
- Linearised. . . nonlinear!

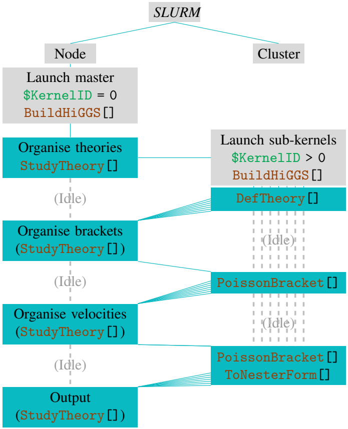


Hamiltonian Gauge Gravity Surveyor (HiGGS)



- Hamiltonian analysis of curvature/torsion 2206.00658
- Mathematica package, based in xAct (xAct.es 0704.1756, 0802.1274, 0803.0862, 0807.0824, 1302.6174, 1308.3493)
- Can use on a desktop, but parallelised for supercomputers
- Clone the repository at `github.com/wevbarker/HiGGS`

Scaling to supercomputers



- Trick is to parallelise over Poisson brackets
- Model as an Amdahl task David P. Rodgers (1985) for n-core speedup $s(n)$

$$S(n) = \frac{1}{1 - p + \frac{p}{n}},$$

- Parallel fraction p limited by a few ‘hard’ brackets

Scaling to supercomputers

- Why are Poisson brackets 'hard'? Formula is for *functionals*

$$\left\{\mathcal{A},\mathcal{B}\right\}\equiv\int\mathrm{d}^3x\left[\frac{\delta\mathcal{A}}{\delta b^i{}_\mu}\frac{\delta\mathcal{B}}{\delta\pi_i{}^\mu}+\frac{\delta\mathcal{A}}{\delta A^{ij}{}_\mu}\frac{\delta\mathcal{B}}{\delta\pi_{ij}{}^\mu}-\frac{\delta\mathcal{A}}{\delta\pi_i{}^\mu}\frac{\delta\mathcal{B}}{\delta b^i{}_\mu}-\frac{\delta\mathcal{A}}{\delta\pi_{ij}{}^\mu}\frac{\delta\mathcal{B}}{\delta A^{ij}{}_\mu}\right]$$

- But *local* quantities are functionals via Dirac distributions

$$\begin{aligned}\left\{\mathcal{A}(\boldsymbol{x}_1),\mathcal{B}(\boldsymbol{x}_2)\right\}\equiv\int\mathrm{d}^3x\big[&J_1(\boldsymbol{x})\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)\delta^3(\boldsymbol{x}-\boldsymbol{x}_2)J_2{}^\alpha(\boldsymbol{x})\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)\partial_\alpha\delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\\&+J_3{}^\alpha(\boldsymbol{x})\partial_\alpha\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)\delta^3(\boldsymbol{x}-\boldsymbol{x}_2)J_4{}^{\alpha\beta}(\boldsymbol{x})\partial_\alpha\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)\partial_\beta\delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\big]\end{aligned}$$

- Thus local Poisson bracket can be a *differential operator*

$$\begin{aligned}\int\mathrm{d}^3x_2\{\mathcal{A}_{\acute{u}}(\boldsymbol{x}_1),\mathcal{B}_{\acute{v}}(\boldsymbol{x}_2)\}\mathcal{C}^{\acute{v}}(\boldsymbol{x}_2)\equiv\mathcal{J}_{1\acute{v}}(\boldsymbol{x}_1)\mathcal{C}^{\acute{v}}(\boldsymbol{x}_1)+\mathcal{J}_{2\acute{v}}{}^\alpha(\boldsymbol{x}_1)D_\alpha\mathcal{C}^{\acute{v}}(\boldsymbol{x}_1)\\+\mathcal{J}_{3\acute{v}}{}^{\alpha\beta}(\boldsymbol{x}_1)D_\alpha D_\beta\mathcal{C}^{\acute{v}}(\boldsymbol{x}_1)\end{aligned}$$

Lagrange multipliers

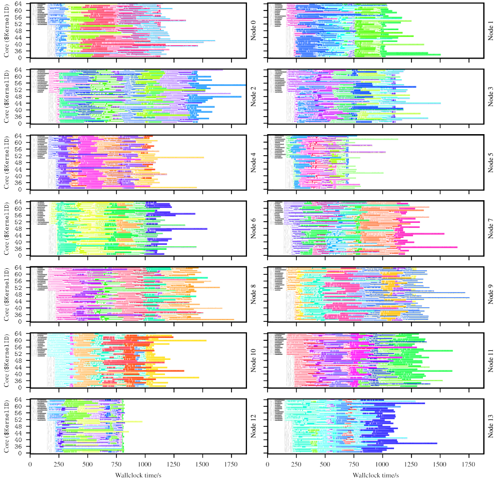
- Remember GR Lagrangian

$$L_{\mathrm{G}} = -\frac{1}{2\kappa}R + \frac{1}{\kappa}\lambda_{\mu}{}^{\rho\sigma}T^{\mu}{}_{\rho\sigma} + \frac{1}{\kappa}\hat{\lambda}_{\mu}{}^{\rho\sigma}Q^{\mu}{}_{\rho\sigma}$$

- Now extend use of multipliers 2205.13534

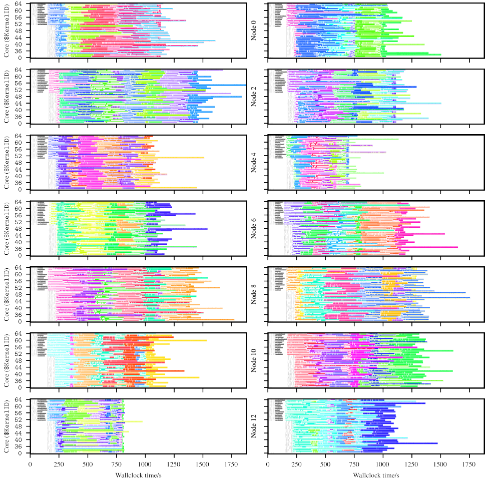
$$L_{\mathrm{G}} = -\frac{1}{2\kappa}R + \sum_{I=1}^{6}\left(\hat{\alpha}_I R^{\mu\nu}{}_{\sigma\rho} + \bar{\alpha}_I \lambda^{\mu\nu}{}_{\sigma\rho}\right) {}^{I}\hat{P}_{\mu\nu}{}^{\sigma\rho}{}_{\kappa\pi}{}^{\xi\zeta} R^{\kappa\pi}{}_{\xi\zeta}$$
$$+ \frac{1}{\kappa}\sum_{M=1}^{3}\left(\hat{\beta}_M T^{\mu}{}_{\nu\sigma} + \bar{\beta}_M \lambda^{\mu}{}_{\nu\sigma}\right) {}^{M}\hat{P}_{\mu}{}^{\nu\sigma}{}_{\pi}{}^{\xi\zeta} T^{\pi}{}_{\xi\zeta} + \frac{1}{\kappa}\hat{\lambda}_{\mu}{}^{\rho\sigma}Q^{\mu}{}_{\rho\sigma}$$

Supercomputer survey



- Start with strongly coupled theory
- Brute-force the Hamiltonian analysis of $3 \times 2^3 \times 2^3 = 192$ variant theories
- About one hour using the *Peta-4* supercomputer
- One line per core, one colour per theory

Supercomputer survey

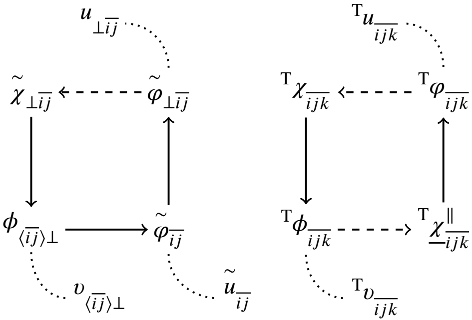


- Special case identified

$$L_{\mathrm{G}} = -\frac{1}{2\kappa}R + \alpha R_{[\mu\nu]}R^{[\mu\nu]} + \frac{1}{\kappa}\lambda_{\mu}{}^{\rho\sigma}\Big(T^{\mu}{}_{\rho\sigma} + T_{\rho}{}^{\mu}{}_{\sigma} + \delta^{\mu}_{\rho}T^{\xi}{}_{\sigma\xi}\Big) + \frac{1}{\kappa}\hat{\lambda}_{\mu}{}^{\rho\sigma}Q^{\mu}{}_{\rho\sigma}$$

- No new d.o.f beyond GR graviton
- Not even strongly coupled d.o.f

Supercomputer survey



- Special case identified

$$L_{\mathrm{G}} = -\frac{1}{2\kappa}R + \alpha R_{[\mu\nu]}R^{[\mu\nu]} + \frac{1}{\kappa}\lambda_{\mu}{}^{\rho\sigma}\Big(T^{\mu}{}_{\rho\sigma} + T_{\rho}{}^{\mu}{}_{\sigma} + \delta^{\mu}_{\rho}T^{\xi}{}_{\sigma\xi}\Big) + \frac{1}{\kappa}\hat{\lambda}_{\mu}{}^{\rho\sigma}Q^{\mu}{}_{\rho\sigma}$$

- Nontrivial correction to GR from an action
- Strictly no new particles (even in strong gravity)

The 'contact' theory

- Correction to Einstein field equations

$$\kappa\tau_{\mu\nu} = G_{\mu\nu} + \alpha\kappa\Big[2\Big(2R_{\lambda[\mu}R_{\nu]}{}^{\lambda} + \big(2R_{(\mu[\lambda\sigma]\nu)} + g_{\mu\nu}R_{[\lambda\sigma]}\big)R^{[\lambda\sigma]}\Big) + \big(T^2 + \nabla\cdot T + T\cdot\nabla + \nabla^2\big)_{\mu\nu[\lambda\sigma]}R^{[\lambda\sigma]}\Big]$$

- Where *all* non-metricity and *part* of the torsion is disabled

$$T^{\mu}{}_{[\rho\sigma]} + T_{[\rho}{}^{\mu}{}_{\sigma]} + \delta^{\mu}_{[\rho}T^{\xi}{}_{\sigma]\xi} = 0, \quad Q^{\rho}{}_{\mu\nu} = 0$$

- And remaining torsion subject to auxiliary constraint

$$T = \alpha\kappa\big(T + \nabla\big)_{[\lambda\sigma]}R^{[\lambda\sigma]}$$

Summary

1. Perturbations around exact solutions *meaningless* if you don't know the nonlinear d.o.f (unless you are doing EFT)
2. Hamiltonian analysis can be done *on silicon*, and *at scale*
3. HiGGS package available for curvature and/or torsion gravity
4. Nontrivial, action-derived modification to GR with no new d.o.f and no strong coupling (awaits further study)

$$\kappa\tau_{\mu\nu} = G_{\mu\nu} + \alpha\kappa\Big[2\Big(2R_{\lambda[\mu}R_{\nu]}{}^{\lambda} + \big(2R_{(\mu[\lambda\sigma]\nu)} + g_{\mu\nu}R_{[\lambda\sigma]}\big)R^{[\lambda\sigma]}\Big) + \big(T^2 + \nabla\cdot T + T\cdot\nabla + \nabla^2\big)_{\mu\nu[\lambda\sigma]}R^{[\lambda\sigma]}\Big]$$