Multi-field inflation in supergravity: rare and tachyonic?

Cosmology from Home 2022

Vikas Aragam

aragam@utexas.edu

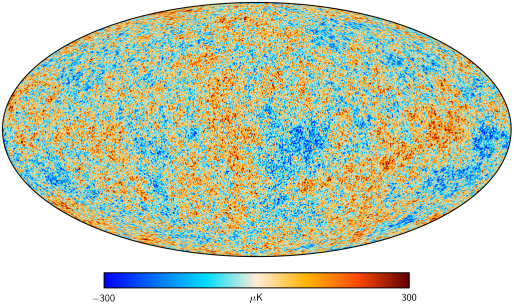
The University of Texas at Austin

July 2022

Acknowledgments

- Based on "Rapid-turn inflation in supergravity is rare and tachyonic," JCAP 03 (2022) 03, 002, [2110.05516]
- Co-authors:
  - Roberta Chiovoloni (Swansea)
  - Sonia Paban (UT Austin)
  - Robert Rosati (UT Austin, NASA Marshall)
  - Ivonne Zavala (Swansea)

Inflation



ESA and the Planck Collaboration

- Observations: CMB+LSS large-scale uniformity, anisotropies
- Single-field, slow-roll solution: nearly flat potential

Multiple fields: why care?

- Single field models... pretty good
  - Can have good phenomenology
  - Some UV motivations (natural inflation, DBI, etc.)
- UV theories: lots of scalars
  - Supergravity, string compactifications
  - Steep potentials, tachyonic directions $\Rightarrow$ dS swampland conjecture [Obied+ 1806.08362; Garg+ 1807.05193]
  - Often light $\Rightarrow$ can't integrate out
- How to formalize this?

Multiple-field inflation

- N real scalars $\phi^a = \phi^a(t)$ in Einstein frame:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} G_{ab}(\phi) g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right]$$

- Equations of motion:

$$3M_P^2 H^2 = \frac{1}{2} G_{ab} \dot{\phi}^a \dot{\phi}^b + V(\phi)$$
$$D_t \dot{\phi}^a + 3H\dot{\phi}^a + G^{ab} V_{,b} = 0$$

  - Here, $D_t \dot{\phi}^a \equiv \ddot{\phi}^a + \Gamma^a_{bc} \dot{\phi}^b \dot{\phi}^c$

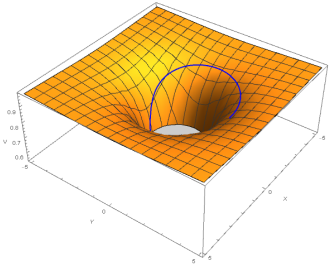
Multi-field trajectories

- Solution to classical EoMs is trajectory in field space
  - Unit tangent vector:

$$T^a \equiv \frac{\dot{\phi}^a}{|\dot{\phi}^a|}$$

  - Slow-roll along T^a:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_P^2}{2}\left(\frac{T^a V_{,a}}{V}\right)^2 \ll 1$$



Multi-field trajectories

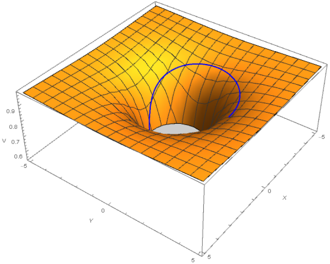
- Solution to classical EoMs is trajectory in field space
  - Unit normal vector:

$$N^a \parallel -D_t T^a$$

  - Turning rate:

$$\Omega \equiv |D_t T^a|$$

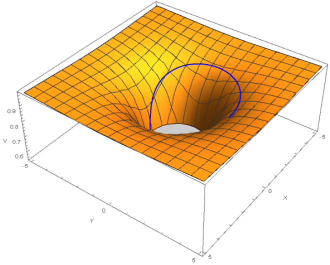
  - High turning: $\frac{\Omega}{H} \gtrsim 1$
  - Non-geodesic motion!



Allows for inflation in steep potential gradients often seen in UV theories! [Achúcarro+ 1807.04390]

Multi-field turning: significance

- Ω couples adiabatic and entropic modes
- Unique signatures in primordial power spectrum
  - Primordial black holes [Fumagalli+ 2004.08369; Braglia+ 2005.02895; Palma+ 2004.06106; Anguelova 2012.03705]
  - Stochastic gravitational waves [Fumagalli+ 2012.02761; Domènech 2109.01398]



Turning is indicative of genuinely multi-field dynamics and phenomenology!

Primer: inflation with two fields

- Field basis: $\phi^a = \{r, \theta\}$
  - $\theta \leftrightarrow$ axion, $r \leftrightarrow$ saxion
- Field-space metric $G_{ab} = f^2(r)\delta_{ab}$
  - Equivalent to other forms under coordinate transformation
- Equations of motion become:

$$r'' + (3-\epsilon)\, r' - \frac{f_{,r}}{f}\left(\theta'^2 - r'^2\right) + \frac{V_{,r}}{H^2 f^2} = 0,$$

$$\theta'' + (3-\epsilon)\,\theta' + 2\frac{f_{,r}}{f}\theta' r' + \frac{V_{,\theta}}{H^2 f^2} = 0$$

  - $(') \equiv \frac{d}{dN} = \frac{1}{H}\frac{d}{dt}$

Inflationary solutions

- $\theta = \text{constant}$, $r = r(t) \Rightarrow$ geodesic motion
  - *Saxion inflation* is effectively single-field motion!
- Genuine multi-field dynamics: motion in the axion direction
- For simplicity, take $r = \text{constant}$, $\theta = \theta(t) \Rightarrow$ *axion inflation*
  - Slow-roll in axion direction, $\theta'' \ll \theta'$:

$$-\frac{f_{,r}}{f}\theta'^2 + \frac{V_{,r}}{H^2 f^2} = 0,$$
$$(3-\epsilon)\,\theta' + \frac{V_{,\theta}}{H^2 f^2} = 0$$

  - Encompasses angular/orbital/sidetracked/hyperinflation [Christodoulidis+ 1803.09841; Achúcarro+ 1907.02020; Garcia-Saenz+ 1804.11279; Brown 1705.03023]
  - Allow $r = r(t)$ in numerical scans

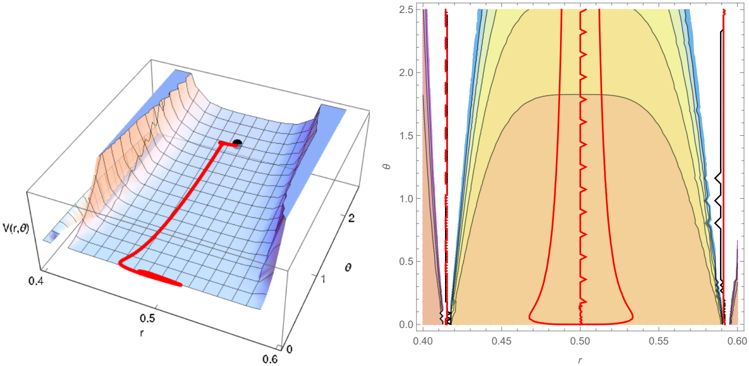
Inflationary solutions

- Equations of motion yield consistency condition:

$$\theta'^2 = \frac{V_\theta^2}{(3-\epsilon)^2 H^4 f^4} = \frac{V_r}{H^2 f\, f_r}$$
$$\Rightarrow \frac{V_\theta^2}{(3-\epsilon) V\, V_r} = \frac{f^3}{f_r}$$

  - Depends only on potential and field-space geometry
  - Independent of initial conditions!
  - Restricts regions of field space admitting both slow-roll *and* high-turn trajectories

Inflationary solutions: consistency condition



[Ellis+ 1405.0271; VA+ 2110.05516]

Turning in multi-field axion inflation

- Under these assumptions:

$$T^a = (0, 1/f)\,, \qquad \text{s.t.} \qquad T^a T_a = N^a N_a = 1$$
$$N^a = (1/f, 0) \qquad\qquad\qquad T^a N_a = 0$$

- Slow-roll and turning rate parameters:

$$\epsilon = \frac{M_P^2}{2}\frac{1}{f^2}\left(\frac{V_{,\theta}}{V}\right)^2$$
$$\frac{\Omega}{H} = \left|M_P\sqrt{2\epsilon}\frac{f_{,r}}{f^2}\right|$$

  - Can be computed *globally* across entire field space for any model!

Multi-field masses

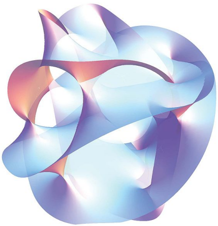
- Mass matrix:

$$\mathbb{M} = \frac{M_P^2}{V} \begin{pmatrix} V_{;TT} & V_{;TN} \\ V_{;NT} & V_{;NN} \end{pmatrix}$$

  - Slow-roll: $V_{;TT} \simeq \Omega^2 + \mathcal{O}(\epsilon), \quad V_{;TN} \simeq 3H\Omega + \mathcal{O}(\epsilon)$
  - Eigenvalues $\leftrightarrow$ field masses
- In slow-roll, all eigenvalues positive when: $V_{;NN} > 9H^2$
  - $V_{;NN} < 9H^2$: at least one tachyonic mass!
    - Exponential growth in primordial power spectrum

Inflation in supergravity

- Now we know what multi-field inflation looks like, how does this fit into fundamental physics?
- Supergravity: low-energy effective description of various string theories
    - Many (complex) scalars from string compactifications



Supergravity: basic ingredients

- Action of $d = 4$ supergravity with complex superfields Φ^i:

$$S = \int d^4x\sqrt{-g}\left[M_P^2\frac{R}{2} - K_{i\bar{\jmath}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{\jmath}} - V(\Phi^k, \bar{\Phi}^{\bar{k}})\right]$$

  - K = Kahler potential
  - $K_{i\bar{\jmath}} = \partial_i\partial_{\bar{\jmath}}K$ = Kahler metric on complex field space

- Scalar potential:

$$V(\Phi^k, \bar{\Phi}^{\bar{k}}) = e^{K/M_P^2}\left(K^{i\bar{\jmath}}D_iW\, D_{\bar{\jmath}}\bar{W} - 3|W|^2M_P^{-2}\right)$$

  - $W = W(\Phi^i)$ = superpotential
  - $D_iW \equiv W_{,i} + K_{,i}W\, M_P^{-2}$ = Kahler covariant derivative

Supergravity: dynamics

- For the inflaton superfield Φ, decompose $\Phi = r + i\theta$
- Enforce multi-field axion inflation: $r' \approx 0$, $\theta = \theta(t)$:

$$\epsilon = \frac{M_P^2}{2}\frac{1}{f^2}\left(\frac{V_{,\theta}}{V}\right)^2 = \frac{1}{2}\left|\frac{1}{2K_{\Phi\bar{\Phi}}}\right|\left(\frac{V_\Phi - V_{\bar{\Phi}}}{V}\right)^2$$

$$\frac{\Omega}{H} = \left|M_P\sqrt{2\epsilon}\frac{f_{,r}}{f^2}\right| = \sqrt{2\epsilon_T}\left|\frac{\left(K_{\Phi\bar{\Phi},\Phi} + K_{\Phi\bar{\Phi},\bar{\Phi}}\right)}{(2K_{\Phi\bar{\Phi}})^{3/2}}\right|$$

No-scale inspired model

- Given by:

$$K = -3\,\alpha \log[(\Phi + \bar{\Phi})/M_P] + S\bar{S}, \quad W = S\,F(\Phi)$$

  - Superfields: Φ = inflaton, S = goldstino
  - $S \to 0$ in the potential via nilpotent condition: $S^2 = 0$ [Ferrara+ 1408.4096]
  - Many forms of $F(\Phi)$ admit inflation
  - $\alpha = 1$: no-scale supergravity

- Field space curvature: $\mathcal{R} = -\frac{4}{3\alpha}$

No-scale inspired model

- $F(\Phi) = p_0 + p_1\Phi, \quad \Phi = r + i\theta$:

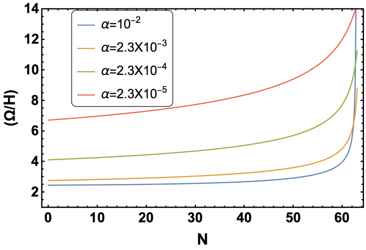
$$V = M_P^{3\alpha}\,\frac{\left[p_1^2\theta^2 + (p_0 + p_1\,r)^2\right]}{8^{\alpha}\,r^{3\alpha}}, \quad G_{ab} = \frac{3\,\alpha\,M_P^2}{2r^2}\delta_{ab}$$

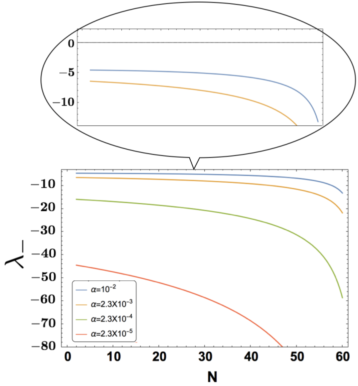
- Turning rate: $\frac{\Omega}{H} = \frac{2\sqrt{\epsilon}}{\sqrt{3\alpha}}$
  - High turning rate at small $\alpha \leftrightarrow$ large field space curvature
- Mass matrix element for $\alpha \ll 1$, $\epsilon \ll 1$:

$$V_{;NN} \simeq -\frac{9\,r}{\theta}\frac{\Omega H}{\epsilon} < 0$$

  - $V_{;NN} < 9H^2$: tachyonic mass along trajectory!

No-scale inspired model





[VA+ 2110.05516]

Other models

- Same analytical analysis for EGNO model: [Ellis+ 1405.0271]

$$K = -3\alpha \log \left[\Phi + \bar{\Phi} - c \left[(\Phi + \bar{\Phi} - 1) \cos(p) - i(\Phi - \bar{\Phi}) \sin(p) \right]^4 \right] + \frac{S\bar{S}}{(\Phi + \bar{\Phi})^3},$$

$$W = SF(\Phi), \quad F(\Phi) = \sqrt{\frac{3}{4}} \frac{M}{a} (\Phi - a)$$

- (Almost) same story: long-lasting inflation with $\epsilon \ll 1$, $\Omega/H \gtrsim 1$, $\alpha \leq 1$
  - Large (positive) curvature + tachyonic mass along trajectory

Other models

- Numerically scanned field and parameter space of 13 other models (Robert Rosati 2020, https://github.com/robertfeldt/BlackBoxOptim.jl)
  - Evolve equations of motion while minimizing cost function:

$$\text{cost}(\phi^a, \dot{\phi}^a, \text{params}) \supset \frac{1}{N_{end}} + A\epsilon + \frac{B}{\Omega_{end}}$$

  - Allowed for saxion evolution, $r = r(t)$; no impact on results
- None supported long-lasting $\Omega/H \gtrsim 1$
  - Some had effectively single-field dynamics, with $\epsilon \ll 1$ over 60+ e-folds

Summary

- Multi-field inflation well motivated from fundamental physics
- Multi-field dynamics and phenomenology $\leftrightarrow$ high turning rates
- Analytically and numerically, supergravity rarely yields slow-roll, high-turning inflation
- When it does happen:
  - Tachyonic masses $\Rightarrow$ interesting phenomenology, theoretical motivations
  - Price to pay: large field-space curvature $\Rightarrow$ difficult in string theory
- Thank you!