A theory-independent framework for testing gravity on all scales in cosmology

Cosmology from Home 2022

Theo Anton, Queen Mary University of London

Based on: Anton & Clifton (arXiv:2111.10860), Clifton & Sanghai (arXiv:1803.01157), Sanghai & Clifton (arXiv:1610.08039)

Why test gravity?

- The standard ΛCDM model of cosmology is built on the General Theory of Relativity (GR).
- This model has been **very successful**. However…

i. GR requires dark matter and dark energy in order to account for cosmological observations, and the fundamental nature of both remains unknown.

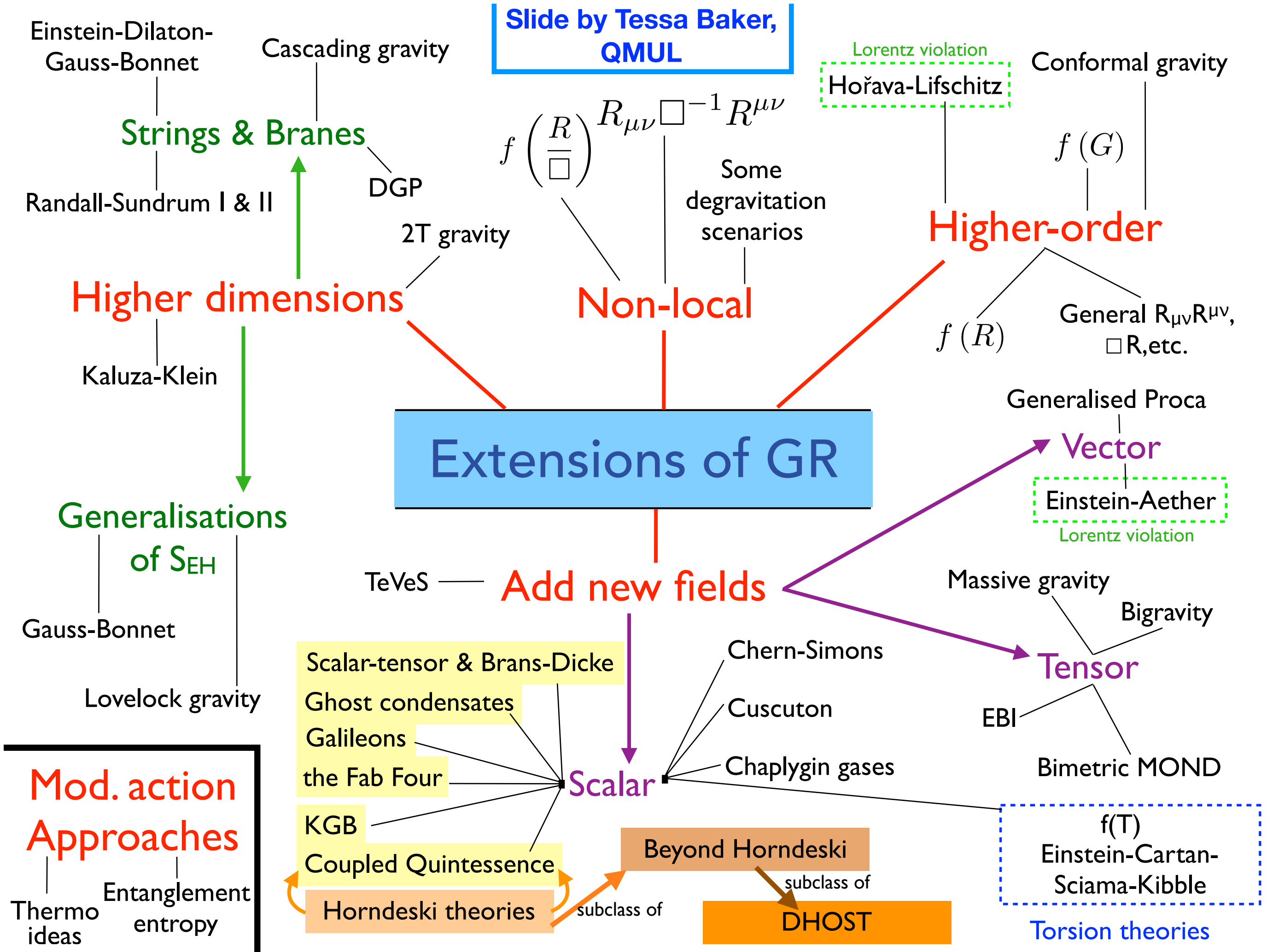
ii. Aspects of cosmological data, such as the Hubble tension, hint at potential shortcomings of ΛCDM.

iii. There remains an ultimate need for a quantum theory of gravity.

These considerations motivate us to consider modified theories of gravity.

- The landscape of modified gravity is very vast.
- It would be useful to **constrain large regions of the landscape** at once, rather than just test individual theories.

Schematic by Tessa Baker (QMUL).



How to test gravity?

Small scales

- Solar System tests
- Laboratory tests
- Binary pulsars
- Gravitational waves

Large scales

- Weak lensing
- Galaxy clustering
- Background expansion
- CMB (ISW effect)

The language used to study gravity in these two disparate regimes can be very different.

How can we make the tests conducted in such different regimes talk to each other?

Theory-independent frameworks

Aim

Provide a framework for **consistent tests of gravity** over a wide range of length scales.

Requirements

- Should apply in a **wide range of physical settings**.
- Should allow us to constrain the overall theory landscape, rather than just one particular model.
- Should be relatively insensitive to finer theoretical details.
- Should be simple (more subjective...)!

N.B. Some theory-independent frameworks for cosmology do already exist, but have some drawbacks.

The parameterised post-Newtonian (PPN) framework

- Originally built to study astrophysical tests of gravity (Will & Nordtvedt 1972, Will 1993 etc.)
- Has placed **very tight constraints on modified gravity** in the Solar System, encoded in a small, physically motivated set of parameters.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

e.g. $$\nabla^2 h_{ij} = \left(-8\pi\gamma\rho + 2\gamma_c\right) \delta_{ij}$$

- Because we build our cosmology from PPN, **results in the cosmological regime will automatically be able to talk to results on small scales**.
- PPN is valid for arbitrarily non-linear densities.

Why PPN needs adapting

- The classic PPN formalism is unsuitable for cosmology, because it assumes that

i. spacetime is asymptotically flat.

ii. All velocity scales are small compared to the speed of light.

iii. the PPN parameters are constant in time.

- **None of these things are true** on cosmological length or time scales.

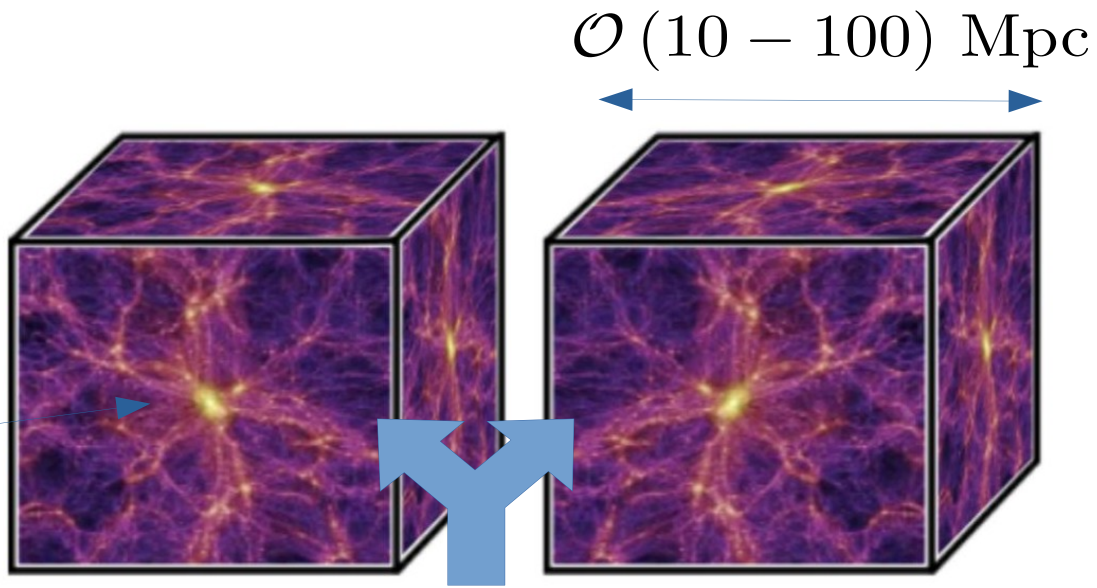
Building a cosmological model

Start from the description we have of gravity on small scales. Then **stitch many small regions together** to construct a cosmological model.

We use the PPN formalism to describe small-scale gravity.

The matter content within all the small regions gives rise to global expansion at a rate given by generalised Friedmann equations.

Inhomogeneous matter



Stitch using Israel junction conditions. Repeat for many regions

Starting from the small scales

Small-scale perturbations to the Minkowski metric are related to post-Newtonian potentials by the PPN parameters.

Transform metric to FRW + Poisson gauge perturbations

$$\eta_{\mu\nu} + \{h_{00}, h_{ij}, h_{0i}\} \longrightarrow \bar{g}^{\mathrm{FRW}}_{\mu\nu} + \{\Phi, \Psi, B_i\}$$

Construct cosmological model by stitching many such small regions together

Generalised Friedmann equations

$$\mathcal{H}^2 = \frac{8\pi}{3}\gamma\bar{\rho}\,a^2 - \frac{2\gamma_c a^2}{3},$$

$$\mathcal{H}' = -\frac{4\pi}{3}\alpha\bar{\rho}\,a^2 + \frac{\alpha_c a^2}{3}.$$

Parameterised equations for small-scale perturbations, e.g.

$$\nabla^2\Psi = -4\pi\gamma\,\delta\rho\,a^2$$

N.B. unlike in classic PPN, ɣ may be a function of time.

Cosmological gravity explicitly related to post-Newtonian gravity via the PPN parameters

Accessing the large scales

- On scales larger than the horizon, an FRW metric with linear perturbations is equivalent to the metric of another exact FRW universe with perturbed coordinates (Bertschinger 2006).
- This observation gives generalised equations for large-scale perturbations, e.g.

$$\Psi'' + \mathcal{H}\Psi' + \mathcal{H}\Phi' + 2\mathcal{H}'\Phi = \frac{\delta\rho}{3\bar{\rho}}\left(\frac{2\mathcal{H}\mathcal{H}' - \mathcal{H}''}{\mathcal{H}}\right)$$

$$= -\frac{4\pi\,\delta\rho\,a^2}{3}\left(\alpha - \frac{1}{3}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln a} + \frac{1}{12\pi\bar{\rho}}\frac{\mathrm{d}\alpha_c}{\mathrm{d}\ln a}\right)$$

All theory-independent

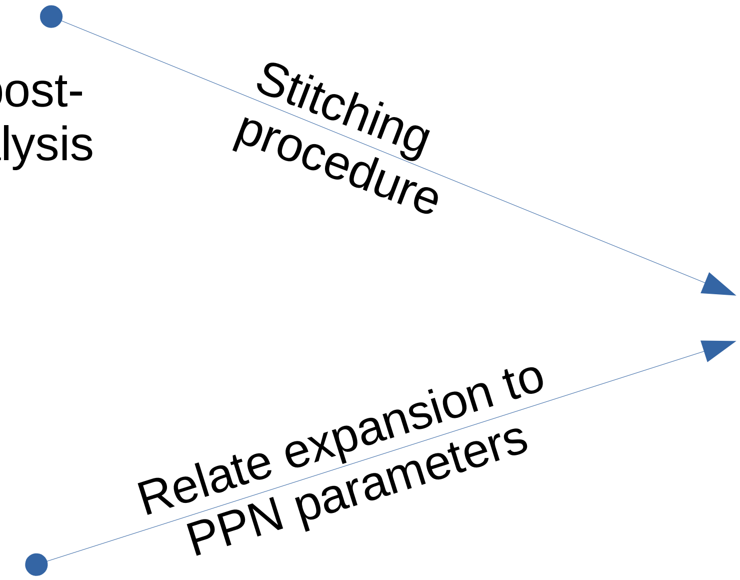
Theory of gravity only enters here

- Perturbation evolution equations directly specified by the background equations. These are already obtained in our theory-independent framework in terms of PPN parameters.

Putting it all together

Small scales: post-Newtonian analysis

Large scales: separate-universe analysis



- Generalised Friedmann equations
- Perturbation equations **valid on all scales**

Parametrisation functions have small and large-scale limits given precisely in terms of (observationally accessible) PPN parameters

Cosmological expansion

$$\mathcal{H}^2 = \frac{8\pi}{3}\gamma\bar{\rho}\,a^2 - \frac{2\gamma_c a^2}{3},$$

$$\mathcal{H}' = -\frac{4\pi}{3}\alpha\bar{\rho}\,a^2 + \frac{\alpha_c a^2}{3}.$$

Perturbation equations can all be related to PPN parameters in both the small and large-scale limits

Smoothly join the limits together via parametrisation functions to get equations valid on all scales

Perturbations

$$\frac{1}{3}\nabla^2\Psi - \mathcal{H}^2\Phi - \mathcal{H}\Psi' = -\frac{4\pi}{3}\mu\,\delta\rho\,a^2.$$

$$\frac{1}{3}\nabla^2\Phi + 2\mathcal{H}'\Phi + \mathcal{H}\Phi' + \Psi'' + \mathcal{H}\Psi' = -\frac{4\pi}{3}\nu\,\delta\rho\,a^2.$$

$$\Psi'_{,i} + \mathcal{H}\Phi_{,i} = 4\pi\mu\,[\rho v_i]^{\mathrm{S}}\,a^2 + \mathcal{G}\mathcal{H}\Psi_{,i}.$$

$$2\left(\mathcal{H}' - \mathcal{H}^2\right)B_i + \frac{1}{2}\nabla^2 B_i = 8\pi\left(\mu + \mathcal{Q}\right)[\rho v_i]^{\mathrm{V}}\,a^2 + \alpha_1\pi\,[\rho w_i]^{\mathrm{V}}\,a^2.$$

Modified-gravity “momentum” parameterisation functions (0 in GR)

Preferred-frame effect term (0 in GR)

Parametrisation functions transition smoothly between **limits given by PPN parameters**

$$\mu = \begin{cases} \gamma & k \longrightarrow \infty \\ \gamma - \frac{1}{3}\frac{\mathrm{d}\gamma}{\mathrm{d}\ln a} + \frac{1}{12\pi\bar{\rho}}\frac{\mathrm{d}\gamma_c}{\mathrm{d}\ln a} & k \longrightarrow 0 \end{cases} \qquad \mathcal{G} = \begin{cases} \frac{\mathrm{d}\ln\gamma}{\mathrm{d}\ln a} + \frac{\alpha - \gamma}{\gamma} & k \longrightarrow \infty \\ 0 & k \longrightarrow 0 \end{cases}$$

$$\nu = \begin{cases} \alpha & k \longrightarrow \infty \\ \alpha - \frac{1}{3}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln a} + \frac{1}{12\pi\bar{\rho}}\frac{\mathrm{d}\alpha_c}{\mathrm{d}\ln a} & k \longrightarrow 0 \end{cases} \qquad \mathcal{Q} = \begin{cases} \frac{\alpha - \gamma}{2} + \frac{\alpha_1}{8} & k \longrightarrow \infty \\ 0 & k \longrightarrow 0 \end{cases}$$

Perturbations

$$\frac{1}{3}\nabla^2\Psi - \mathcal{H}^2\Phi - \mathcal{H}\Psi' = -\frac{4\pi}{3}\mu\,\delta\rho\,a^2.$$

$$\frac{1}{3}\nabla^2\Phi + 2\mathcal{H}'\Phi + \mathcal{H}\Phi' + \Psi'' + \mathcal{H}\Psi' = -\frac{4\pi}{3}\nu\,\delta\rho\,a^2.$$

$$\Psi'_{,i} + \mathcal{H}\Phi_{,i} = 4\pi\mu\left[\rho v_i\right]^{\mathrm{S}} a^2 + \mathcal{G}\mathcal{H}\Psi_{,i}.$$

$$2\left(\mathcal{H}' - \mathcal{H}^2\right)B_i + \frac{1}{2}\nabla^2 B_i = 8\pi\left(\mu + \mathcal{Q}\right)\left[\rho v_i\right]^{\mathrm{V}} a^2 + \alpha_1\pi\left[\rho w_i\right]^{\mathrm{V}} a^2.$$

A concrete example

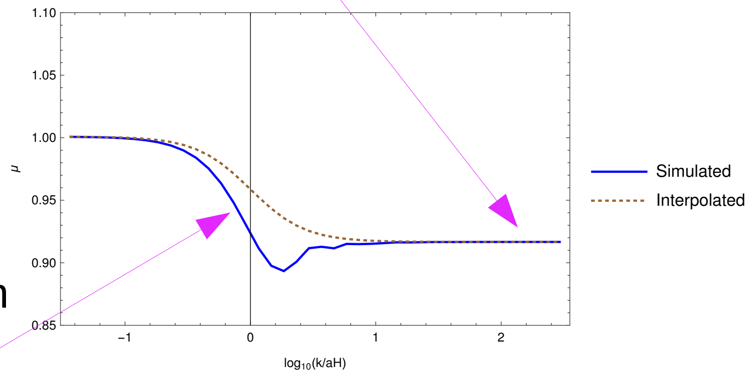
PPNC equations explicitly calculated for

- Brans-Dicke theory
- Vector-tensor theories
- Quintessence DE models

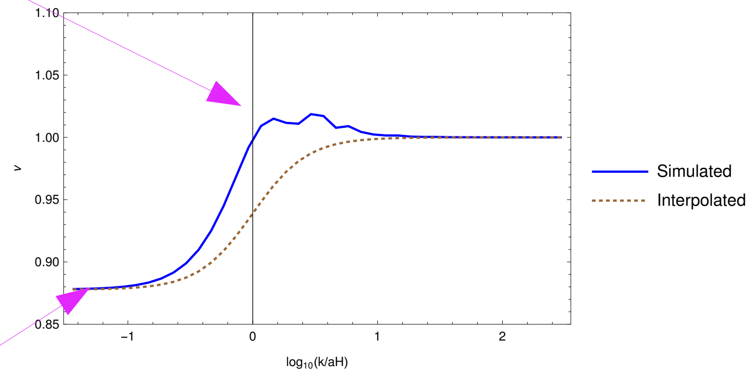
see arXiv:2111.10860

- Testing the validity of the formalism via simulations (Daniel Thomas, paper in prep).
- Plots shown: Brans-Dicke theory evolved to the present day, with a constant potential and ω = 10.

PPNC small-scale limit satisfied



Simple smooth interpolation breaks down around horizon where scalar field oscillations are significant



PPNC large-scale limit satisfied

Plots by Daniel Thomas

Summary

- There is a huge landscape of modified gravity theories one might wish to test.
- Theory-independent frameworks allow us to do this systematically.
- **We have developed such a framework** that is suitable for use in cosmological tests of gravity.
- It is self-consistent, valid in the non-linear regime, and tied to PPN constraints.
- Future/possible extensions:
  - Incorporation of theories with screening mechanisms
  - PPNC equation for the "shear" $\Phi - \Psi$
  - Further study of the PPNC interpolation procedure
- Thanks for listening! Get in touch: Slack/t.j.anton@qmul.ac.uk/@TheoJAnton on Twitter

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Technical details: Post-Newtonian formalism

$$\frac{\frac{\partial}{\partial t}}{\frac{\partial}{\partial x}} \sim |v| \ll 1$$

In a region far smaller than the horizon, one can safely apply post-Newtonian expansion about Minkowski spacetime

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)\delta_{ij}dx^i dx^j + 2B_i dt dx^i$$

$$\Phi \sim \Psi \sim v^2, \ |B_i| \sim v^3$$

$$\nabla^2\Phi = -4\pi\alpha\,\rho + \alpha_c \, ; \, \nabla^2\Psi = -4\pi\gamma\,\rho + \gamma_c$$

$$\nabla^2 B_i = 8\pi\left[\alpha + \gamma + \frac{1}{4}\alpha_1\right]\rho\, v_i - \left[\alpha + \alpha_2 - \zeta_1 + 2\xi\right]\dot{U}_{,i} + 2\pi\alpha_1 w_i\,\rho + 2\alpha_2 w^j U_{,ij}$$

Technical details: stitching procedure

Consider the timelike boundaries of each region. The intrinsic and extrinsic curvatures of those boundary surfaces must satisfy the Israel junction conditions:

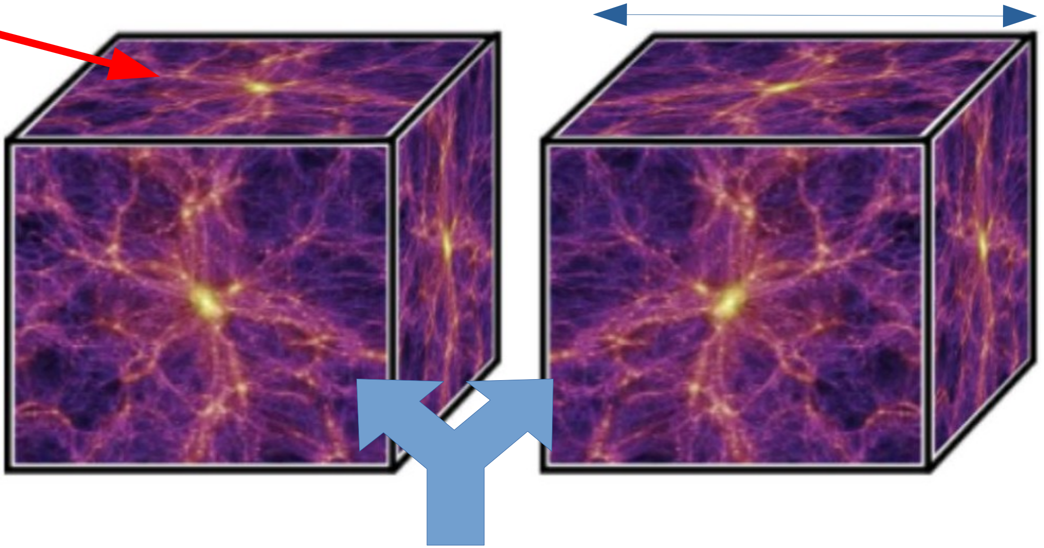
$$[h_{ij}]^{+}_{-} = 0$$

$$[K_{ij}]^{+}_{-} = 0$$

Can be related to the post-Newtonian potentials, and therefore to the matter content

See arXiv:1503.08747 for rigorous details

$$\mathcal{O}\,(10 - 100)\ \mathrm{Mpc}$$



Stitch using Israel junction conditions. Repeat for many regions

Technical details: super-horizon scales

Approach used by Bertschinger (2006), arXiv:astro-ph/0604485

A universe consisting of an FRW background + small super-horizon (i.e. time-dependent only) perturbations is equivalent to a different exact FRW universe with perturbed coordinates.

$$\tau \longrightarrow \tau + A(\tau);\; x^i \longrightarrow x^i \left(1 + \beta(\tau)\right)$$

Compare the line elements of the two “universes” to relate A and β to FRW metric perturbations

$$\Rightarrow\; \Phi = -A' - \mathcal{H}A;\; \Psi = \beta + \mathcal{H}A$$

Perturb the density in the same way to find $\delta\rho = -3\Psi\bar{\rho}$, and matter 3-velocity $v_i = -\dfrac{\delta\rho_{,i}}{\bar{\rho}'}$

This gives Hamiltonian constraint, Raychaudhuri and momentum constraint equations for the scalar perturbations (similar treatment gives a momentum constraint for the divergenceless vector perturbation)

e.g. Raychaudhuri equation $\Psi^{''} + \mathcal{H}\Psi' + \mathcal{H}\Phi' + 2\mathcal{H}'\Phi = \frac{\delta\rho}{3\bar{\rho}}\left(\frac{2\mathcal{H}\mathcal{H}' - \mathcal{H}^{''}}{\mathcal{H}}\right)$

$$= -\frac{4\pi\,\delta\rho\,a^2}{3}\left(\alpha - \frac{1}{3}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln a} + \frac{1}{12\pi\bar{\rho}}\frac{\mathrm{d}\alpha_c}{\mathrm{d}\ln a}\right)$$

Technical details: from perturbed Minkowski to perturbed FRW

$$t = \hat{t} + \frac{a^2 H}{2}\hat{r}^2 + T(\hat{t}, \hat{\mathbf{x}}) + \mathcal{O}(v^5),\ T \sim v^3$$

$$x^i = a\,\hat{x}^i \left[1 + \frac{a^2 H^2}{4}\hat{r}^2\right] + \mathcal{O}(v^4)$$

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)\delta_{ij}dx^i dx^j + 2B_i dt dx^i = -(1-2\hat{\Phi})d\hat{t}^2 + a^2(\hat{t})(1+2\hat{\Psi})\delta_{ij}d\hat{x}^i d\hat{x}^j + 2a(\hat{t})\hat{B}_i d\hat{t} d\hat{x}^i$$

$$\Phi = \Phi + \frac{\ddot{a}\,a}{2}\hat{r}^2$$

$$\Psi = \hat{\Psi} - \frac{\dot{a}^2}{4}\hat{r}^2$$

$$B_i = \hat{B}_i - 2\dot{a}\,\hat{x}^j \delta_{ij}\left(\hat{\Phi} + \hat{\Psi}\right) - a\,\dot{a}\,\ddot{a}\,\hat{r}^2 \hat{x}^j \delta_{ij} + \frac{1}{a}T_{,i}$$

where T is chosen such that the Poisson gauge condition is satisfied: $\hat{B}_{i,i} = 0$

Technical details: Brans-Dicke example

$$I = \frac{1}{16\pi}\int \mathrm{d}^4x\,\sqrt{-g}\left[\varphi R - \frac{\omega}{\varphi}\nabla^{\mu}\varphi\nabla_{\mu}\varphi\right] + I_{\mathrm{m}}$$

Vary action → field equations → take post-Newtonian limit → obtain PPN parameters (and their PPNC extensions)

$$\alpha = \frac{4+2\omega}{3+2\omega}\,\frac{1}{\bar{\varphi}}\,, \quad \gamma = \frac{2+2\omega}{3+2\omega}\,\frac{1}{\bar{\varphi}}\,, \quad \alpha_1 = \alpha_2 = \xi = \zeta_1 = 0$$

$$\alpha_c = \frac{1}{a^2}\left[-\frac{\bar{\varphi}''}{\bar{\varphi}} + \mathcal{H}\frac{\bar{\varphi}'}{\bar{\varphi}} - \omega\left(\frac{\bar{\varphi}'}{\bar{\varphi}}\right)^2\right]\,, \quad \gamma_c = -\frac{1}{2a^2}\left[\frac{\bar{\varphi}''}{\bar{\varphi}} - \mathcal{H}\frac{\bar{\varphi}'}{\bar{\varphi}} + \frac{\omega}{2}\left(\frac{\bar{\varphi}'}{\bar{\varphi}}\right)^2\right]\,.$$

Plug these into the PPNC equations to get the Friedmann equations, and governing equations for the scalar and vector metric perturbations.

“Sanity check”: linearise the equations. Can then verify that the linearised equations are identical to those obtained using the full cosmological perturbation theory machinery. But we know the PPNC equations are also valid on non-linear scales.