Dynamical Stability of Bouncing Cosmology in Extended Gravity

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Cosmology From Home - 2022

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Outline of Presentation

- Introduction
- Basic Formalism.
- Dynamical Parameters.
- Energy Conditions.
- Cosmographic Parameter.
- Stability of The Model.
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Introduction

- Now, It is well known fact (due to Edwin Hubble) and observational study [1,2,3] that the Universe is expanding. The expansion rate is determined by the time evolution of the scale factor.
- Kretschmann Scalar $\left(k = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} \propto \frac{1}{a^4}\right)$
- A quantum theory of gravity may avoid such a initial cosmological singularity. but we don't know what is the correct quantum theory of gravity.
- However, In the absence of fully accepted quantum gravity, bounce cosmology is the most promising one that allows a non-singular Universe.
- At the time of bounce, the following conditions has to be hold

$$a \neq 0 \qquad \dot{a} = 0 \qquad \ddot{a} > 0$$

- Among various bounce models, the Matter Bounce Scenario (MBS) is the most popular one.

[1] A.G. Riess et al., *The Astronomical Journal*, **116**, 1009 (1998).
[2] S. Perlmutter, M. S. Turner, M. White *Phys. Rev. Lett.*, **83**, 670 (1999).
[3] M. Tegmark et al., *Phys. Rev. D*, **69**, 103501 (2004).

Introduction

- However, MBS suffers from the following problems.

i) Anisotropic issue (BKL instability) : During the cotracting universe the anisotropic energy density grows ($\propto \frac{1}{a^6}$) and it lead to an instability in the background evolution.

ii) Explaining the dark energy era : The MBS leads to a matter dominated era at late epoch, that means the deceleration epoch.

- Thus we have considered a singular free scale factor $a(t) = \left(\frac{\alpha}{\chi} + t^2\right)^{\frac{1}{2\chi}}$ [4]. We will analyse the nature of dynamical variables in the extended theory of gravity.

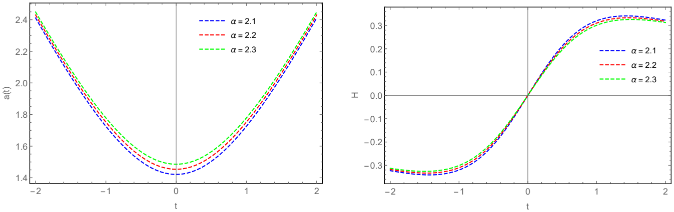


Figure: Scale Factor (left panel) and Hubble Parameter (right panel) vs. cosmic time t.

[4] Abdussattar, S.R. Prajapati, *Astrophys Space Sci.* **331**, 657–663 (2011).

Basic Formalism

The action of $f(R,T)$ gravity,

$$S = \int \left[\frac{f(R,T)}{16\pi} + \mathcal{L}_m\right]\sqrt{-g}d^4x, \tag{1}$$

The field equation can be defined as

$$f_R(R)R_{ij} - \frac{1}{2}f(R)g_{ij} - (\nabla_i\nabla_j - g_{ij})f_R(R) = 8\pi T_{ij} + f_T(T)T_{ij} + [f_T(T)p + \frac{1}{2}f(T)]g_{ij} \tag{2}$$

There are three different forms of $f(R,T)$ gravity as
I) $f(R,T) = R + 2f(T)$,
II) $f(R,T) = f_1(R) + f_2(T)$
III) $f(R,T) = f_1(R) + f_2(R)f_3(T)$

We have considered, $f(R,T) = R + 2f(T)$[5], such that $f(R,T) = R + 2\beta T + 2\Lambda_0$[6].
The space time is considered as,

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \tag{3}$$

[5] T. Harko, F.S.N. Lobo, S. Nojiri, S.D. Odintsov, *Phys. Rev. D* **84**, 024020 (2011).
[6] B. Mishra, S. Tarai, S.K. Tripathy, *Mod. Phys. Lett. A*, **33**, 1850170 (2018).

We have assumed the matter field as the perfect fluid,

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} \tag{4}$$

The field equations of $f(R,T)$ gravity in Hubble term H can be can be obtained as,

$$\begin{aligned} 2\dot{H} + 3H^2 &= -\eta p + \beta\rho + \Lambda_0 \qquad (5) \\ 3H^2 &= \eta\rho - \beta p + \Lambda_0 \qquad (6) \end{aligned}$$

where $\eta = 8\pi + 3\beta$.

$$\begin{aligned} p &= -\frac{1}{(\eta^2-\beta^2)}\left[2\eta\dot{H} + 3(\eta-\beta)H^2 - (\eta-\beta)\Lambda_0\right] \qquad (7) \\ \rho &= \frac{1}{(\eta^2-\beta^2)}\left[-2\beta\dot{H} + 3(\eta-\beta)H^2 - (\eta-\beta)\Lambda_0\right] \qquad (8) \\ \omega &= -1 + \left[\frac{2(\eta+\beta)\dot{H}}{2\beta\dot{H} - 3(\eta-\beta)H^2 + (\eta-\beta)\Lambda_0}\right] \qquad (9) \end{aligned}$$

Dynamical Parameters

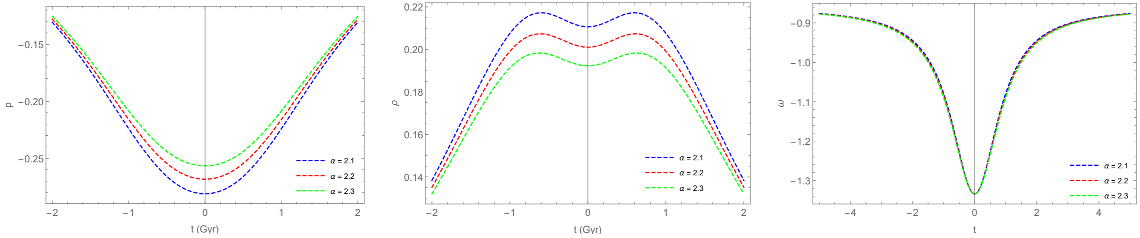


Figure: Variation of pressure energy density and EoS parameter in cosmic time t with varying α.

Energy Conditions

The energy conditions[7] can be defined as,

$$\rho + p = -\frac{2}{(\eta-\beta)}\left[\frac{\alpha-\chi t^2}{(\alpha+\chi t^2)^2}\right] \tag{10}$$

$$\rho + 3p = \frac{1}{(\eta^2-\beta^2)}\left[\frac{(-2\beta-6\eta)(\alpha-\chi^2 t)-6(\eta-\beta)t^2}{(\alpha+\chi t^2)^2}\right] + \frac{2\Lambda_0}{(\eta+\beta)} \tag{11}$$

$$\rho - p = \frac{1}{(\eta^2-\beta^2)}\left[\frac{(-2\beta+2\eta)(\alpha-\chi^2 t)+6(\eta-\beta)t^2}{(\alpha+\chi t^2)^2}\right] - \frac{2\Lambda_0}{(\eta+\beta)} \tag{12}$$

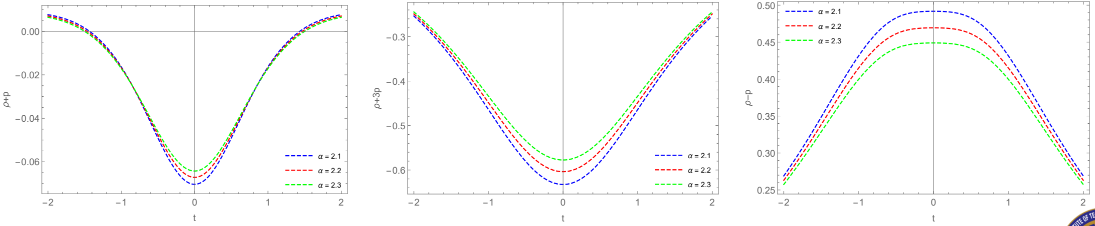


Figure: Variation of NEC, SEC, and DEC in cosmic time t with varying α.

[7] S. Hawking and G. F. R. Ellis, Cambridge University Press (1973).

Cosmographic Parameters

$$
\begin{aligned}
q &= -1 - \frac{\alpha}{t^2} + \chi, \\
j &= \frac{(2\chi - 1)[t^2(\chi - 1) - 3\alpha]}{t^2}, \\
s &= -\frac{(2\chi - 1)[3\alpha^2 + t^4(\chi - 1)(3\chi - 1) + 6\alpha t^2(1 - 3\chi)]}{t^4}
\end{aligned}
$$

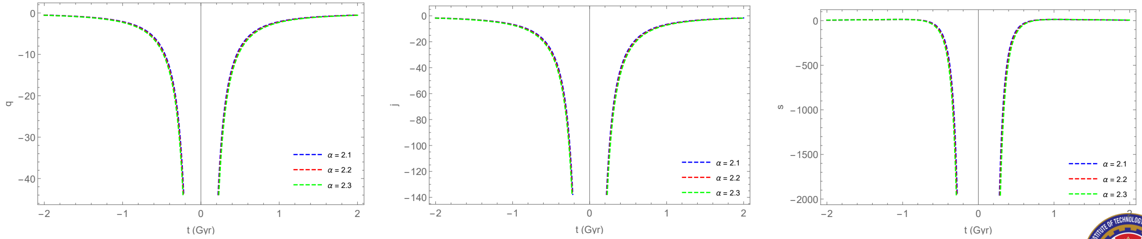


Figure: Variation of Deceleration Parameter, Jerk Parameter and Snap Parameter in cosmic time t with varying α.

Stability of The Model

We consider a pressure-less dust FRW background whose general solution may be $H(t_0) = H_b(t)$[8].

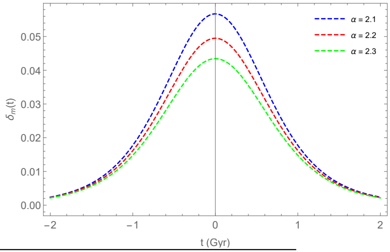
$$H(t) = H_b\left(1 + \delta(t)\right), \qquad \rho(t) = \rho_b\left(1 + \delta_m(t)\right), \tag{13}$$

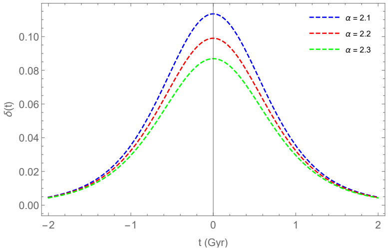
Using the perturbative approach in the equivalent FRW equation, we obtain

$$6H_b^2\delta(t) = \eta\rho_b\delta_m(t), \tag{14}$$

For a bouncing scenario as prescribed above, we obtain the geometrical perturbation and matter perturbation as

$$\delta(t) = C_k\left(\alpha + \chi t^2\right)^{-\frac{3\eta}{4\chi(\eta+\beta)}}, \qquad \delta_m(t) = \frac{2C_k\left(\eta+\beta\right)}{\eta}\left(\alpha + \chi t^2\right)^{-\frac{3\eta}{4\chi(\eta+\beta)}}. \tag{15}$$





[8] E.J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Modern Phys. D*, **15**, 1753 (2006).

Results and Discussion

- The bouncing scenario of the Universe has been analysed in the modified $f(R,T)$ gravity.
- From the behaviour of the scale factor and dynamical parameters, we have noticed the bounce at the epoch $t = 0$.
- At the bounce, since $\omega < -1$, the model is experiencing phantom behaviour, however when it evolves out, passes through ΛCDM line and subsequently to the quintessence phase.
- Another criteria for the bouncing scenario is the violation of NEC and we have obtained the violation of NEC in the range where the bounce occurs.
- The behaviour of the Hubble parameter and the other cosmographic parameters confirms the bouncing scenario of the Universe.
- On the stability feature of the model, we have observed that the model remains stable throughout the evolution that includes the bouncing epoch.
- However, it is scope to study the matter bounce scenario with late-time cosmic acceleration.


Thank you for your attention