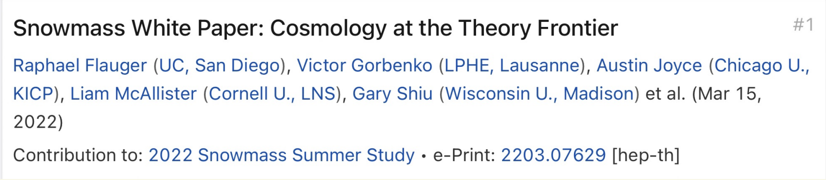
Formal Developments in Cosmology

Victor Gorbenko

EPFL

Outline:

- Motivation
- Inflation and de Sitter space
- Analytic techniques for cosmological correlators (bootstrap, unitarity...)
- Taming of the infrared divergences
- Non-perturbative gravitational effects



Snowmass White Paper: Cosmology at the Theory Frontier

Raphael Flauger (UC, San Diego), Victor Gorbenko (LPHE, Lausanne), Austin Joyce (Chicago U., KICP), Liam McAllister (Cornell U., LNS), Gary Shiu (Wisconsin U., Madison) et al. (Mar 15, 2022)

Contribution to: 2022 Snowmass Summer Study • e-Print: 2203.07629 [hep-th]

Motivation

- Why care about purely theoretical developments in cosmology?
- We know that our current theory [Standard model of particle physics + ΛCDM] is not complete both theoretically [UV completion of GR] and experimentally [Dark Matter].
- We do not know where and when new physics will show up.

- There are no experiments that are guaranteed to see any signs of new physics ☹

⌜Last such experiments

Particle phys.: Higgs boson 2012 (theory 1967)
Astro: Gravitational waves 2016 (theory 1916)⌟

- But we may hope for a surprise: primordial non-Gaussianities and GW, fraction of DM is not cold, light relics, $w \neq -1$ ...

- A nice example is a discovery of $\Lambda > 0$ (1998)

- One could argue that it was, in fact, predicted by Weinberg in 1987 (at least by an order of magnitude). However, Weinbergs prediction (based on anthropic principle) was by far not as robust as Higgs or GW.

- This is because we do not have a complete enough theory (even now).

- Situation in theory is similar. We have SM+ΛCDM+ Inflation which works as an Effective theory and explains all observations.

- Unlike SM and GR, for Inflation, our understanding of the allowed parameters of the theory is not as complete.

- One may hope that there exist purely theoretical principles that constrain the models in a way that leads to predictions for observations.

- Bound on the # of e-foldings in terms of $\frac{H}{M_{pl}}$, and/or slow-roll parameters, or on the size of NG.

- So far there are no sharp results, however, we will see something of this sort in the last part of the talk [in a very remote-from-reality toy model]

Inflation and de Sitter space

- The simplest known way to generate the initial conditions for hot Big Bang.

- Main features of the theory:

1. Quasi-de Sitter spacetime

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 \qquad a(t) \approx e^{Ht}$$

H ↓ scale of Inflation, so far not known, related to the amplitude of tensor modes

2. Clock field (inflaton) determines the end of inflation, breaks dS isometries, generates scalar perturbations

$$\langle \zeta_k \zeta_{-k} \rangle \approx \underbrace{\frac{1}{c_s} \cdot \frac{H^2}{M_{pl}^2} \cdot \frac{H^2}{\dot{H}}}_{10^{-10}} \frac{1}{k^{3+(1-\underset{\substack{\downarrow \\ 0.965\pm0.004}}{n_s})}}$$

$$\varepsilon = \frac{\dot{H}}{H^2} \sim n_s - 1$$ natural but not necessary

$a \sim e^{Ht}$ $(H = \text{const} \sim dS)$

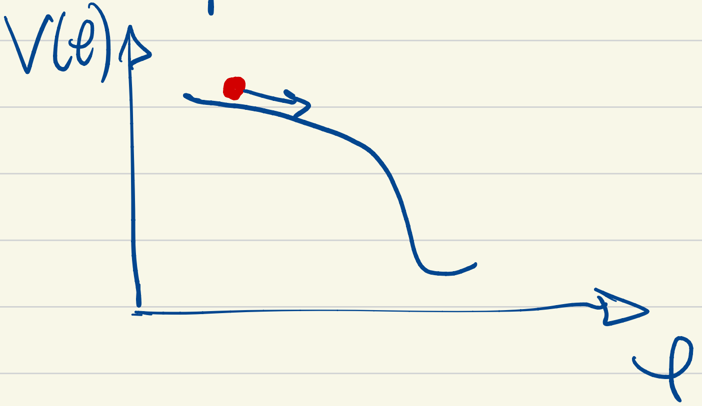
3. Non-gaussianities of perturbations

$$\langle \zeta\zeta\zeta \rangle \neq 0 \quad \text{and} \quad \langle \zeta\zeta\zeta\zeta \rangle_c \neq 0$$

However, from effective theory point of view they can be very small.

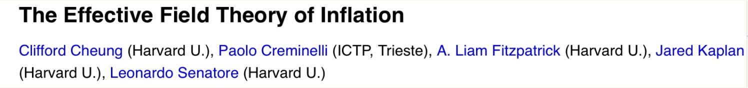
- Two approaches to inflation

Explicit potential



Effective field theory

$$\mathcal{L}(\zeta) \sim \dot{H}(\partial\zeta)^2 + \dot{\zeta}^3 + \dots$$



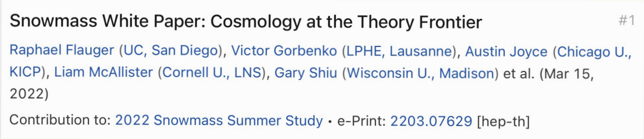
The Effective Field Theory of Inflation

Clifford Cheung (Harvard U.), Paolo Creminelli (ICTP, Trieste), A. Liam Fitzpatrick (Harvard U.), Jared Kaplan (Harvard U.), Leonardo Senatore (Harvard U.)

- EFT approach is more general, however, it likely includes theories that are not consistent. Potential is also only a partial UV completion.

Two topics of ongoing research I'm not going to discuss

- Embedding of inflation in String theory

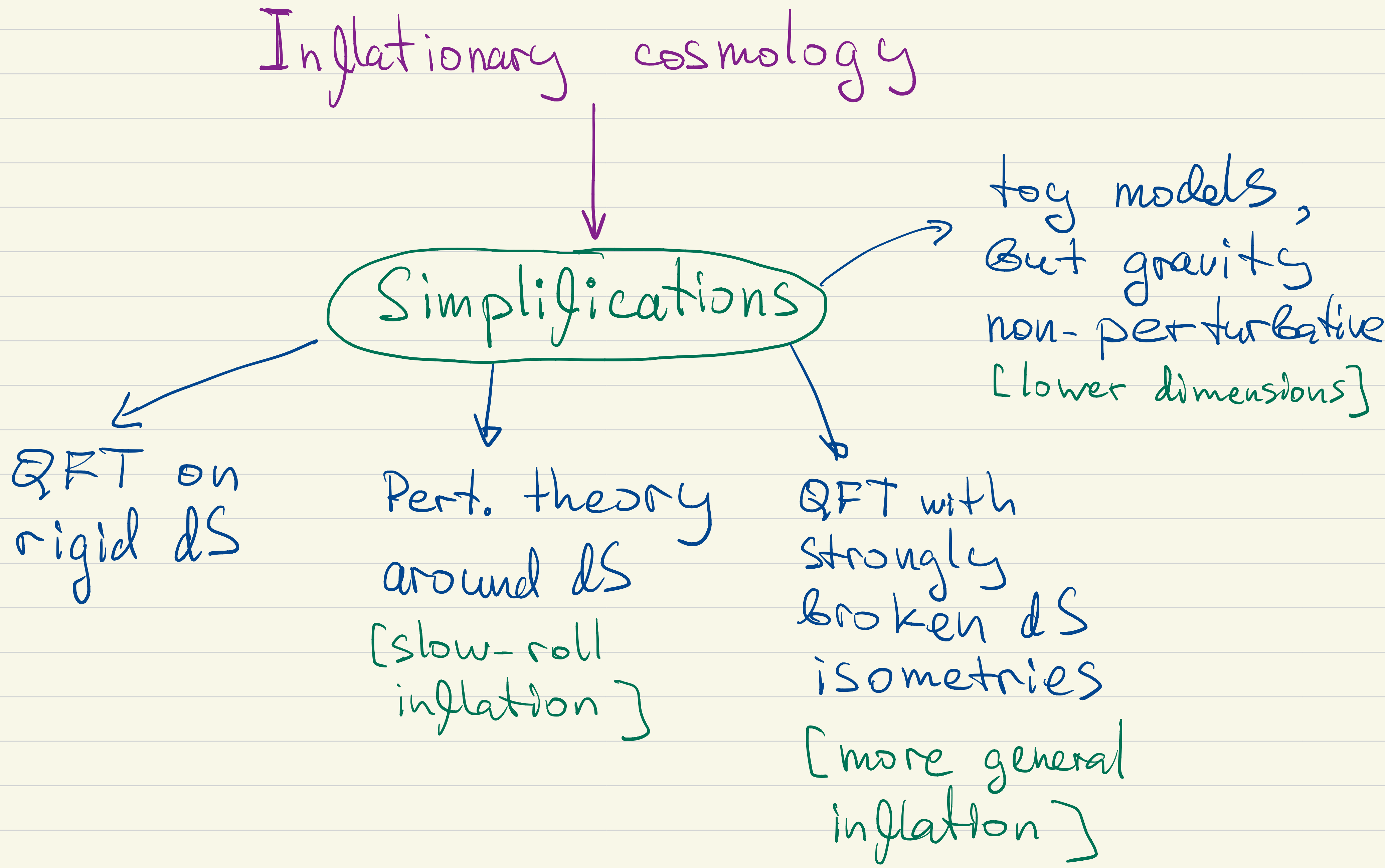


Snowmass White Paper: Cosmology at the Theory Frontier #1

Raphael Flauger (UC, San Diego), Victor Gorbenko (LPHE, Lausanne), Austin Joyce (Chicago U., KICP), Liam McAllister (Cornell U., LNS), Gary Shiu (Wisconsin U., Madison) et al. (Mar 15, 2022)

Contribution to: 2022 Snowmass Summer Study • e-Print: 2203.07629 [hep-th]

- Genericity of initial conditions [within strongly coupled classical GR]



Inflationary cosmology

Simplifications

toy models,
but gravity
non-perturbative
[lower dimensions]

QFT on
rigid dS

Pert. theory
around dS
[slow-roll
inflation]

QFT with
strongly
broken dS
isometries
[more general
inflation]

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Analytic techniques for cosmological correlators (bootstrap, unitarity...)

- Higher-point statistics of primordial perturbations on the reheating surface [future boundary of dS]
- Two approaches, both inspired by the corresponding developments in flat space and in AdS.

dS AdS/Flat

Cosmological Bootstrap ~ Amplitudes

Calculating on-shell observables by understanding singularities and building blocks of perturbative diagrams

Daniel Baumann (Amsterdam U.), Daniel Green (UC, San Diego), Thomas Hartman (Cornell U., Phys. Dept.) (Jun 24, 2019)
Published in: *JHEP* 12 (2019) 134 • e-Print: 1906.10226 [hep-th]

Matthijs Hogervorst (EPFL, Lausanne, FSL), João Penedones (EPFL, Lausanne, FSL), Kamran Salehi Vaziri (EPFL, Lausanne, FSL) (Jul 29, 2021)
e-Print: 2107.13871 [hep-th]

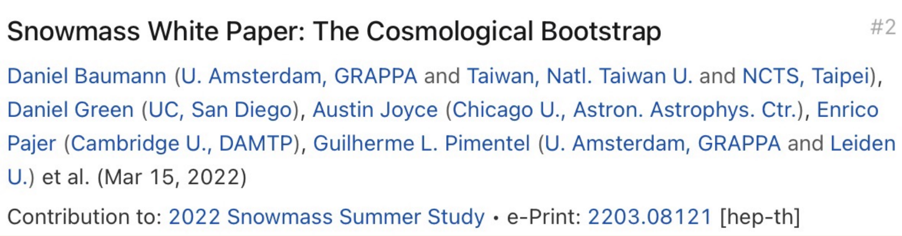
Lorenzo Di Pietro (Trieste U. and INFN, Trieste), Victor Gorbenko (Stanford U., ITP), Shota Komatsu (CERN) (Aug 3, 2021)
Published in: *JHEP* 03 (2022) 023 • e-Print: 2108.01695 [hep-th]

~ Conformal/S-matrix Bootstrap

Calculating (placing bounds) on onshell observables using non-perturbative phys. principles, valid in the UV to constrain the IR.

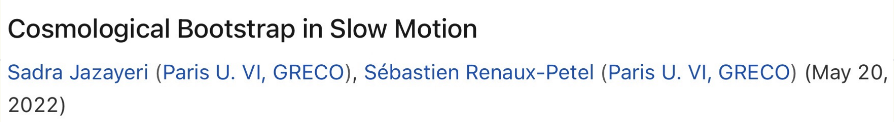
- Cosmological Bootstrap was discussed in the plenary talk By Gui Pimentel last year, there are also several talks this year, so I will not talk about it in details.

- Recent developments focused on strong breaking of dS isometries.


Snowmass White Paper: The Cosmological Bootstrap #2

Daniel Baumann (U. Amsterdam, GRAPPA and Taiwan, Natl. Taiwan U. and NCTS, Taipei), Daniel Green (UC, San Diego), Austin Joyce (Chicago U., Astron. Astrophys. Ctr.), Enrico Pajer (Cambridge U., DAMTP), Guilherme L. Pimentel (U. Amsterdam, GRAPPA and Leiden U.) et al. (Mar 15, 2022)

Contribution to: 2022 Snowmass Summer Study • e-Print: 2203.08121 [hep-th]


Cosmological Bootstrap in Slow Motion

Sadra Jazayeri (Paris U. VI, GRECO), Sébastien Renaux-Petel (Paris U. VI, GRECO) (May 20, 2022)

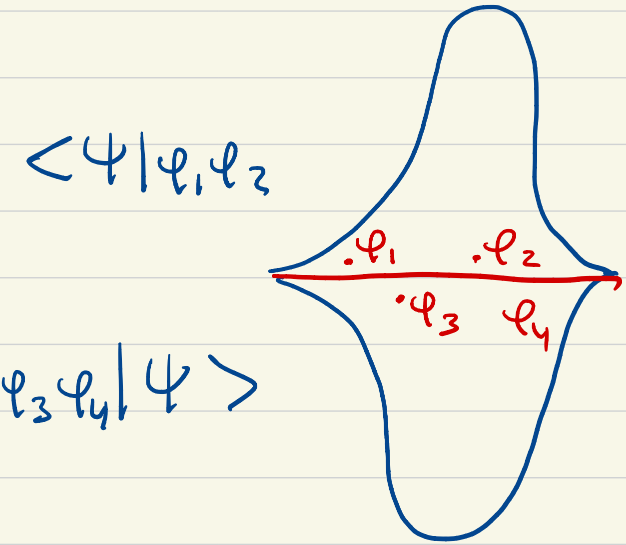

Boostless Cosmological Collider Bootstrap

Guilherme L. Pimentel (Leiden U. and U. Amsterdam, IHEF), Dong-Gang Wang (Cambridge U., DAMTP) (Apr 29, 2022)

Non-perturbative analytic techniques
[so far with dS isometries]

- Unitarity: using Conformal Partial Wave [CPW] expansion we can transorm an abstract notion of positivity of cosmological measure into an equation on correlators:

$$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle \sim$$



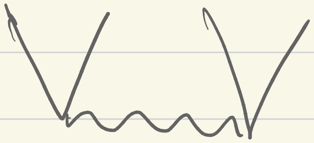
$$P \sim \Psi \Psi^* \geq 0$$

$$\rho_J(\nu) = \int dx_1 dx_2 dx_3 dx_4 \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle \underbrace{\mathcal{F}^{\Delta_\varphi}_{J,\nu}(x_i)}_{\text{CPW}}$$

↳ spectral density

SO(3)

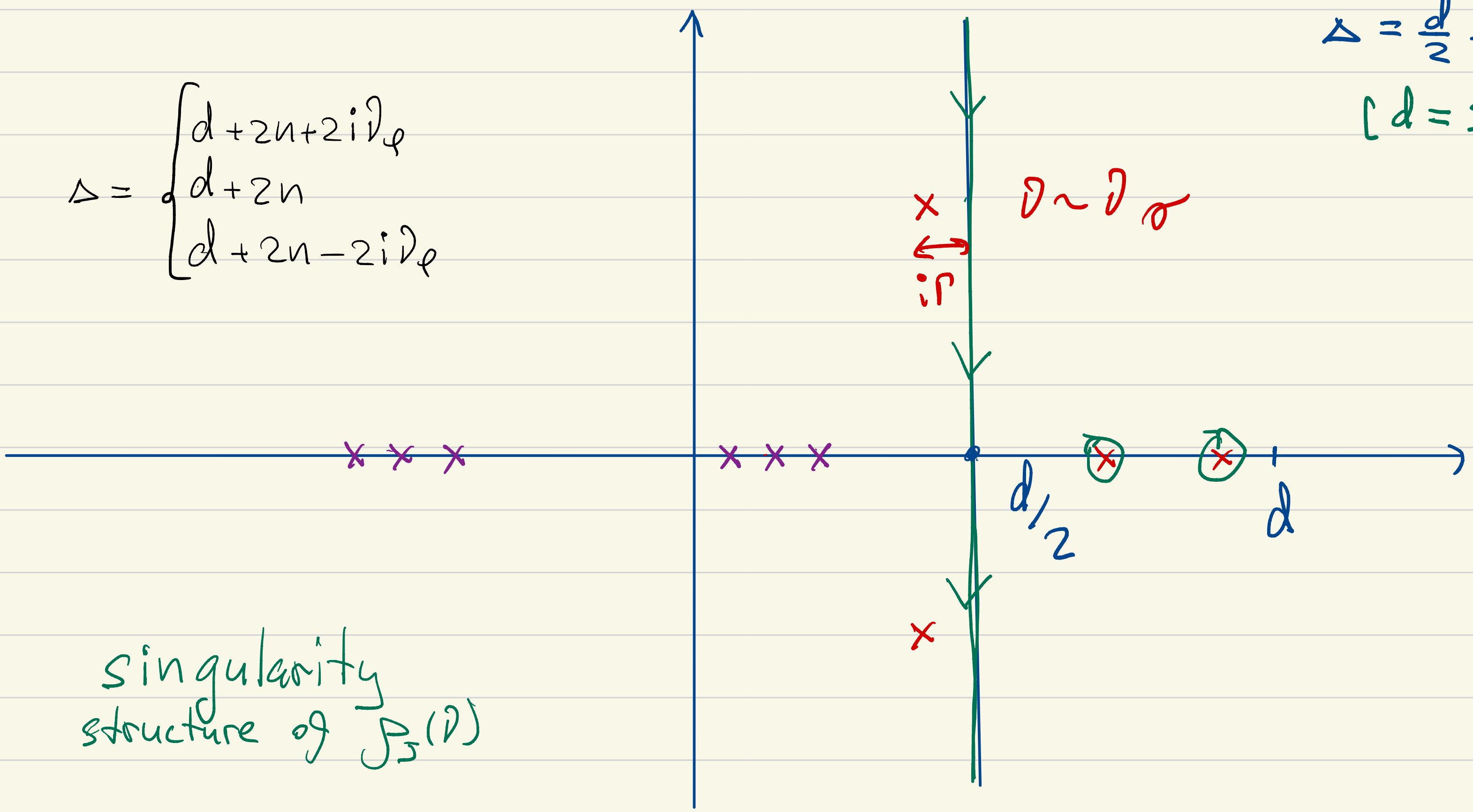
[like $Y_{\ell,m}$, but for SO(1,4)]



$\Delta = \frac{d}{2} - i\nu$

$[d=3]$

$$\Delta = \begin{cases} d+2n+2i\nu_\varphi \\ d+2n \\ d+2n-2i\nu_\varphi \end{cases}$$

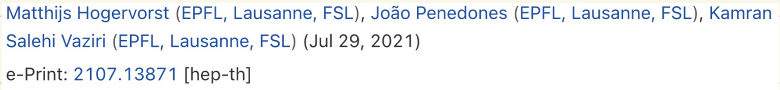


singularity structure of $\rho_J(\nu)$

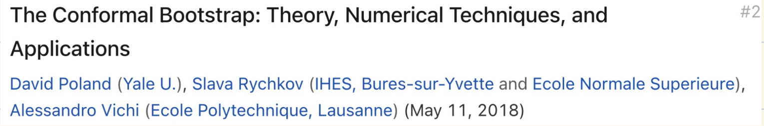
- Unitarity $\rho_J(\nu) > 0$

- Matching of expansions in two channels often leads to non-trivial constraints

$$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle = \langle \varphi_1 \varphi_3 \varphi_2 \varphi_4 \rangle$$


Matthijs Hogervorst (EPFL, Lausanne, FSL), João Penedones (EPFL, Lausanne, FSL), Kamran Salehi Vaziri (EPFL, Lausanne, FSL) (Jul 29, 2021)
e-Print: 2107.13871 [hep-th]

[This is the best method to compute critical exponents in the 3D Ising model]


The Conformal Bootstrap: Theory, Numerical Techniques, and Applications #2
David Poland (Yale U.), Slava Rychkov (IHES, Bures-sur-Yvette and Ecole Normale Superieure), Alessandro Vichi (Ecole Polytechnique, Lausanne) (May 11, 2018)

- To do list: Bounds on $(\partial\varphi)^4$ coupling in dS; Bounds on $\dot{\zeta}^3$, $(\partial_i \zeta)^2 \dot{\zeta}$, etc. couplings in the EFT of inflation [need breaking of dS isometries]

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Taming of the infrared divergences

- Punchline: there are no problems or surprises in QFT on dS, or in inflation, away from the eternal inflation regime.
- Why expect a problem?
- Consider first a light scalar field on rigid dS

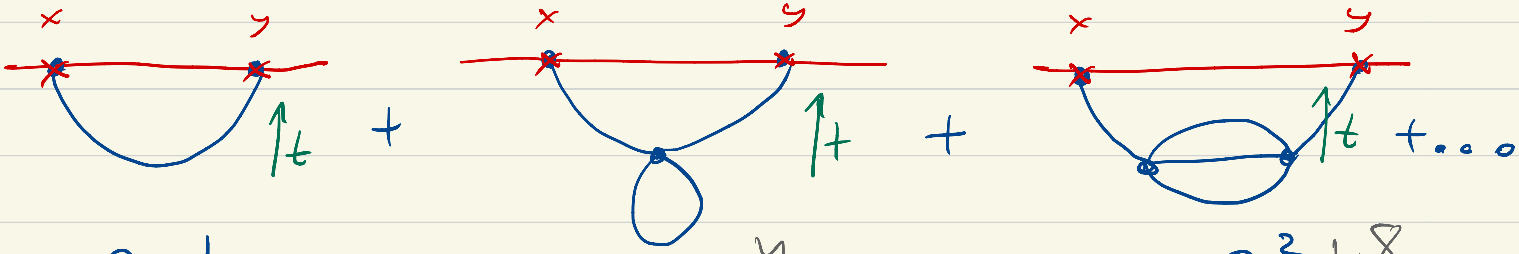
$$\mathcal{L} = (\partial\varphi)^2 - V(\varphi) \qquad ds^2 = -dt^2 + e^{2Ht} d\vec{x}^{\,2}$$

e.g. $V(\varphi) \simeq m^2\varphi^2 + \lambda\varphi^4$ $\qquad M_{pl} \to \infty$, $H = \text{const}$

focus on $m^2 \ll H^2$, $\lambda \ll 1$

- Lets attempt a perturbative calculation of power-spectrum.

$\langle \varphi(x)\varphi(y)\rangle \simeq$



~ 1 $\qquad \sim \frac{\lambda H^4}{m^4}$ $\qquad \sim \frac{\lambda^2 H^8}{m^8}$

$\left(\bullet\xrightarrow{\vec{k}}\bullet \sim \frac{a^{-m^2}}{k^{3-m^2}} \right)$

both $\vec{k}$ and t integrals lead to $\frac{1}{m^2}$

$m^2 \ll H^2 \lambda^{1/2}$

- If mass is small enough, perturbation theory is badly divergent!

- It doesn't mean there is a physical problem, it means this method of doing computations is bad.

- With Senatore (1911.00022) we carefully developed such formalism, although the idea was known since Starobinsky '84

- Define long field in position space:

$$\varphi_i = \int_0^{\Lambda(t)} d^3k\, e^{ikx_i} \varphi_{\vec{k}} \qquad \Lambda = \varepsilon a(t) H$$

- P_n's generate correlators of φ_ℓ: $\qquad e^{-\frac{1}{\sqrt{\lambda}}} \ll \varepsilon \ll \lambda$

$$\langle \varphi(x_1) \dots \varphi(x_n) \rangle = \int d\varphi_1 \dots d\varphi_n\, \varphi_1 \dots \varphi_n\, P_n(\varphi_1 \dots \varphi_n, \vec{x}_{ij}, t) + \text{short modes}$$

- They satisfy a system of PDE's:

$$\Gamma_i = \frac{\partial^2}{\partial \varphi_i^2} + \frac{\partial}{\partial \varphi_i} V'(\varphi_i) + O(\lambda, \varepsilon)$$

"Diffusion" "Drift"

$$\Gamma_{ij} = \frac{\sin \varepsilon a x_{ij}}{\varepsilon a x_{ij}} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} + O(\lambda, \varepsilon)$$

$$\partial_t P_1(\varphi_1, t) = \underline{\Gamma_1 P_1} + D_{12} P_2 + \dots \qquad D_{nn+1} \sim \int d\varphi_{n+1}\, \Omega\, P_{n+1} \sim \lambda$$

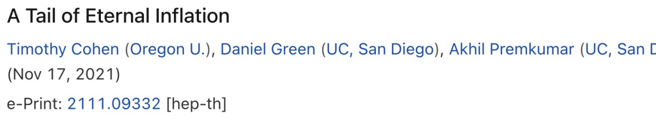
$$\partial_t P_2(\varphi_1, \varphi_2; x_{12}, t) = \underline{(\Gamma_1 + \Gamma_2 + \Gamma_{12}) P_2} + D_{23} P_3 + \dots$$

$$\dots$$

$$\partial_t P_n(\{\varphi_i\}; \{x_{ij}\}, t) = \underline{\left(\sum_i^n \Gamma_i + \sum_{i \neq j}^n \Gamma_{ij} \right) P_n} + D_{nn+1} P_{n+1} + \dots$$

- No need to evaluate Feynman diagrams for long modes.

- Inflation and perturbative gravity can be easily incorporated in this framework, see, e.g.



A Tail of Eternal Inflation

Timothy Cohen (Oregon U.), Daniel Green (UC, San Diego), Akhil Premkumar (UC, San

(Nov 17, 2021)

e-Print: 2111.09332 [hep-th]

- We add the inflaton perturbations to our P_n's:

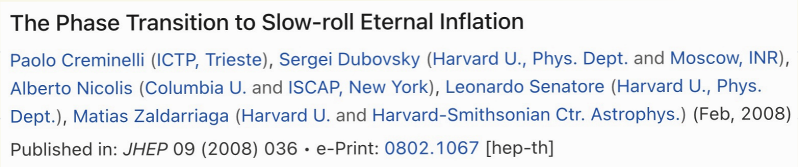
$$\partial_t P_1(\zeta_1, \varphi_1 ..) = \left(\frac{\partial^2}{\partial \zeta_1^2} + \frac{\partial^2}{\partial \varphi_1^2} + \frac{\partial}{\partial \varphi_1} V' + \dots \right) P_1$$

- Difference is that there is no potential for ζ, so it never reaches a stationary solution:

$$P_1(\zeta_1, \varphi_1) \simeq e^{-\frac{\zeta_1^2}{\sigma t}} \cdot e^{-\frac{V(\varphi_1)}{H^4}}$$

- ζ-distribution gets broader with time, however, due to the bound on the length of inflation it never becomes too large, unless we are in the eternal inflation regime

$$\frac{H}{M_{pl}\sqrt{\epsilon}} \sim 1 \Rightarrow \frac{\delta\rho}{\rho} \sim 1$$



The Phase Transition to Slow-roll Eternal Inflation

Paolo Creminelli (ICTP, Trieste), Sergei Dubovsky (Harvard U., Phys. Dept. and Moscow, INR), Alberto Nicolis (Columbia U. and ISCAP, New York), Leonardo Senatore (Harvard U., Phys. Dept.), Matias Zaldarriaga (Harvard U. and Harvard-Smithsonian Ctr. Astrophys.) (Feb, 2008)

Published in: *JHEP* 09 (2008) 036 • e-Print: 0802.1067 [hep-th]

- Eternal inflation ≈ universe never globally reheats..

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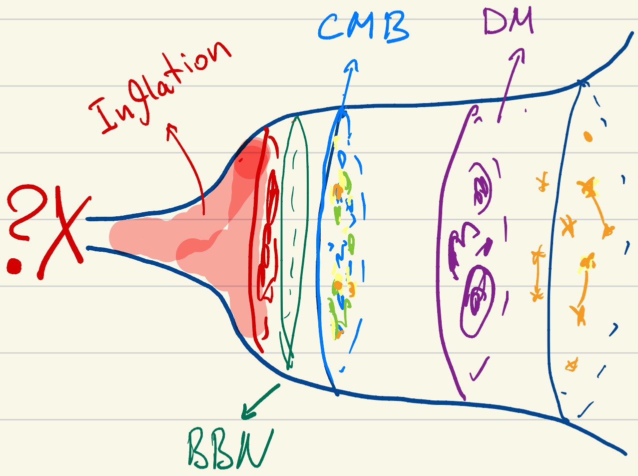
Non-perturbative gravitational effects

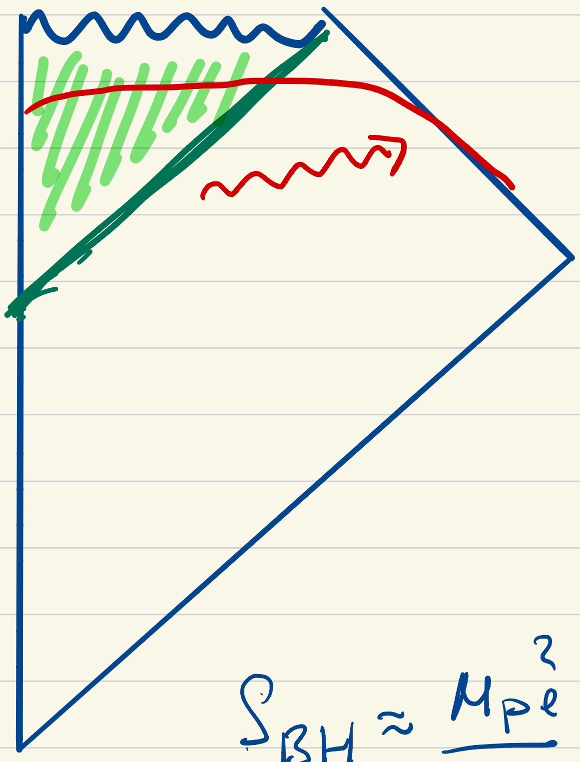
- Why would they be important?

  - Inflation has a feature that it erases information about the initial conditions to some extent



- Nevertheless, a complete theory must explain them as well. Maybe inflation is preceded by a singularity where UV QG effects become important...

- However, recent studies of Black hole evaporation, in the framework of AdS/CFT revealed (somewhat unexpectedly) that non-perturbative gravitational effects can also be dominant in the IR



$$S_{BH} \approx \frac{M_{pl}^2}{H^2}$$

- Classically black holes can exist forever.

Black hole complementarity:

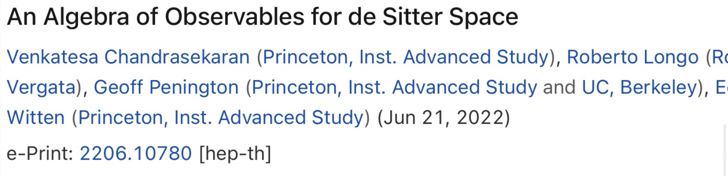
At late times interior of a black hole is not an independent set of degrees of freedom from the exterior

[Non-perturbative gravitational effect]

- Most sharply formulated in AdS/CFT [emergent spacetime]

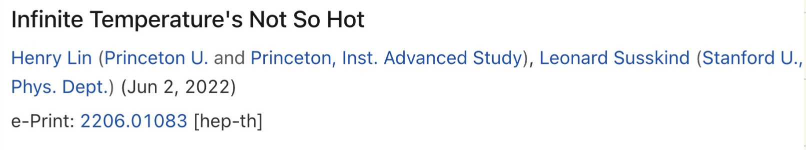
Almheiri, Engelhardt, Marolf, Maxfield
Penington
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini
Penington, Shenker, Stanford, Yang
`19...

- For cosmology we do not have an analogous description with emergent spacetime, however, there is a spike of recent activity:


An Algebra of Observables for de Sitter Space

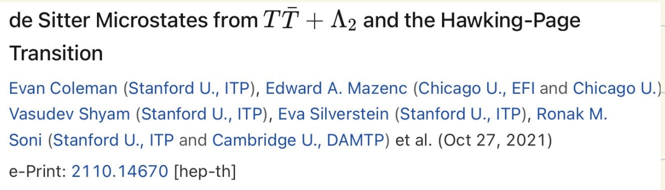
Venkatesa Chandrasekaran (Princeton, Inst. Advanced Study), Roberto Longo (Rome U., Tor Vergata), Geoff Penington (Princeton, Inst. Advanced Study and UC, Berkeley), Edward Witten (Princeton, Inst. Advanced Study) (Jun 21, 2022)

e-Print: 2206.10780 [hep-th]


Infinite Temperature's Not So Hot

Henry Lin (Princeton U. and Princeton, Inst. Advanced Study), Leonard Susskind (Stanford U., Phys. Dept.) (Jun 2, 2022)

e-Print: 2206.01083 [hep-th]


de Sitter Microstates from $T\bar{T}+\Lambda_2$ and the Hawking-Page Transition

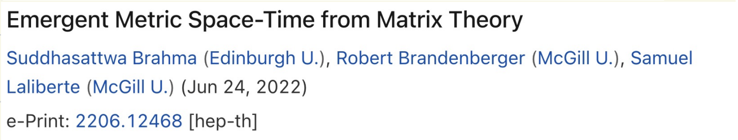
Evan Coleman (Stanford U., ITP), Edward A. Mazenc (Chicago U., EFI and Chicago U.) Vasudev Shyam (Stanford U., ITP), Eva Silverstein (Stanford U., ITP), Ronak M. Soni (Stanford U., ITP and Cambridge U., DAMTP) et al. (Oct 27, 2021)

e-Print: 2110.14670 [hep-th]


Cosmology from the vacuum

Stefano Antonini (Maryland U.), Petar Simidzija (British Columbia U.), Brian Swingle (Maryland U. and Brandeis U.), Mark Van Raamsdonk (British Columbia U.) (Mar 21, 2022)

e-Print: 2203.11220 [hep-th]


Emergent Metric Space-Time from Matrix Theory

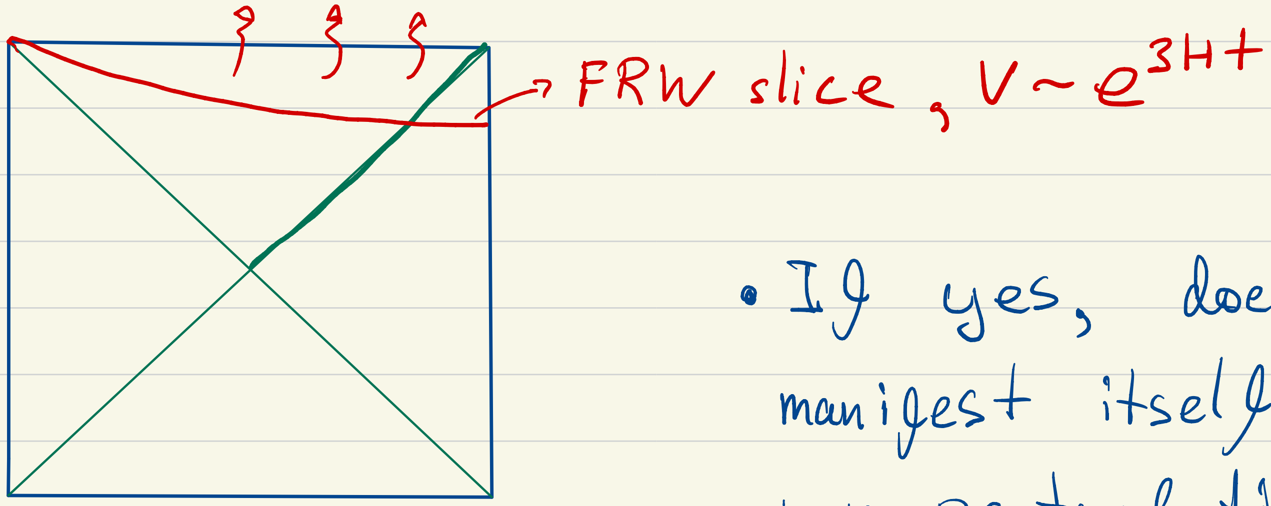
Suddhasattwa Brahma (Edinburgh U.), Robert Brandenberger (McGill U.), Samuel Laliberte (McGill U.) (Jun 24, 2022)

e-Print: 2206.12468 [hep-th]

- One is tempted to speculate, that de Sitter (Inflationary) spacetime can also be described with a finite number of degrees of freedom

- This leads to a puzzle given that clasically the volume grows exponentially.

- Is there a notion of complementarity in cosmology?



Penrose diagram of global de Sitter

$$S_{GH} \simeq \frac{M_{pl}^2}{H^2}$$

- If yes, does it manifest itself as a non-perturbative gravitational effect, leading to observable consequences?

- We do not know the answer yet, however, there are some very prelimenary, yet encouraging results.

with J. Maldacena and Y. Chen, 2007.16091

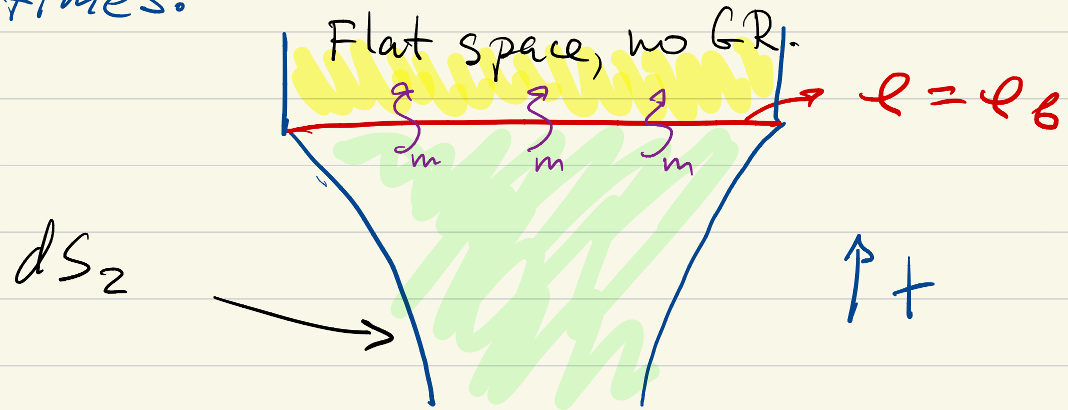
- Consider the dS version of 2D gravity.

$$S^{dS} \simeq \varphi_0 \int R + \int \varphi (R-2) + S_m \qquad \varphi_0 \gg 1$$

It is a toy model for inflation, dilaton $\varphi \sim$ inflaton, matter CFT $\sim$ CMB radiation

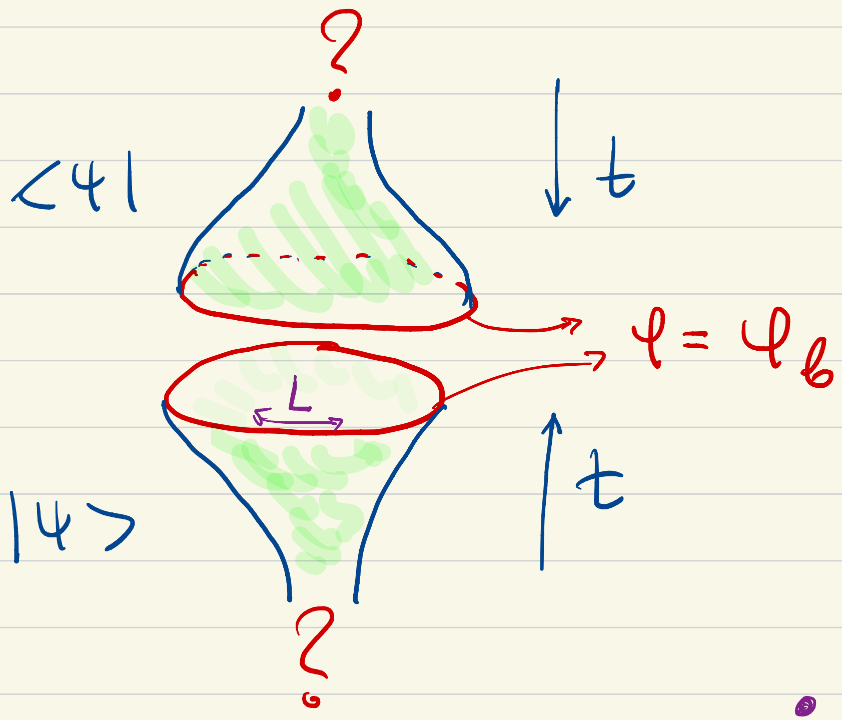
- Generic solution at late times:

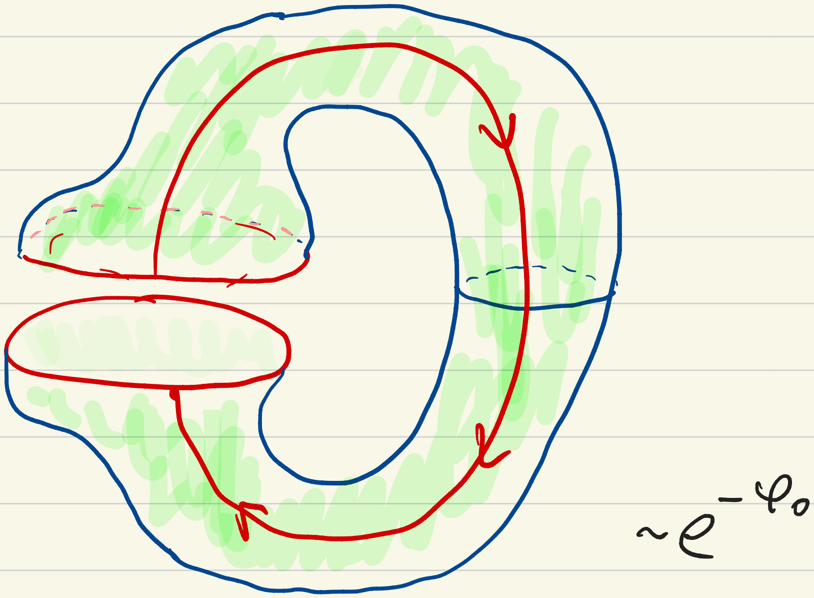
$$ds^2 = -dt^2 + e^{2t} dx^2$$
$$\varphi = \varphi_r e^t$$



- To calculate cosmological correlators we need two "copies of spacetime" for bra and ket of the wave function:

Bra-Ket wormhole:





- For a large universe the wormhole will dominate the state of the universe.

- Simple observables, like the power spectrum of "CMB" radiation, are sensitive to the thermal nature of the solution, at low k

$$P_\zeta(k) \approx \frac{T}{k^2}$$, instead of scale-invariant $\frac{1}{k}$

[$T \ll H$ depends on parameters of the model]

Conclusions

- Inflation is the dominant theory for initial conditions consistent at low energies $\sim H$, however, we do not yet know how to promote it to a non-perturbative complete theory.

- There are no perturbative IR instabilities, contrary to some claims in the literature

- Analytic methods for cosmological correlators are being actively developed, providing efficient computational tools, and, possibly, will lead to constraints on the space of inflationary models

- There is an exciting opportunity that non-perturbative quantum gravity effects play an important role, and can be seen, in the early universe cosmology.