Positivity Bounds for Cosmology

Cosmology from Home

05 -16 July 2021

Scott Melville

What are Positivity Bounds?

Motivation

Sketch of Derivation

How to apply Positivity in Cosmology?

Strategy 1: Covariant Theories — Dark Energy

Strategy 2: Subhorizon Amplitudes — Inflation

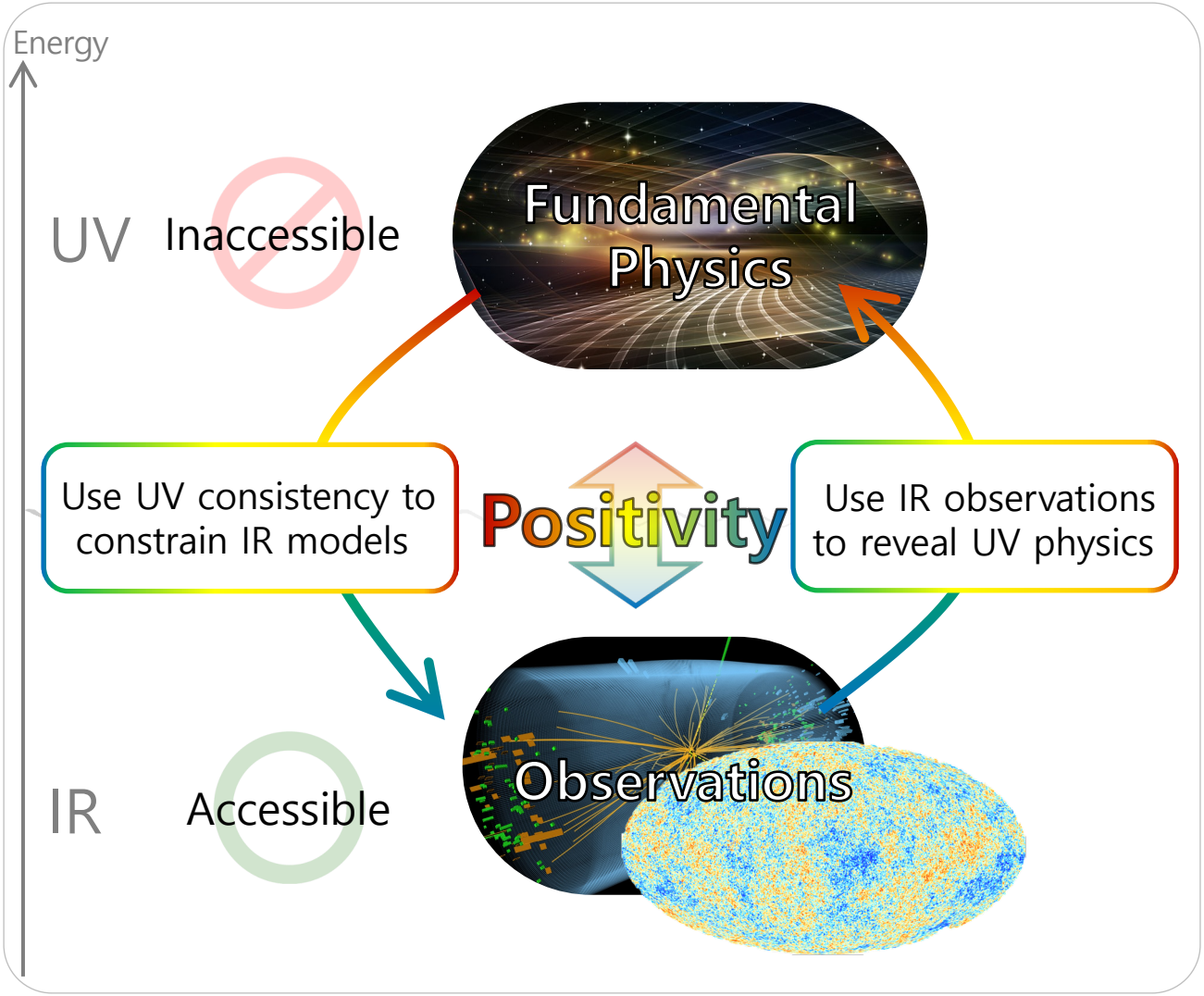
Where do we go from here?

Strategy 3? Amplitudes → Wavefunctions

A Positive Outlook

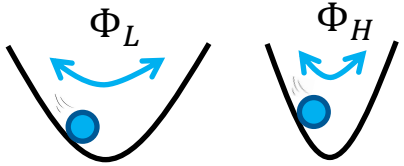
What are Positivity Bounds?

Motivation


Energy
UV
Inaccessible
Fundamental Physics
Use UV consistency to constrain IR models
Positivity
Use IR observations to reveal UV physics
Observations
IR
Accessible

Energy

UV Inaccessible



All degrees of freedom

$\mathcal{L}_{UV}[\Phi_L, \Phi_H]$

Fundamental

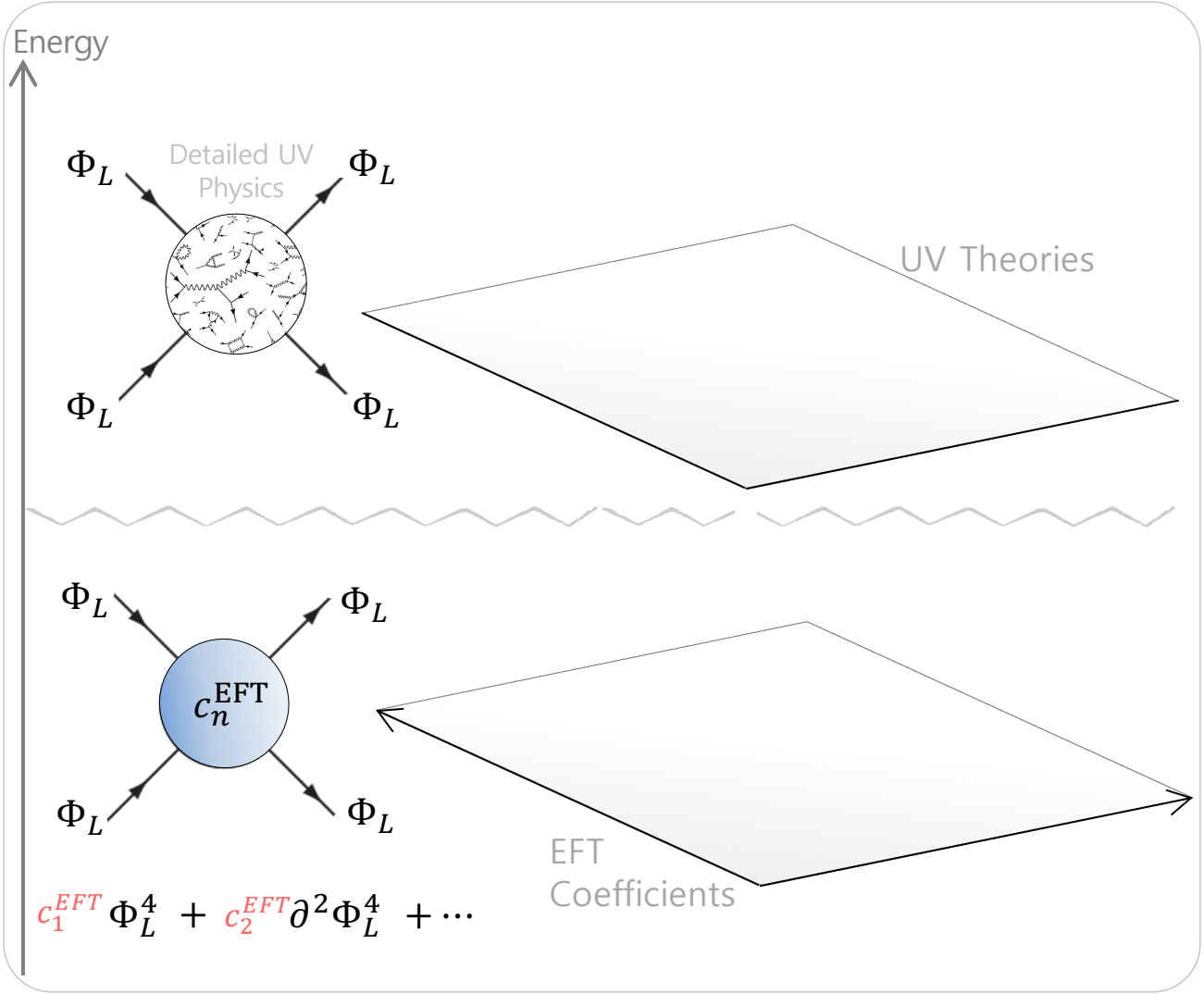
Positivity

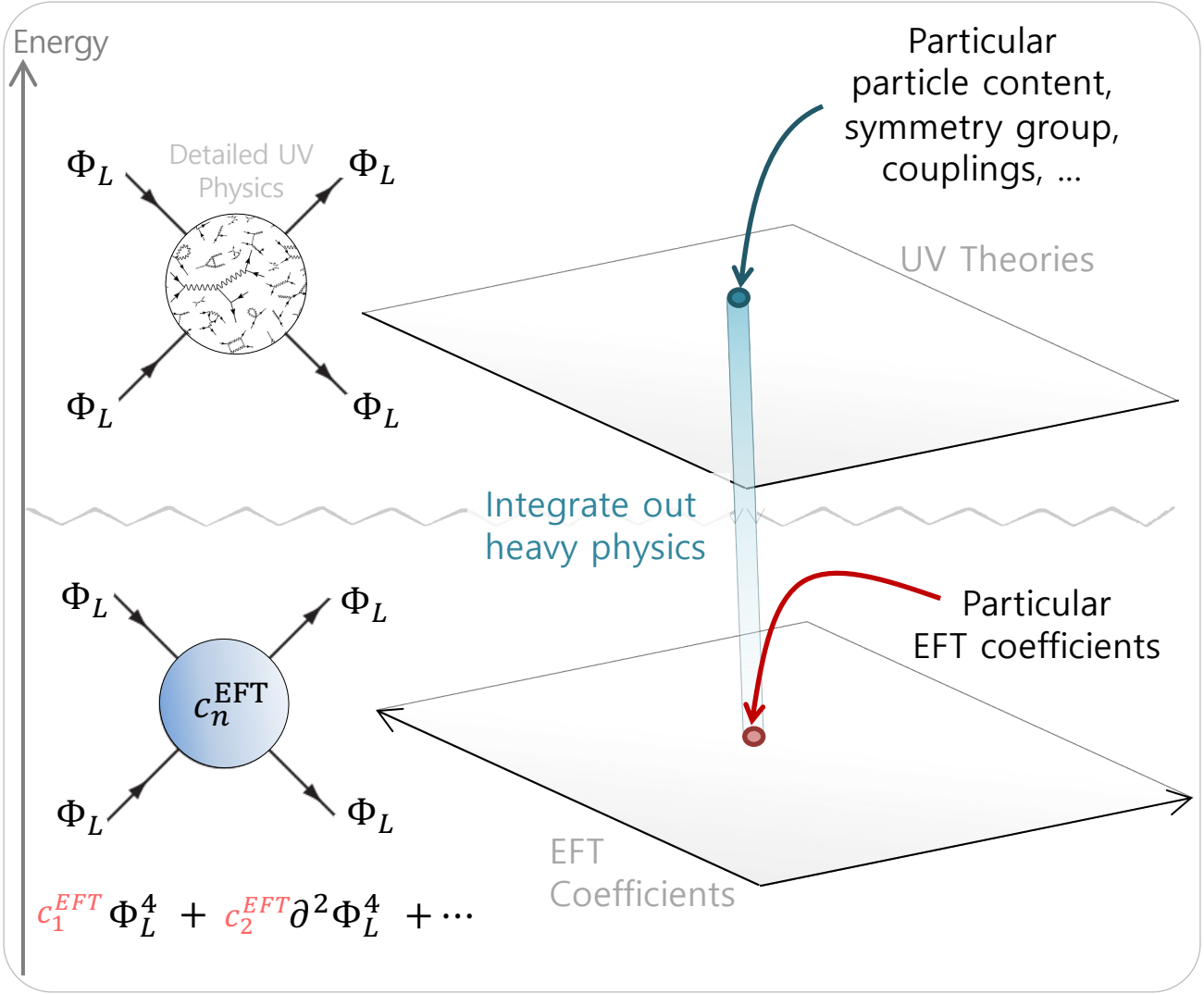
Phenomenological

$\mathcal{L}_{EFT}[\Phi_L]$

Only light degrees of freedom

IR Accessible


Energy
Φ_L
Φ_L
Detailed UV Physics
Φ_L
Φ_L
UV Theories
Φ_L
Φ_L
c_n^{EFT}
Φ_L
Φ_L
EFT Coefficients
$c_1^{EFT}\,\Phi_L^4 + c_2^{EFT}\partial^2\Phi_L^4 + \cdots$


Energy
Detailed UV Physics
Φ_L
Φ_L
Φ_L
Φ_L
Particular particle content, symmetry group, couplings, ...
UV Theories
Integrate out heavy physics
c_n^{EFT}
Φ_L
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Particular EFT coefficients
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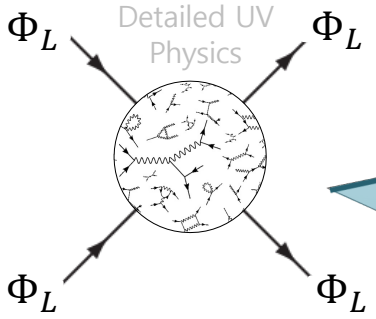
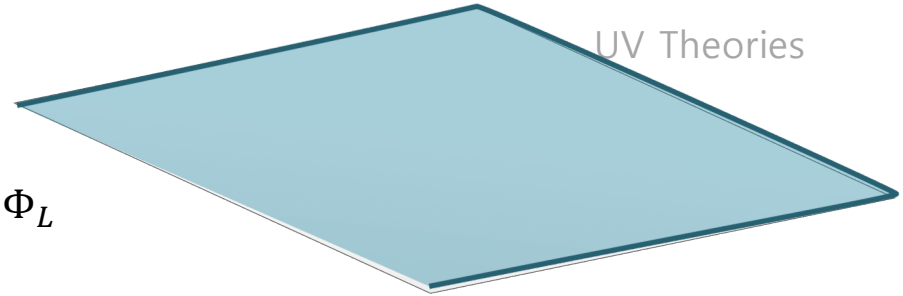
Energy

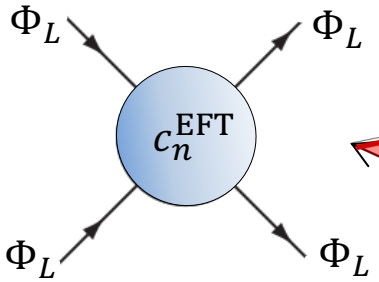
Particular particle content, symmetry group, couplings, …

?

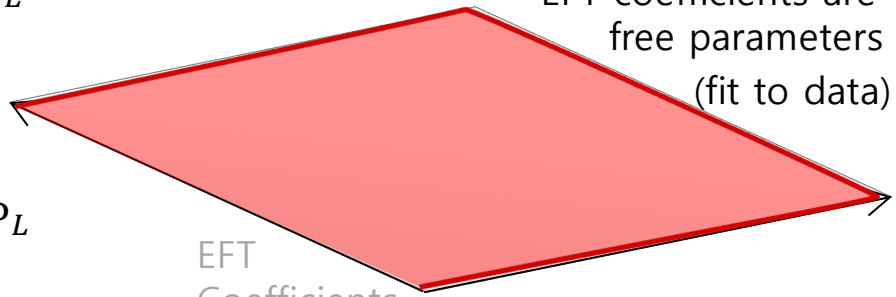


UV Theories



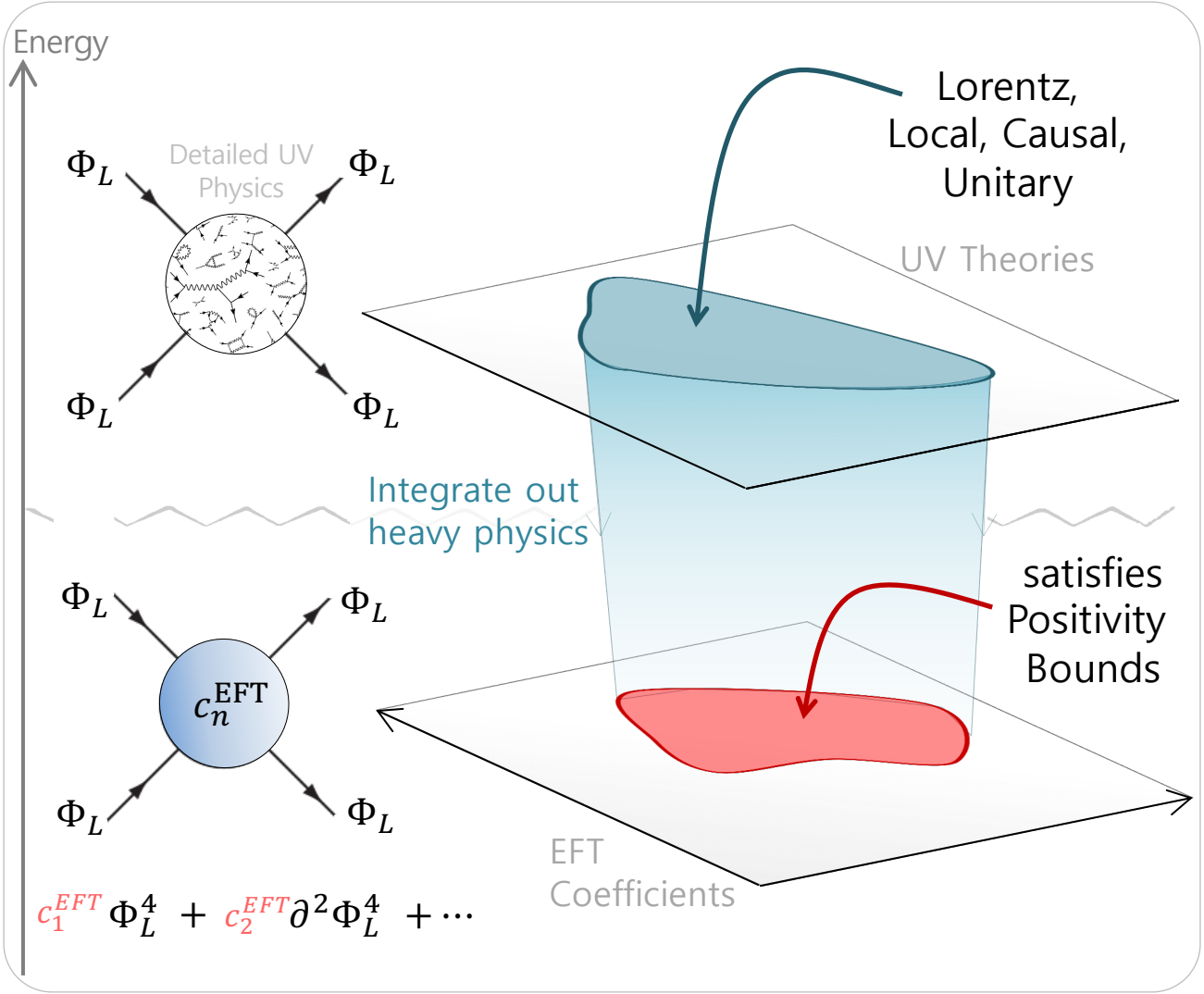


EFT coefficients are free parameters (fit to data)



EFT Coefficients

$c_1^{EFT}\,\Phi_L^4 + c_2^{EFT}\partial^2\Phi_L^4 + \cdots$


Energy
Detailed UV Physics
Φ_L
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Lorentz, Local, Causal, Unitary
UV Theories
Integrate out heavy physics
satisfies Positivity Bounds
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EFT Coefficients
$c_1^{EFT}\,\Phi_L^4 + c_2^{EFT}\partial^2\Phi_L^4 + \cdots$

What are Positivity Bounds?

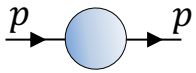
Motivation

Sketch of Derivation

[Adams+Arkani Hamed+Dubovsky +Nicolis+Rattazzi 2006]

... and many other recent works

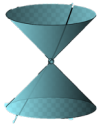
Lorentz Invariance

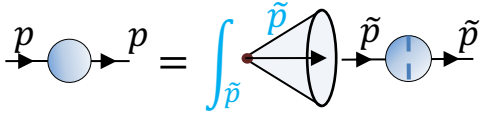


$= A(p^2)$

Fixes kinematics

(Locality) + Causality

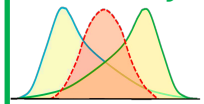


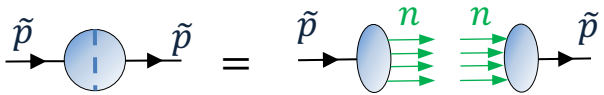


$$A(p^2) = \int_0^\infty \frac{d\tilde{p}^2}{\tilde{p}^2 - p^2} \operatorname{Im} A(\tilde{p}^2)$$

Relates IR to UV
(via dispersion relation)

Unitarity





$$\operatorname{Im} A(\tilde{p}^2) = \frac{1}{2} \sum_n A_{1\to n}(\tilde{p}) A^*_{1\to n}(\tilde{p})$$

UV is positive
(via optical theorem)

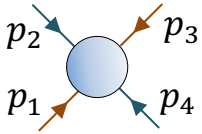
Positivity

EFT expansion, $A(p^2) = \sum_j p^{2j} c_j^{EFT}$

has $c_j^{EFT} = \int_0^\infty \frac{d\tilde{p}^2}{\tilde{p}^{2j+2}} \sum_n |A_{1\to n}(\tilde{p})|^2 > 0$

IR coefficients are positive

Lorentz Invariance

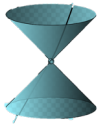


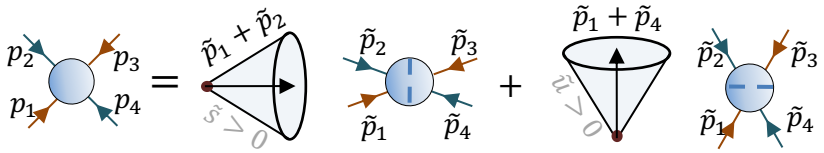
$$A(p_1, p_2, p_3, p_4) = A(s, t)$$

function of only two variables

$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_3)^2$$
$$u = 4m^2 - s - t$$

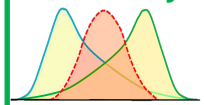
(Locality) + Causality

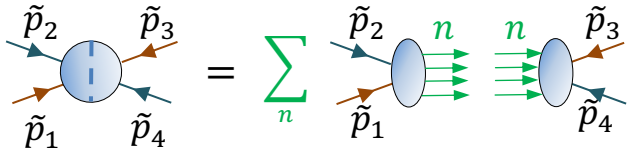




$$\partial_s^2 A(s,t) = \int_0^\infty \frac{d\tilde{s}}{(\tilde{s}-s)^3} \operatorname{Im} A(\tilde{s},t) + \int_0^\infty \frac{d\tilde{u}}{(\tilde{u}-u)^3} \operatorname{Im} A(\tilde{u},t)$$

Unitarity





$$\operatorname{Im} A_{12\to 34} = \frac{1}{2} \sum_n A_{12\to n} A^*_{34\to n} > 0$$

in forward limit ($t = 0 \Rightarrow 12 = 34$)

Positivity

$$A_{EFT}(s,t) = \sum_{a,b} s^a t^b c_{ab} \text{ has } \underline{\text{bounded}} \ c_{ab}$$

e.g. $c_{20} > 0$, $c_{21} > \frac{3}{2} c_{20}$, ...

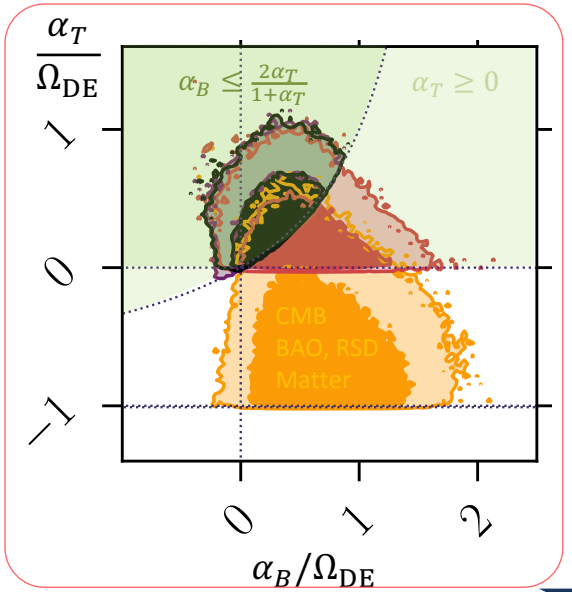
[Adams++ 2006] [Nicolis+Rattazzi+Trincherini 2010] [de Rham+SM+Tolley+Zhou 2017]

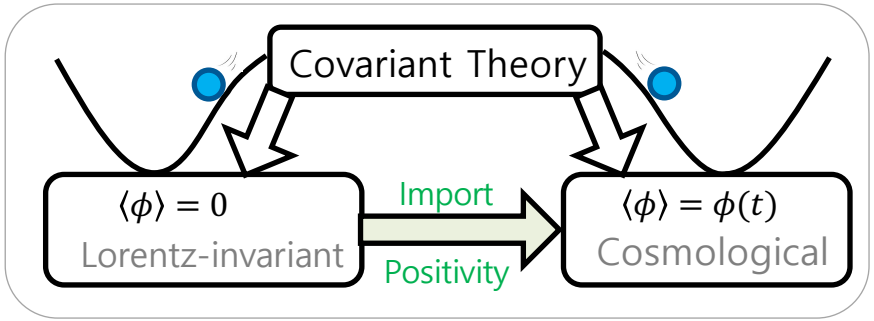
How to apply Positivity Bounds in Cosmology?

Strategy 1: Covariant Theory

[SM+Noller 2019]

[de Rham+SM+Noller 2021]



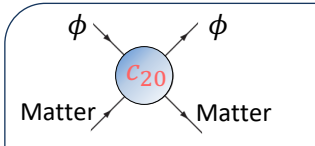


$(X = \frac{1}{2}(\nabla\phi)^2)$ $P(X)$ Dark Energy Scalar-Tensor

$$\int d^4x\sqrt{-g}\left[G_2(X) + G_4(X)R + G_4'(X)\left((\Box\phi)^2 - (\nabla\nabla\phi)^2\right)\right]$$

$\langle\phi\rangle = 0$
$g_{\mu\nu} = \eta_{\mu\nu}$

$\langle\phi\rangle = \phi(t)$
$g_{\mu\nu} = \text{FLRW}$



$G_4' \geq 0$ $\quad G_4'^2 \leq G_4 G_4''$

[Bellini+Sawicki 2014]

Tensor Speed $\alpha_T = 4XG_4'$ $\quad$ Braiding (DE Clustering) $\alpha_B = 8X(G_4' + 2XG_4'')$

$\alpha_T \geq 0$ $\quad \alpha_B \leq \frac{2\alpha_T}{1+\alpha_T}$

Requires
(i) Both solutions exists and are stable
(ii) $G_{4,X}(\dot{\phi}^2) - G_{4,X}(0)$ small

How to apply Positivity Bounds in Cosmology?

Strategy 1: Covariant Theory

[SM+Noller 2019]

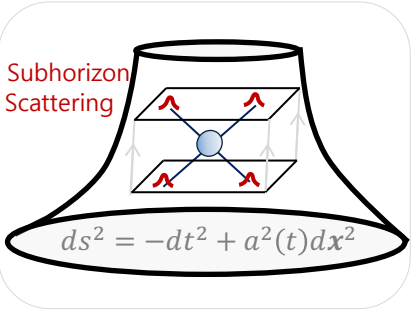
[de Rham+SM+Noller 2021]

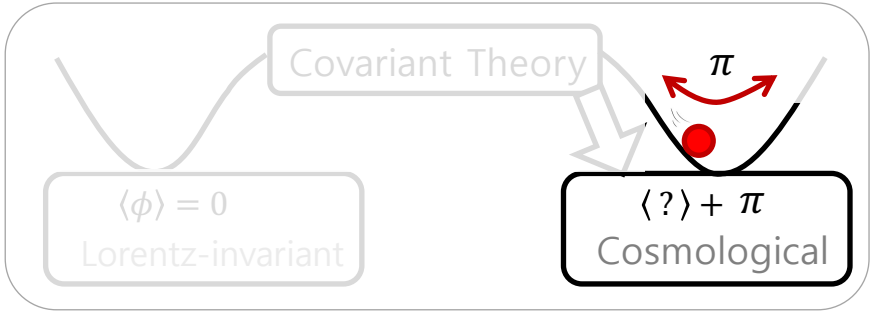
Strategy 2: Subhorizon Scattering

[Baumann+Green+Lee+Porto 2016]

[Grall+SM 2020]

[Grall+SM 2021]





$A(s,t,\omega_1,\omega_2,\omega_3)$

Explicit energy dependence

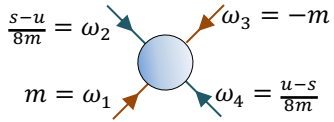
Scatter with $\dot{H}^2, H^4 \ll \omega^4 \ll M_P^2\dot{H}$

$\Rightarrow$ Only boosts are broken

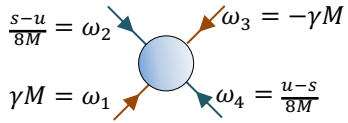
~~Lorentz Invariance~~

$\partial_s^2 A_{\rm EFT} \propto \int_0^\infty \frac{d\tilde{s}}{\tilde{s}^3}\,\mathrm{Im}\,A_{\rm UV}$

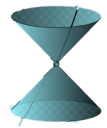
with $t=0$, $\omega_1 = -\omega_3$ and $\frac{s-u}{\omega_2}$ held fixed



$\boldsymbol{p}_1 + \boldsymbol{p}_3 = 0$ $\rightarrow$



$\boldsymbol{p}_1 + \boldsymbol{p}_3 = \boldsymbol{p}_{\rm Breit}(\gamma, M)$



$\mathrm{Im}\,A_{UV} > 0$

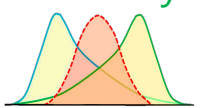
in forward limit $(t=0,\ \omega_3 = -\omega_1)$

$\boldsymbol{p}_1 + \boldsymbol{p}_2 = 0$ $\rightarrow$ $\boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{p}_{\rm CM}$

$A(s,\theta) = \sum_\ell P_\ell(\theta) a_\ell(s)$

$A(s, |\boldsymbol{p}_{\rm CM}|, \theta_1, \varphi_1, \theta_3, \varphi_3) = \sum_{\ell_i m_i} Y_{\ell_1}^{m_1}(\theta_1,\varphi_1) Y_{\ell_3}^{m_3 *}(\theta_3,\varphi_3) a_{\ell_1\ell_3}^{m_1 m_3}(s, |\boldsymbol{p}_{\rm CM}|)$

Unitarity



$\partial_s^2 A_{\rm EFT} > 0$

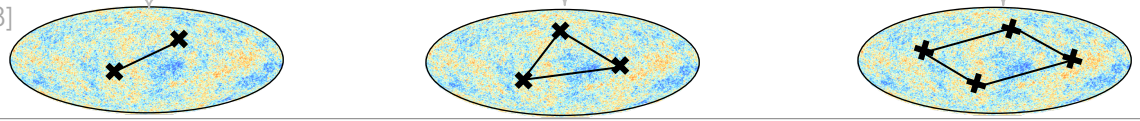
with $t=0$, $\omega_1 = -\omega_3$ and $\frac{s-u}{\omega_2}$ held fixed

$A_{\rm EFT} = \sum_{a,b} (\omega_2+\omega_4)^a t^b C_{ab}$ has bounded C_{ab}

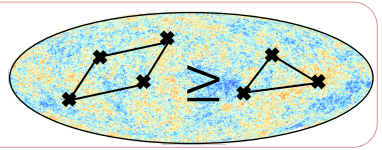
Positivity

$$\mathcal{L}_{\rm EFT} = \dot\pi^2 - c_s^2(\partial_i\pi)^2 + \alpha_1\,\dot\pi^3 + \alpha_2\dot\pi(\partial\pi)^2 + \beta_1\,\dot\pi^4 + \beta_2\dot\pi^2(\partial\pi)^2 + \beta_3(\partial\pi)^4$$

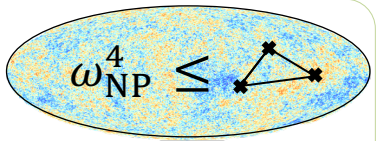
[Cheung++ 2008]

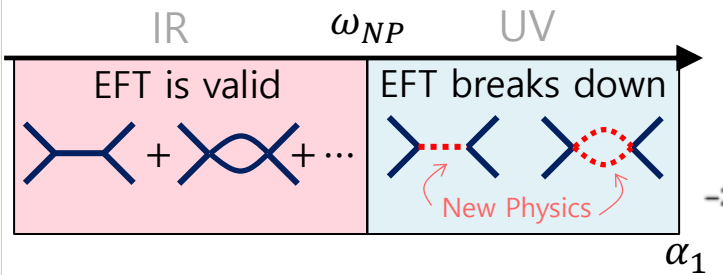


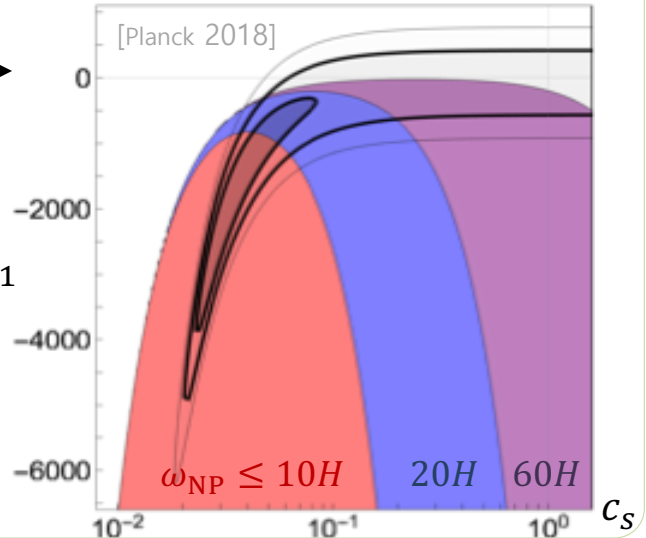
$$\partial_s^2 A_{EFT} > 0 \quad \Rightarrow \quad \beta_1 > \frac{3}{2}\alpha_1^2 - 2\alpha_1 - \frac{1}{3}\frac{1-c_s^2}{c_s^4}$$



$$\partial_s^2 A_{EFT} > \int^{\omega_{NP}} \mathrm{Im}\, A_{\rm EFT} \;\Rightarrow\; \frac{\omega_{\rm NP}^4}{M_P^2|\dot H|} \le \frac{30\pi^2 c_s^4}{\left|1 - c_s^2 + \frac{3}{2}\alpha_1 c_s^2\right|}$$







Positivity requires new physics (beyond single-field weakly-coupled inflation) on subhorizon scales!

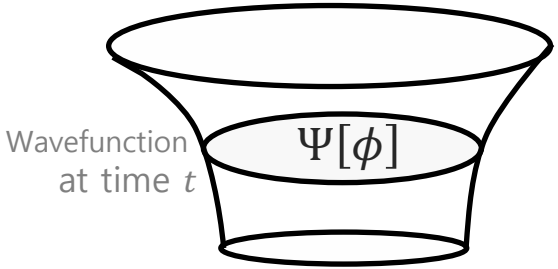
Where do we go from here?

Strategy 3? Amplitudes → Wavefunction

[Pajer+Jazayeri+Goodhew 2021]
[Cespedes+Davis+SM 2021]
[SM+Pajer 2021]
[Pajer+Jazayeri+Goodhew+Lee 2021]**

(**also Arkani Hamed+Baumann+Duaso Pueyo+Joyce+Lee+Pimentel+Stefanyzyn and others)

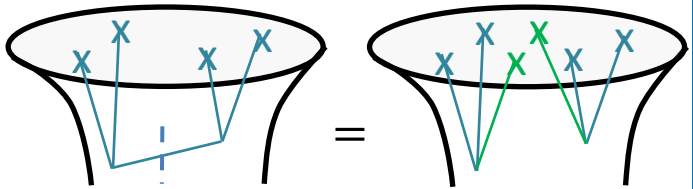
Wavefunction Coefficient



$$\psi_n = \frac{\delta}{\delta\phi_1} \cdots \frac{\delta}{\delta\phi_n} \Psi[\phi] \Big|_{\phi=0}$$

Calculable on horizon scales

Well-defined boundary observable

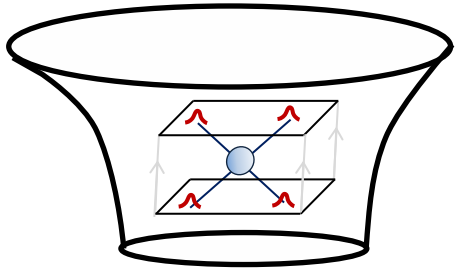


Unitarity ⇒ Cosmological Cutting Rules

Causality ⇒ ???

Need a cosmological dispersion relation

Subhorizon Amplitude

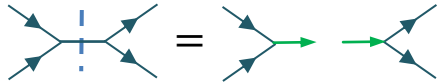


$$A(s, t, \omega_1, \omega_2, \omega_3)$$

Easy to compute (subhorizon)

Analogy with particle physics

e.g. Positivity ingredients:



Unitarity ⇒ Cutting Rules

Causality ⇒ Dispersion relation

(Analytic in complex s)

Where do we go from here?

Strategy 3? Amplitudes → Wavefunction

[Pajer+Jazayeri+Goodhew 2021]
[Cespedes+Davis+SM 2021]
[SM+Pajer 2021]
[Pajer+Jazayeri+Goodhew+Lee 2021]

A Positive Outlook

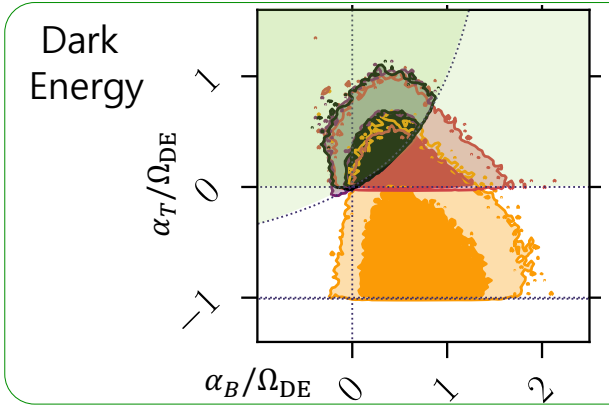
Small-scale (UV)

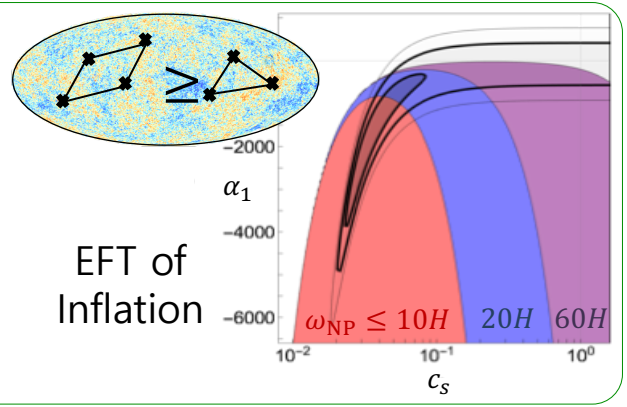
Large-scale (IR)

Unitary + Causal + Local

$$A_{EFT}(s,t) = \sum_{a,b} s^a t^b c_{ab}$$

$c_{20} > 0\,, \quad c_{21} > \frac{3}{2} c_{20}\,, \ \ldots$





Apply Existing Tools

Beyond Horndeski / DHOST

Vector-tensor Theories, …

EFT of Dark Energy

Develop New Tools

Multiple species

Include gravity

Moments/Crossing/EFThedron

Wavefunction Positivity