Curing inflationary degeneracies using Reheating predictions and relic Gravitational Waves

Cosmology from Home 2021

Swagat Saurav Mishra (SRF CSIR)
Inter-University Centre for Astronomy and Astrophysics(**IUCAA**), Pune.

Collaborators: Prof. Varun Sahni (IUCAA) and Prof. Alexei Starobinsky (Landau Institute, Moscow)

July 2021

Paper

"***Curing inflationary degeneracies using reheating predictions and relic gravitational waves***",
S. S. Mishra, V. Sahni and A.A Starobinsky,
JCAP 05 (2021) 075
[arXiv:2101:00271 [gr-qc]]

Prologue

- Theoretical and observational developments in the past three decades $\Rightarrow$ standard model of Cosmology – **flat ΛCDM model**.

Prologue

- Theoretical and observational developments in the past three decades ⇒ standard model of Cosmology – **flat ΛCDM model**.
  1. **Homogeneity and Isotropy on large length scales**
  2. **Spatial flatness**
  3. **Almost scale invariant, nearly Gaussian and adiabatic initial density fluctuations**
- **Cosmic Inflation** as the leading paradigm of the early universe

Prologue

- Theoretical and observational developments in the past three decades ⇒ standard model of Cosmology – **flat ΛCDM model**.
  1. **Homogeneity and Isotropy on large length scales**
  2. **Spatial flatness**
  3. **Almost scale invariant, nearly Gaussian and adiabatic initial density fluctuations**
- **Cosmic Inflation** as the leading paradigm of the early universe ⇒ appropriate initial conditions for hot Big Bang phase.

Inflationary Dynamics of a Scalar Field

Standard scenario – A single canonical scalar field minimally coupled to gravit

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

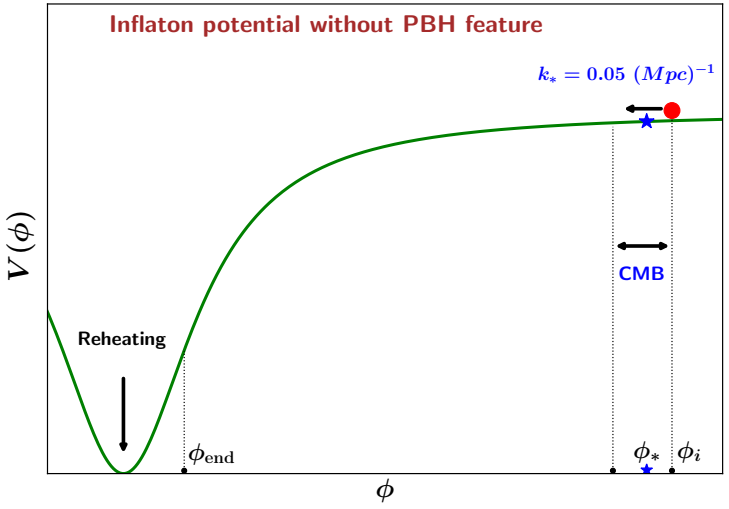
And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi,$$

$$\dot{H} = -\frac{1}{2m_p^2}\dot{\phi}^2$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)\left(\rho_\phi + 3\,p_\phi\right)$$

$$\frac{\ddot{a}}{a} = H^2\left(1 - \epsilon_H\right)$$



Condition for Inflation

$$\epsilon_H = -\frac{\dot{H}}{H^2} < 1 \Rightarrow \dot{\phi}^2 < V(\phi)$$

The dynamics of the scalar field is governed by

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

Slow-roll Inflation

Slow-roll Regime of Inflation

Under slow-roll approximation –

$$\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1 \ , \quad \eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

Slow-roll Inflation

Slow-roll Regime of Inflation

Under slow-roll approximation –

$$\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1 \ , \quad \eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

Power-spectrum is

$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2} \left|\mathcal{R}_k\right|^2_{k \ll aH}$$

Slow-roll Inflation

Slow-roll Regime of Inflation

Under slow-roll approximation –

$$\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1 \ , \quad \eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

Power-spectrum is

$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2} \left|\mathcal{R}_k\right|^2_{k \ll aH}$$

During the slow-roll regime, $\epsilon_H, \ \eta_H \ll 1$,

$$P_{\mathcal{R}} = \frac{1}{8\pi^2} \left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_H}\bigg|_{k=aH} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{m_p}\right)^2 \frac{m_p^2}{\dot{\phi}^2}$$

$$P_{\mathcal{T}} = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2$$

Inflationary Power-spectrum

During slow-roll $\epsilon_H, \eta_H \ll 1$
Primordial Power-spectra at least at large scales –

$$P_{\mathcal{R}} = A_s \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$$P_t = A_t \left(\frac{k}{k_*}\right)^{n_t}$$

CMB pivot scale $k_* = 0.05$ Mpc^{-1}

Inflationary Power-spectrum

During slow-roll $\epsilon_H, \eta_H \ll 1$
Primordial Power-spectra at least at large scales –

$$P_{\mathcal{R}} = A_s \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$$P_t = A_t \left(\frac{k}{k_*}\right)^{n_t}$$

CMB pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$

$$A_s = \frac{1}{8\pi^2}\left(\frac{H_*}{m_p}\right)^2 \frac{1}{\epsilon_H}$$

$$A_t = \frac{2}{\pi^2}\left(\frac{H_*}{m_p}\right)^2$$

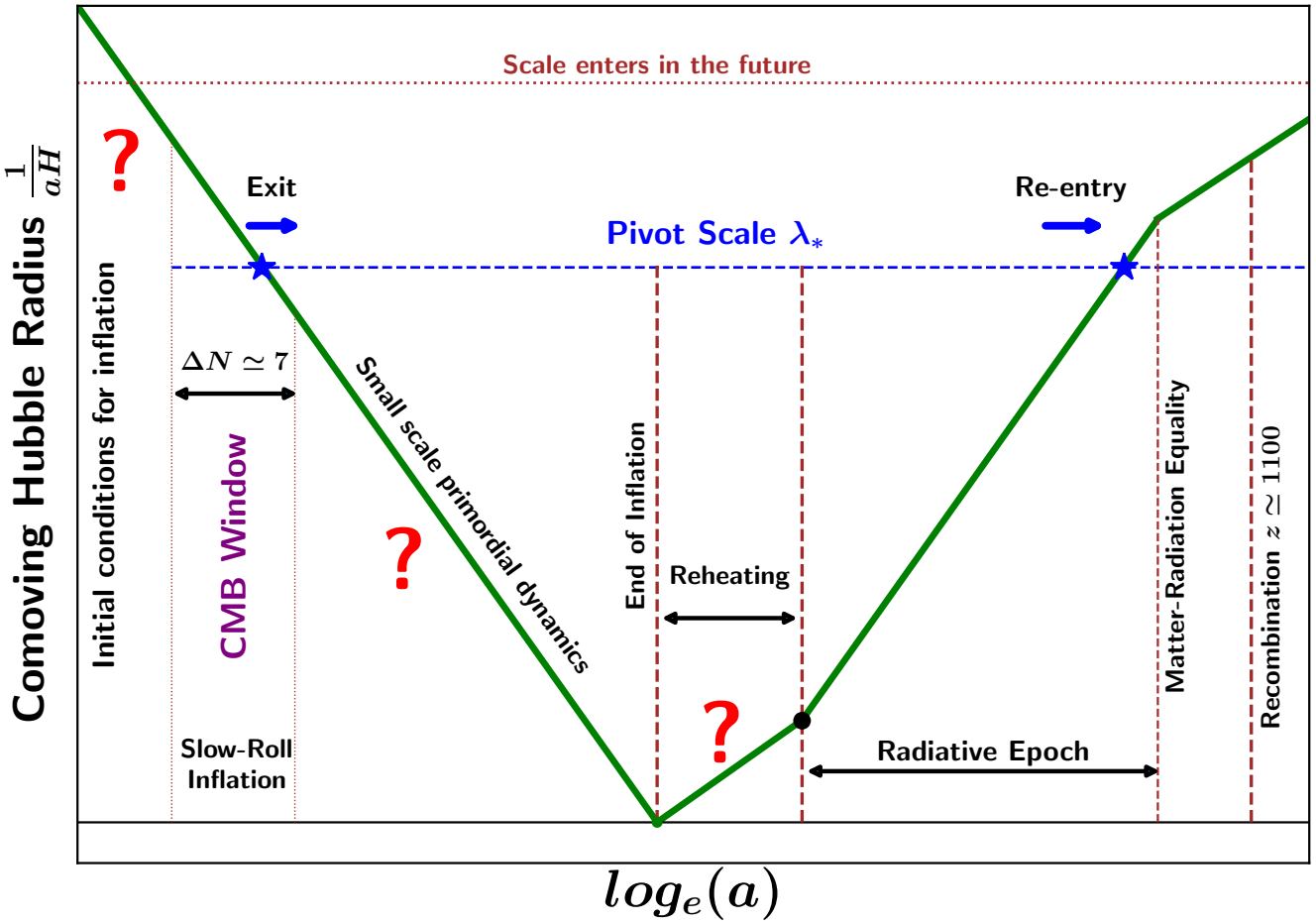
Scalar Spectral Index

$$n_S = 1 + 2\eta_H(k_*) - 4\epsilon_H(k_*)$$

Tensor to Scalar Ratio

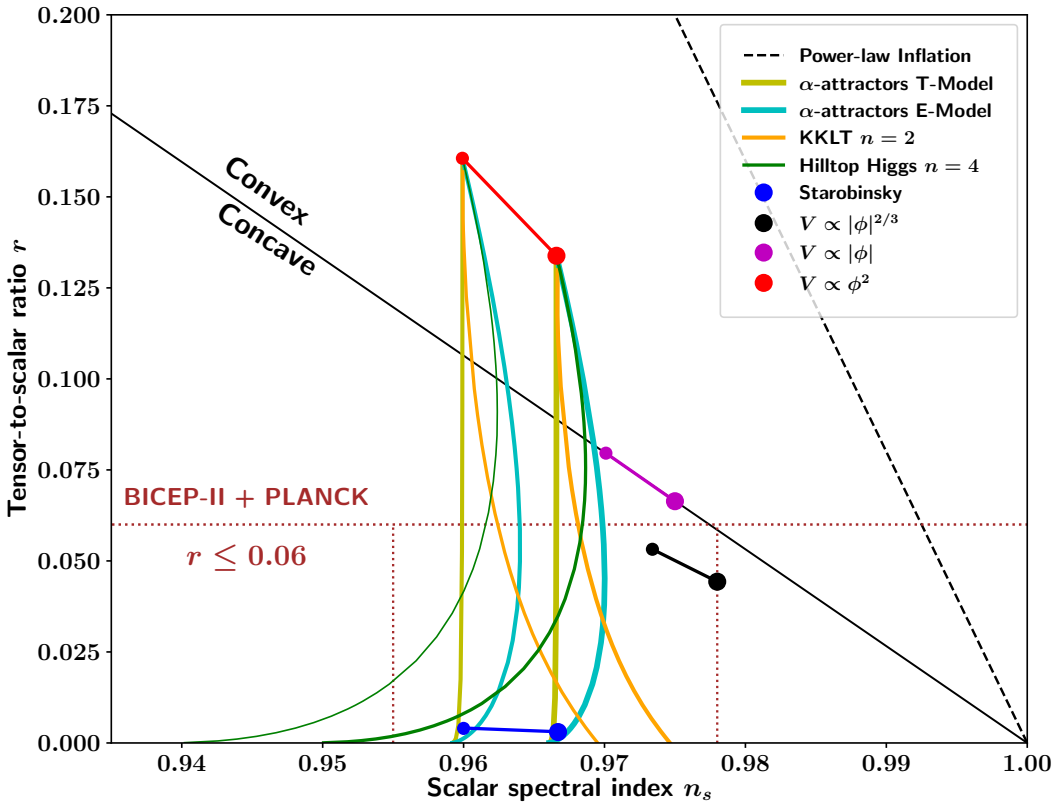
$$r = 16\epsilon_H(k_*)$$

Behaviour of fluctuations

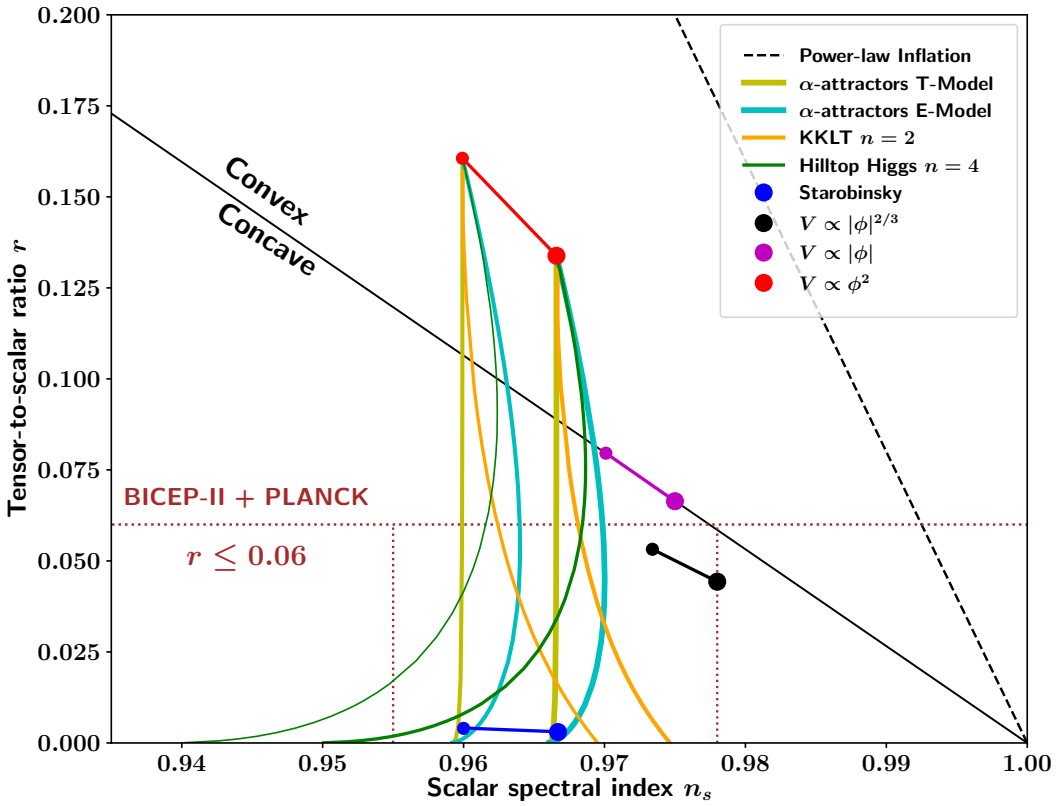


What we know from Observations

What we know from Observations



What we know from Observations



$r \leq 0.06 \Rightarrow$**Asymptotically flat** Concave potentials, nearly scale-invariant but slightly red tilted scalar power spectrum with $n_s \simeq 0.967$ near the CMB pivot scale.

$$H_k^{\rm inf} \leq 2.5 \times 10^{-5}\ m_p = 6.1 \times 10^{13}\ \text{GeV}$$

Inflationary Degeneracies

- Inflationary paradigm often displays degeneracies in its predictions of $\{n_S, r\}$.

Inflationary Degeneracies

- Inflationary paradigm often displays degeneracies in its predictions of $\{n_s, r\}$.
- Either because several models predict same values of $\{n_s, r\}$

Inflationary Degeneracies

- Inflationary paradigm often displays degeneracies in its predictions of $\{n_s, r\}$.
- Either because several models predict same values of $\{n_s, r\}$ or for a given model, $\{n_s, r\}$ insensitive to some model parameter λ.

Inflationary Degeneracies

- Inflationary paradigm often displays degeneracies in its predictions of $\{n_S, r\}$.
- Either because several models predict same values of $\{n_S, r\}$ or for a given model, $\{n_S, r\}$ insensitive to some model parameter λ.
- $\Rightarrow$ CMB observations can not constitute a unique probe of the physics during inflation.

Inflationary Degeneracies

- Inflationary paradigm often displays degeneracies in its predictions of $\{n_s, r\}$.
- Either because several models predict same values of $\{n_s, r\}$ or for a given model, $\{n_s, r\}$ insensitive to some model parameter λ.
- $\Rightarrow$ CMB observations can not constitute a unique probe of the physics during inflation.
- Let's see a demonstration of such degeneracies.

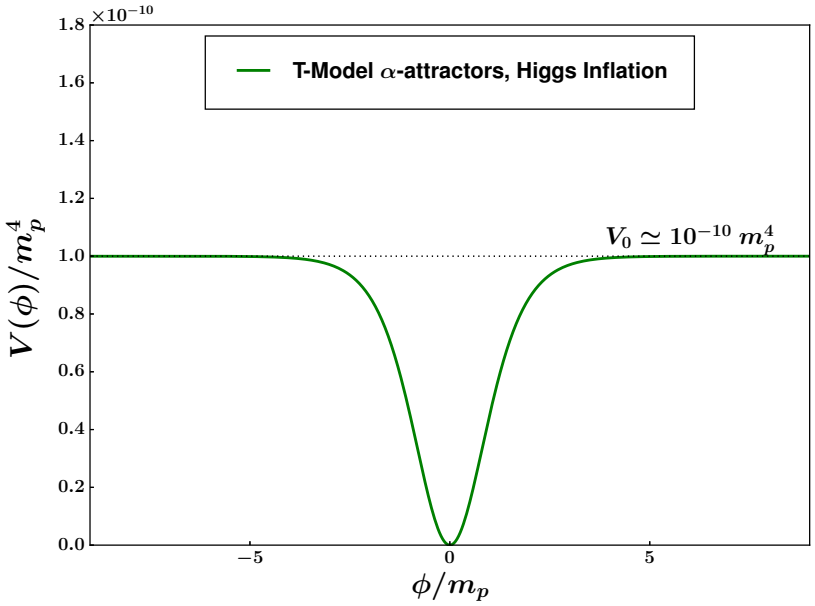
Degeneracy in the T-model

T-model potential $V(\phi) = V_0 \tanh^{2p}(\lambda\phi/m_p)\ ; \quad p = 1, 2, 3....$

$$V(\phi) \simeq V_0,\ |\lambda\phi| \gg m_p$$

.

$$V(\phi) \simeq V_0 \left(\frac{\lambda\phi}{m_p}\right)^{2p},\ |\lambda\phi| \ll m_p$$



Mean EOS

$$\langle w_\phi \rangle = \frac{p-1}{p+1}\ .$$

Degeneracy in the T-model

T-model potential $V(\phi) = V_0 \tanh^{2p}(\lambda\phi/m_p)\ ; \quad p = 1, 2, 3....$

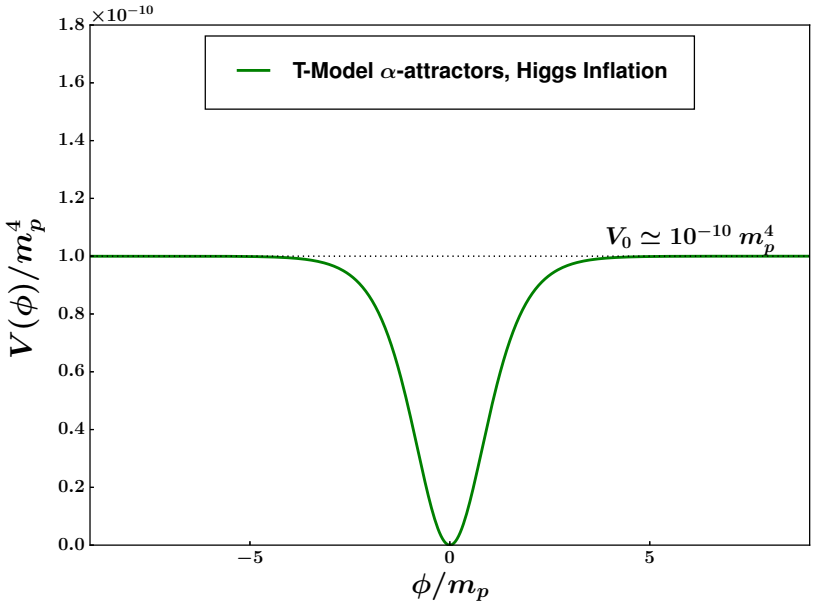
$$V(\phi) \simeq V_0,\ |\lambda\phi| \gg m_p$$

.

$$V(\phi) \simeq V_0 \left(\frac{\lambda\phi}{m_p}\right)^{2p},\ |\lambda\phi| \ll m_p$$

Mean EOS

$$\langle w_\phi \rangle = \frac{p-1}{p+1}\ .$$



When $\lambda > 0.1$,

$$n_s - 1 \simeq -\frac{2}{N_k},\ r \simeq \frac{2}{N_k^2\,\lambda^2}$$

Degeneracy in the T-model

T-model potential $V(\phi) = V_0 \tanh^{2p}(\lambda\phi/m_p)\ ; \quad p = 1, 2, 3....$

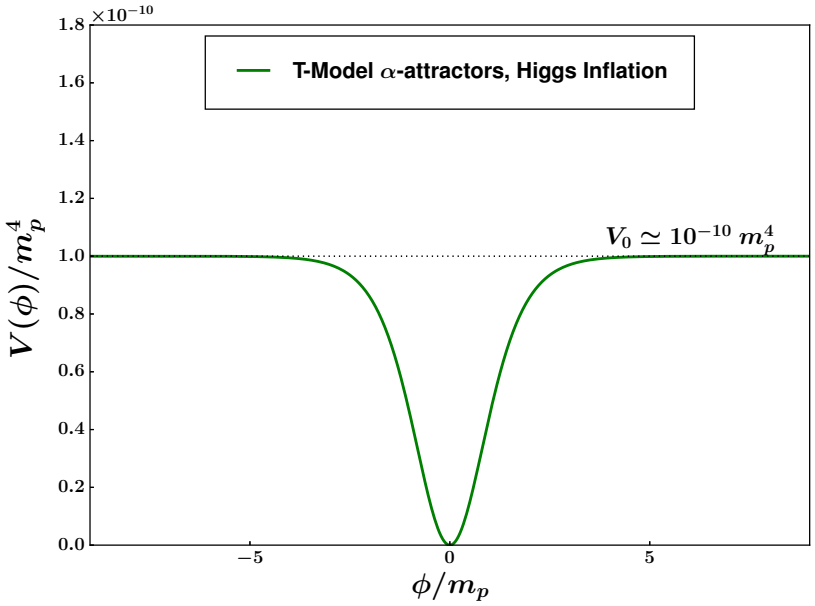
$$V(\phi) \simeq V_0,\ |\lambda\phi| \gg m_p$$

.

$$V(\phi) \simeq V_0 \left(\frac{\lambda\phi}{m_p}\right)^{2p},\ |\lambda\phi| \ll m_p$$

Mean EOS

$$\langle w_\phi \rangle = \frac{p-1}{p+1}\ .$$



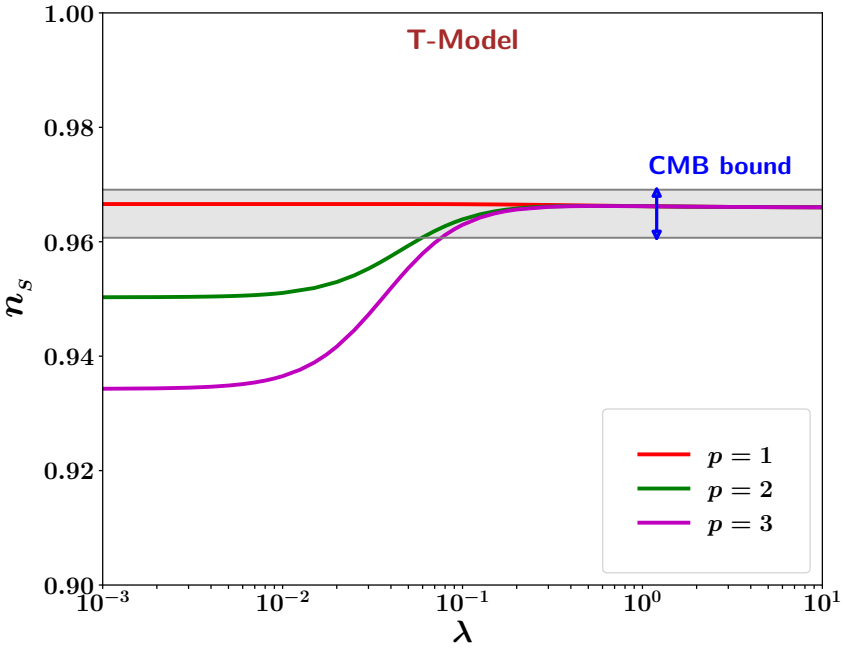
When $\lambda > 0.1$,

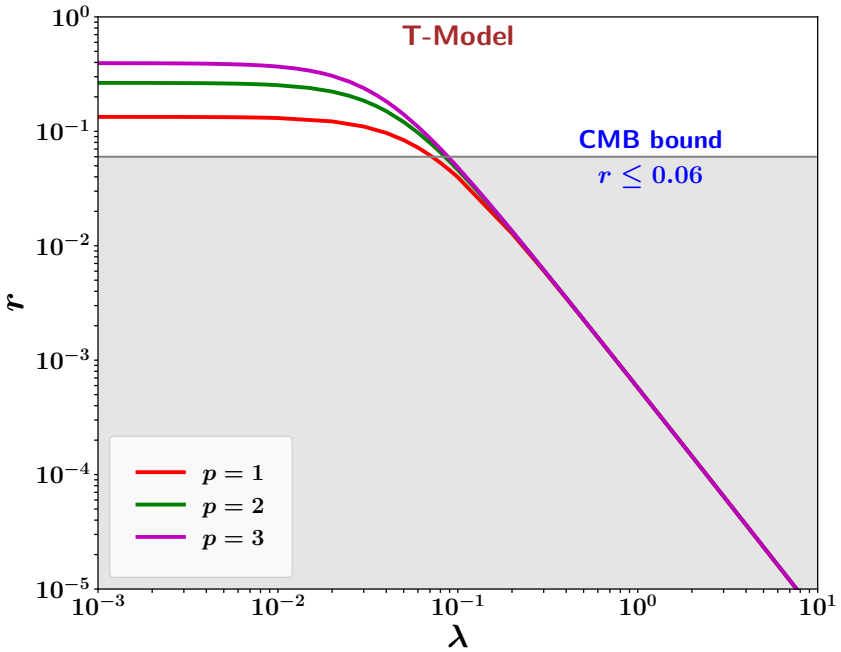
$$n_s - 1 \simeq -\frac{2}{N_k},\ r \simeq \frac{2}{N_k^2\,\lambda^2}$$

Independent of 'p'

Degeneracy in the T-model

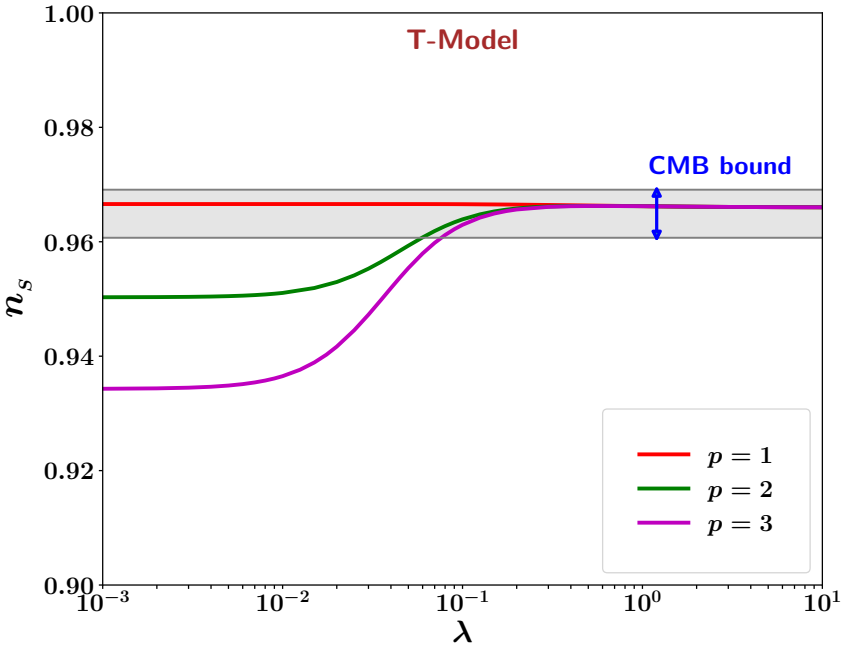
T-model potential $V(\phi) = V_0 \tanh^{2p}(\lambda\phi/m_p)\ ; \quad p = 1, 2, 3....$

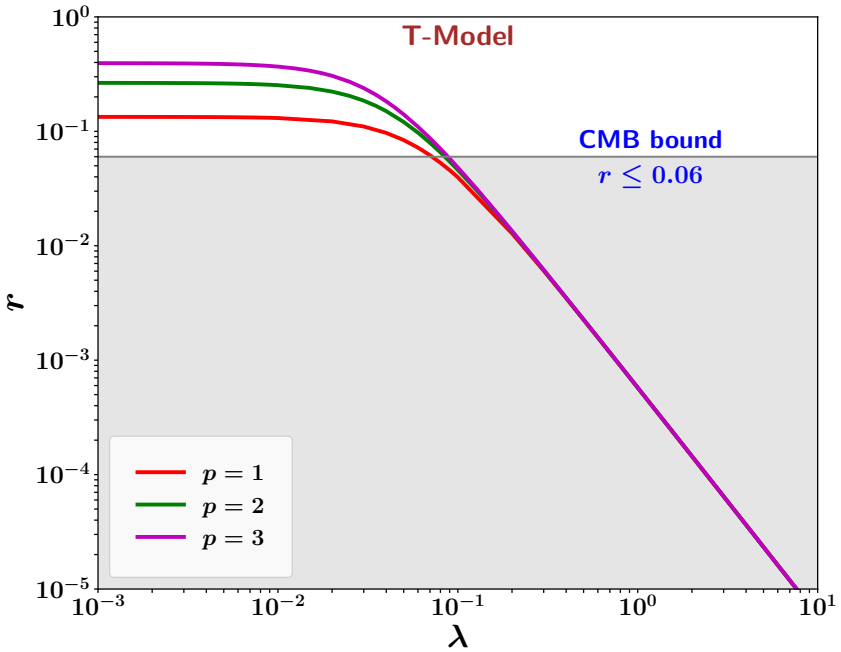




Degeneracy in the T-model

T-model potential $V(\phi) = V_0 \tanh^{2p}(\lambda\phi/m_p)\ ; \quad p = 1, 2, 3....$





$\{n_S, r\}$ **independent of 'p'**

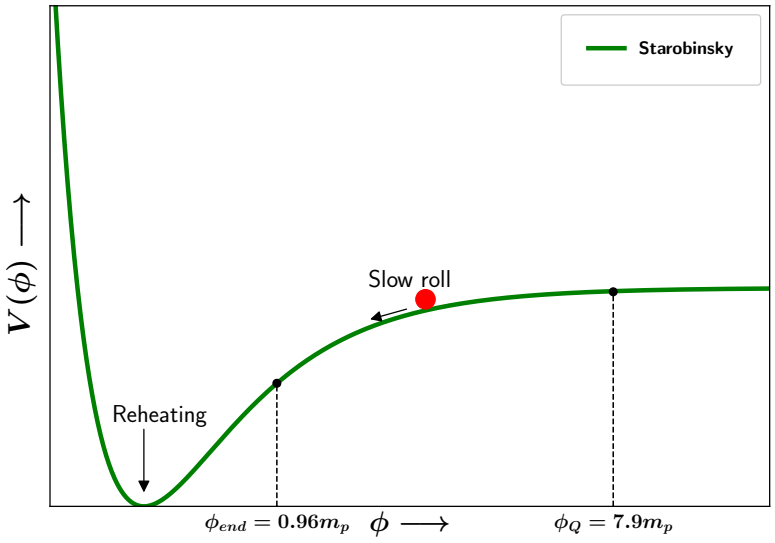
Degeneracy in the E-model

E-model potential $V(\phi) = V_0 \left[1 - \exp\left(-\lambda \frac{\phi}{m_p}\right)\right]^{2p}$

$$V(\phi) \simeq V_0, \ \lambda\phi \gg m_p$$

.

$$V(\phi) \simeq V_0 \left(\frac{\lambda\phi}{m_p}\right)^{2p}, \ |\lambda\phi| \ll m_p$$



Mean EOS

$$\langle w_\phi \rangle = \frac{p-1}{p+1} \ .$$

Degeneracy in the E-model

E-model potential $V(\phi) = V_0 \left[1 - \exp\left(-\lambda \frac{\phi}{m_p}\right)\right]^{2p}$

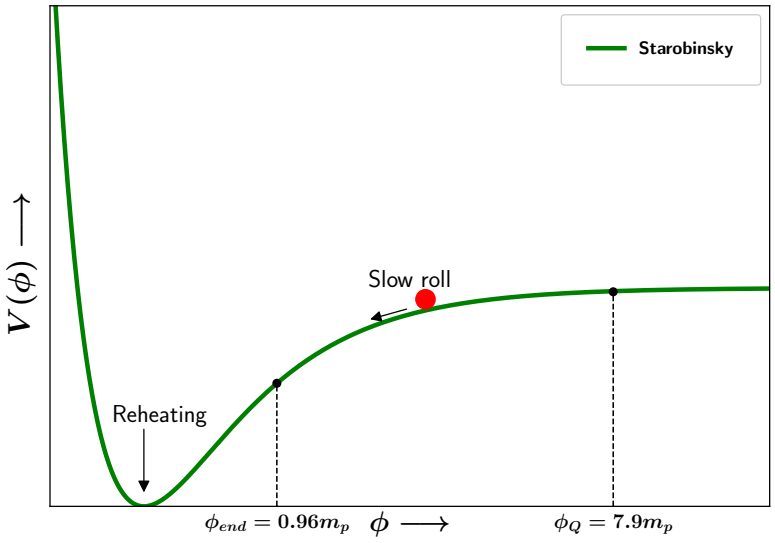
$$V(\phi) \simeq V_0, \ \lambda\phi \gg m_p$$

.

$$V(\phi) \simeq V_0 \left(\frac{\lambda\phi}{m_p}\right)^{2p}, \ |\lambda\phi| \ll m_p$$

Mean EOS

$$\langle w_\phi \rangle = \frac{p-1}{p+1} \ .$$



When $\lambda > 0.5$,

$$n_s - 1 \simeq -\frac{2}{N_k}, \ r \simeq \frac{8}{N_k^2 \lambda^2}$$

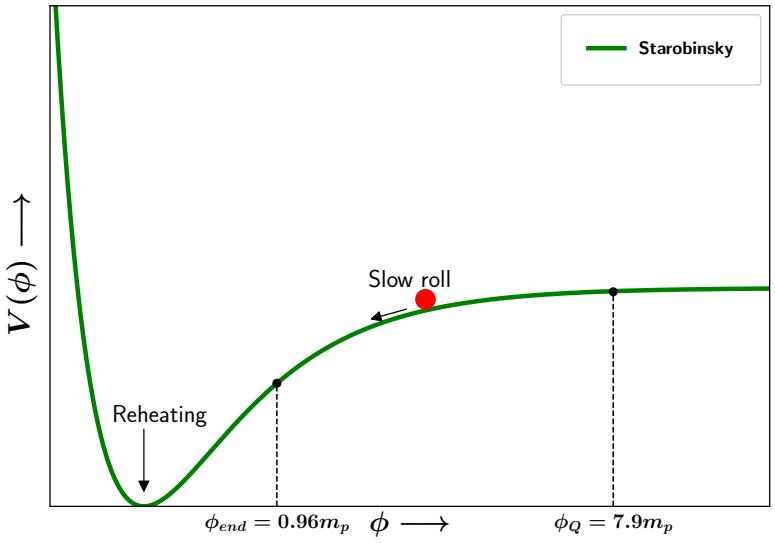
Degeneracy in the E-model

E-model potential $V(\phi) = V_0 \left[1 - \exp\left(-\lambda \frac{\phi}{m_p}\right)\right]^{2p}$

$$V(\phi) \simeq V_0, \ \lambda\phi \gg m_p$$

.

$$V(\phi) \simeq V_0 \left(\frac{\lambda\phi}{m_p}\right)^{2p}, \ |\lambda\phi| \ll m_p$$



Mean EOS

$$\langle w_\phi \rangle = \frac{p-1}{p+1} \ .$$

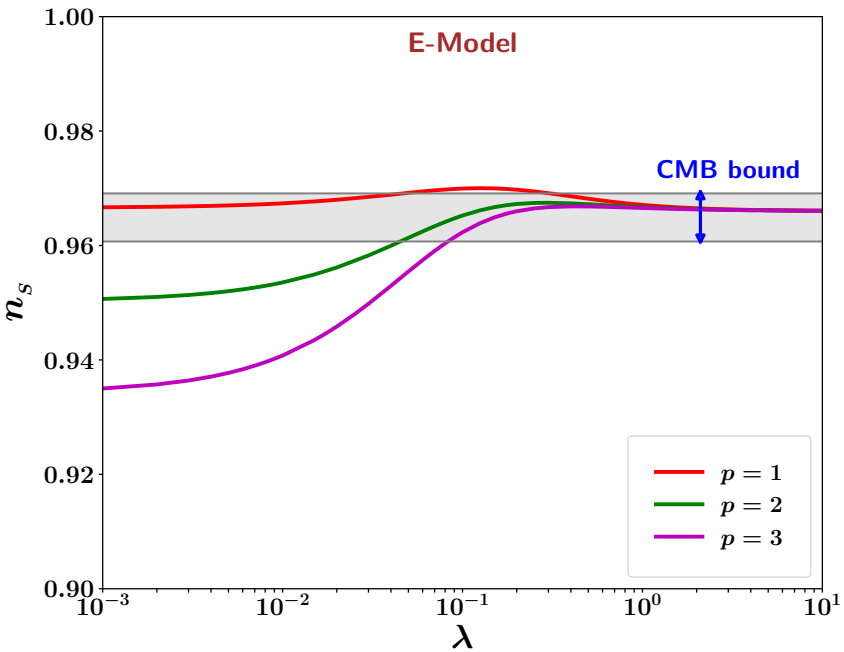
When $\lambda > 0.5$,

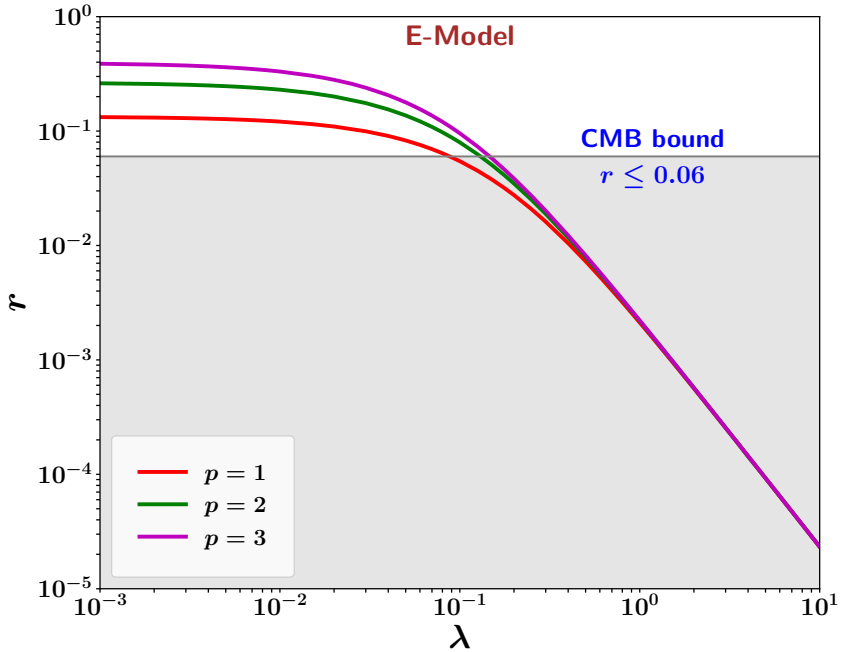
$$n_s - 1 \simeq -\frac{2}{N_k}, \ r \simeq \frac{8}{N_k^2 \lambda^2}$$

Independent of 'p'

Degeneracy in the E-model

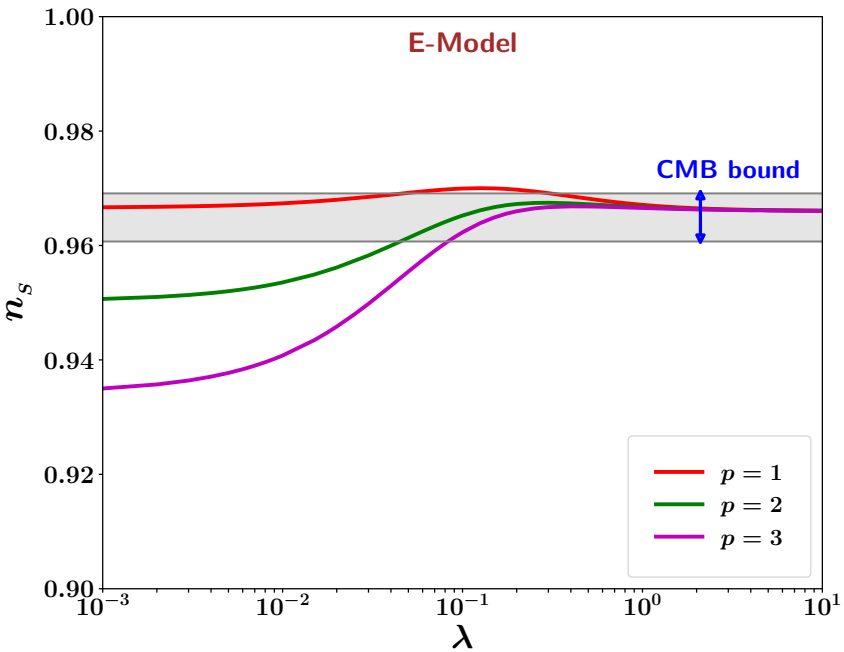
E-model potential $V(\phi) = V_0 \left[1 - \exp\left(-\lambda \frac{\phi}{m_p}\right)\right]^{2p}$

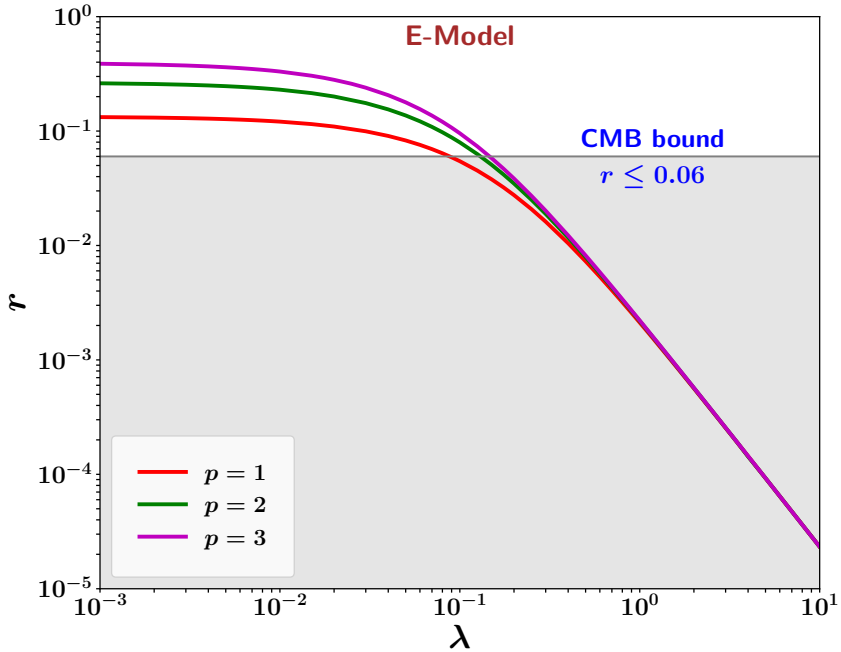




Degeneracy in the E-model

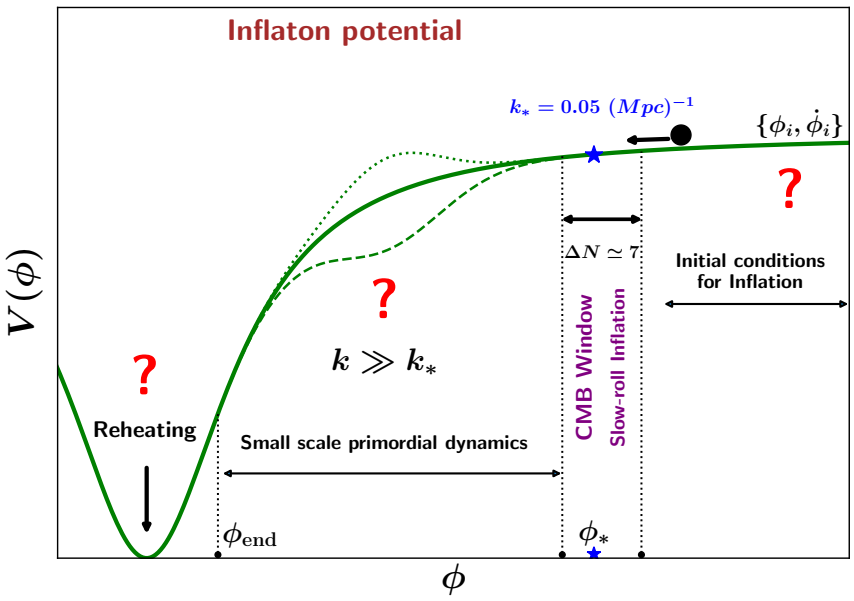
E-model potential $V(\phi) = V_0 \left[1 - \exp\left(-\lambda \frac{\phi}{m_p}\right)\right]^{2p}$

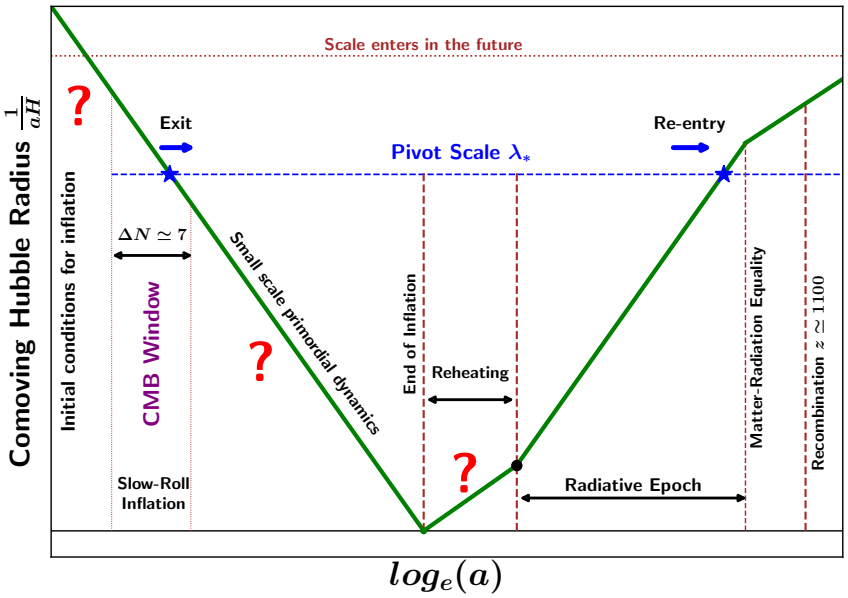




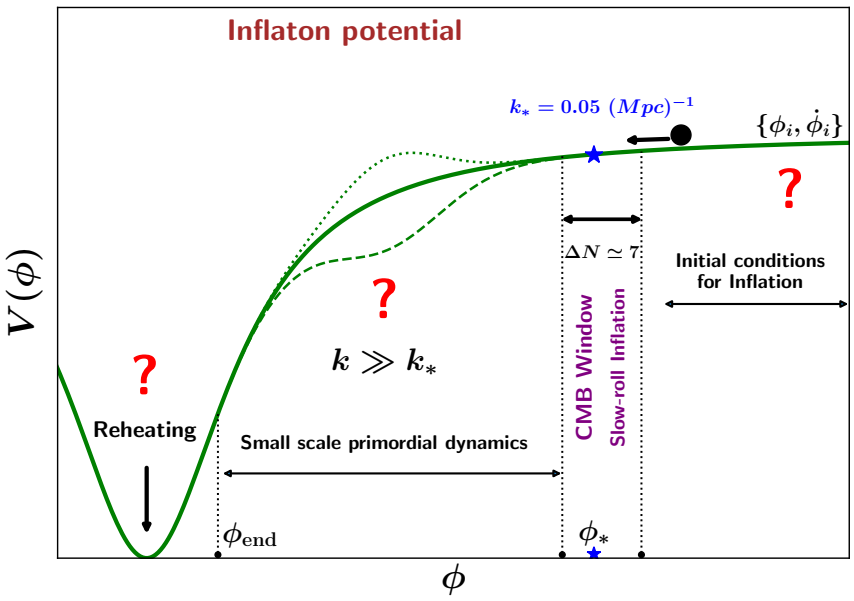
$\{n_S, r\}$ **independent of 'p'**

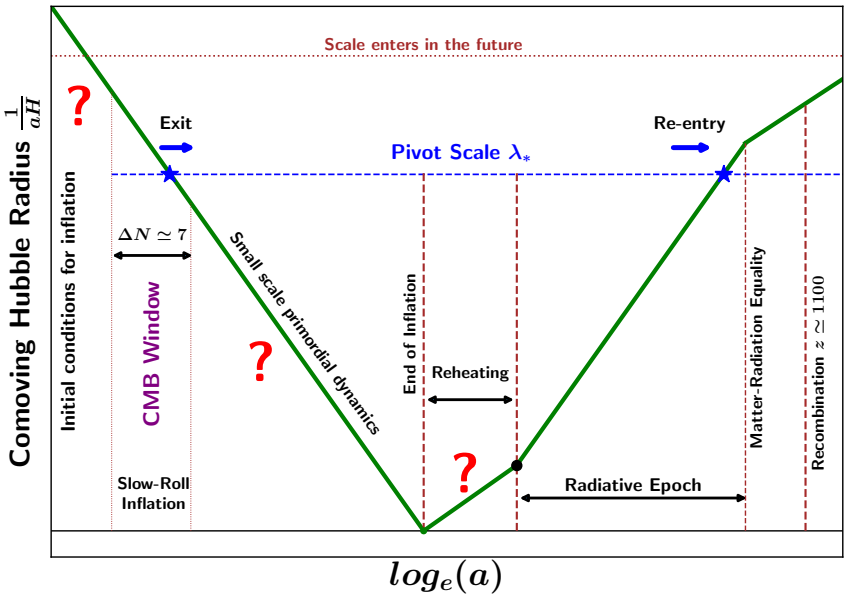
Post-inflationary Evolution





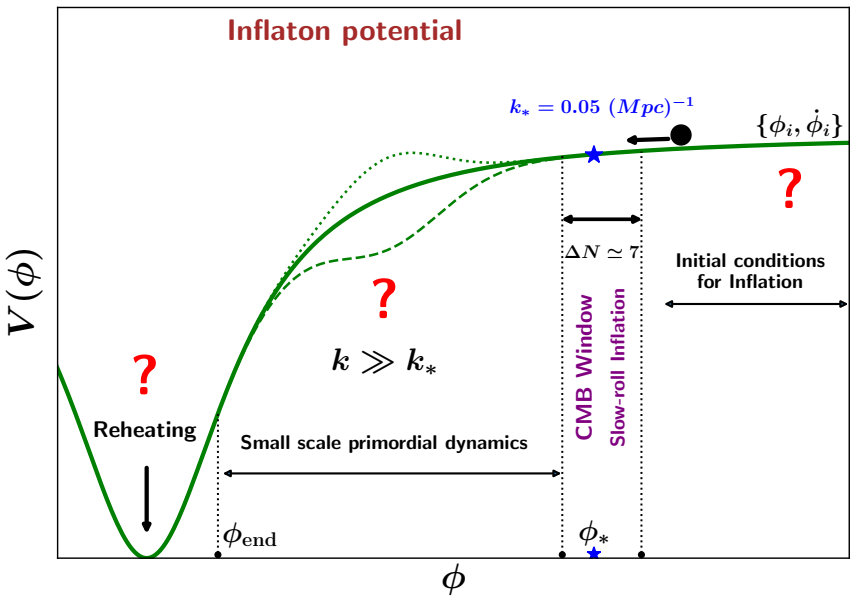
Post-inflationary Evolution

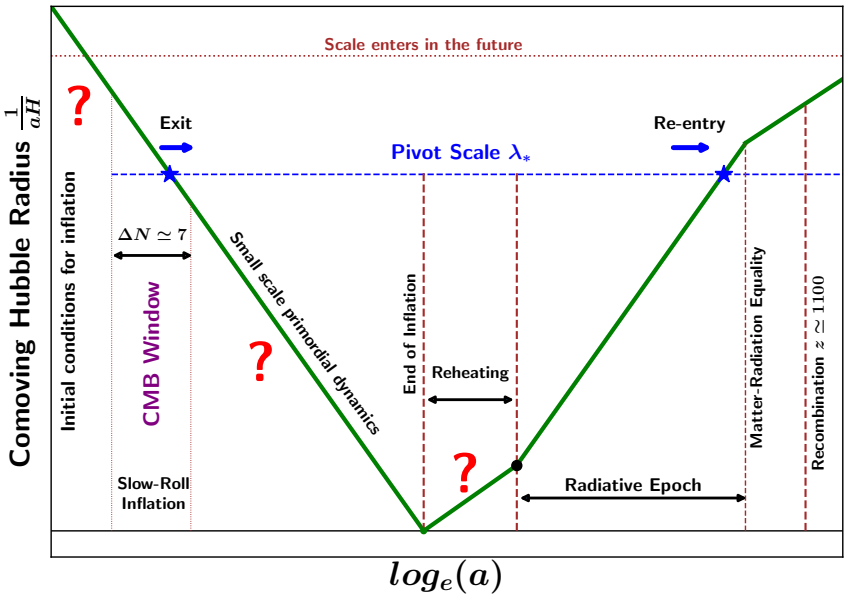




Inflation $\rightarrow$ Reheating $\rightarrow$ hot Big Bang

Post-inflationary Evolution

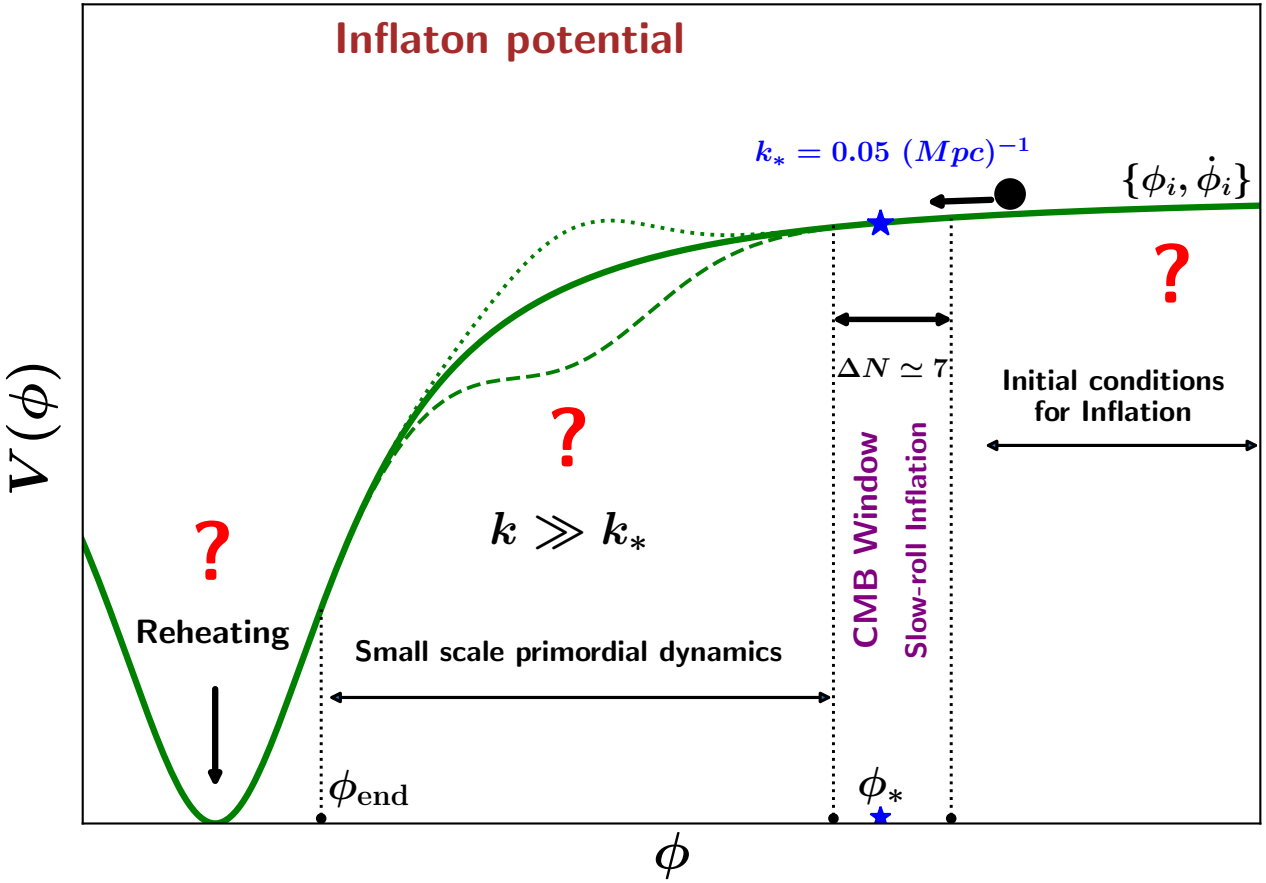




Inflation $\rightarrow$ Reheating $\rightarrow$ hot Big Bang

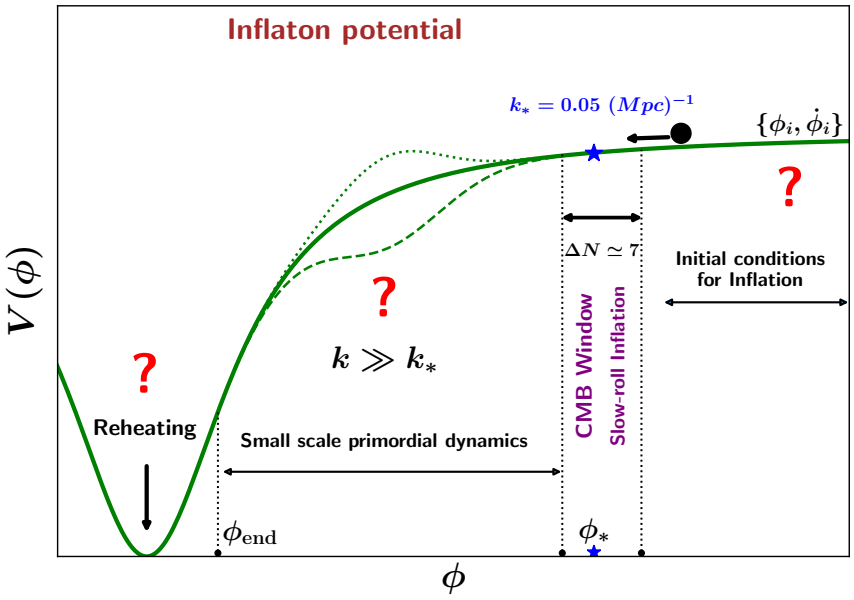
Two implications

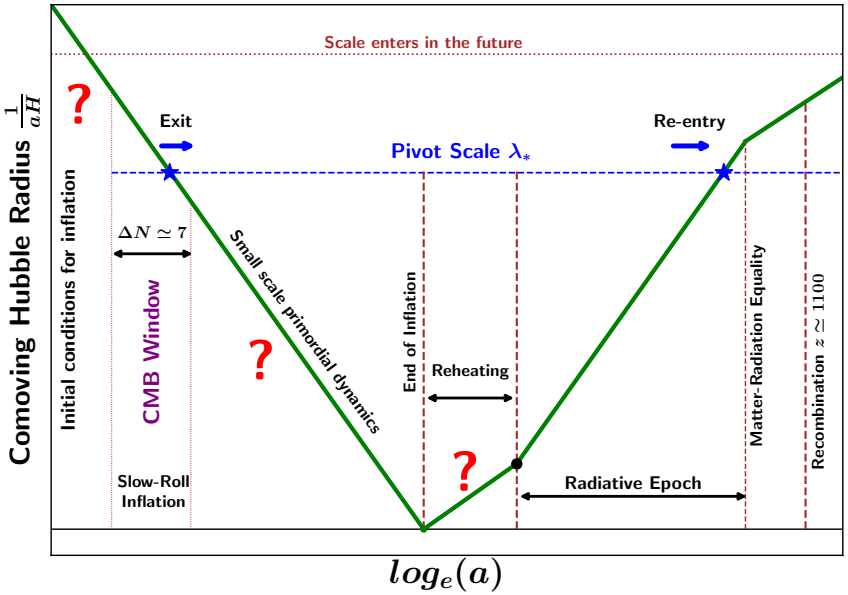
Post-inflationary Evolution



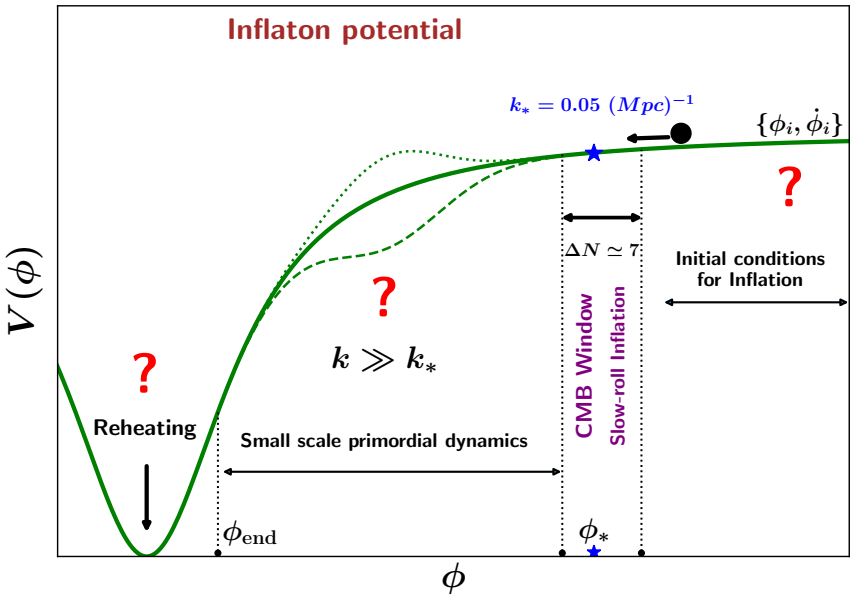
1) Where is ϕ_*

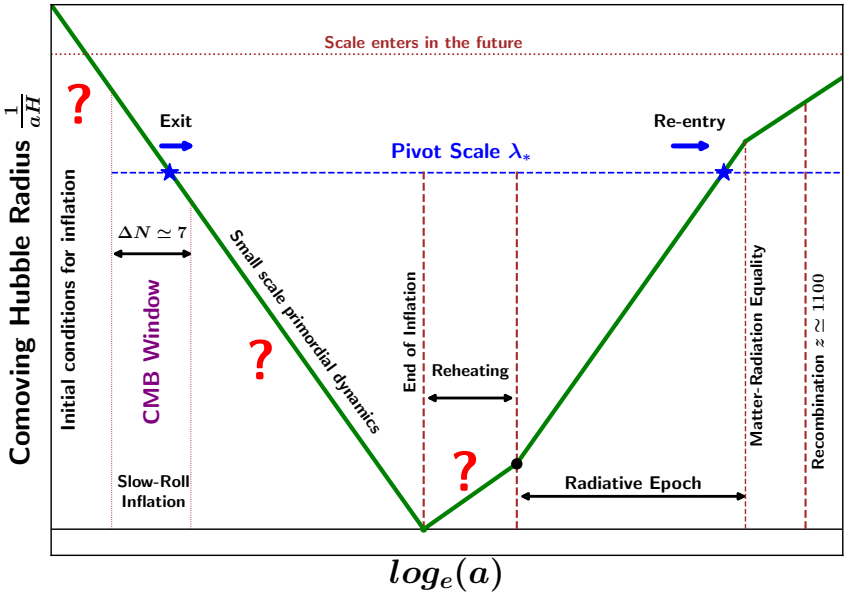
Post-inflationary Evolution





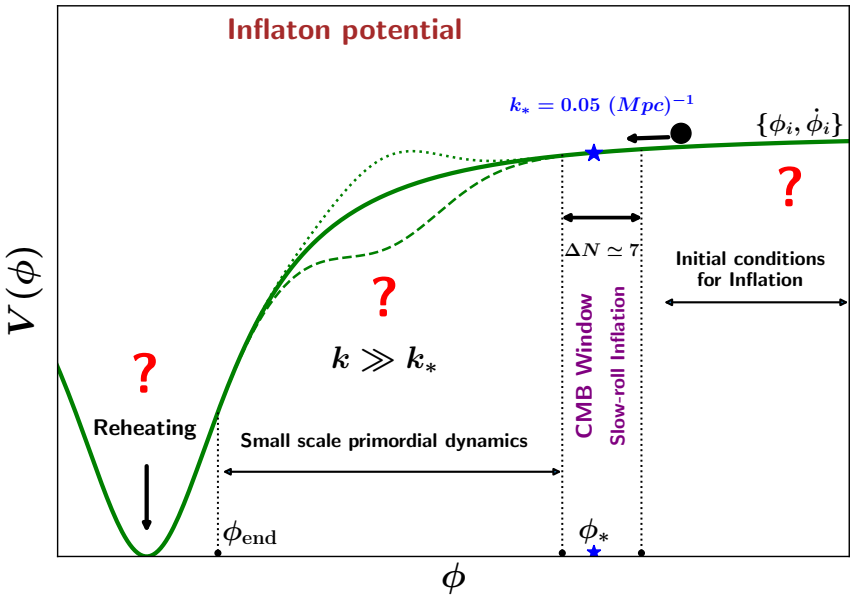
Post-inflationary Evolution

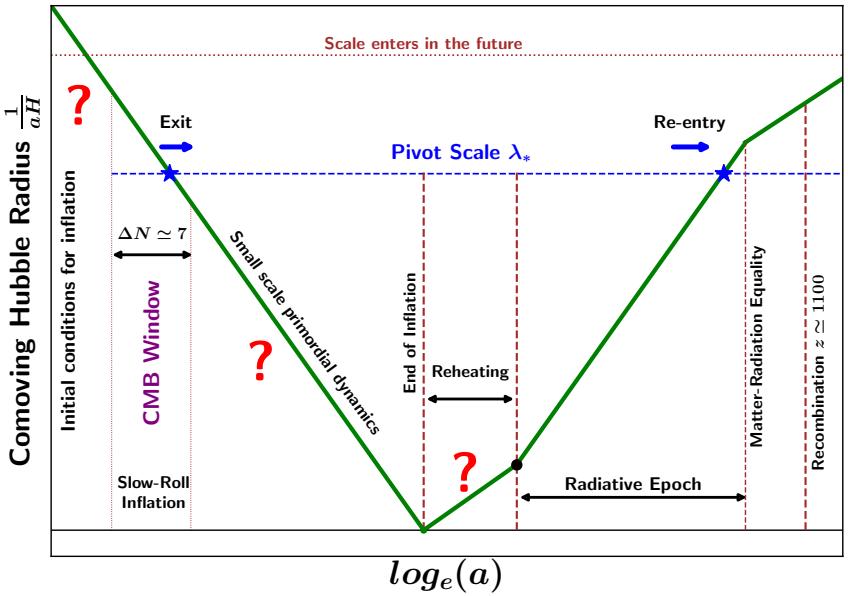




Two implications **1) Where is ϕ_* 2) Breaking degeneracies**

Post-inflationary Evolution





Two implications **1) Where is ϕ_* 2) Breaking degeneracies**

Physics of Reheating

Physics of Reheating

Decay of inflaton field ultimately to the standard model d.o.f and beginning on 'Hot Big Bang' phase.
$\phi \longrightarrow \chi, \psi, \delta\phi$ etc.

- **Perturbative Reheating**

Physics of Reheating

Decay of inflaton field ultimately to the standard model d.o.f and beginning on 'Hot Big Bang' phase.
$\phi \longrightarrow \chi, \psi, \delta\phi$ etc.

- **Perturbative Reheating**
  Elementary theory of decay of Inflaton to other matter fields ($m_\phi \gg m_\chi$, m_ψ)

Physics of Reheating

Decay of inflaton field ultimately to the standard model d.o.f and beginning on 'Hot Big Bang' phase.
$\phi \longrightarrow \chi, \psi, \delta\phi$ etc.

- **Perturbative Reheating**
  Elementary theory of decay of Inflaton to other matter fields ($m_\phi \gg m_\chi$, m_ψ)
  Reheating ends when $\Gamma = H$ and reheating temperature is

$$T_{\rm re} \simeq 0.1\sqrt{\Gamma\, m_p}$$

Physics of Reheating

Decay of inflaton field ultimately to the standard model d.o.f and beginning on 'Hot Big Bang' phase.
$\phi \longrightarrow \chi, \psi, \delta\phi$ etc.

- **Perturbative Reheating**
  Elementary theory of decay of Inflaton to other matter fields ($m_\phi \gg m_\chi$, m_ψ)
  Reheating ends when $\Gamma = H$ and reheating temperature is

$$T_{\rm re} \simeq 0.1\sqrt{\Gamma\, m_p}$$

Typically $T_{\rm re} \leq 10^9$ GeV

Physics of Reheating

Decay of inflaton field ultimately to the standard model d.o.f and beginning on 'Hot Big Bang' phase.
$\phi \longrightarrow \chi, \psi, \delta\phi$ etc.

- **Perturbative Reheating**
  Elementary theory of decay of Inflaton to other matter fields ($m_\phi \gg m_\chi$, m_ψ)
  Reheating ends when $\Gamma = H$ and reheating temperature is

$$T_{\rm re} \simeq 0.1\sqrt{\Gamma\, m_p}$$

  Typically $T_{\rm re} \leq 10^9$ GeV
- **Non-perturbative Reheating**

  Parametric Resonance,
  Instant Preheating (Non-oscillatory potentials)

Perturbative Reheating

Perturbative Reheating

If the inflaton is oscillating around

$$V(\phi) \propto \phi^{2p}$$

then the average equation of state of universe during reheating is

$$w_{\text{re}} = \frac{p-1}{p+1}$$

Perturbative Reheating

If the inflaton is oscillating around

$$V(\phi) \propto \phi^{2p}$$

then the average equation of state of universe during reheating is

$$w_{\rm re} = \frac{p-1}{p+1}$$

$$w_{\rm re} = \begin{cases} 0\,, & p = 1\,, \\ \frac{1}{3}\,, & p = 2\,, \\ \frac{1}{2}\,, & p = 3 \end{cases}$$

Perturbative Reheating

If the inflaton is oscillating around

$$V(\phi) \propto \phi^{2p}$$

then the average equation of state of universe during reheating is

$$w_{\rm re} = \frac{p-1}{p+1}$$

$$w_{\rm re} = \begin{cases} 0\,, & p = 1\,, \\ \frac{1}{3}\,, & p = 2\,, \\ \frac{1}{2}\,, & p = 3 \end{cases}$$

$w_{\rm re} < 1/3$ if $p < 2$ **(shallow)**

Perturbative Reheating

If the inflaton is oscillating around

$$V(\phi) \propto \phi^{2p}$$

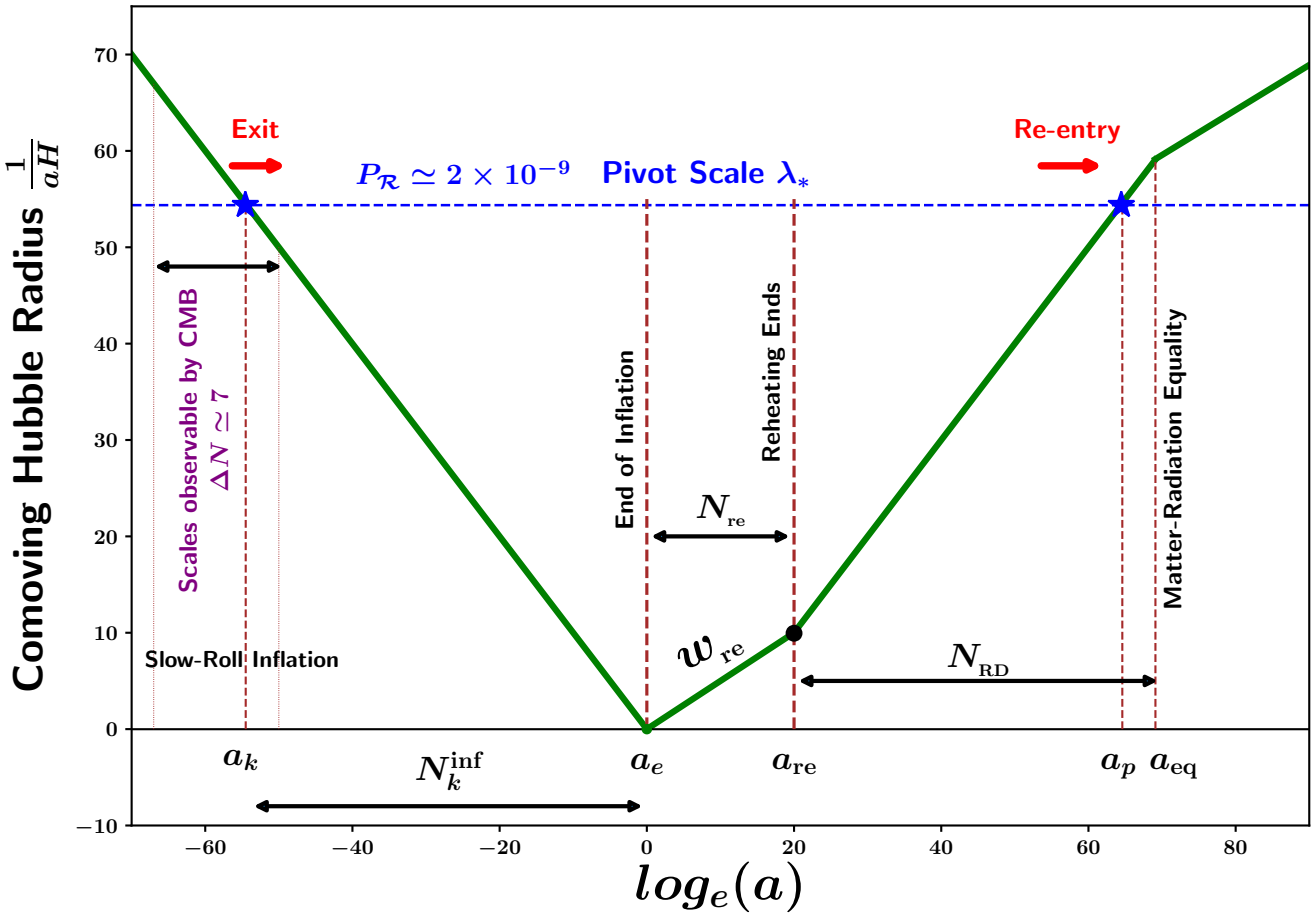
then the average equation of state of universe during reheating is

$$w_{\rm re} = \frac{p-1}{p+1}$$

$$w_{\rm re} = \begin{cases} 0\,, & p = 1\,, \\ \frac{1}{3}\,, & p = 2\,, \\ \frac{1}{2}\,, & p = 3 \end{cases}$$

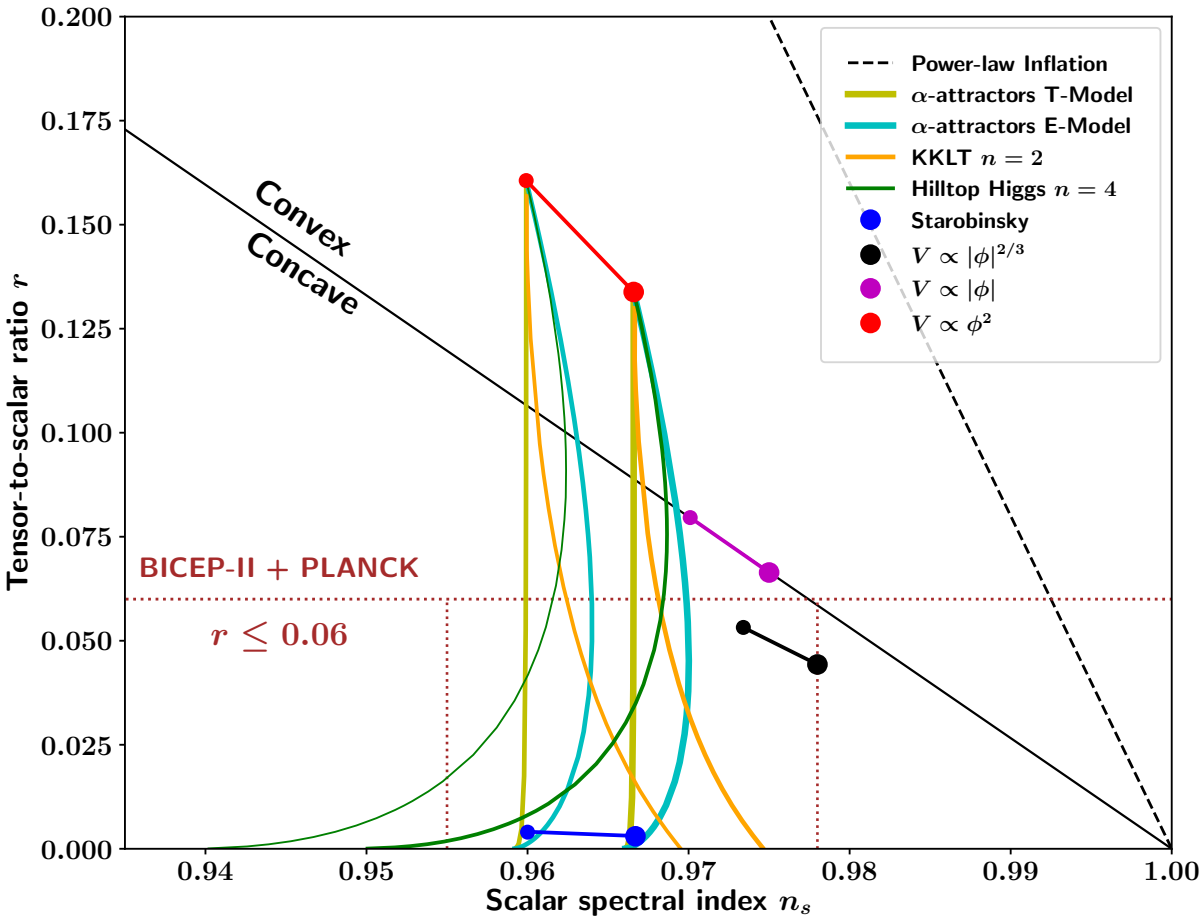
$w_{\rm re} < 1/3$ if $p < 2$ **(shallow)** $\quad w_{\rm re} > 1/3$ if $p > 2$ **(stiff)**

Reheating



Reheating parameters $\{w_{\rm re}, N_{\rm re}, T_{\rm re}\}$

The usual 50–60 e-folds



Reheating parameters $\{w_{\rm re}, N_{\rm re}, T_{\rm re}\}$

Relation between reheating parameters

If $w_{\rm re} \neq \frac{1}{3}$,

Relation between reheating parameters

If $w_{\rm re} \neq \frac{1}{3}$, the **duration of reheating**

$$N_{\rm re} = \frac{4}{1-3w_{\rm re}}\left[61.55 - N_k^{\rm inf} - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}}\right)\right]$$

Relation between reheating parameters

If $w_{\rm re} \neq \frac{1}{3}$, the **duration of reheating**

$$N_{\rm re} = \frac{4}{1-3w_{\rm re}}\left[61.55 - N_k^{\rm inf} - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}}\right)\right]$$

and **reheating Temperature**

$$T_{\rm re} = \left(\frac{45}{\pi^2 g_{\rm re}}\right)^{\frac{1}{4}} V_e^{\frac{1}{4}}\, e^{-\frac{3}{4}(1+w_{\rm re})N_{\rm re}}$$

Relation between reheating parameters

If $w_{\rm re} \neq \frac{1}{3}$, the **duration of reheating**

$$N_{\rm re} = \frac{4}{1-3w_{\rm re}} \left[61.55 - N_k^{\rm inf} - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}} \right) \right]$$

and **reheating Temperature**

$$T_{\rm re} = \left(\frac{45}{\pi^2 g_{\rm re}} \right)^{\frac{1}{4}} V_e^{\frac{1}{4}} \, e^{-\frac{3}{4}(1+w_{\rm re})N_{\rm re}}$$

If $w_{\rm re} = \frac{1}{3}$, then $N_{\rm re}$ and $T_{\rm re}$ are **undefined**,

Relation between reheating parameters

If $w_{\rm re} \neq \frac{1}{3}$, the **duration of reheating**

$$N_{\rm re} = \frac{4}{1-3w_{\rm re}}\left[61.55 - N_k^{\rm inf} - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}}\right)\right]$$

and **reheating Temperature**

$$T_{\rm re} = \left(\frac{45}{\pi^2 g_{\rm re}}\right)^{\frac{1}{4}} V_e^{\frac{1}{4}}\, e^{-\frac{3}{4}(1+w_{\rm re})N_{\rm re}}$$

If $w_{\rm re} = \frac{1}{3}$, then $N_{\rm re}$ and $T_{\rm re}$ are **undefined**,we get

$$N_k^{\rm inf} = 61.55 - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}}\right)$$

Relation between reheating parameters

If $w_{\rm re} \neq \frac{1}{3}$, the **duration of reheating**

$$N_{\rm re} = \frac{4}{1-3w_{\rm re}}\left[61.55 - N_k^{\rm inf} - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}}\right)\right]$$

and **reheating Temperature**

$$T_{\rm re} = \left(\frac{45}{\pi^2 g_{\rm re}}\right)^{\frac{1}{4}} V_e^{\frac{1}{4}}\, e^{-\frac{3}{4}(1+w_{\rm re})N_{\rm re}}$$

If $w_{\rm re} = \frac{1}{3}$, then $N_{\rm re}$ and $T_{\rm re}$ are **undefined**,we get

$$N_k^{\rm inf} = 61.55 - \log\left(\frac{V_e^{\frac{1}{4}}}{H_k^{\rm inf}}\right)$$

$\Rightarrow$ **sharp predictions for** $\{n_s, r\}$

Incorporating Reheating Constraints

Tensor-to-scalar ratio $r \leq 0.06 \Rightarrow$

Incorporating Reheating Constraints

Tensor-to-scalar ratio $r \leq 0.06 \Rightarrow$

$$H_k^{\rm inf} \leq 2.5 \times 10^{-5}\ m_p = 6.1 \times 10^{13}\ \text{GeV}$$

and

$$T_{\rm inf} \equiv \left(3\, m_p^2 \left(H_k^{\rm inf}\right)^2\right)^{1/4} \leq 1.6 \times 10^{16}\ \text{GeV}$$

Incorporating Reheating Constraints

Tensor-to-scalar ratio $r \leq 0.06 \Rightarrow$

$$H_k^{\rm inf} \leq 2.5 \times 10^{-5}\ m_p = 6.1 \times 10^{13}\ \text{GeV}$$

and

$$T_{\rm inf} \equiv \left(3\, m_p^2 \left(H_k^{\rm inf}\right)^2\right)^{1/4} \leq 1.6 \times 10^{16}\ \text{GeV}$$

The most conservative constraints on the **reheating Temperature**

Incorporating Reheating Constraints

Tensor-to-scalar ratio $r \leq 0.06 \Rightarrow$

$$H_k^{\rm inf} \leq 2.5 \times 10^{-5}\ m_p = 6.1 \times 10^{13}\ \text{GeV}$$

and

$$T_{\rm inf} \equiv \left(3\, m_p^2 \left(H_k^{\rm inf}\right)^2\right)^{1/4} \leq 1.6 \times 10^{16}\ \text{GeV}$$

The most conservative constraints on the **reheating Temperature**

$$1\,\text{MeV} \leq T_{\rm re} \leq 10^{16}\,\text{GeV}$$

Incorporating Reheating Constraints

Tensor-to-scalar ratio $r \leq 0.06 \Rightarrow$

$$H_k^{\rm inf} \leq 2.5 \times 10^{-5}\ m_p = 6.1 \times 10^{13}\ \text{GeV}$$

and

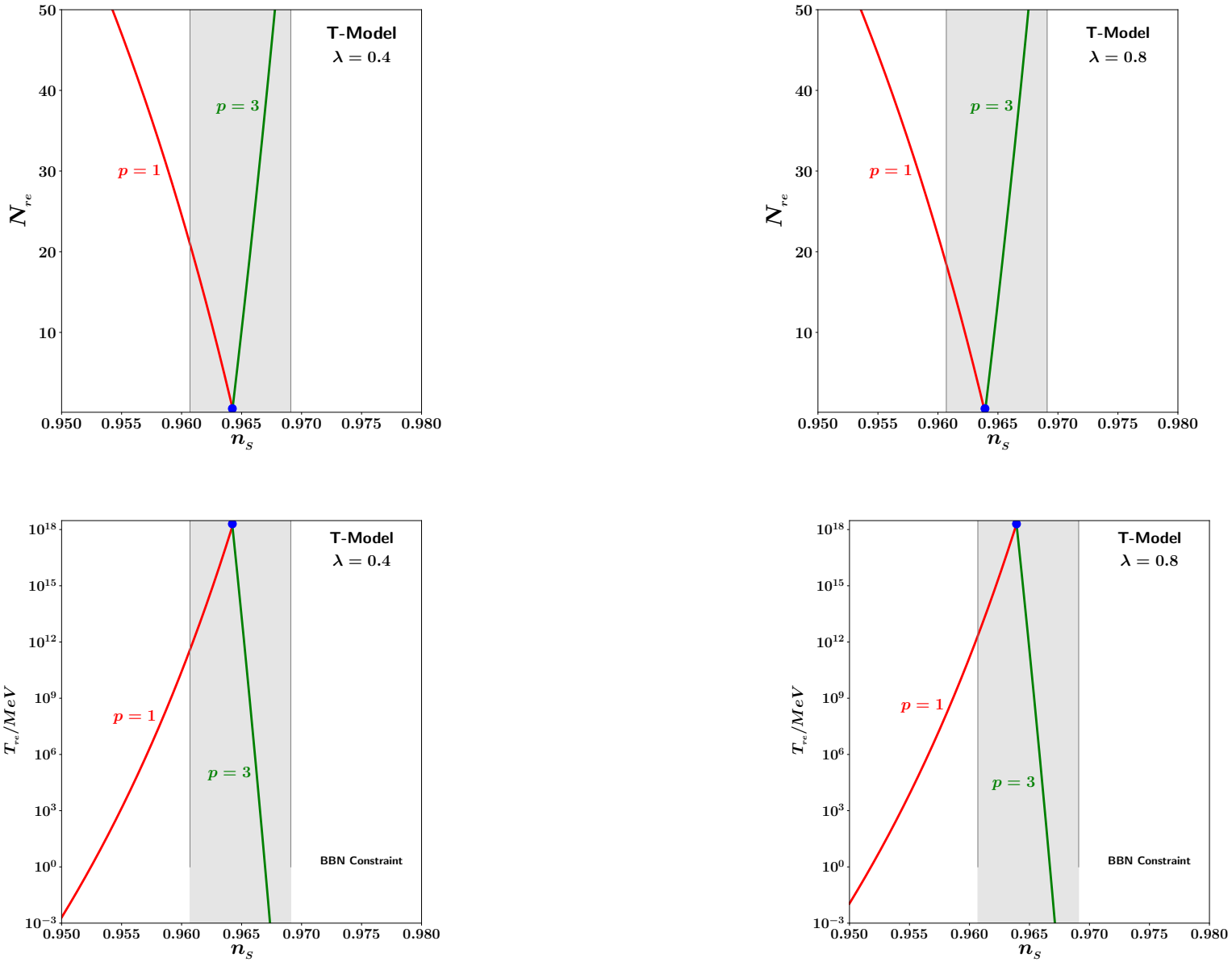
$$T_{\rm inf} \equiv \left(3\, m_p^2 \left(H_k^{\rm inf}\right)^2\right)^{1/4} \leq 1.6 \times 10^{16}\ \text{GeV}$$

The most conservative constraints on the **reheating Temperature**

$$1\,\text{MeV} \leq T_{\rm re} \leq 10^{16}\,\text{GeV}$$

Good enough to break inflationary degeneracies

Breaking degeneracies in the T-Model

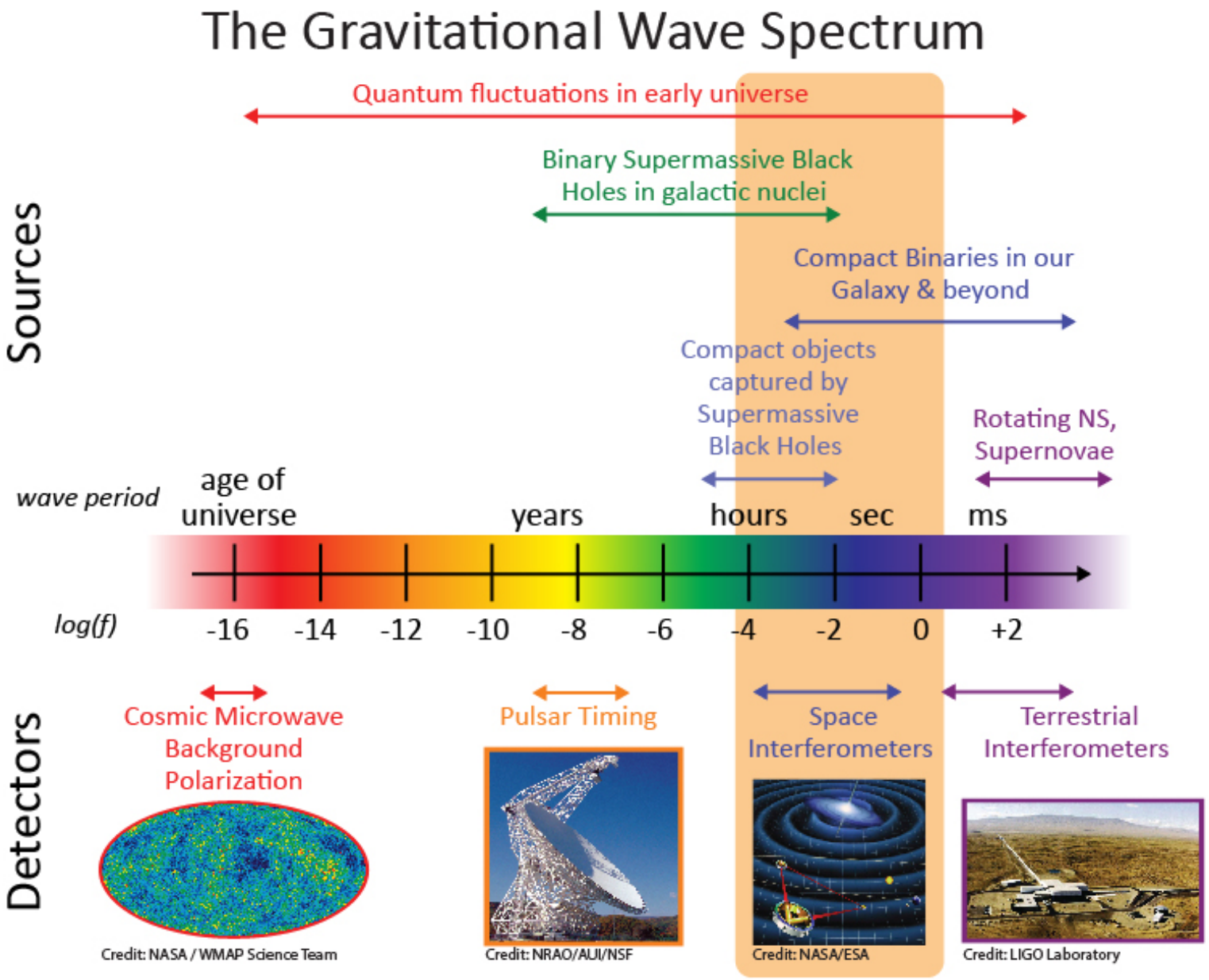


Segregation of Inflationary Observables

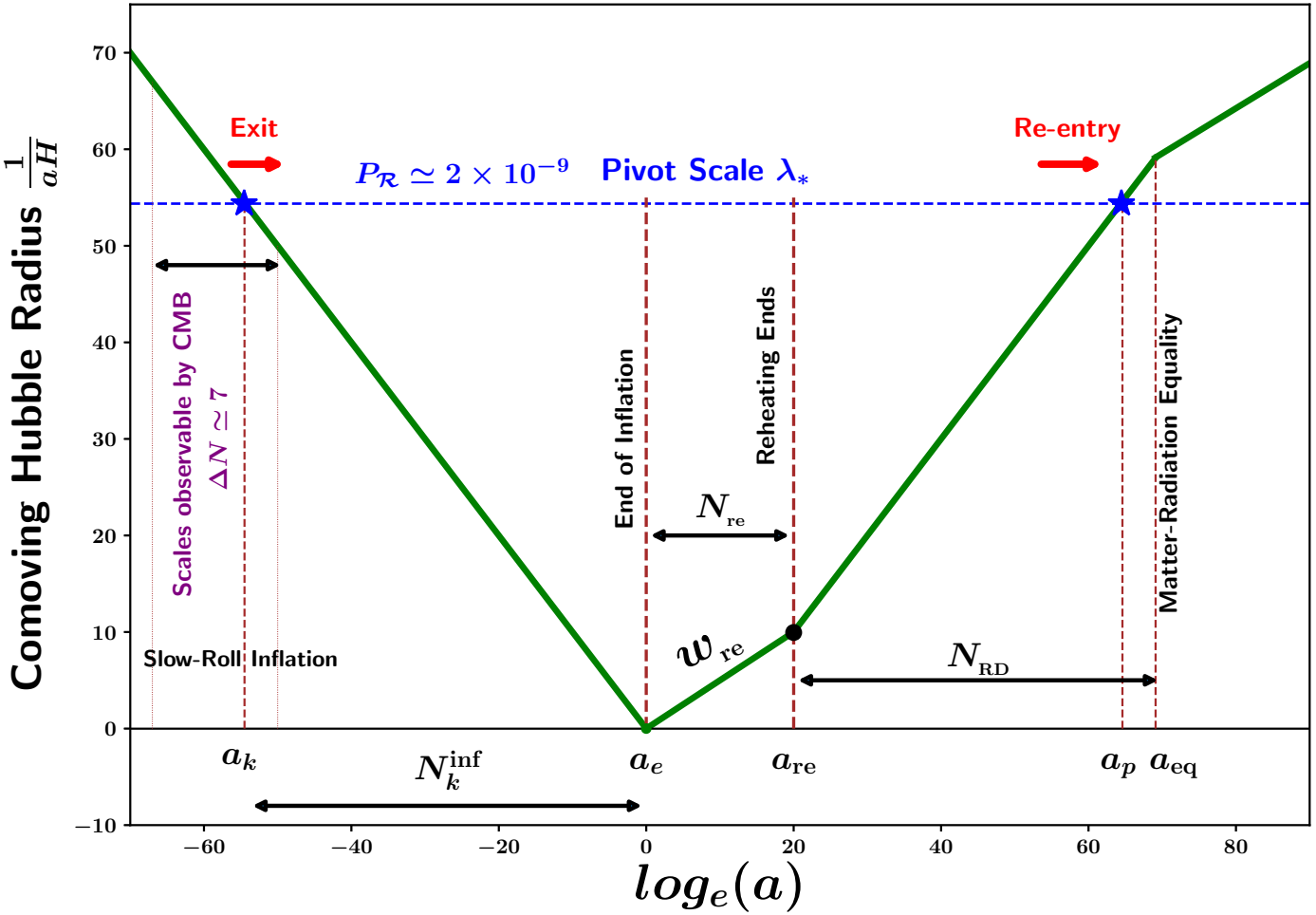
λ	Observables	$p = 1,\ w_{\rm re} = 0$	$p = 2,\ w_{\rm re} = 1/3$	$p = 3,\ w_{\rm re} = 1/2$
0.4	$N_k^{\rm inf}$	[50.4931, 55.686]	55.72	[55.724, 60.383]
	n_S	[0.9607, 0.9643]	0.9643	[0.96435, 0.967]
	r	[0.003867, 0.004683]	0.003899	[0.003326, 0.003896]
	$N_{\rm re}$	[0, 20.9624]	0	[0, 37.5836]
	$T_{\rm re}$	$[3.9 \times 10^8,\ 2.4 \times 10^{15}]$ GeV	2.4×10^{15} GeV	$[10^{-3},\ 2.4 \times 10^{15}]$ GeV
0.8	$N_k^{\rm inf}$	[50.6026, 55.17]	55.1749	[55.1761, 59.8198]
	n_S	[0.9607, 0.9639]	0.9639	[0.9639, 0.966706]
	r	[0.001009, 0.001198]	0.00101	[0.0008604, 0.00101]
	$N_{\rm re}$	[0, 18.4401]	0	[0, 37.4696]
	$T_{\rm re}$	$[2.2 \times 10^9,\ 2.1 \times 10^{15}]$ GeV	2.1×10^{15} GeV	$[10^{-3},\ 2.1 \times 10^{15}]$ GeV

$\Rightarrow$ **Degeneracies Broken**

Primordial Gravitational Waves



Tensor fluctuations → GWs (Physically)



Present-day Frequency of GWs

$$f = \frac{1}{2\pi}\left(\frac{k}{a_0}\right) = \frac{1}{2\pi}\left(\frac{a}{a_0}\right) H$$

$$f = 7.36 \times 10^{-8}\,\mathrm{Hz}\left(\frac{g_0^s}{g_T^s}\right)^{\frac{1}{3}}\left(\frac{g_T}{90}\right)^{\frac{1}{2}}\left(\frac{T}{\mathrm{GeV}}\right)$$

Epoch	**Temperature T**	**GW f (in Hz)**
Matter-radiation equality	~ 1 eV	1.7×10^{-17}
CMB pivot scale re-entry	~ 5 eV	8.5×10^{-17}
Big Bang Nucleosynthesis	~ 1 MeV	1.8×10^{-11}
Electro-weak symmetry breaking	~ 100 GeV	2.7×10^{-6}

Table:

Gravitational Wave Spectrum

Definition:

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{0c}} \frac{d\rho^0_{\rm GW}(f)}{d\log f}$$

Gravitational Wave Spectrum

Definition:

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{0c}} \frac{d\rho^0_{\rm GW}(f)}{d\log f}$$

Radiative epoch: $\Omega^{(\rm RD)}_{\rm GW}(f) = \left(\frac{1}{24}\right) r\, A_S \left(\frac{f}{f_*}\right)^{n_T} \Omega_{0r}\ ,\ \ f_{\rm eq} < f \leq f_{\rm re}\ ,$

During reheating: $\Omega^{(\rm re)}_{\rm GW}(f) = \Omega^{(\rm RD)}_{\rm GW}(f) \left(\frac{f}{f_{\rm re}}\right)^{2\left(\frac{w-1/3}{w+1/3}\right)}\ ,\ \ f_{\rm re} < f \leq f_{\rm e}\ ,$

Gravitational Wave Spectrum

Definition:

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{0c}} \frac{d\rho^0_{\rm GW}(f)}{d\log f}$$

Radiative epoch: $\Omega^{(\rm RD)}_{\rm GW}(f) = \left(\frac{1}{24}\right) r\, A_S \left(\frac{f}{f_*}\right)^{n_T} \Omega_{0r}\,, \quad f_{\rm eq} < f \leq f_{\rm re}\,,$

During reheating: $\Omega^{(\rm re)}_{\rm GW}(f) = \Omega^{(\rm RD)}_{\rm GW}(f) \left(\frac{f}{f_{\rm re}}\right)^{2\left(\frac{w-1/3}{w+1/3}\right)}\,, \quad f_{\rm re} < f \leq f_{\rm e}\,,$

$$n^{\rm PI}_{\rm GW} = \frac{d\log\Omega_{\rm GW}(k)}{d\log k} = \frac{d\log\Omega_{\rm GW}(f)}{d\log f}$$

$$n^{\rm PI}_{\rm GW} = n_T + 2\left(\frac{w-1/3}{w+1/3}\right)$$

GW Spectrum – General Behaviour

$$n_{\rm GW}^{\rm PI} = n_T + 2\left(\frac{w - 1/3}{w + 1/3}\right)$$

GW Spectrum – General Behaviour

$$n_{\rm GW}^{\rm PI} = n_T + 2\left(\frac{w - 1/3}{w + 1/3}\right)$$

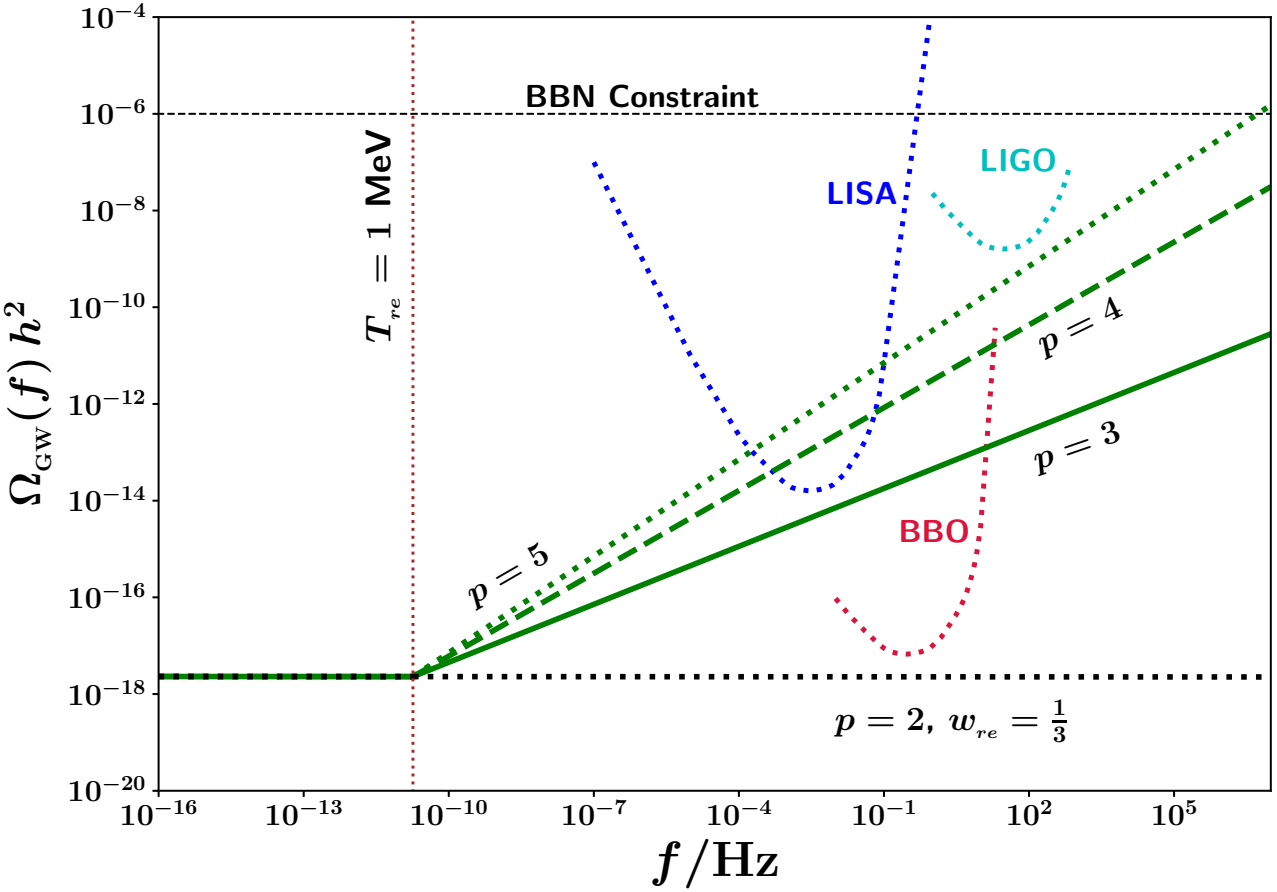
$$n_{\rm GW}^{\rm PI} \leq 0 \;\; \text{for} \;\; w < 1/3 \;\Rightarrow\; \textbf{Red Tilt}$$

$$n_{\rm GW}^{\rm PI} \simeq 0 \;\; \text{for} \;\; w = 1/3$$

$$n_{\rm GW}^{\rm PI} \geq 0 \;\; \text{for} \;\; w > 1/3 \;\Rightarrow\; \textbf{Blue Tilt}$$

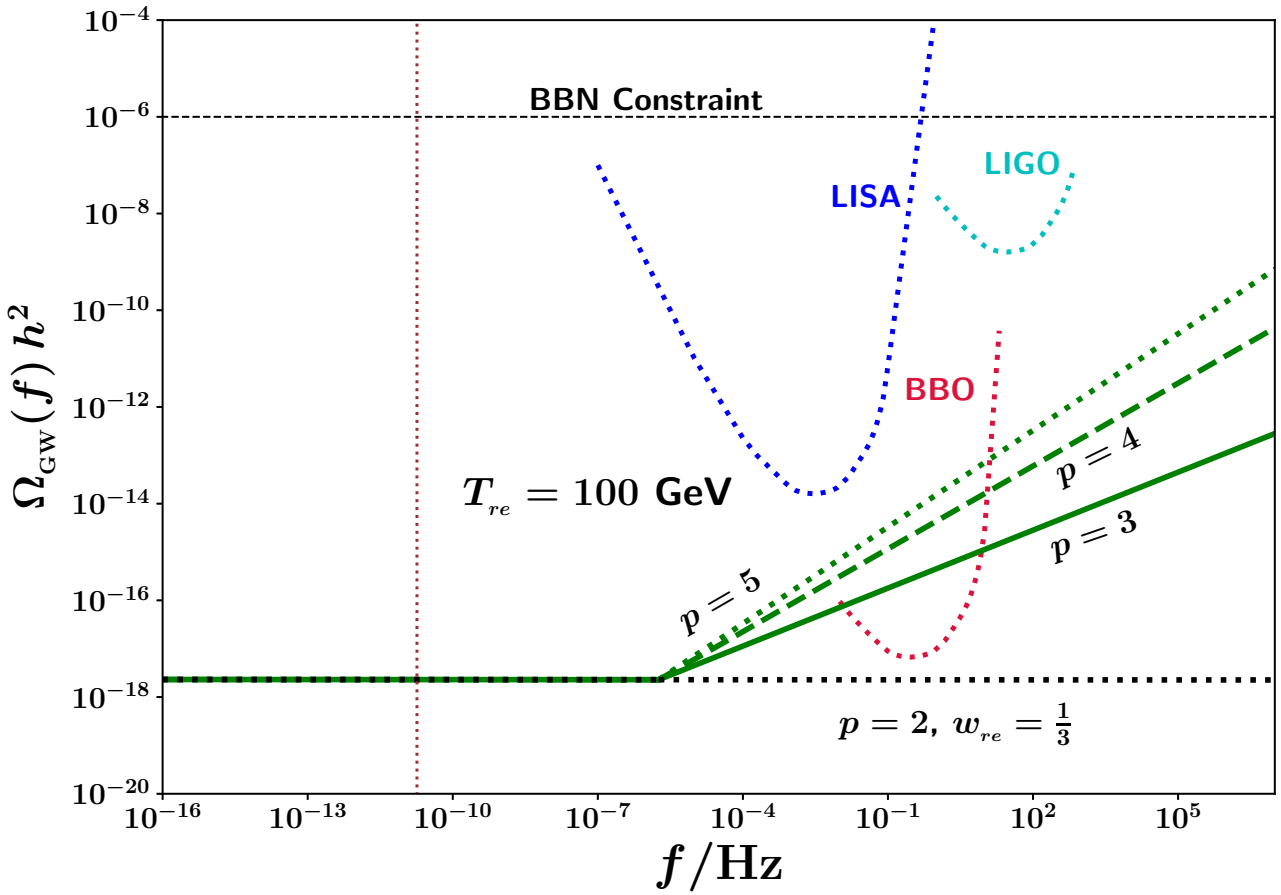
GW Spectrum – General Behaviour

$$n_{\rm GW}^{\rm PI} = n_T + 2\left(\frac{w - 1/3}{w + 1/3}\right)$$

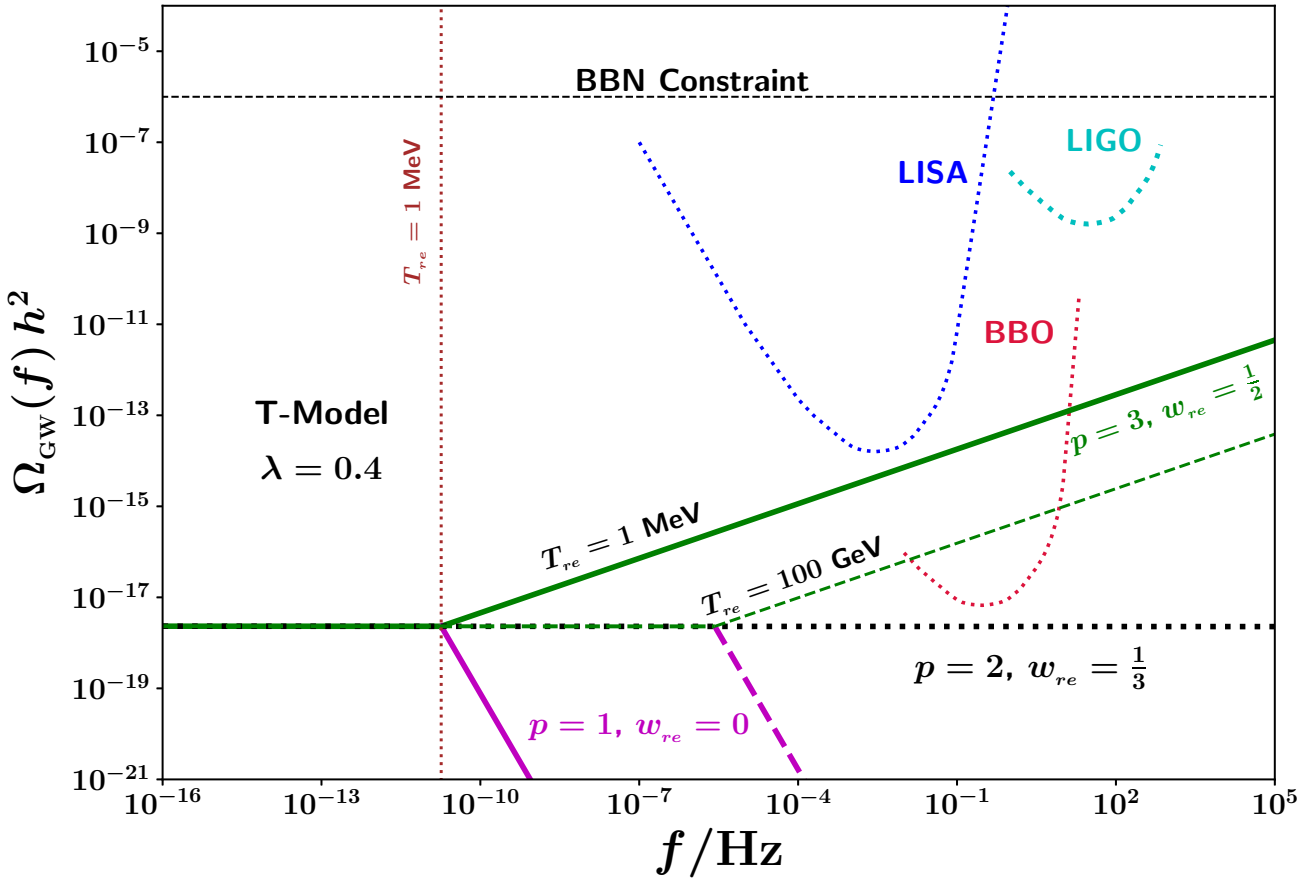


GW Spectrum – General Behaviour

$$n_{\rm GW}^{\rm PI} = n_T + 2\left(\frac{w - 1/3}{w + 1/3}\right)$$



GW Spectrum – Breaking Degeneracies



Summary

- Epoch of Reheating – Unexplored/inaccessible at present

Summary

- Epoch of Reheating – Unexplored/inaccessible at present
- Extremely interesting high energy physics

Summary

- Epoch of Reheating – Unexplored/inaccessible at present
- Extremely interesting high energy physics and breaking inflationary degeneracies

Summary

- Epoch of Reheating – Unexplored/inaccessible at present
- Extremely interesting high energy physics and breaking inflationary degeneracies
- T-Model and E-Model α-attractors : Degeneracies

Summary

- Epoch of Reheating – Unexplored/inaccessible at present
- Extremely interesting high energy physics and breaking inflationary degeneracies
- T-Model and E-Model α-attractors : Degeneracies
- Degeneracies easily broken by demanding $N_{\rm re} > 0$ and

$$1\,\mathrm{MeV} \leq T_{\rm re} \leq 10^{16}\,\mathrm{GeV}$$

Summary

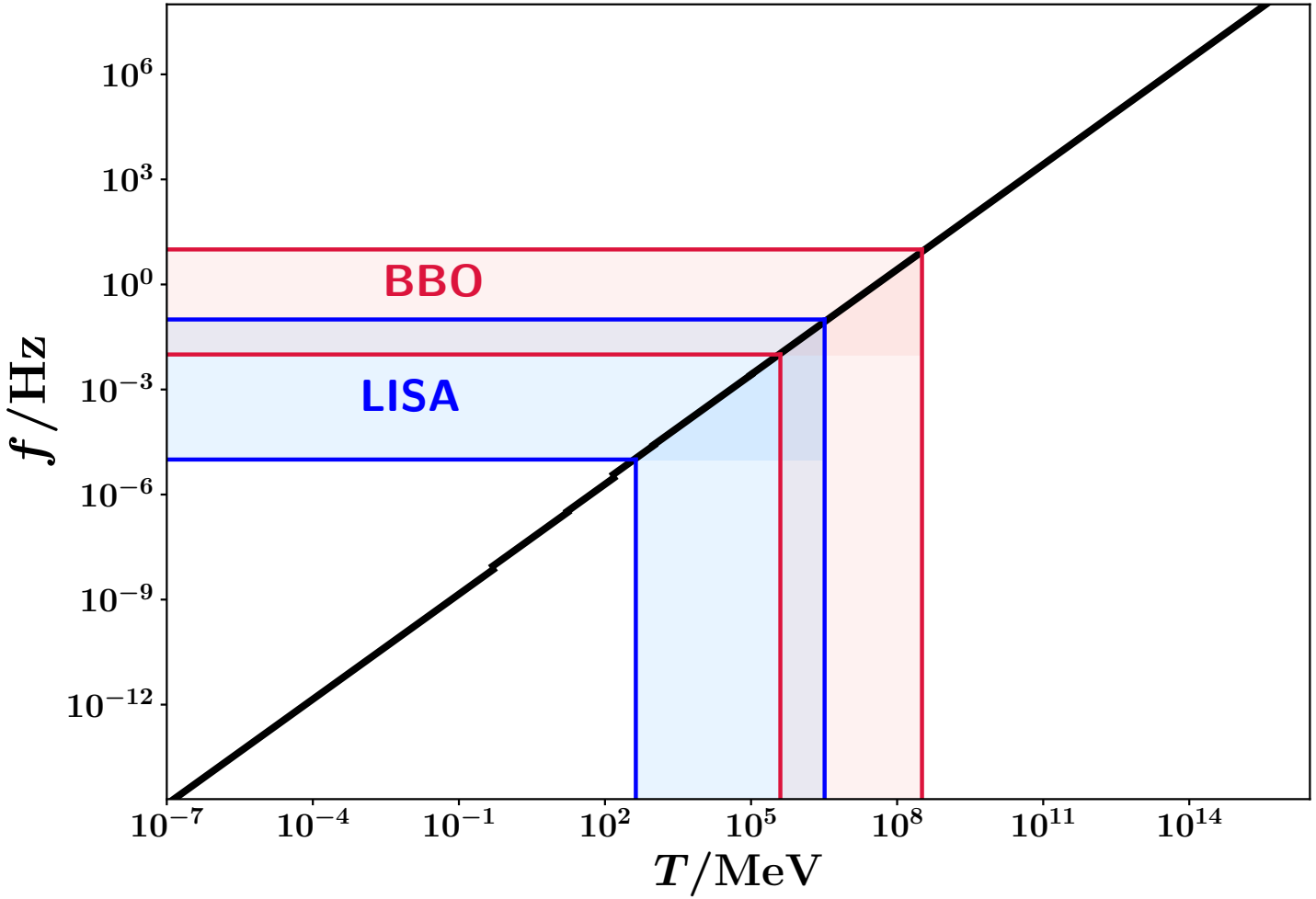
- Epoch of Reheating – Unexplored/inaccessible at present
- Extremely interesting high energy physics and breaking inflationary degeneracies
- T-Model and E-Model α-attractors : Degeneracies
- Degeneracies easily broken by demanding $N_{\rm re} > 0$ and

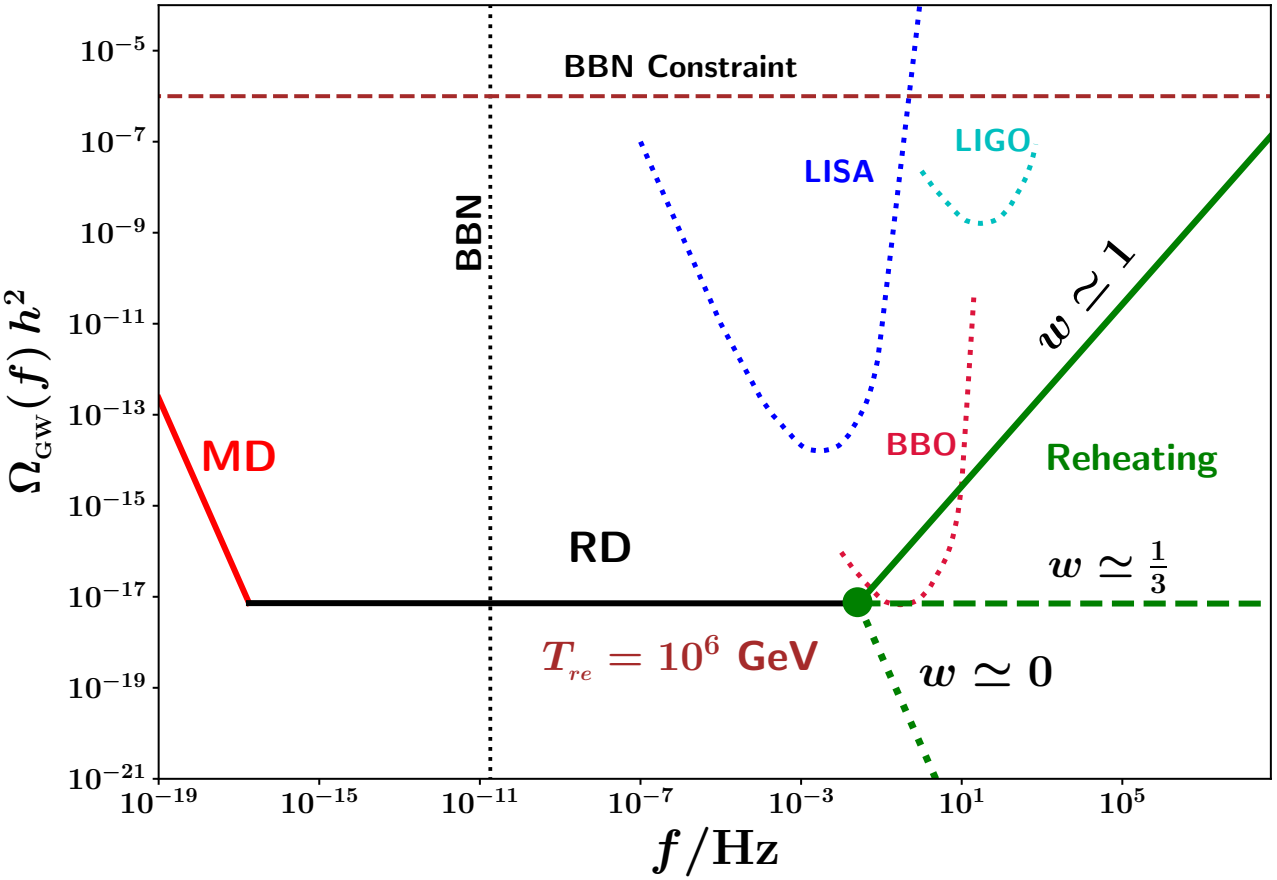
$$1\,\mathrm{MeV} \leq T_{\rm re} \leq 10^{16}\,\mathrm{GeV}$$

- Relic Gravitational Waves as the cleanest probe of the Early Universe including the epoch of reheating

Extra Slides

Present-day Frequency of GWs



GW Spectrum – General Behaviour

$$n_{\rm GW}^{\rm PI} = n_T + 2\left(\frac{w - 1/3}{w + 1/3}\right)$$