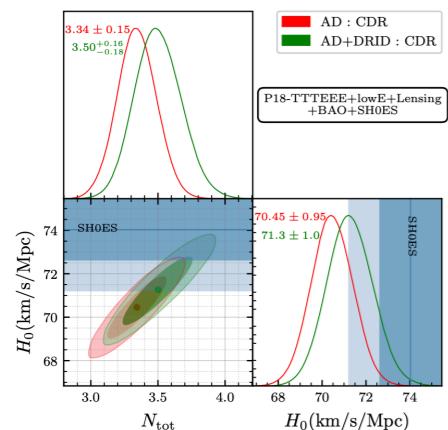
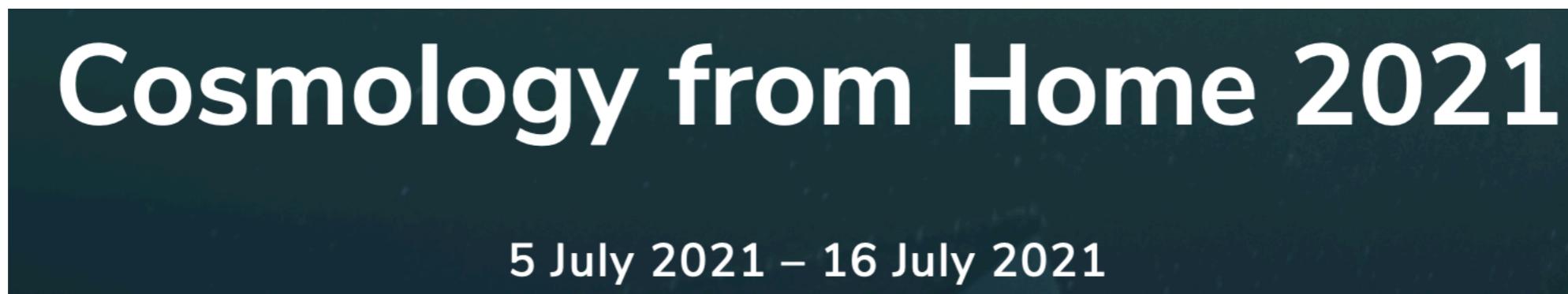
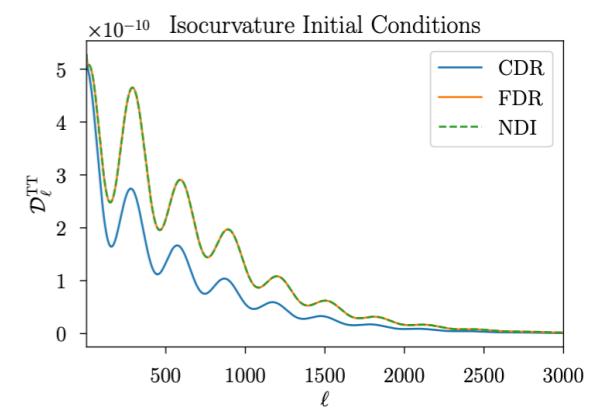


Dark Radiation Isocurvature: Constraints and Application to the H_0 Tension



Soubhik Kumar
UC Berkeley and LBL

w/ Subhajit Ghosh and Yuhsin Tsai, in progress



Physics of the Early Universe

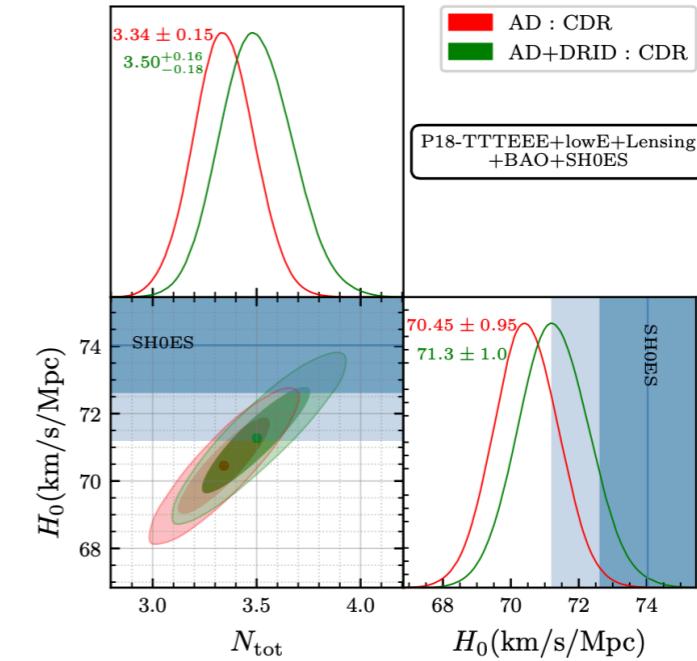
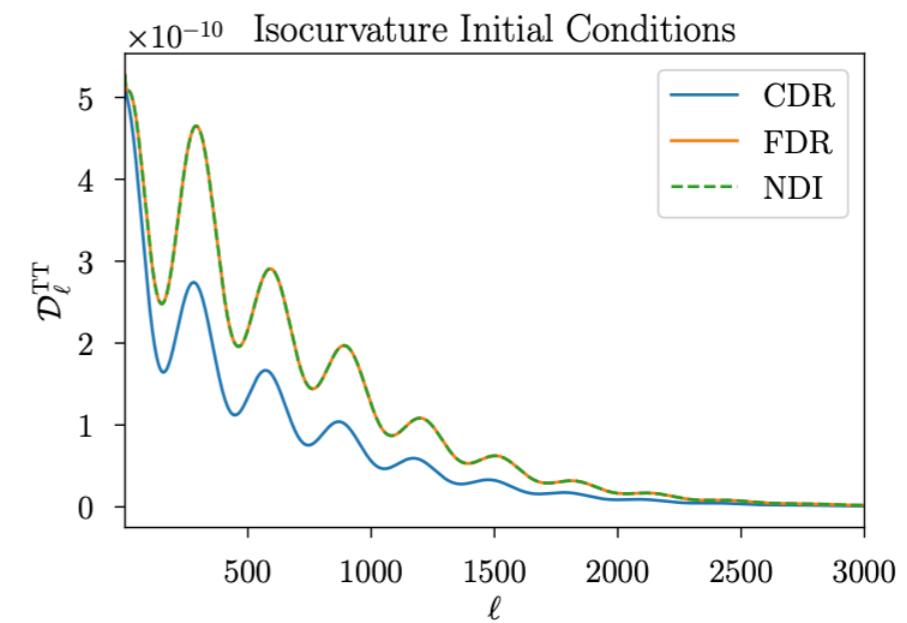
- Primordial perturbations as inferred through Planck are adiabatic, (almost) scale invariant and Gaussian
Planck '18
- → single-field slow-roll inflation, however, very plausible to have other fields during inflation
- Extra light fields can give rise to isocurvature perturbations → do not contribute to total curvature
- e.g. axion DM isocurvature
 $\frac{I}{\mathcal{R}} \lesssim 10\%$
baryon, DM, ν
Planck '18

Isocurvature Perturbations in Dark Radiation

- Dark radiation (DR) commonplace in extensions of Standard Model, e.g., dark photon, ultralight axions, gravitational waves...
Bashinsky, Seljak '03
Hu, Sugiyama '95 ...
- Parametrized by ΔN_{eff} , adiabatic case extensively studied
e.g. damping, phase shift
- They are “dark” → can easily come from a separate field during inflation, e.g., curvaton
Coupled DR
- New consequences depending on their nature
similar to $\nu \leftarrow$ Free-Streaming DR

Summary

- For **isocurvature initial conditions**, FDR leads to more CMB anisotropies than CDR—opposite to well-known scenario with adiabatic initial conditions.
- Presence of **blue-tilted isocurvature perturbations** compensates extra Silk damping due to $\Delta N_{\text{eff}} \rightarrow$ higher value of H_0



Constraints on CDR (new) and FDR (updated) Isocurvature

Outline

- Setting up Isocurvature Initial Conditions
- Analytical Comparison of FDR and CDR
- MCMC Results
- Conclusion

Conventions

- “Iso”curvature: perturbations that do not give rise to curvature

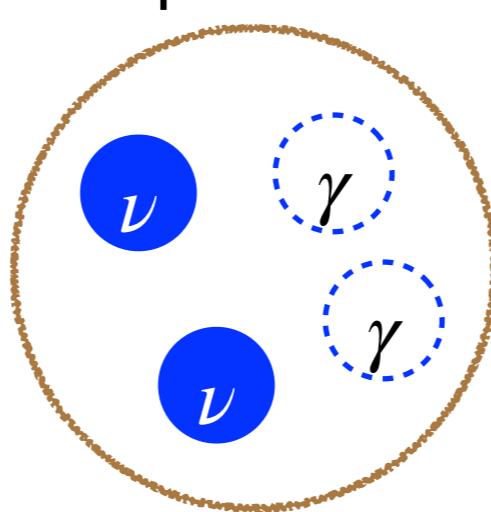
- Example, neutrino density isocurvature:
Bucher et al. '99 *synchronous gauge

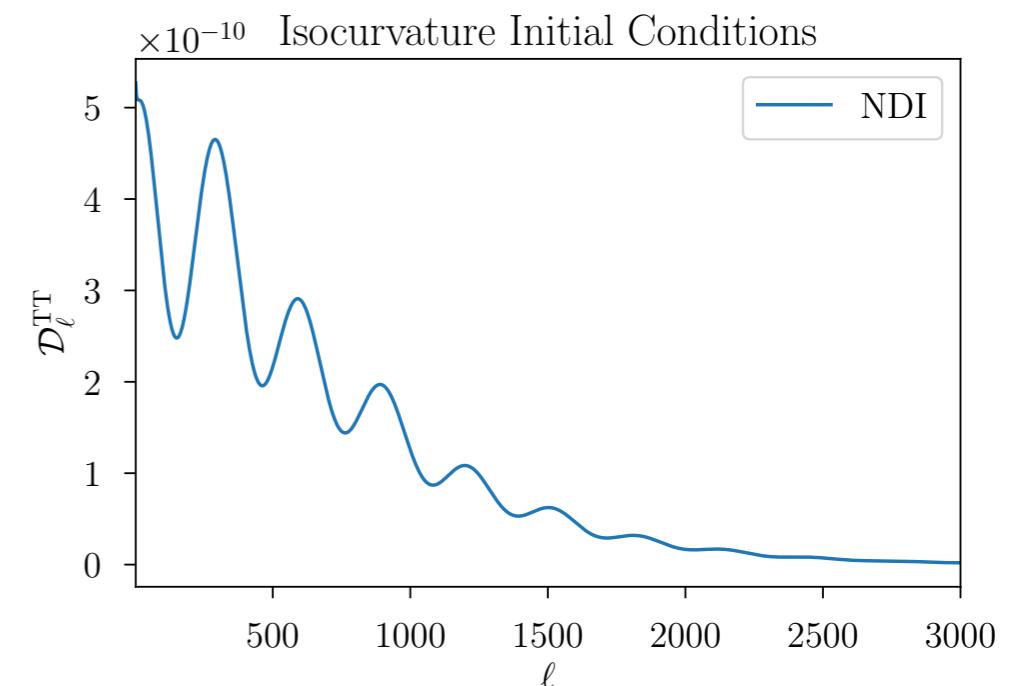
$$(\delta\rho_\nu + \delta\rho_\gamma) \Big|_{\tau \rightarrow 0} = 0 \quad \text{and vanishing metric pert.}$$

*suppressing $\sim 10^{-5}$

$$\delta_\nu \equiv \frac{\delta\rho_\nu}{\rho_\nu} = 1, \frac{\delta\rho_\gamma}{\rho_\gamma} = -\frac{\rho_\nu}{\rho_\gamma}$$

$I_\nu \equiv \frac{3}{4} (\delta_\nu - \delta_\gamma) \neq 0$





Dark Radiation Isocurvature

- Proceed analogously,

$$(\delta\rho_{\text{DR}} + \delta\rho_\gamma + \delta\rho_\nu) \Big|_{\tau \rightarrow 0} = 0$$



$$R_i \equiv \frac{\bar{\rho}_i}{\bar{\rho}_\gamma + \bar{\rho}_\nu + \bar{\rho}_{\text{DR}}}$$

$$\delta_{\text{DR}} = 1, \quad \delta_\gamma = \delta_\nu = -\frac{R_{\text{DR}}}{1 - R_{\text{DR}}}$$

- Above gives initial conditions for density perturbations, and identical for CDR and FDR
- However, important difference once anisotropic stress σ comes into play!

Deriving Initial Conditions

- Need to set superhorizon initial conditions for other quantities such as, $\theta_\gamma, \delta_{b,c} \dots$
- Solve coupled Boltzmann Einstein equations order-by-order in $k\tau$ and $\omega\tau$: $\omega \equiv \mathcal{H}_0 \Omega_{m,0} / \sqrt{\Omega_{r,0}}$

$$\dot{\delta}_{\text{DR}} = -\frac{4}{3}\theta_{\text{DR}} - \frac{2}{3}\dot{h},$$

$$\dot{\theta}_{\text{DR}} = k^2 \left(\frac{1}{4}\delta_{\text{DR}} - \sigma_{\text{DR}} \right),$$

~~$$\dot{\sigma}_{\text{DR}} = \frac{4}{15}\theta_{\text{DR}} + \frac{2}{15}\dot{h} + \frac{4}{5}\dot{\eta},$$~~

CDR

+ standard
equations

$$\begin{aligned} k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} &= 4\pi G a^2 \delta T_0^0(\text{Syn}), \\ k^2\dot{\eta} &= 4\pi G a^2 (\bar{\rho} + \bar{P})\theta(\text{Syn}), \\ \ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta &= -8\pi G a^2 \delta T_i^i(\text{Syn}), \\ \ddot{h} + 6\ddot{\eta} + 2\frac{\dot{a}}{a}(\dot{h} + 6\dot{\eta}) - 2k^2\eta &= -24\pi G a^2 (\bar{\rho} + \bar{P})\sigma(\text{Syn}) \\ \dot{\delta}_\gamma &= -\frac{4}{3}\theta_\gamma - \frac{2}{3}\dot{h}, \\ \dot{\theta}_\gamma &= k^2 \left(\frac{1}{4}\delta_\gamma - \sigma_\gamma \right) + a n_e \sigma_T (\theta_b - \theta_\gamma) \\ \dot{\delta}_b &= -\theta_b - \frac{1}{2}\dot{h} \\ \dot{\delta}_c &= -\frac{1}{2}\dot{h} \end{aligned}$$

e.g., Ma & Bertschinger, '95

Superhorizon Initial Conditions

variable	$\mathcal{O}(0)$	$\mathcal{O}(k\tau)$	$\mathcal{O}((k\tau)^2)$	$\mathcal{O}(k\omega\tau^2)$	$\mathcal{O}((k\tau)^3)$	$\mathcal{O}(\omega k^2\tau^3)$
δ_γ	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	0	0	
θ_γ/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	$\frac{3R_{\text{DR}}R_b}{16(1-R_{\text{DR}})(1-R_{\text{DR}}-R_\nu)}$	$\frac{R_{\text{DR}}}{72(1-R_{\text{DR}})}$	
δ_ν	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	0	0	
θ_ν/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	0	α^f	
σ_ν	0	0	$-\frac{19R_{\text{DR}}}{30(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	0	0	
δ_{DR}	1	0	$-\frac{1}{6}$	0	0	
θ_{DR}/k	0	$\frac{1}{4}$	0	0	β^f	
σ_{DR}	0	0	$\frac{15-15R_{\text{DR}}+4R_\nu}{30(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	0	0	
η	0	0	$\frac{-R_{\text{DR}}+R_{\text{DR}}^2+R_{\text{DR}}R_\nu}{6(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	0	0	
h	0	0	0	0	$\frac{R_{\text{DR}}R_b}{40(1-R_{\text{DR}})}$	
δ_b	0	0	$\frac{R_{\text{DR}}}{8(1-R_{\text{DR}})}$	0	0	
δ_c	0	0	0	0	$-\frac{R_{\text{DR}}R_b}{80(1-R_{\text{DR}})}$	

matches with
NDI after

$$R_\nu \rightarrow 0; R_{\text{DR}} \rightarrow R_\nu; \{\delta, \theta, \sigma\}_{\text{DR}} \rightarrow \{\delta, \theta, \sigma\}_\nu$$

FDR

can not be matched!

qualitatively new

because $\sigma_{\text{DR}} = 0$



variable	$\mathcal{O}(0)$	$\mathcal{O}(k\tau)$	$\mathcal{O}((k\tau)^2)$	$\mathcal{O}(k\omega\tau^2)$	$\mathcal{O}((k\tau)^3)$	$\mathcal{O}(\omega k^2\tau^3)$
δ_γ	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	0	0	
θ_γ/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	$\frac{3R_{\text{DR}}R_b}{16(1-R_{\text{DR}})(1-R_{\text{DR}}-R_\nu)}$	$\frac{R_{\text{DR}}}{72(1-R_{\text{DR}})}$	
δ_ν	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	0	0	
θ_ν/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	0	α^c	
σ_ν	0	0	$-\frac{R_{\text{DR}}}{2(1-R_{\text{DR}})(15+4R_\nu)}$	0	0	
δ_{DR}	1	0	$-\frac{1}{6}$	0	0	
θ_{DR}/k	0	$\frac{1}{4}$	0	0	β^c	
η	0	0	$\frac{R_{\text{DR}}R_\nu}{6(1-R_{\text{DR}})(15+4R_\nu)}$	0	0	
h	0	0	0	0	0	$\frac{R_{\text{DR}}R_b}{40(1-R_{\text{DR}})}$
δ_b	0	0	$\frac{R_{\text{DR}}}{8(1-R_{\text{DR}})}$	0	0	
δ_c	0	0	0	0	0	$-\frac{R_{\text{DR}}R_b}{80(1-R_{\text{DR}})}$

$$ds^2 = a^2(\tau) (-d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j)$$

CDR

$$h_{ij}(\tau, \vec{x}) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \left(\hat{k}_i \hat{k}_j h(\tau, \vec{k}) + \left(\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) 6\eta(\tau, \vec{k}) \right)$$

Outline

- Setting up Isocurvature initial conditions 
- Analytical understanding of FDR vs. CDR properties
- MCMC results
- Conclusion

Adiabatic Initial Conditions

- All the species follow the same density perturbations:

$$\delta_\gamma = \delta_\nu = \delta_{\text{DR}} \dots$$

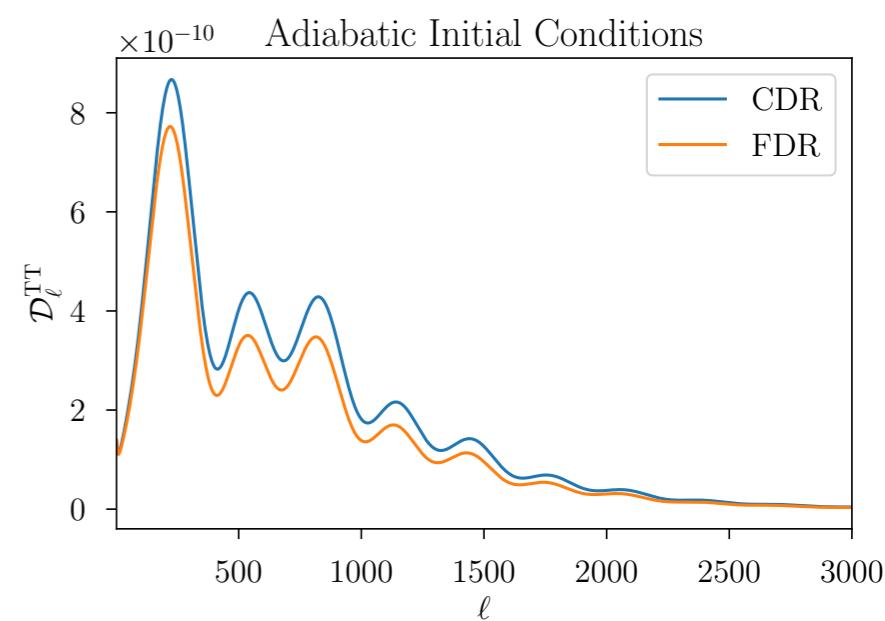
- FDR \rightarrow free stream out of potential wells \rightarrow smaller metric potential \rightarrow smaller CMB anisotropy

Bashinsky, Seljak '03
Hu, Sugiyama '95 ...

SW term

$$\frac{1}{4}\delta_\gamma^{\text{con}} + \psi \Big|_{\tau \rightarrow 0} \approx \begin{cases} \frac{5}{15 + 4R_\nu} & \text{CDR} \\ \frac{5}{15 + 4R_\nu + 4R_{\text{DR}}} & \text{FDR} \end{cases}$$

- CDR \rightarrow no free streaming \rightarrow larger CMB anisotropy



Isocurvature Initial Conditions: Shear

- Primary difference between CDR and FDR is σ_{DR}

$$\sigma = \frac{1}{2(15 + 4R_{\text{FS}})} (k\tau)^2 \delta_{\text{FS}}$$

$$\delta_{\text{FS}} = \sum_{i=\text{FS}} R_i \delta_i$$

$$R_{\text{FS}} = \rho_{\text{FS}} / \rho_{\text{tot}}$$

σ sourced only
by free-streaming (FS)
radiation

CDR: only ν free-streams $\rightarrow \delta_{\text{FS}} = \delta_\nu < 0 \Rightarrow \sigma < 0$

$$\delta_{\text{DR}} = 1, \quad \delta_\gamma = \delta_\nu = -\frac{R_{\text{DR}}}{1 - R_{\text{DR}}}$$

FDR: both ν , DR free-streams $\rightarrow \delta_{\text{FS}} = \sum_{i=\nu, \text{DR}} R_i \delta_i > 0 \Rightarrow \sigma > 0$

Effect on the Metric Perturbations

- Convenient to go to Newtonian gauge:

similar to Einstein eq.

$$\phi + \psi \approx -\frac{2\sigma}{(k\tau)^2}$$

$$\phi - \psi \approx \frac{6}{(k\tau)^2}\sigma$$

- For isocurvature initial conditions, metric perturbations determined by σ to the leading order



CDR and FDR sources
different metric perturbations

CDR: $\sigma < 0$

$$\phi + \psi > 0$$

FDR: $\sigma > 0$

$$\phi + \psi < 0$$

Implications on CMB spectrum

- Metric potentials directly affect the CMB via **Sachs-Wolfe redshifting**,

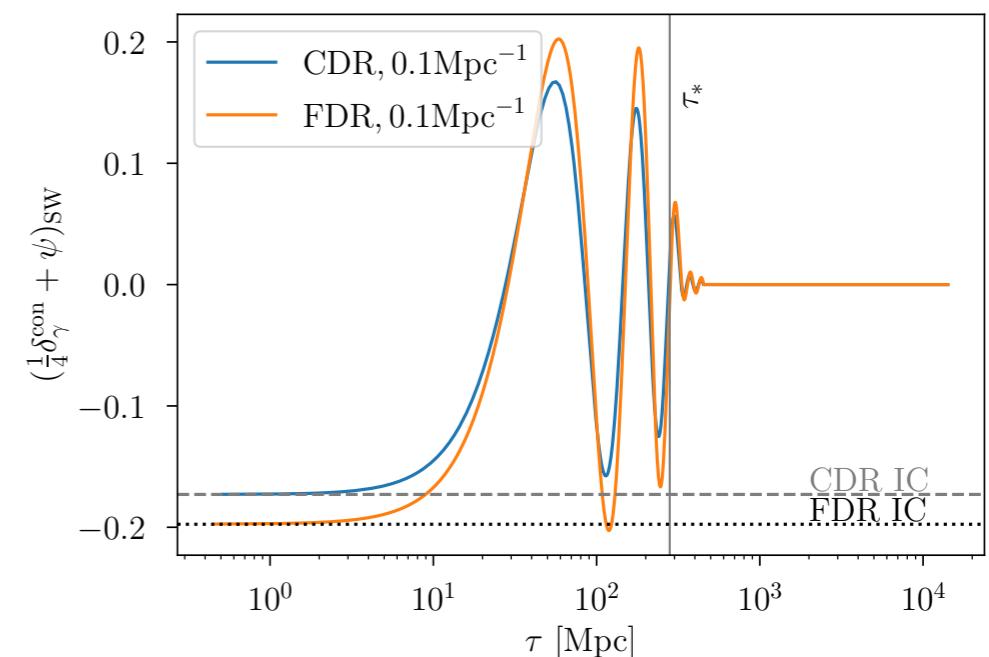
$$\frac{1}{4}\delta_\gamma^{\text{con}} + \psi = \zeta_\gamma + \phi + \psi$$

$$\delta_{\text{DR}} = 1, \quad \delta_\gamma = \delta_\nu = -\frac{R_{\text{DR}}}{1 - R_{\text{DR}}}$$

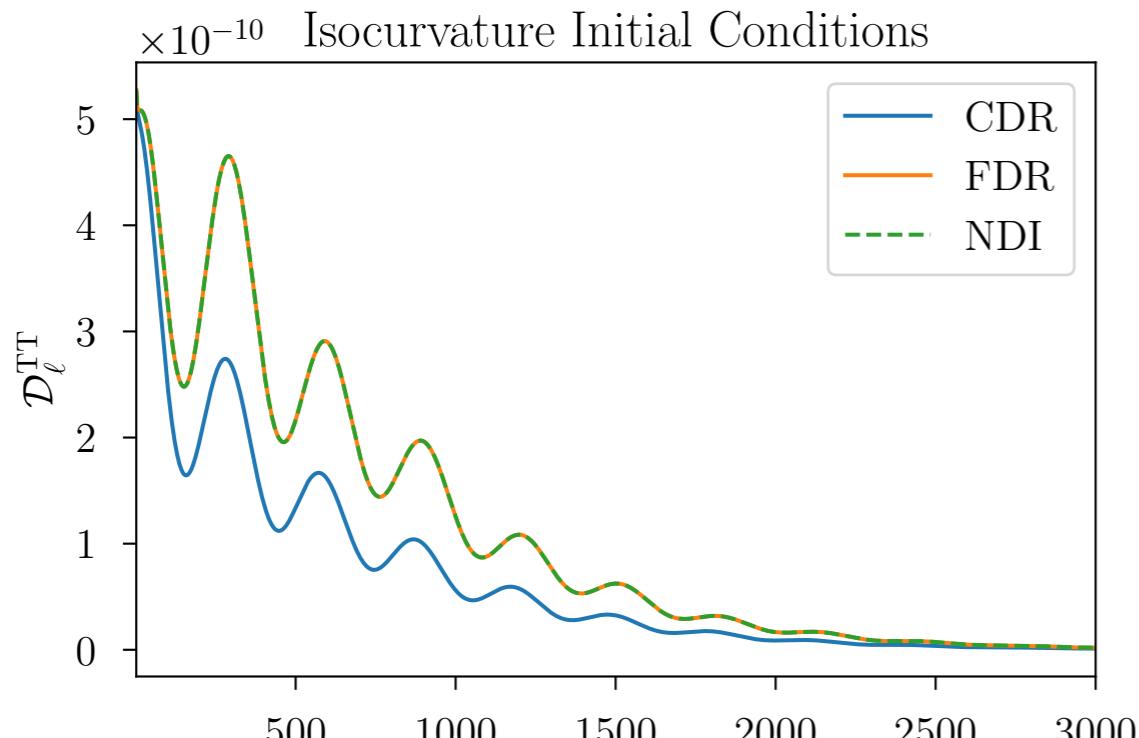
- Isocurvature initial condition has $\zeta_\gamma \approx \delta_\gamma < 0$

CDR
 $\phi + \psi > 0$
smaller $|\zeta_\gamma + \phi + \psi|$
less anisotropy

FDR
 $\phi + \psi < 0$
larger $|\zeta_\gamma + \phi + \psi|$
more anisotropy

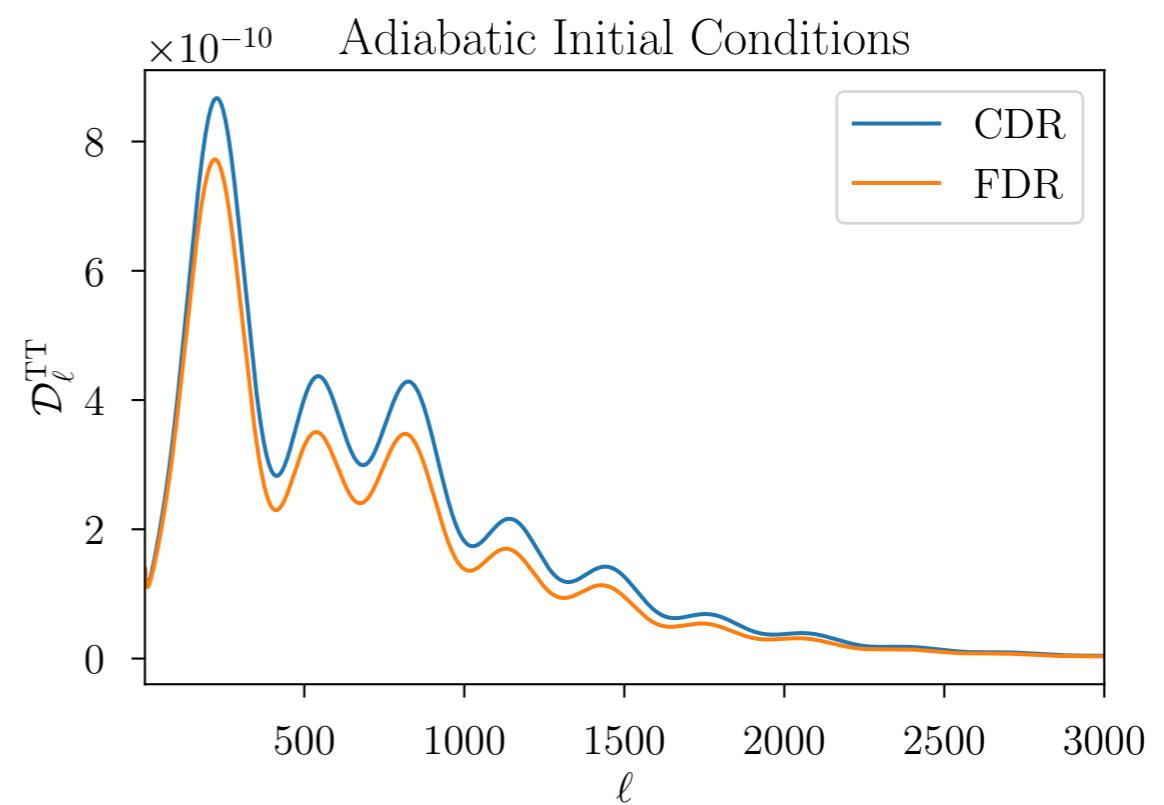


CMB Spectrum



- FDR sources more anisotropy
- FDR matches with NDI

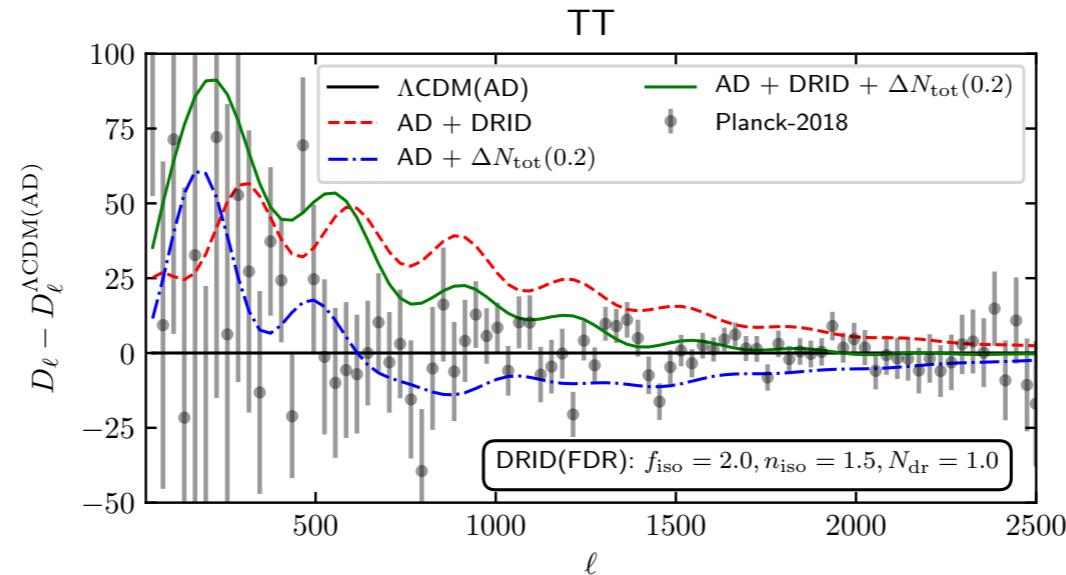
- CDR \rightarrow more anisotropy
- FDR gets Silk-damped



Expect to hide more CDR Isocurvature in data \rightarrow larger ΔN_{eff}

Application to the H_0 Tension

- Usually larger N_{eff} implies smaller damping scale θ_d (keeping sound horizon θ_s the same) \rightarrow more Silk damping
- However, isocurvature spectrum can have a **tilt different from adiabatic**, and **blue tilt** can **compensate Silk damping**



- Thus can allow a **larger N_{eff}** \rightarrow **larger H_0**

Outline

- Setting up Isocurvature initial conditions 
- Analytical understanding of FDR vs. CDR properties 
- MCMC results
- Conclusion

Choice of Isocurvature Parameters

- Due to the isocurvature initial conditions,

$$\delta_\gamma, \theta_\gamma, \eta, h \propto \frac{R_{\text{DR}}}{1 - R_{\text{DR}}} \approx R_{\text{DR}} \propto N_{\text{dr}}$$

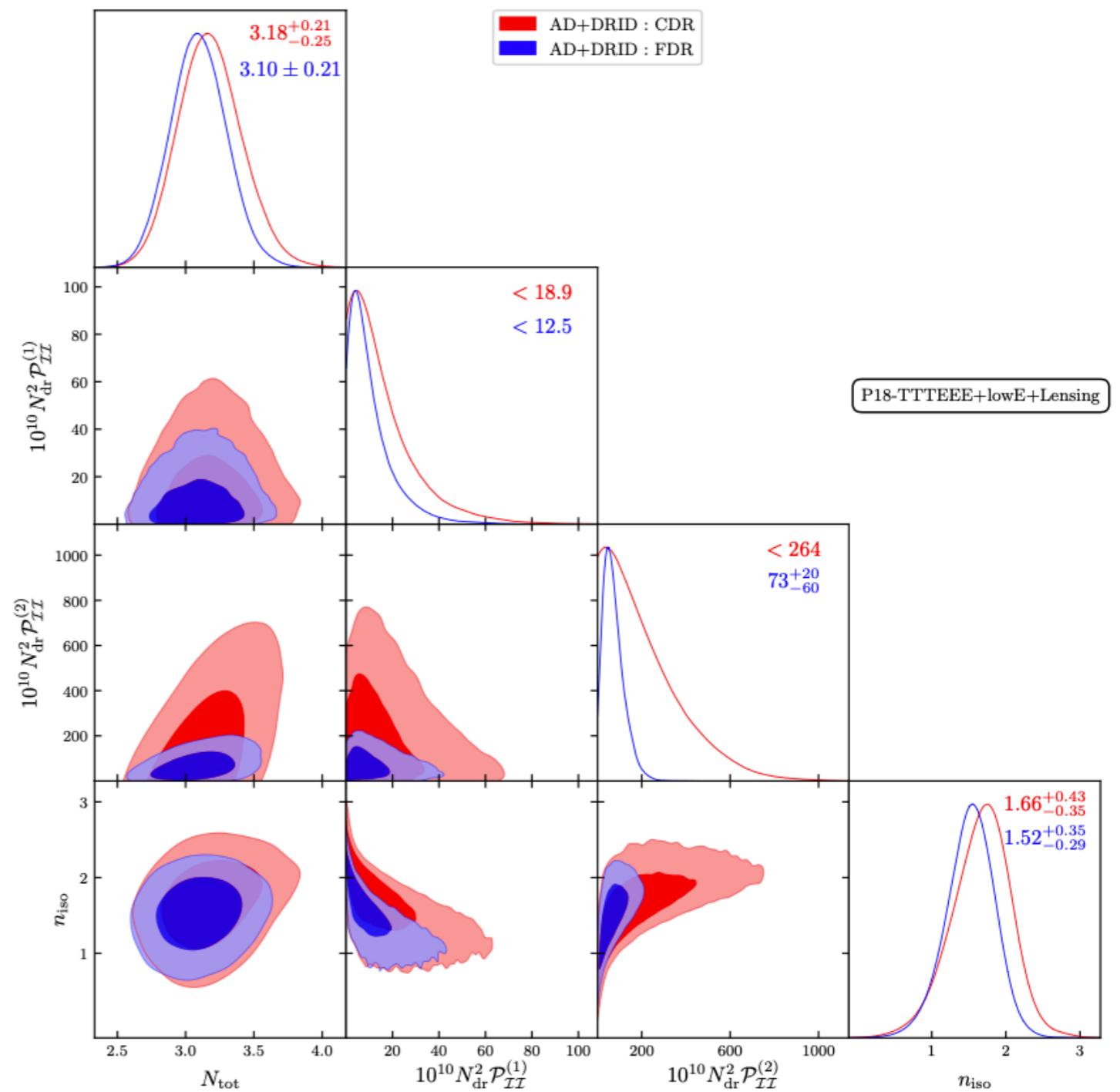
$$C_{\ell, \text{DRID}} \propto A_{\text{iso}} N_{\text{dr}}^2$$

$$A_{\text{iso}} = \mathcal{P}_{II}^{(1)} \left(\frac{k_*}{k_1} \right)^{(n_{\text{iso}} - 1)}$$

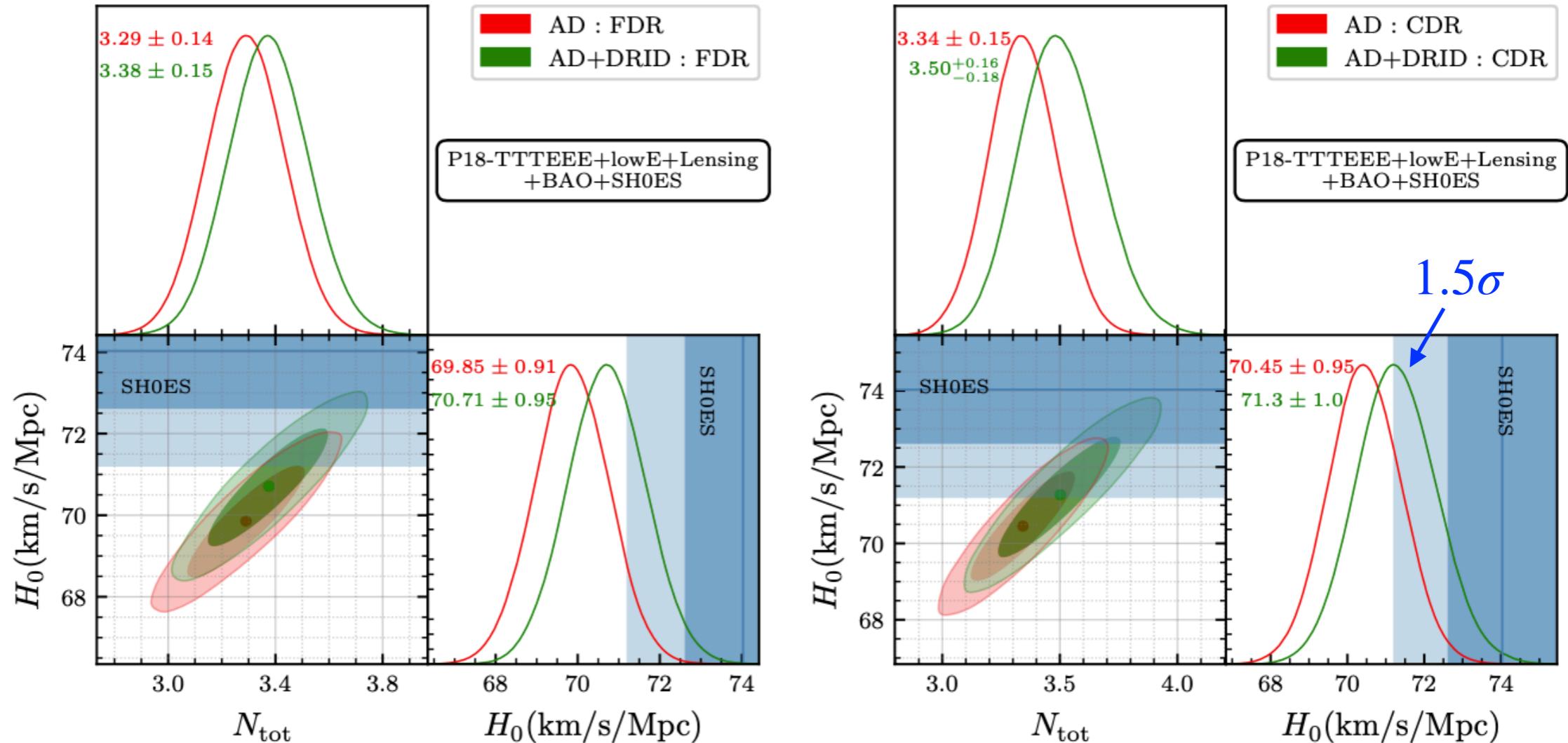
- Therefore $N_{\text{dr}}^2 \mathcal{P}_{II}^{(1)}$ and (2) are well-behaved, physical variables to vary
- In summary, vary four extra parameters compared to ΛCDM
 $N_{\text{ur}}, N_{\text{dr}}, N_{\text{dr}}^2 \mathcal{P}_{II}^{(1)}$ and $N_{\text{dr}}^2 \mathcal{P}_{II}^{(2)}$

New constraints on DR Isocurvature

- FDR gives larger anisotropy \leftrightarrow CDR can have more isocurvature
- For the adiabatic case, CDR allows for larger ΔN_{eff} : feature persists for isocurvature as well.



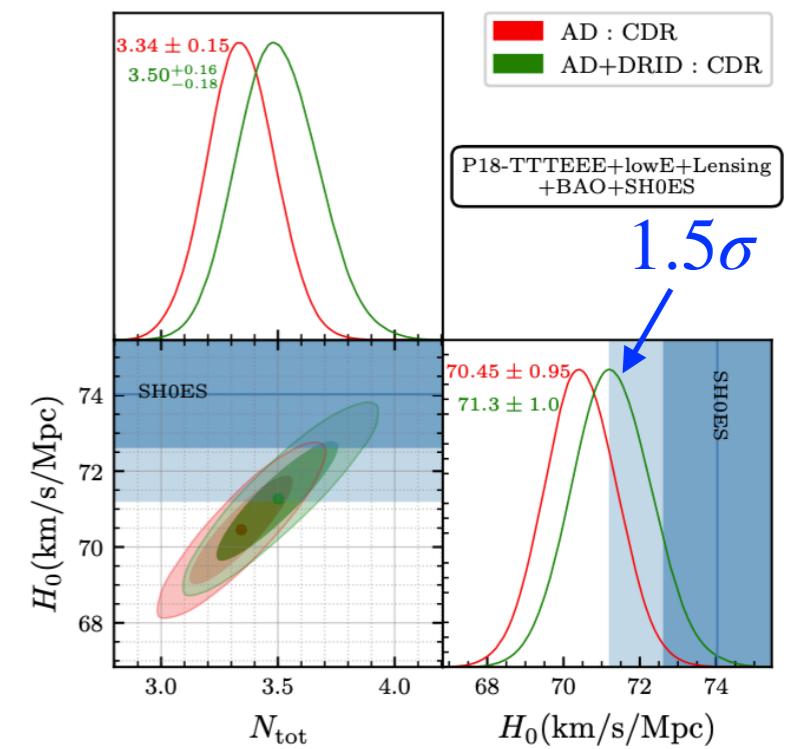
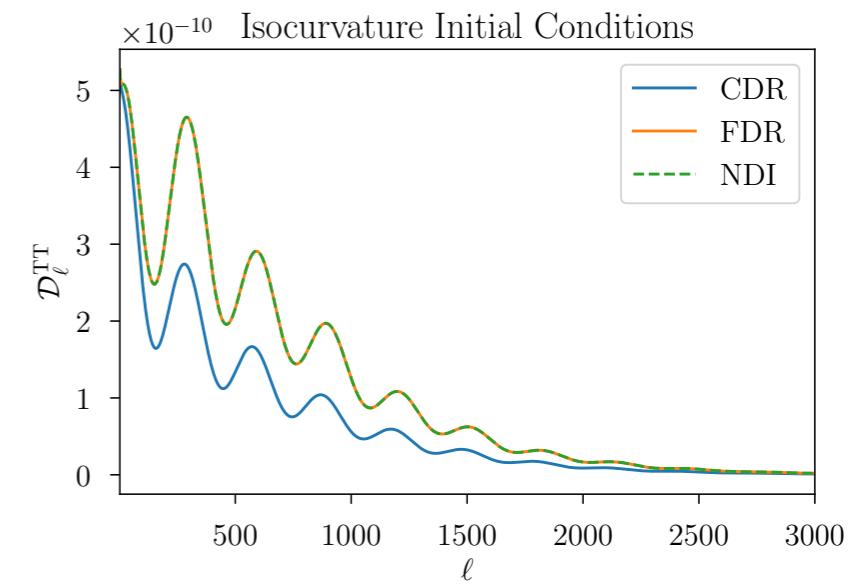
Relaxing the H_0 tension



- Larger N_{eff} and blue-tilted isocurvature: both help

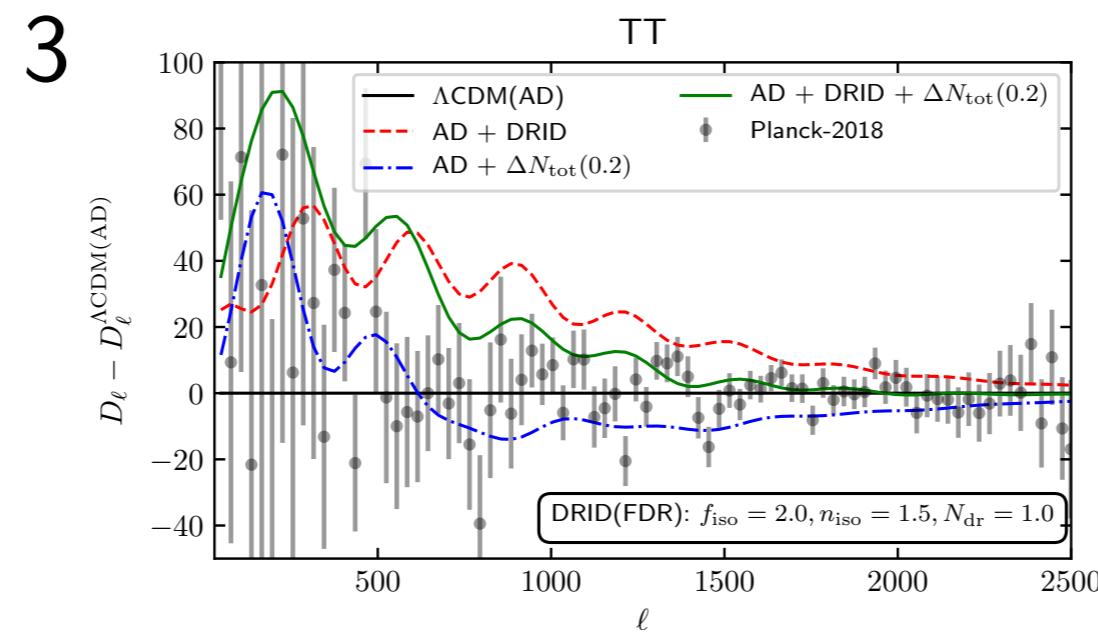
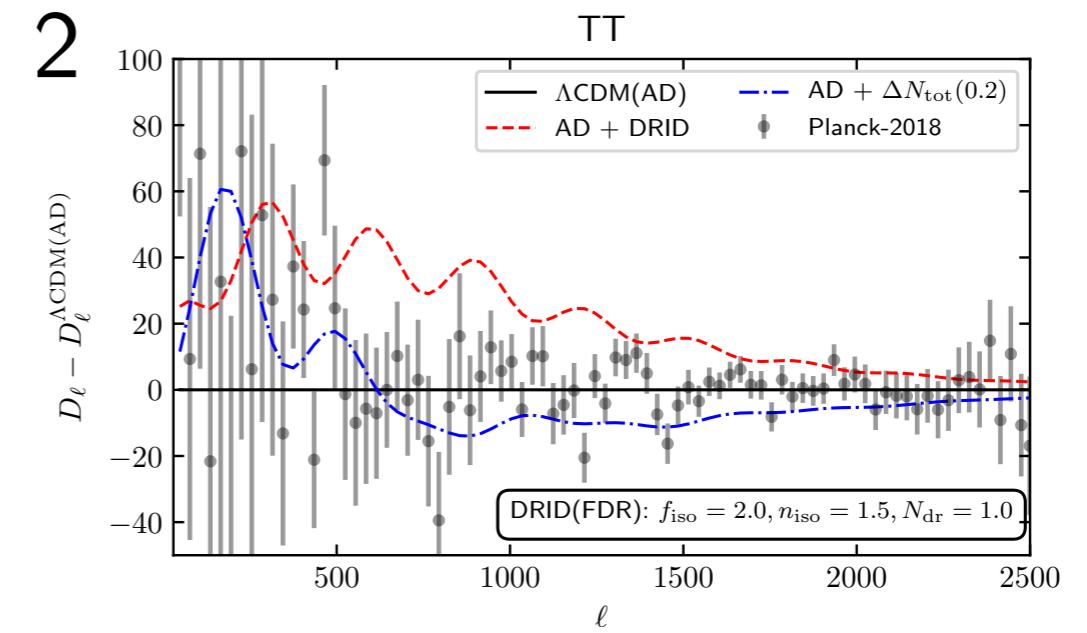
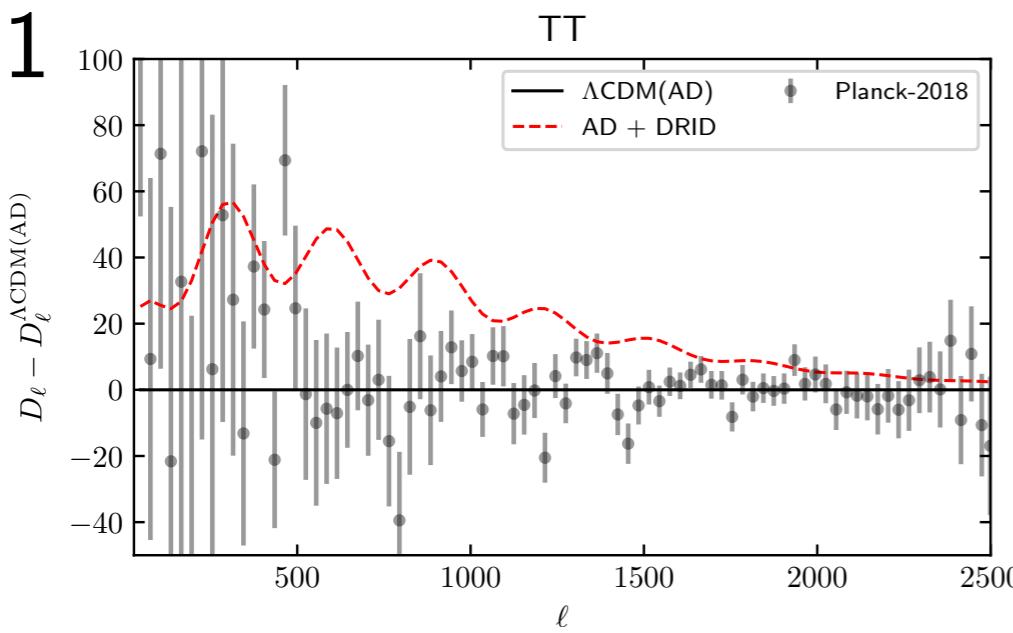
Conclusions

- In the presence of isocurvature initial conditions, FDR gives larger CMB anisotropy compared to CDR.
- Can hide more CDR in the data.
- Blue-tilt of isocurvature perturbation helps relax the H_0 tension.
- Combined, CDR Isocurvature can reduce tension to 1.5σ

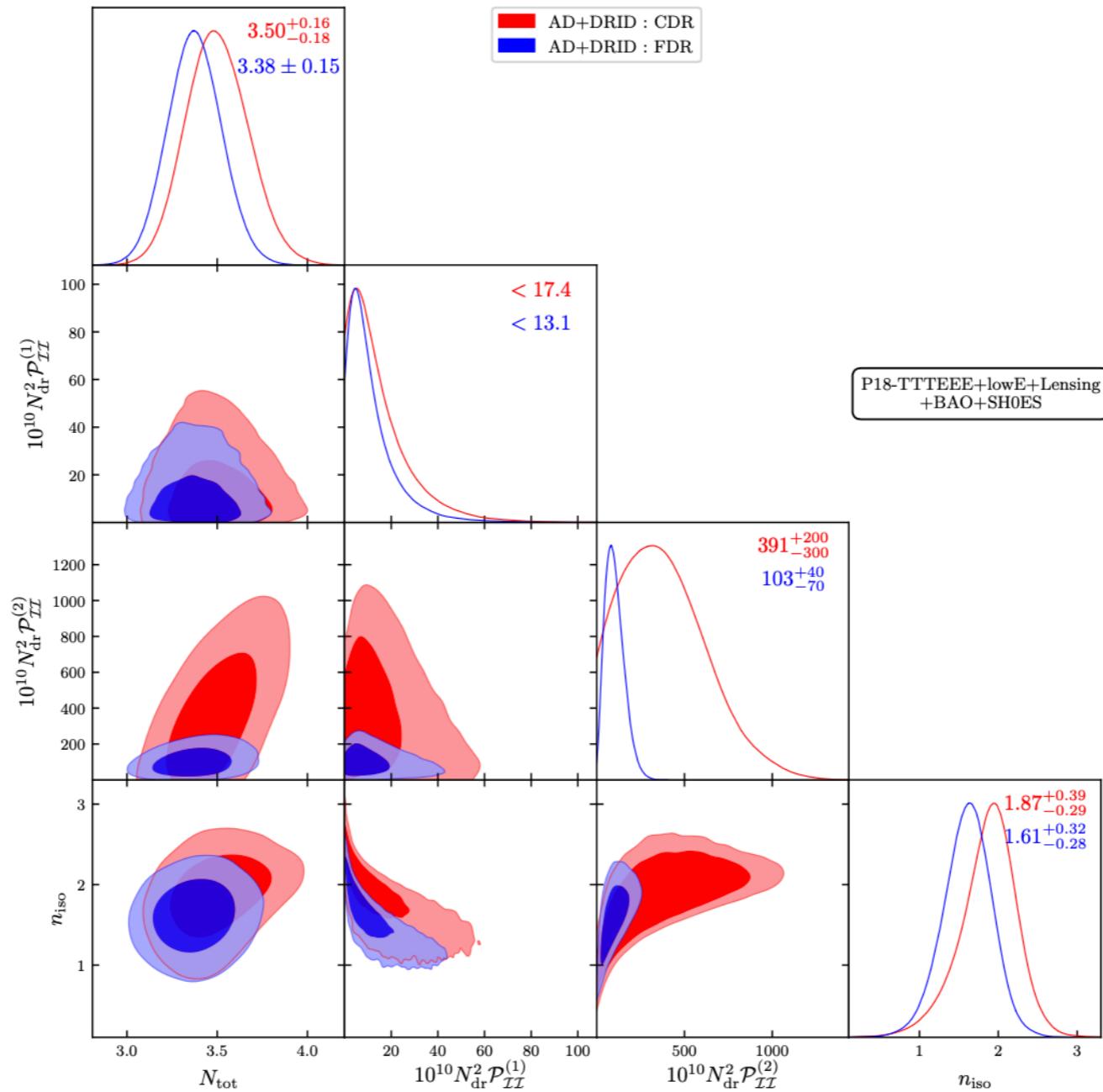


Thanks for your attention!

Backup - Compensating Silk Damping



Adding BAO and SH0ES



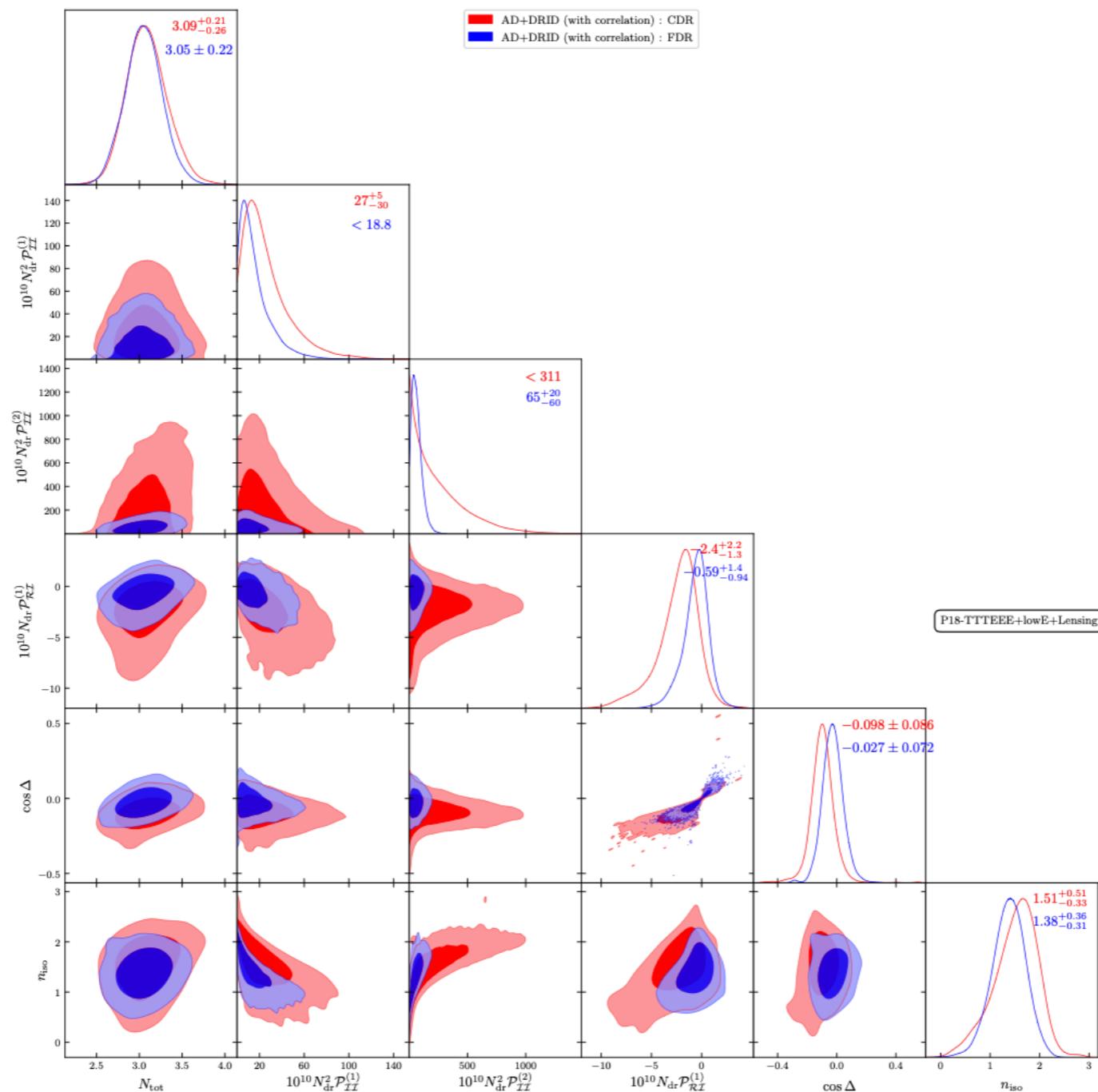
FDR Parameters

FDR	P18-TT+lowE	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES
$100 \omega_b$	$2.3^{+0.052}_{-0.064}$	$2.253^{+0.025}_{-0.026}$	$2.278^{+0.017}_{-0.017}$
ω_{cdm}	$0.1252^{+0.0046}_{-0.0058}$	$0.12^{+0.0031}_{-0.0031}$	$0.1241^{+0.0027}_{-0.0028}$
$100 * \theta_s$	$1.042^{+0.00073}_{-0.00077}$	$1.042^{+0.00052}_{-0.00053}$	$1.042^{+0.0005}_{-0.0005}$
τ_{reio}	$0.05416^{+0.0085}_{-0.0091}$	$0.05534^{+0.0075}_{-0.008}$	$0.05594^{+0.007}_{-0.0075}$
$10^{10} P_{\mathcal{R}\mathcal{R}}^{(1)}$	$22^{+1.1}_{-1.1}$	$23.32^{+0.57}_{-0.57}$	$22.88^{+0.49}_{-0.49}$
$10^{10} P_{\mathcal{R}\mathcal{R}}^{(2)}$	$20.55^{+0.57}_{-0.63}$	$20.37^{+0.43}_{-0.46}$	$20.68^{+0.37}_{-0.39}$
$10^{10} N_{\text{dr}}^2 P_{\mathcal{I}\mathcal{I}}^{(1)}$	$17.48^{+2.1}_{-17}$	$11.22^{+1.7}_{-11}$	$11.84^{+1.7}_{-12}$
$10^{10} N_{\text{dr}}^2 P_{\mathcal{I}\mathcal{I}}^{(2)}$	$228.9^{+61}_{-2.2e+02}$	73.91^{+26}_{-60}	102.1^{+37}_{-67}
N_{ur}	$2.469^{+1.3}_{-0.79}$	$2.031^{+1.1}_{-0.49}$	$2.265^{+1.1}_{-0.47}$
N_{dr}	$1.19^{+0.34}_{-1.2}$	$1.066^{+0.32}_{-1.1}$	$1.111^{+0.33}_{-1.1}$
H_0	$74.03^{+3.9}_{-5.1}$	$68.8^{+1.6}_{-1.7}$	$70.71^{+0.97}_{-0.98}$
σ_8	$0.8231^{+0.015}_{-0.015}$	$0.82^{+0.01}_{-0.01}$	$0.8302^{+0.009}_{-0.0092}$
$10^{+9} A_s$	$2.079^{+0.045}_{-0.049}$	$2.087^{+0.036}_{-0.038}$	$2.105^{+0.033}_{-0.034}$
n_s	$0.9828^{+0.017}_{-0.017}$	$0.9654^{+0.0091}_{-0.0092}$	$0.9741^{+0.0068}_{-0.0068}$
n_{iso}	$1.72^{+0.36}_{-0.32}$	$1.52^{+0.35}_{-0.29}$	$1.61^{+0.32}_{-0.28}$
f_{iso}	$17.4^{+7.0}_{-17}$	$11.9^{+5.2}_{-11}$	$13.0^{+5.3}_{-12}$
N_{tot}	$3.66^{+0.4}_{-0.54}$	$3.097^{+0.21}_{-0.21}$	$3.376^{+0.15}_{-0.16}$
f_{dr}	$0.3285^{+0.097}_{-0.33}$	$0.3444^{+0.1}_{-0.34}$	$0.3293^{+0.095}_{-0.33}$
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	-0.36	-3.54	-9.24

CDR Parameters

CDR	P18-TT+lowE	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES
$100 \omega_b$	$2.267^{+0.039}_{-0.05}$	$2.257^{+0.026}_{-0.028}$	$2.285^{+0.018}_{-0.018}$
ω_{cdm}	$0.1301^{+0.0053}_{-0.011}$	$0.122^{+0.0034}_{-0.004}$	$0.1272^{+0.0031}_{-0.0035}$
$100 * \theta_s$	$1.042^{+0.0011}_{-0.0012}$	$1.043^{+0.00064}_{-0.00075}$	$1.042^{+0.00065}_{-0.00081}$
τ_{reio}	$0.05327^{+0.0079}_{-0.0087}$	$0.0561^{+0.0075}_{-0.0084}$	$0.05643^{+0.007}_{-0.0077}$
$10^{10} P_{\mathcal{R}\mathcal{R}}^{(1)}$	$23.06^{+0.93}_{-0.95}$	$23.46^{+0.55}_{-0.57}$	$23.14^{+0.52}_{-0.54}$
$10^{10} P_{\mathcal{R}\mathcal{R}}^{(2)}$	$20.32^{+0.69}_{-0.67}$	$20.19^{+0.45}_{-0.48}$	$20.34^{+0.45}_{-0.44}$
$10^{10} N_{dr}^2 P_{\mathcal{I}\mathcal{I}}^{(1)}$	25.54^{+4}_{-26}	$16.43^{+2.9}_{-16}$	$15.39^{+2.6}_{-15}$
$10^{10} N_{dr}^2 P_{\mathcal{I}\mathcal{I}}^{(1)}$	$662.7^{+91}_{-6.6e+02}$	$218.7^{+50}_{-2.2e+02}$	$390.6^{+1.6e+02}_{-3.2e+02}$
N_{ur}	$3.408^{+0.46}_{-0.72}$	$2.938^{+0.24}_{-0.26}$	$3.164^{+0.27}_{-0.24}$
N_{dr}	$0.2589^{+0.051}_{-0.26}$	$0.2444^{+0.064}_{-0.24}$	$0.3372^{+0.14}_{-0.27}$
H_0	$71.79^{+3}_{-4.7}$	$69.19^{+1.7}_{-1.9}$	$71.27^{+1}_{-1.1}$
σ_8	$0.8341^{+0.017}_{-0.023}$	$0.8205^{+0.01}_{-0.011}$	$0.8298^{+0.0096}_{-0.0096}$
$10^{+9} A_s$	$2.077^{+0.054}_{-0.051}$	$2.073^{+0.038}_{-0.04}$	$2.081^{+0.038}_{-0.038}$
n_s	$0.9677^{+0.016}_{-0.016}$	$0.9617^{+0.0092}_{-0.0094}$	$0.9671^{+0.0086}_{-0.0079}$
n_{iso}	$1.83^{+0.45}_{-0.41}$	$1.66^{+0.43}_{-0.35}$	$1.87^{+0.39}_{-0.29}$
f_{iso}	< 31.7	58^{+22}_{-53}	49^{+23}_{-44}
N_{tot}	$3.666^{+0.37}_{-0.71}$	$3.182^{+0.22}_{-0.26}$	$3.501^{+0.17}_{-0.19}$
f_{dr}	$0.07186^{+0.014}_{-0.072}$	$0.07591^{+0.021}_{-0.076}$	$0.09615^{+0.04}_{-0.077}$
$\chi^2 - \chi^2_{\Lambda CDM}$	2.72	0.46	-5.8

Turning on Isocurvature Correlation



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