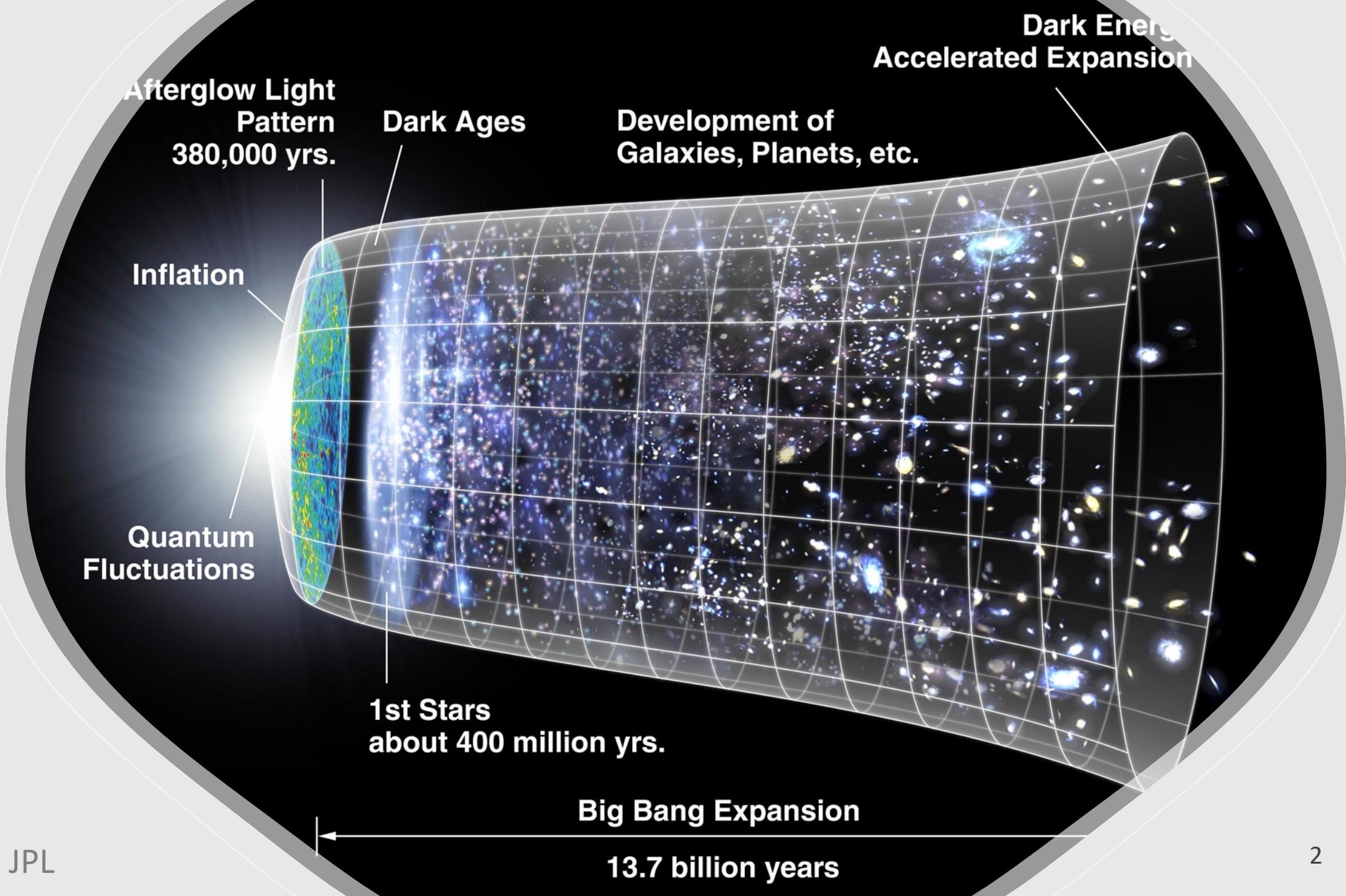


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Based on arXiv:2103.01229 w/ Hayden Lee & Cora Dvorkin



Why cross-correlations between different datasets?

Schmittfull, Seljak (2017)
Yu, Knight, Sherwin+ (2018)

Total Sum of Neutrino Mass

Neutrino oscillation measurements implies

$$m_2^2 - m_1^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2$$

$$|m_3^2 - m_1^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

Normal Hierarchy

$$m_3 > m_2 > m_1$$



$$\sum m_i \gtrsim 60 \text{ meV}$$

Inverted Hierarchy

$$m_2 > m_1 > m_3$$



$$\sum m_i \gtrsim 100 \text{ meV}$$

Planck 2018 sets an 95% upper bound of $\sum m_i \lesssim 120 \text{ meV}$

Primordial Non-Gaussianity

We focus on the local non-Gaussianity

$$\phi(\mathbf{x}) = \phi_g(\mathbf{x}) + f_{\text{NL}}(\phi_g^2(\mathbf{x}) - \langle \phi_g^2 \rangle)$$

$$\sigma(f_{\text{NL}}) \simeq 1$$



Insights into many physically motivated inflationary scenarios

Planck 2018 sets an 68% CL of $\sigma(f_{\text{NL}}) = -0.9 \pm 5.1$

- Momentum Space Spectra
- Harmonic Space Spectra
- Application

Matter Density Field

Some corrections needed in the presence of massive neutrinos

- Amplitude of the total matter density

$$\delta = (1 - f_\nu) \delta_{cb} + f_\nu \delta_\nu \quad f_\nu \equiv \frac{\Omega_\nu}{\Omega_m} \approx \frac{1}{\Omega_{m0} h^2} \frac{M_\nu}{93.14 \text{ eV}}$$

- Scale-dependent growth function

$$k_{\text{fs}} \approx 0.023 \left(\frac{M_\nu}{0.1 \text{ eV}} \right) \left(\frac{2}{1+z} \frac{\Omega_{m0}}{0.23} \right)^{\frac{1}{2}} h \text{Mpc}^{-1}$$

Galaxy Number Density

We consider the EFTofLSS

$$\delta_g = \sum_{\mathcal{Q}} b_{\mathcal{Q}} \mathcal{Q}(\Phi)$$

At one-loop power spectrum, we need up to third-order terms as

$$\begin{aligned} \delta_g = & b_\delta \delta_{cb} + b_{\delta^2} \delta_{cb}^2 + b_{\mathcal{G}_2} \mathcal{G}_2[\Phi_g] + b_{\delta^3} \delta_{cb}^3 \\ & + b_{\mathcal{G}_2 \delta} \mathcal{G}_2[\Phi_g] \delta_{cb} + b_{\mathcal{G}_3} \mathcal{G}_3[\Phi_g] + b_{\Gamma_3} \Gamma_3 + O(\Phi^4) \end{aligned}$$

McDonald (2006)

Chan, Scoccimarro, Sheth (2012)

Assassi, Baumann, Green+ (2014)

Senatore (2014)

Galaxy Power Spectrum and Bispectrum

- Power Spectrum (Up to one-loop)

$$P^{gm} = b_\delta (P_{11}^{cb,m} + P_{13}^{cb,m} + P_{22}^{cb,m}) + b_{\delta^2} \mathcal{I}_{\delta^2}^{cb,m} + b_{\mathcal{G}_2} \mathcal{I}_{\mathcal{G}_2}^{cb,m} + (b_{\mathcal{G}_2} + \frac{2}{5} b_{\Gamma_3}) \mathcal{F}_{\mathcal{G}_2}^{cb,m}$$

$$\begin{aligned} P^{gg} = & b_\delta^2 (P_{11}^{cb,cb} + P_{13}^{cb,cb} + P_{22}^{cb,cb}) + 2b_{\partial^2 \delta} (k/k_*)^2 P_{11}^{cb,cb} + 2b_\delta b_{\delta^2} \mathcal{I}_{\delta^2}^{cb,cb} + 2b_\delta b_{\mathcal{G}_2} \mathcal{I}_{\mathcal{G}_2}^{cb,cb} \\ & + (2b_\delta b_{\mathcal{G}_2} + \frac{4}{5} b_\delta b_{\Gamma_3}) \mathcal{F}_{\mathcal{G}_2}^{cb,cb} + b_{\delta^2}^2 \mathcal{I}_{\delta^2 \delta^2}^{cb,cb} + b_{\mathcal{G}_2}^2 \mathcal{I}_{\mathcal{G}_2 \mathcal{G}_2}^{cb,cb} + 2b_{\delta^2} b_{\mathcal{G}_2} \mathcal{I}_{\delta^2 \mathcal{G}_2}^{cb,cb} \end{aligned}$$

- Bispectrum (Only tree-level)

$$B^{mmm} = 2P_2^{mm} P_3^{mm} F_2(\mathbf{k}_2, \mathbf{k}_3) + 2 \text{ perms}$$

$$B^{mmg} = 2P_2^{mm} P_3^{cb,m} (b_\delta F_2(\mathbf{k}_2, \mathbf{k}_3)) + (1 \leftrightarrow 2) + 2P_1^{cb,m} P_2^{cb,m} (b_{\delta^2} + b_{\mathcal{G}_2} L_2(\mathbf{k}_2, \mathbf{k}_3) + b_\delta F_2(\mathbf{k}_2, \mathbf{k}_3))$$

$$B^{mgg} = 2b_\delta P_1^{cb,m} P_2^{cb,cb} (b_{\delta^2} + b_{\mathcal{G}_2} L_2(\mathbf{k}_1, \mathbf{k}_2) + b_\delta F_2(\mathbf{k}_1, \mathbf{k}_2)) + (2 \leftrightarrow 3) + 2b_\delta^2 P_2^{cb,m} P_3^{cb,m} F_2(\mathbf{k}_2, \mathbf{k}_3)$$

$$B^{ggg} = 2b_\delta^2 P_1^{cb,cb} P_2^{cb,cb} (b_{\delta^2} + b_{\mathcal{G}_2} L_2(\mathbf{k}_1, \mathbf{k}_2) + b_\delta F_2(\mathbf{k}_1, \mathbf{k}_2)) + 2 \text{ perms}$$

Primordial Local Non-Gaussianity

Two kinds of contributions to the galaxy spectra

- Direct relation to the primordial potential

$$B_{\text{PNG}}^{mmm}(\{z_i, k_i\}) = M(z_1, k_1)M(z_2, k_2)M(z_3, k_3)B^{\phi\phi\phi}(k_1, k_2, k_3)$$

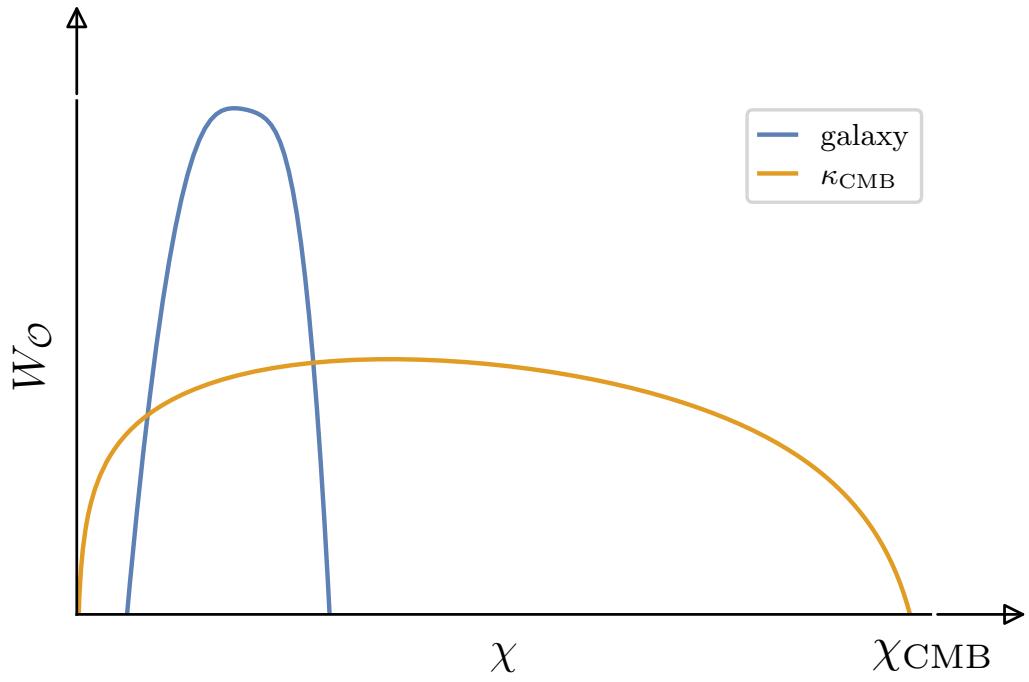
- Scale-dependent galaxy bias

$$\Delta b_\delta(z, k) = \frac{2f_{\text{NL}}(b_\delta(z) - 1)\delta_c}{M(z, k)}$$

- Momentum Space Spectra
- Harmonic Space Spectra
- Application

Projected Observables

$$\mathcal{O}(\hat{\mathbf{n}}) = \int_0^\infty d\chi W_{\mathcal{O}}(\chi) \mathcal{O}(\chi, \chi \hat{\mathbf{n}}) = \sum_{\ell m} \mathcal{O}_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$



Advantages:

- Direct Observables
- Combination with weak lensing

Disadvantages:

- Line of sight integral is time consuming (w/o Limber)
- We may lose some information

Line of Sight Integral

The spherical harmonic coefficient of the projected field is

$$\mathcal{O}_{\ell m} = 4\pi i^\ell \int_0^\infty d\chi W_{\mathcal{O}}(\chi) \int_{\mathbf{k}} j_\ell(k\chi) Y_{\ell m}^*(\hat{\mathbf{k}}) \tilde{\mathcal{O}}(z, \mathbf{k})$$

Then we can write correlation function as

$$\langle \mathcal{O}_{\ell_1 m_1} \cdots \mathcal{O}_{\ell_n m_n} \rangle = \frac{\mathcal{G}_{m_1 \cdots m_n}^{\ell_1 \cdots \ell_n}}{(2\pi^2)^n} \int_0^\infty dr r^2 I_{\ell_1}^{(1)}(r) \cdots I_{\ell_n}^{(n)}(r)$$



$$I_\ell^{(i)}(r) \equiv 4\pi \int_0^\infty d\chi W_{\mathcal{O}}(\chi) \int_0^\infty dk k^2 j_\ell(k\chi) j_\ell(kr) g_i(\chi, k)$$

Problems with Limber Approximation

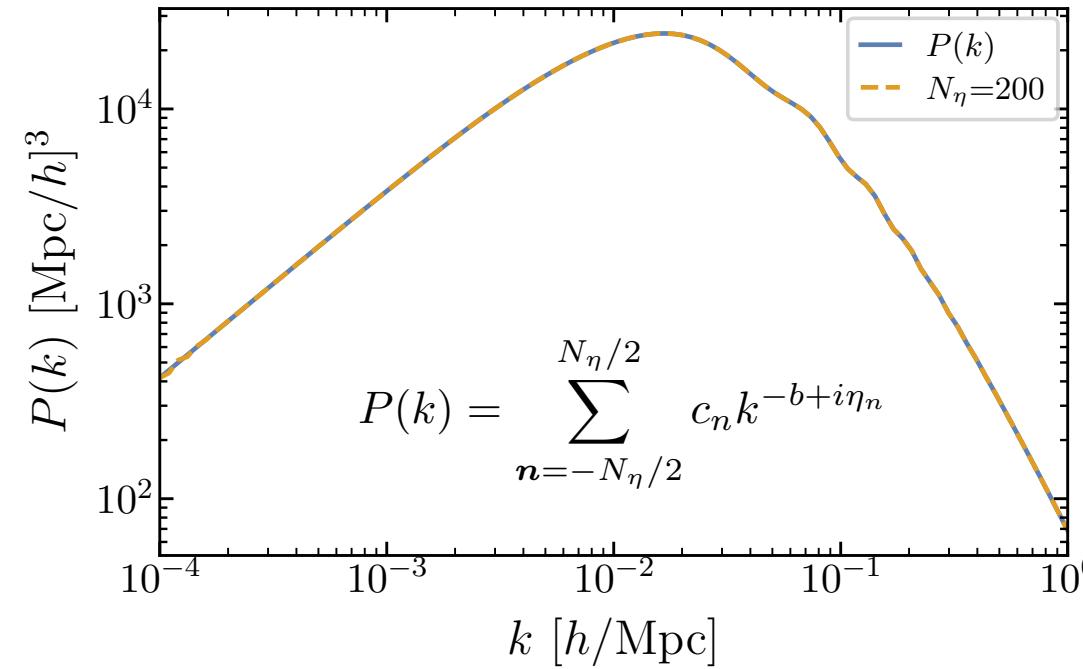
- Fails at large scale, or small ℓ 's
- Fails for non-overlapping tomographic bins
- Wide tomographic bin

$$j_\ell(x) \simeq \sqrt{\frac{\pi}{2\ell}} \delta_D(\ell + \frac{1}{2} - x)$$

FFTLog Algorithm

Fast Fourier Transformation in Log k

$$g_i(\chi, k) \sim \sum_{n=-N_\eta/2}^{N_\eta/2} c_n(\chi) k^{-b+i\eta_n} \quad \text{with} \quad \eta_n \equiv \frac{2\pi n}{\log(k_{\max}/k_{\min})}$$



FFTLog Algorithm

$$I_\ell^{(i)}(r) \equiv 4\pi \int_0^\infty d\chi W_O(\chi) \int_0^\infty dk k^2 j_\ell(k\chi) j_\ell(kr) g_i(\chi, k)$$



Involves growth function and matter power spectrum

$$I_\ell^{(i)}(r) \simeq \sum_n \int_0^\infty d\chi c_n(\chi) W_O(\chi) \chi^{-\nu_n} I_\ell(\nu_n, \frac{r}{\chi}), \text{ with } \nu_n \equiv 3 - b + i\eta_n$$

$$I_\ell(\nu, w) = \frac{2^{\nu-1} \pi^2 \Gamma(\ell + \frac{\nu}{2})}{\Gamma(\frac{3-\nu}{2}) \Gamma(\ell + \frac{3}{2})} w^\ell {}_2F_1 \left[\begin{array}{c} \frac{\nu-1}{2}, \ell + \frac{\nu}{2} \\ \ell + \frac{3}{2} \end{array} \middle| w^2 \right] \quad (|w| \leq 1)$$

FFTLog Algorithm: Take a closer look

$$I_\ell^{(i)}(r) \equiv 4\pi \int_0^\infty d\chi W_O(\chi) \int_0^\infty dk k^2 j_\ell(k\chi) j_\ell(kr) g_i(\chi, k)$$

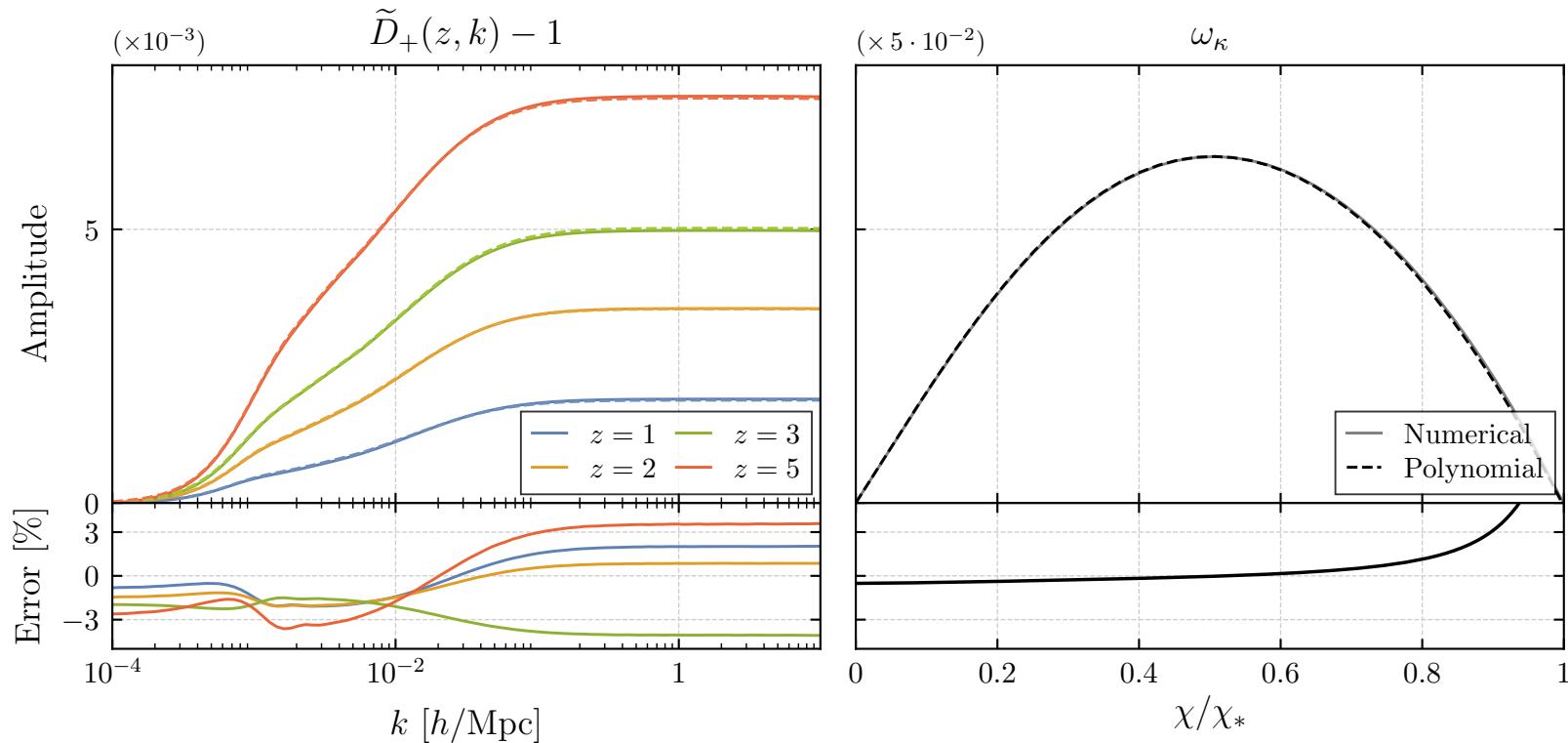
If $g_i(\chi, k) \equiv f(\chi)h(k)$, we can have

$$I_\ell^{(i)}(r) \simeq \sum_n \int_0^\infty d\chi c_n W_\delta(\chi) \chi^{-\nu_n} I_\ell(\nu_n, \frac{r}{\chi})$$


Independent of the comoving distance!

Incorporating Scale-Dependent Growth Function

$$g_i(\chi, k) \simeq \sum_{p=0}^{N_{\text{poly}}} w_p(k) \chi^p$$



Angular Power Spectra and Bispectra

Chen, Lee, Dvorkin (2021)

- Power Spectra

$$C_\ell = \sum_{p_1 p_2} c^{\mathcal{O}_1 \mathcal{O}_2} \int_0^\infty dr^2 r^2 I_\ell^{\mathcal{O}_1 \mathcal{O}_2}(r; 0, p_1, p_2) I_\ell^{\mathcal{O}_2}(r; 0, p_1, p_2)$$

- Bispectra

$$\begin{aligned} b_{\ell_1 \ell_2 \ell_3}^{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3} &= \sum_{p_1 p_2 p_3 p_4} \sum_{n_1 n_2 n_3} c_{n_1 n_2 n_3}^{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3} \\ &\times \int_0^\infty dr^2 r^2 I_{\ell_1}^{\mathcal{O}_1 \mathcal{O}_3}(r; n_1, p_1, p_3) I_{\ell_2}^{\mathcal{O}_2 \mathcal{O}_3}(r; n_2, p_2, p_4) I_{\ell_3}^{\mathcal{O}_3}(r; n_3, p_3, p_4) + 2 \text{ perms} \end{aligned}$$

$$I_\ell^{\mathcal{O} \mathcal{O}'}(r; n, p, q) \equiv 4\pi \int_0^\infty d\chi \chi^p W_{\mathcal{O}}(\chi) \int_0^\infty dk k^{2+n} j_\ell(kr) j_\ell(k\chi) \omega_p(k) \omega_q(k) P^{\mathcal{O} \mathcal{O}'}(k)$$

$$I_\ell^{\mathcal{O}}(r; n, p, q) \equiv 4\pi \int_0^\infty d\chi \chi^{p+q} W_{\mathcal{O}}(\chi) \int_0^\infty dk k^{2+n} j_\ell(kr) j_\ell(k\chi)$$

Can we make it even faster? Take C_ℓ for example

$$C_\ell \simeq \frac{1}{2\pi^2} \sum_{n,p,q} c_{npq} \int_{\mathbb{Z}_i} d\chi \chi^p \int_{\mathbb{Z}_j} d\chi' \chi'^{q-\nu_{npq}} I_\ell(\nu_{npq}, \frac{\chi}{\chi'})$$



$$(\cdots) \times {}_3F_2(\cdots)$$

We only need to evaluate the generalized hypergeometric function!

Can we make it even faster? Take C_ℓ for example

Method	M_ν	N_η	N_χ	N_{poly}	N_ℓ	Time
Numerical	—	100	80	—	30	1 min
Analytical	—	100	—	3	30	10 s
Numerical	✓	100	80	3	30	10 min
Analytical	✓	100	—	3	30	10 s

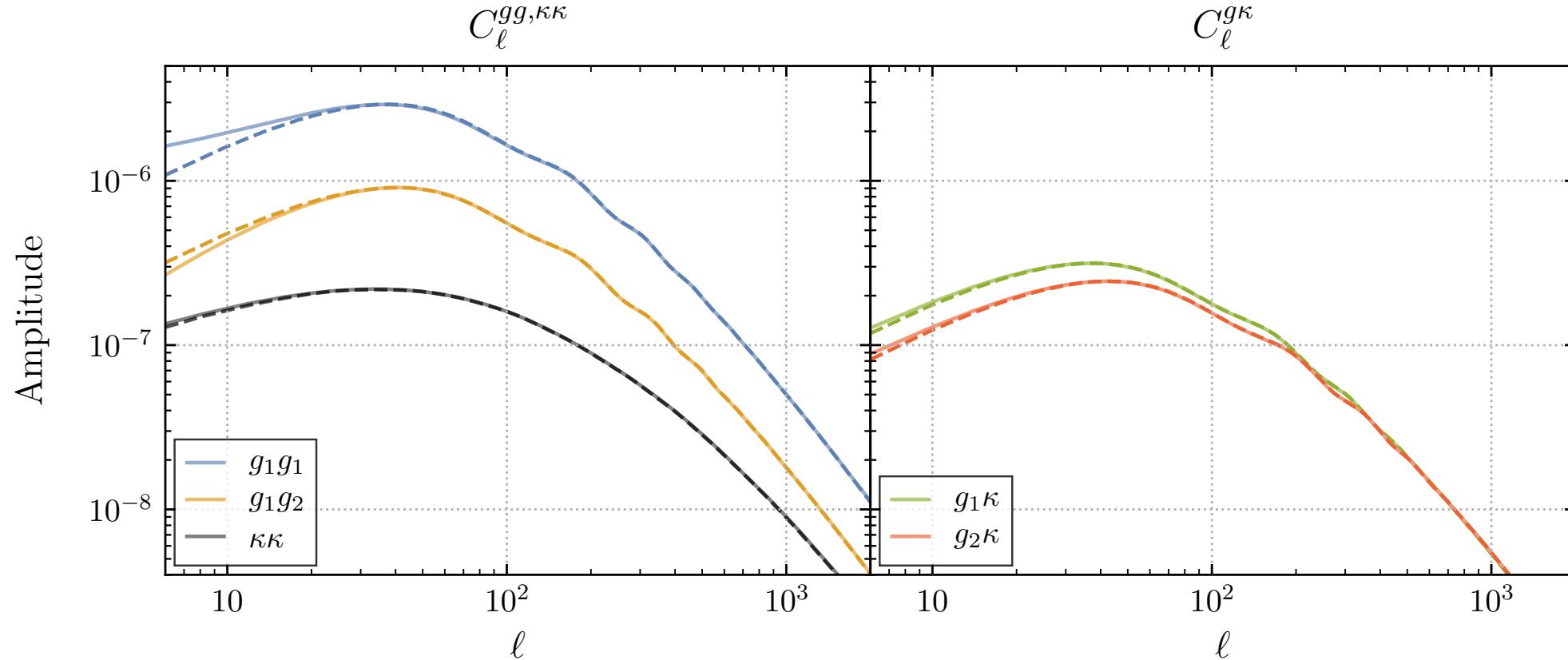
M_ν : whether including the scale-dependent growth function

N_χ : number of sampling points in the integral

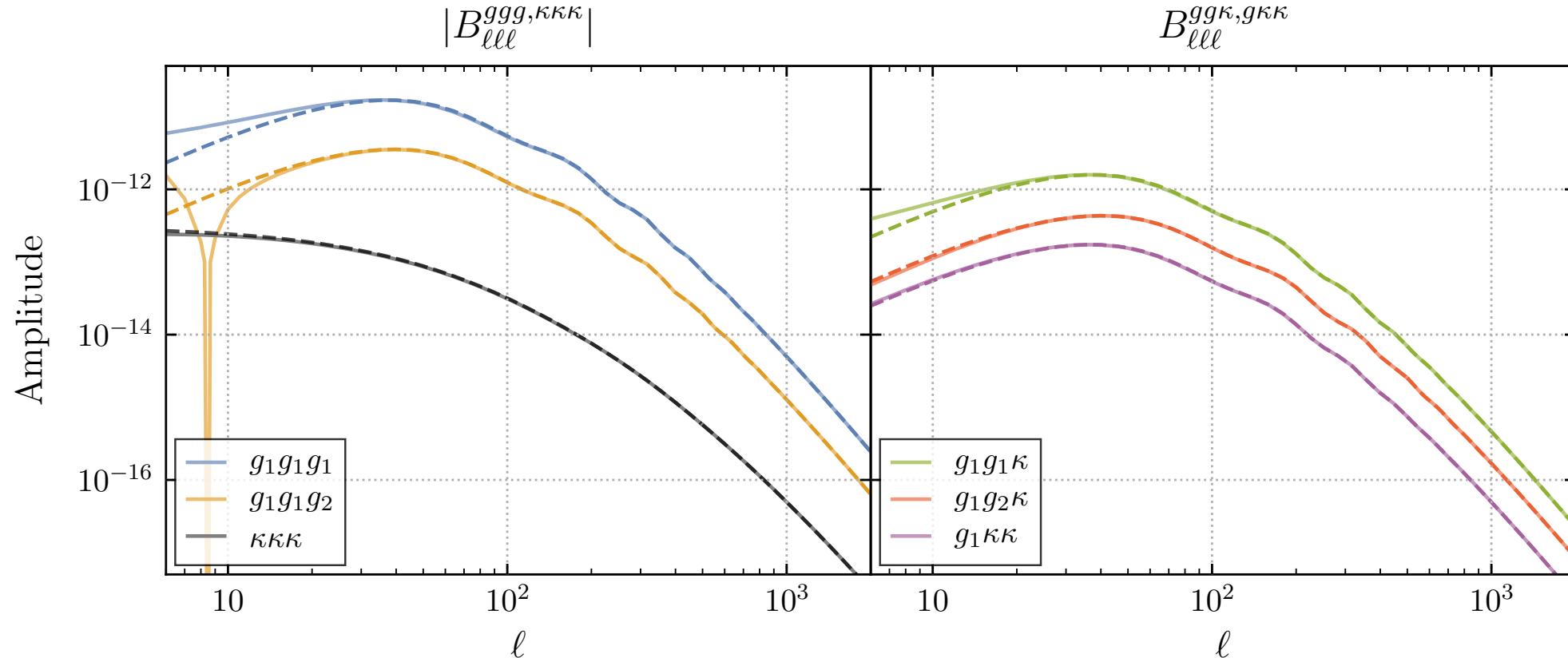
N_η : number of modes for the FFTLog transformation

N_{poly} : degree of the polynomial expansion

Comparison between Limber and FFTLog



Comparison between Limber and FFTLog



- Momentum Space Spectra
- Harmonic Space Spectra
- Application

Experimental Specifications

We consider cross-correlated power spectra and bispectra

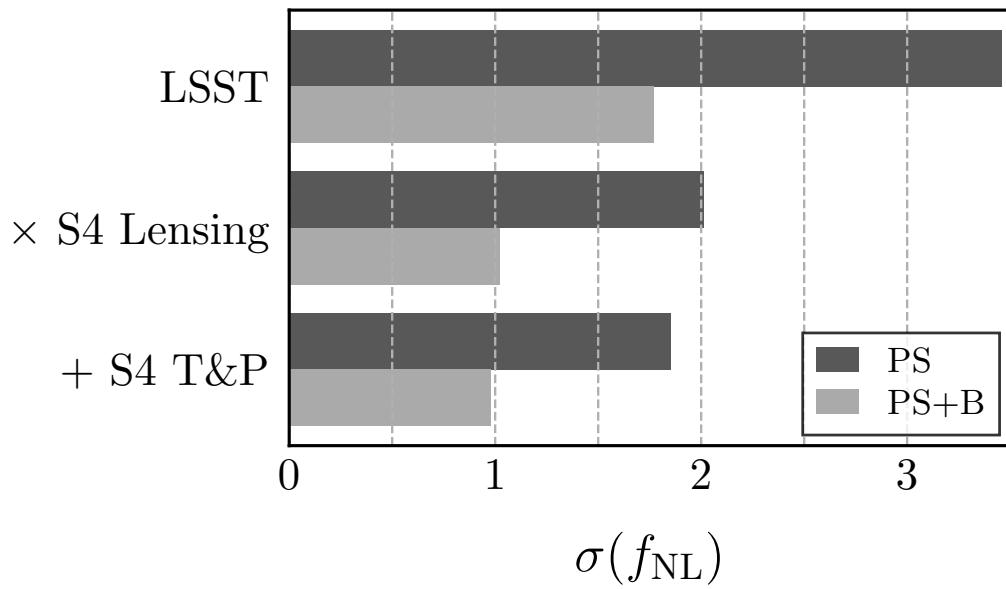
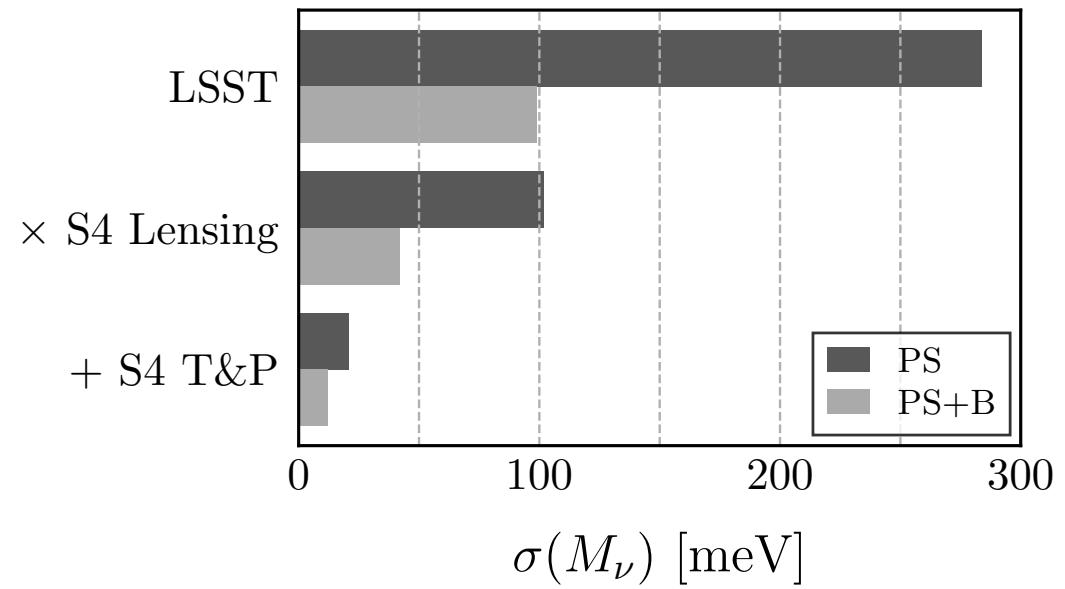
- Power spectra: Tree + One-loop with $k_{\max} = 0.3 h \text{Mpc}^{-1}$
- Bispectra: Tree with $k_{\max} = 0.1 h \text{Mpc}^{-1}$
- Both set $\ell_{\min} = 20$

We split it into 16 tomographic bins up to $z = 7$

We consider the following cosmological parameters

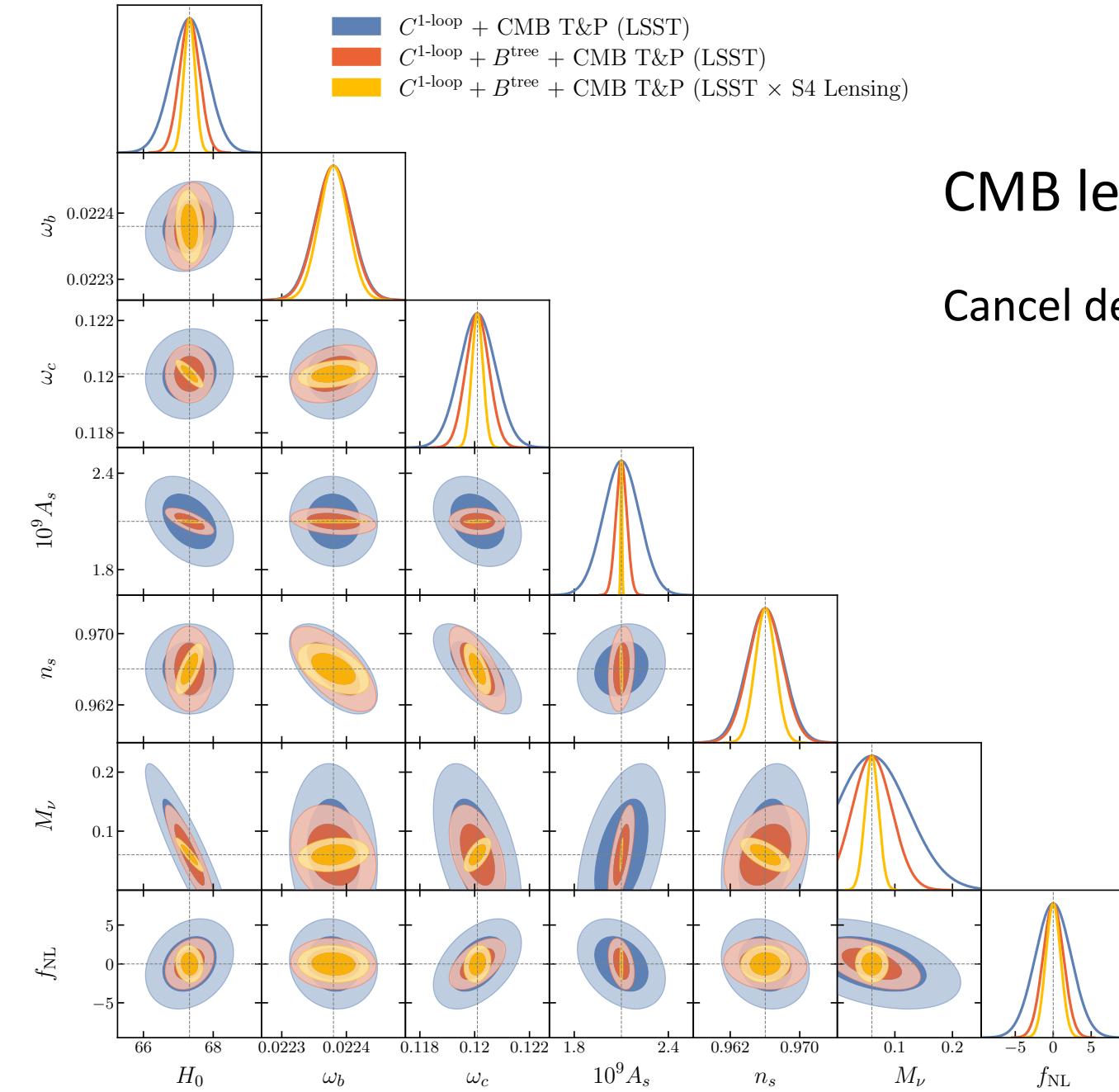
$$\lambda = \underbrace{\{H_0, \omega_c, \omega_b, A_s, n_s, \tau\}}_{\Lambda\text{CDM}} \cup \underbrace{\{M_\nu, f_{\text{NL}}\}}_{\text{non-minimal}} \cup \bigcup_{i=1}^{N_z} \underbrace{\{\bar{b}_\delta^{(i)}, \bar{b}_{\delta^2}^{(i)}, \bar{b}_{\mathcal{G}_2}^{(i)}, \bar{b}_{\partial^2 \delta}^{(i)}\}}_{\text{nuisance}}$$

Results of Fisher Forecast



Discussions on results of Fisher Forecast

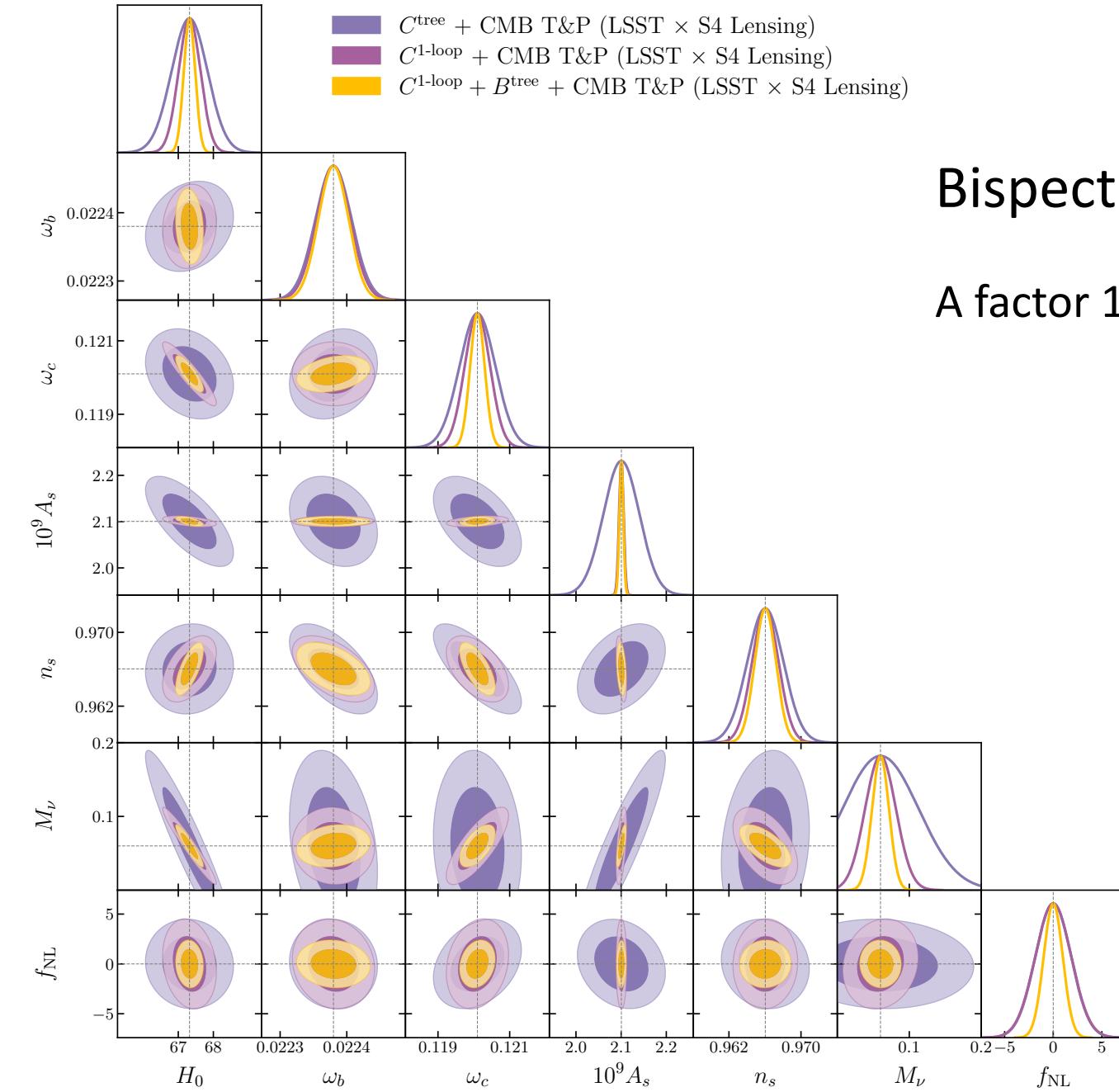
- CMB lensing
- Bispectrum
- One-Loop power spectrum
- Limber v.s. FFTLog



CMB lensing

Cancel degeneracies between amplitude-like parameters

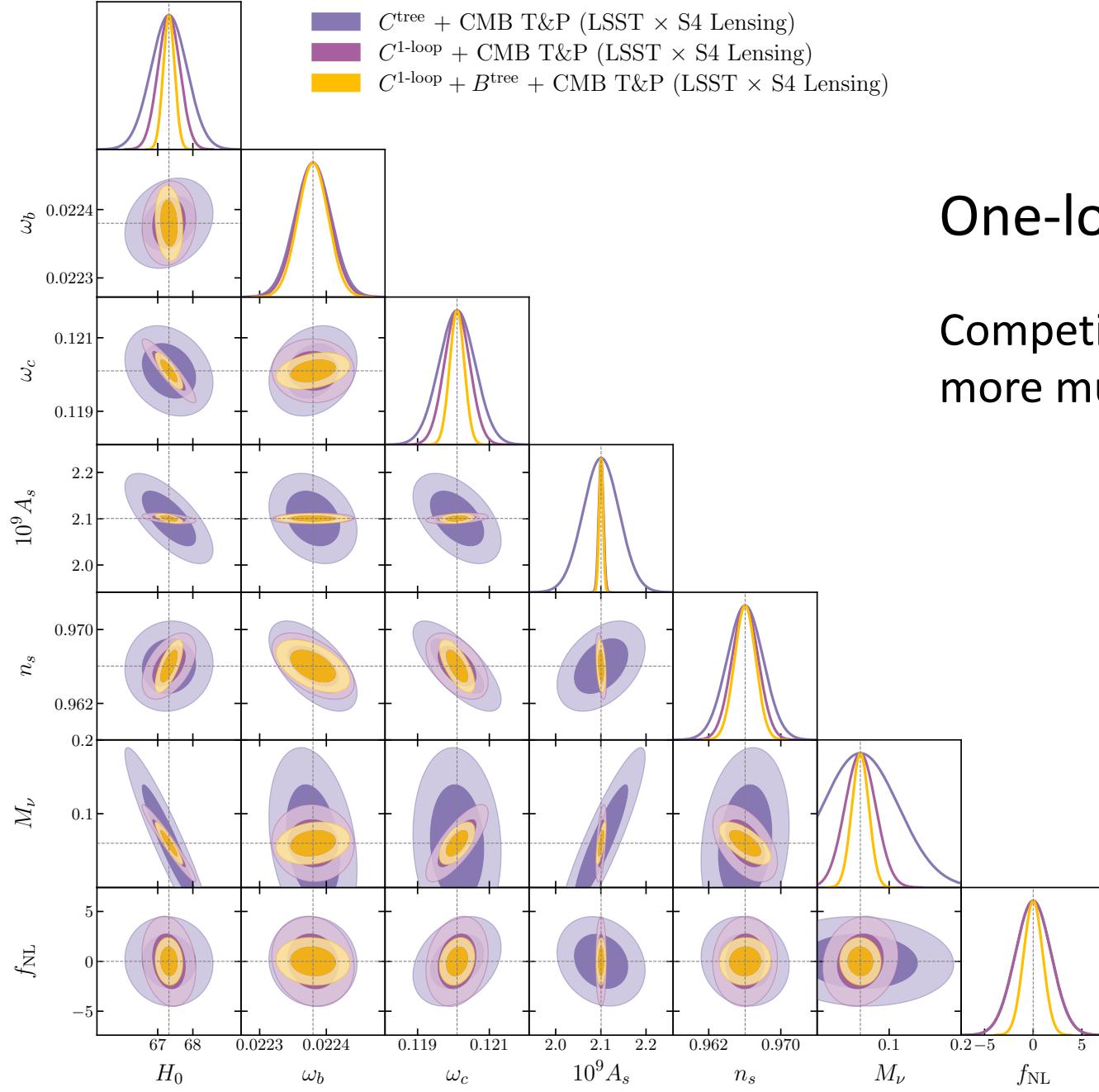
Chen, Lee, Dvorkin (2021)



Bispectrum

A factor 1.5-2.5 improvements on most parameters

Chen, Lee, Dvorkin (2021)



One-loop power spectrum

Competing effects between more nuisance parameters, more multipoles, and less degeneracies

Chen, Lee, Dvorkin (2021)

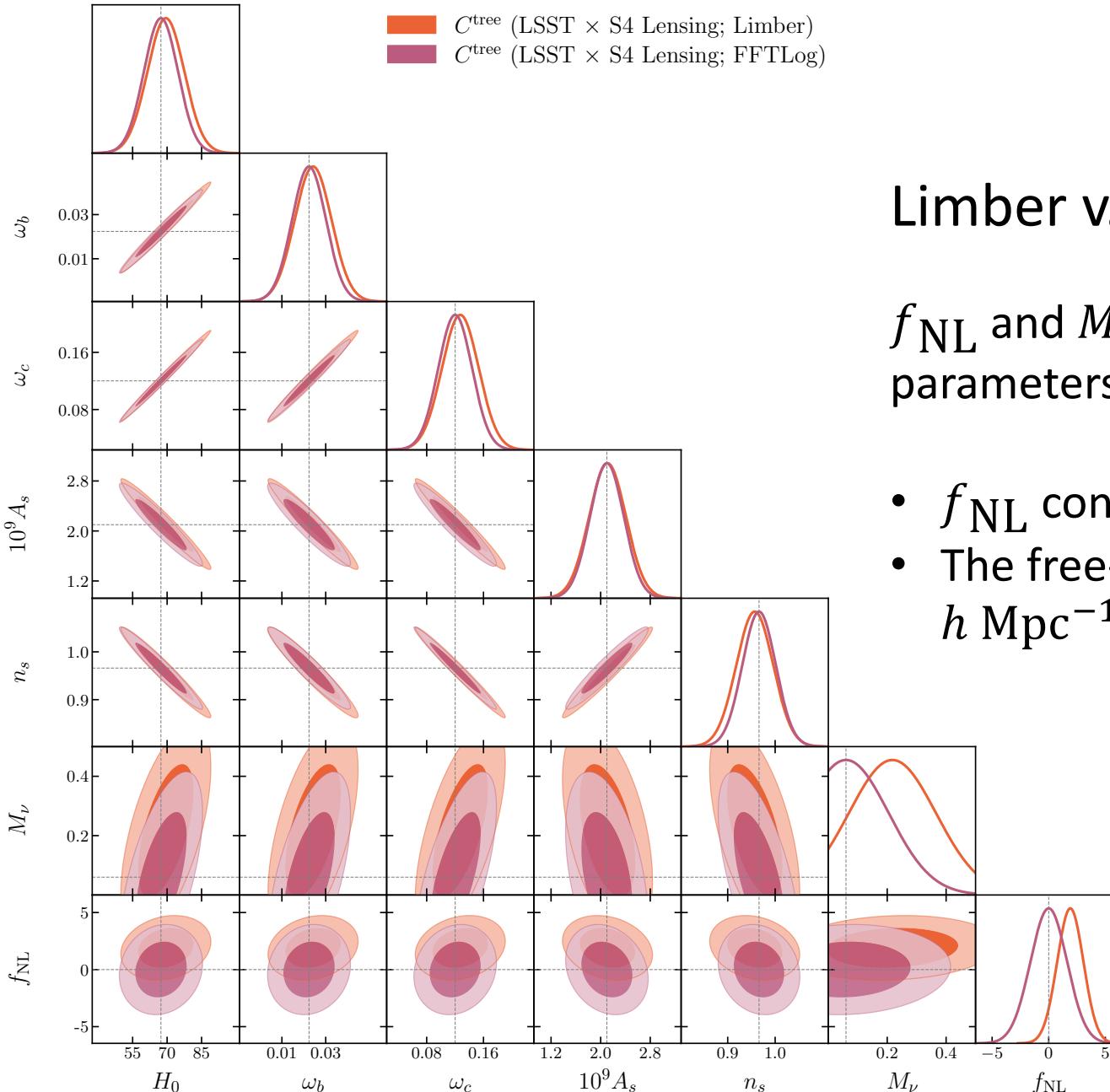
Limber v.s. FFTLog

The width of the constraints are similar between Limber and FFTLog

But there are **biases for the best-fit parameters!**

How do we estimate the bias?

$$\Delta\lambda_\alpha = \sum_\beta (\mathcal{F}_{\text{Limber}}^{\text{2pt}})_{\alpha\beta}^{-1} \left[\sum_{x'y'} \sum_x \sum_y \left(C_\ell^{xy} - C_{\ell,\text{Limber}}^{xy} \right) (C_{\text{Limber}}^{-1})_\ell^{xy,x'y'} \frac{\partial C_{\ell,\text{Limber}}^{x'y'}}{\partial \lambda_\beta} \right]$$



Limber v.s. FFTLog with $\ell_{\min} = 10$

f_{NL} and M_ν show 1 to 2σ shifts on the best-fit parameters!

- f_{NL} contributes to large scale in power spectrum
- The free-streaming scale for neutrino is roughly $0.02 h \text{ Mpc}^{-1}$, but Limber works well above $0.05 h \text{ Mpc}^{-1}$

Chen, Lee, Dvorkin (2021)

Conclusions

- FFTLog algorithm provides a good way for precise computation
- Limber approximation can induce systematic biases on the best-fit parameters
- Adding one-loop corrections, CMB lensing and bispectrum can each improve the overall parameter constraints by a factor of 1-2
- Significant detection of neutrino mass is possible