Dynamical behaviour of accelerating cosmological model $F(R, G)$ gravity

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Outlines

- Introduction and $F(R, G)$ gravity
- Field equations and Basic formalism of $F(R, G)$ gravity
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- Analysis of the model
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Introduction and $F(R, G)$ gravity

• The recent observational data in cosmology seem to indicate that the universe is currently expanding in an accelerated way.

• The present accelerating inflation of the universe has accelerated many investigations into the essence of the dark energy which might be important for this unexpected dynamics. [1]

• The $F(R, G)$ cosmology naturally leads to an effective cosmological constant, quintessence or phantom cosmic acceleration, not without describing the transition from a decelerated stage of the universe expansion.[2]

• A two-scalar field theory with combinations of R and G well represents gravity with second-order curvature invariants.

• Gauss–Bonnet topological invariant G that naturally arises in the process of quantum field theory regularization and re-normalization in curved spacetime. [3]

[1] E.J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15** 1753 (2006)

[2] E. Elizalde et al., Class. Quant. Grav. **27**, 095007 (2010)

[3] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge University Press, Cambridge, England, 1982)

Field equations and Basic formalism of $F(R,G)$ gravity

The most general action for $F(R,G)$ gravity[4,5]

$$S = \int \sqrt{-g}\left[\frac{1}{2k^2}F(R,G) + \mathcal{L}_m\right] d^4x \tag{1}$$

The Gauss-Bonnet curvature term is defined as

$$\mathcal{G} \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \tag{2}$$

In differential geometry G can be described as

$$\int_{\mathcal{M}} \mathcal{G} d^n x = \chi(\mathcal{M}) \tag{3}$$

Where χ is the Euler characteristics of manifold $\mathcal{M}$ in n-dimensions.

[4] M. De Laurentis et al., Phys. Rev. D **91**, 083531 (2015)
[5] S.D. Odintsov et al., Nucl. Phys. B **938**, 935 (2019)

Field Equations in $F(R, G)$ Gravity:

we consider the spatially flat FLRW metric with line element,

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{4}$$

Then R and G become,

$$R = 6(\dot{H} + 2H^2) \qquad G = 24H^2(\dot{H} + H^2) \tag{5}$$

We obtain the field equation from eq. (1) and eq. (4)

$$3H^2 F_R = \kappa^2\rho + \frac{1}{2}[RF_R + GF_G - F] - 3H\dot{F}_R - 12H^3\dot{F}_G \tag{6}$$

$$2\dot{H}F_R + 3H^2 F_R = -\kappa^2\rho + \frac{1}{2}[RF_R + GF_G - F] - 2H\dot{F}_R - \ddot{F}_R - 4H^2\ddot{F}_G - 8H\dot{H}\dot{F}_G - 8H^3\dot{F}_G \tag{7}$$

The Model and Hybrid scale factor

- We extend our analysis by considering a specific form for the $F(R, G)$ function.[6]

$$F(R, G) = R + \alpha R^2 + \beta G^2 \tag{8}$$

where α and β are constants with dimensions ℓ^2 and ℓ^4 respectively.

- Hybrid scale factor (HSF) $a(t) = e^{\eta t} t^{\nu}$, where η and ν are the scale factor parameters and are constrained in the ranges $\eta > 0$ and $0 < \nu < 1$.[7]

[6] M. De Laurentis et al., Phys. Rev. D **91**, 083531 (2015)

[7] B.Mishra and S.K. Tripathy, Mod.Phys. Lett. A, **30**, 1550175 (2015)

Analysis of the Model

I. Physical Parameter

• The energy density ρ and the equation of state (EoS) parameter ω behaviour can be seen in following fig.

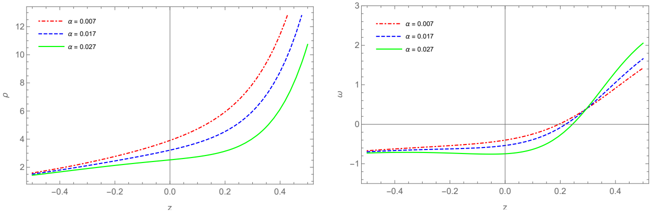

Figure: I

• The energy density decreasing from a higher positive value and approaching to a small value at the late time.

• The EoS parameter that shows us the behaviour of the accelerating universe is found to be evolve from a early positive value and approaching to ≈ -0.7 at late time.

II. Energy Conditions

- The energy conditions are basically the boundary conditions in order to keep the energy density positive.[8]

(**NEC**): $\rho + p \geq 0$; (**WEC**): $\rho + p \geq 0$, $\rho \geq 0$; (**SEC**): $\rho + 3p \geq 0$; (**DEC**): $\rho - p \geq 0$.

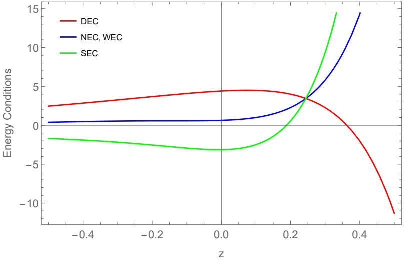

Figure: II

- The SEC starts violating from ($z \approx 0.2$) and was satisfying before that.
- The WEC remains positive from an early time ($z \approx 0.35$) till the late phase.

[8]S. Hawking and G.F.R. Ellis, The Large Scale Structure of Space-Time, Cambridge

III. Cosmographic Parameters

• The parameter set Hubble parameter (H), deceleration parameter (q), jerk parameter (j), snap parameter (s), lerk parameter (l) represents the alphabet of the cosmography.

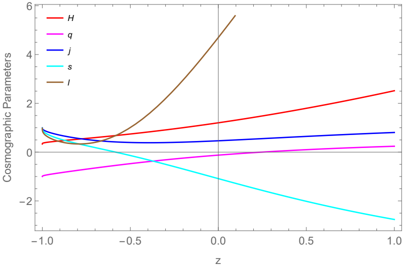

Figure: III

• The deceleration parameter is showing the signature flipping behaviour and at $t \to 0$, $q \simeq -1 + \frac{1}{\nu}$ whereas at $t \to 0$, $q \simeq -1$.

• Our model favours the quintessence like behaviour, $j < 1$ and $s > 0$.

IV. Scalar Field Reconstruction

- The cosmic acceleration phenomena can be modelled through the scalar field ϕ which can either be quintessence like or phantom like.

- The action for the scalar field reconstruction is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} + \frac{\epsilon}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] \tag{9}$$

where, $\epsilon = +1$ for quintessence field and $\epsilon = -1$ for phantom field.

- In a flat Friedman background, the energy density and pressure are derived for quintessence field as

$$\rho_\phi = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) \tag{10}$$

$$p_\phi = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 - V(\phi) \tag{11}$$

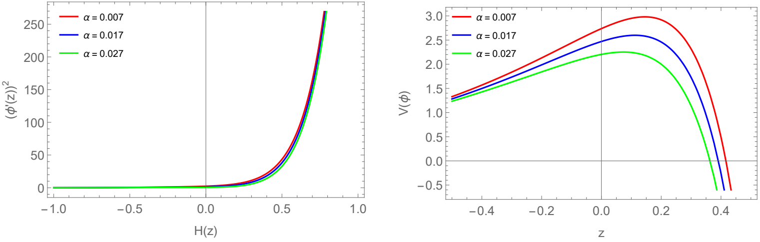

Figure: V

- At late times, the curve approaches to a small value.
- Another observation is that higher in value of α the curve is more steep.

V. Stability Analysis

- The stability of corresponding solutions of the model, $C_s^2 = \frac{\frac{dp}{dt}}{\frac{d\rho}{dt}}$

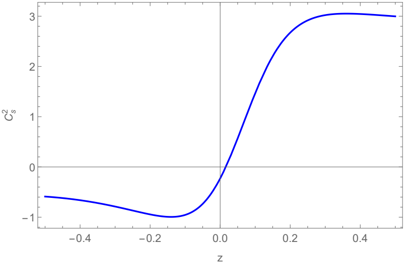

Figure: VI

- For $C_s^2 \geq 0$, the model is stable and for $C_s^2 \leq 0$, the model is unstable.

- The stability of the model can be studied by considering the mechanical stability of the cosmic fluid by calculating the adiabatic speed of sound through the cosmic fluid.

Results and Discussion

• The late time cosmic acceleration issue in $F(R,G)$ theory of gravity in presence of time varying deceleration parameter.

• The model shows quintessence like behaviour and remain stable at early times and unstable at late time.

• The behaviour of the EoS parameter at late time becomes insensitive to the choice of α since all the trajectories of EoS parameter for different choices of α behave alike at late phase.

• The violation of SEC further validates the accelerating behaviour of the model in a modified theory of gravity.

• We obtained on the geometrical parameters, the deceleration parameter approaches to -1 at late times, however the (j,s) pair merging close to $(1,0)$. Since the model favours quintessence behaviour, this value of (j,s) pair is expected.

In Collaboration with

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Thank
You