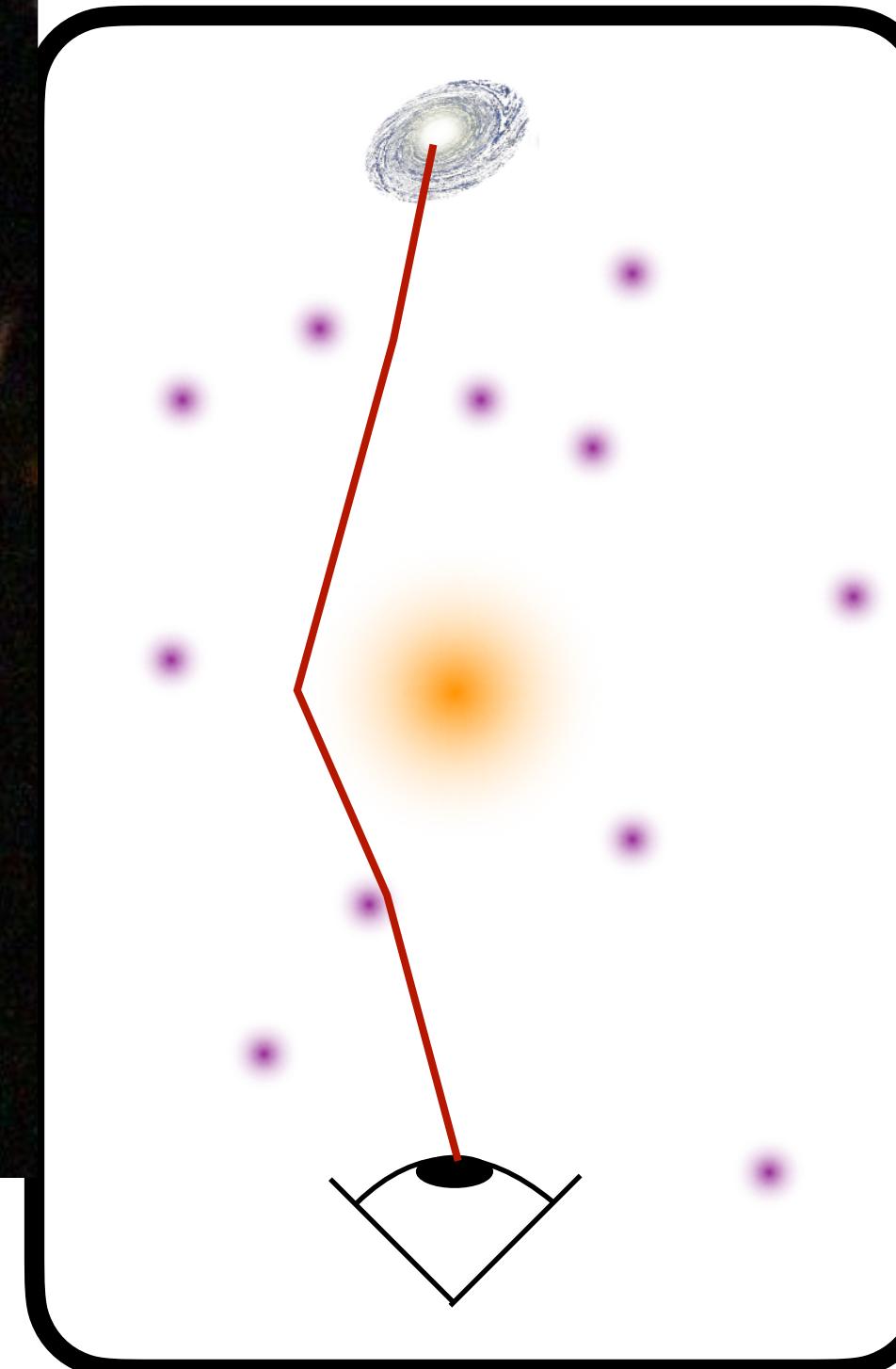


GRAVITATIONAL LENSING



**THE
WEAK**

**THE
STRONG**
**and THE
LINE OF
SIGHT**



directed by
PIERRE FLEURY (IFT)
co-starring
JULIEN LARENA (UCT)
JEAN-PHILIPPE UZAN (IAP)

an exclusive show at
COSMOLOGY FROM HOME
in
JULY 2021

a story based on literature
arXiv:2104.08883

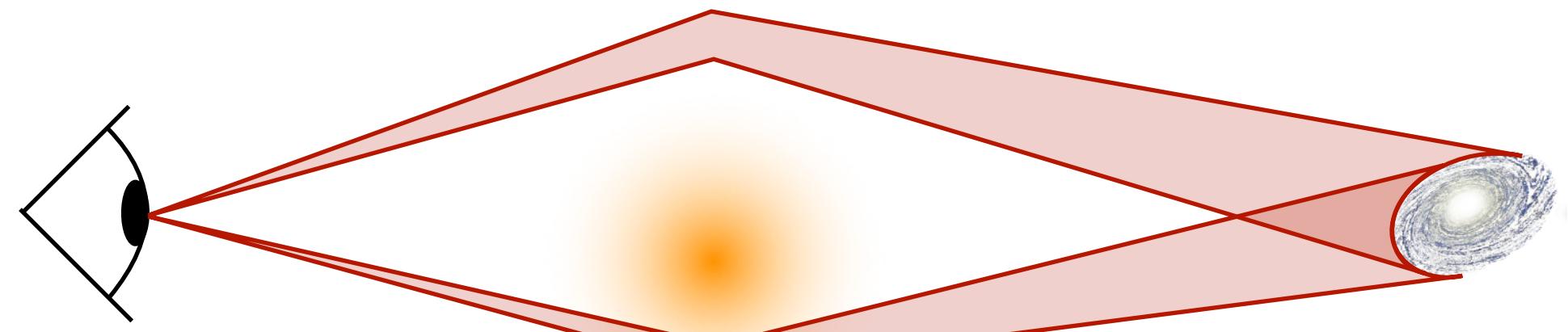
“You see, in this world there’s two kinds of lenses, my friend; those producing multiple images, and those that don’t. You don’t.”

Blondie



Strong and weak lensing

The Strong



- multiple images, strong distortions
- due to **one isolated dense lump**
- cosmology: measures H_0

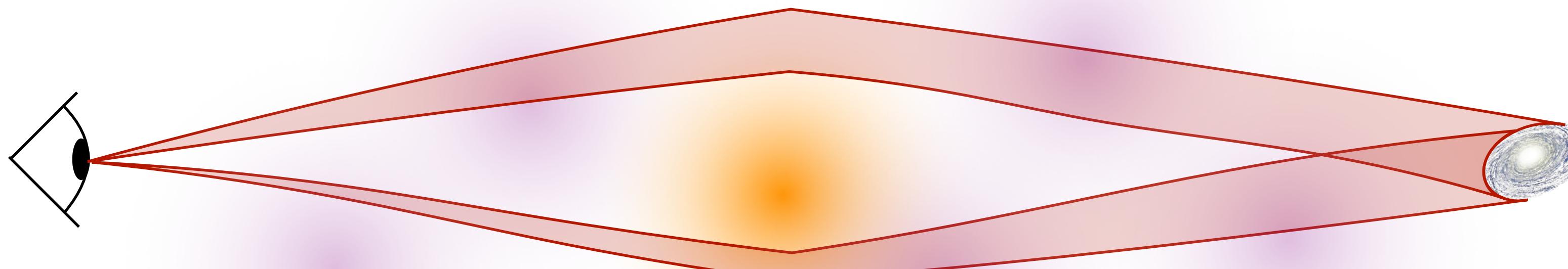
The Weak



- one image, weak distortions
- due to **many diffuse lumps**
- cosmology: measures Ω_m, σ_8

The Line of Sight

Line-of-sight (LOS) effects: the **Weak** perturbs the **Strong**

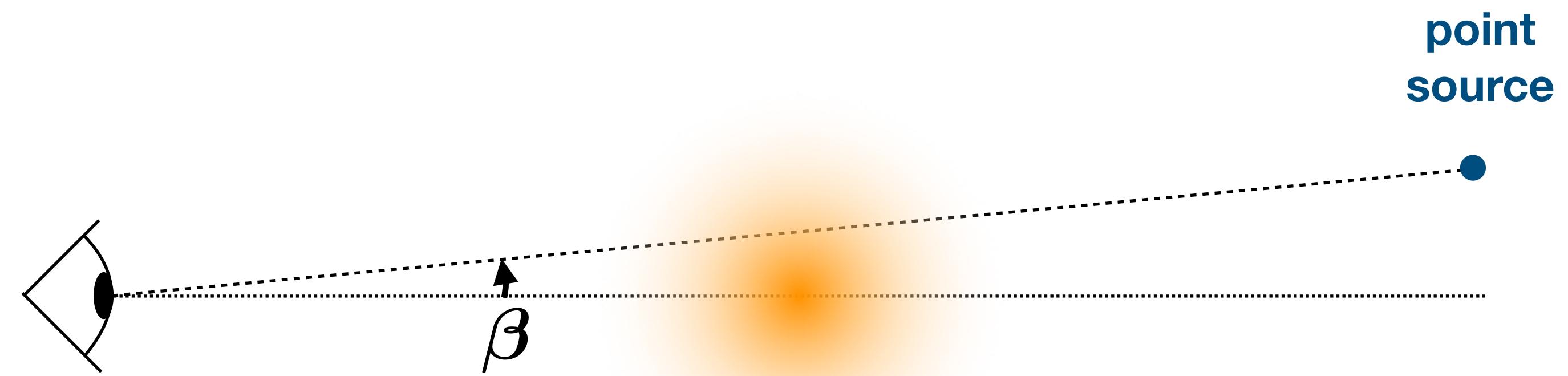


- Source of *uncertainty* in strong lensing [e.g. TDCOSMO]
- New *opportunities* for weak lensing [see also Birrer+2016]

THEORETICAL MODELLING

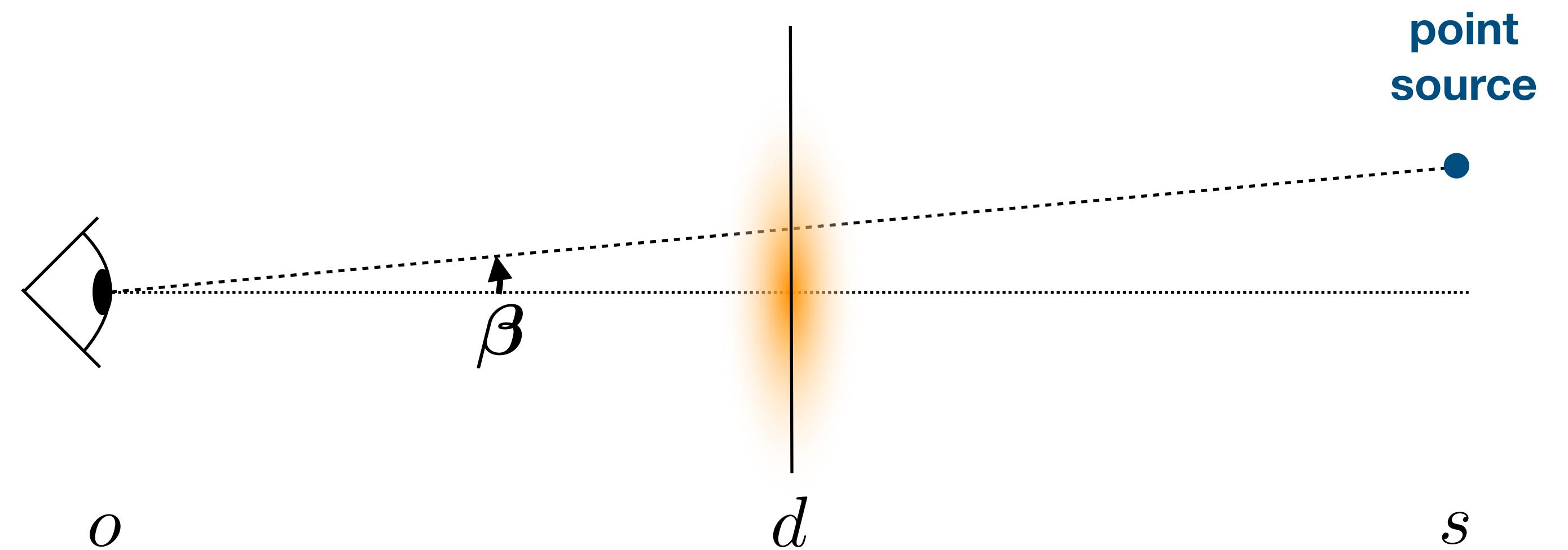
Dominant-lens approximation

NB: differs from Birrer+ (2016)



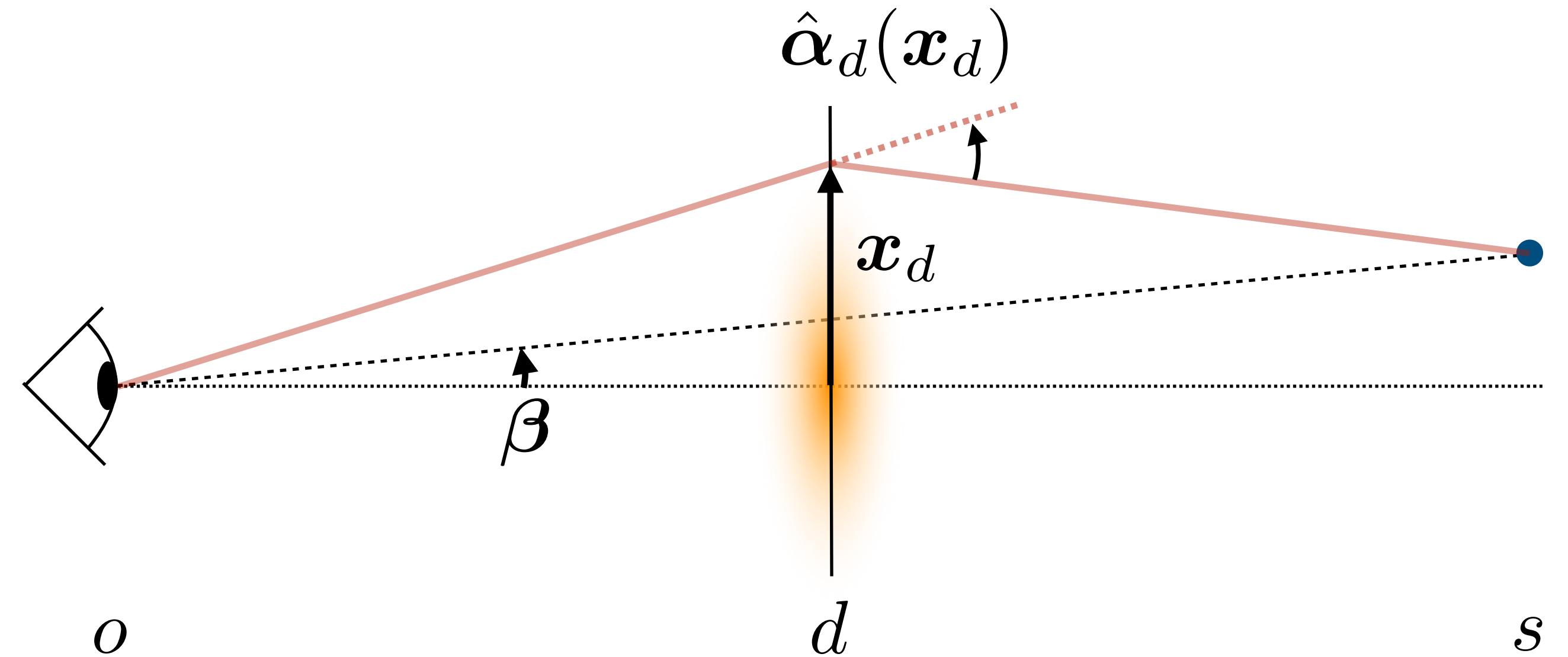
Dominant-lens approximation

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Dominant-lens approximation

NB: differs from Birrer+ (2016)

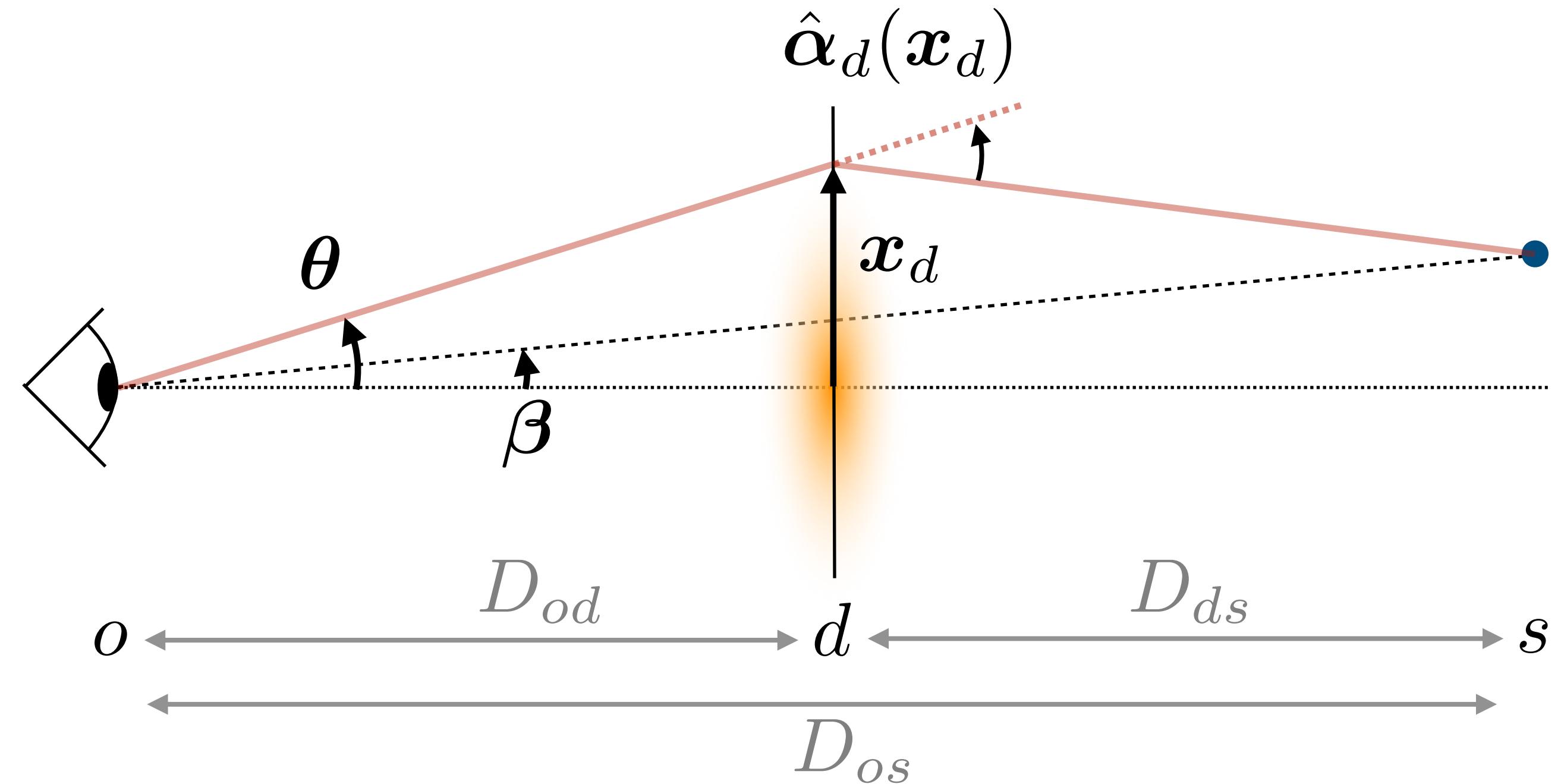


Deflection angle

$$\hat{\alpha}_d(\mathbf{x}_d) = \int d^2\mathbf{x} 4G\Sigma_d(\mathbf{x}) \frac{\mathbf{x}_d - \mathbf{x}}{|\mathbf{x}_d - \mathbf{x}|^2}$$

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle

$$\hat{\alpha}_d(x_d) = \int d^2x \ 4G\Sigma_d(x) \frac{x_d - x}{|x_d - x|^2}$$

Displacement angle

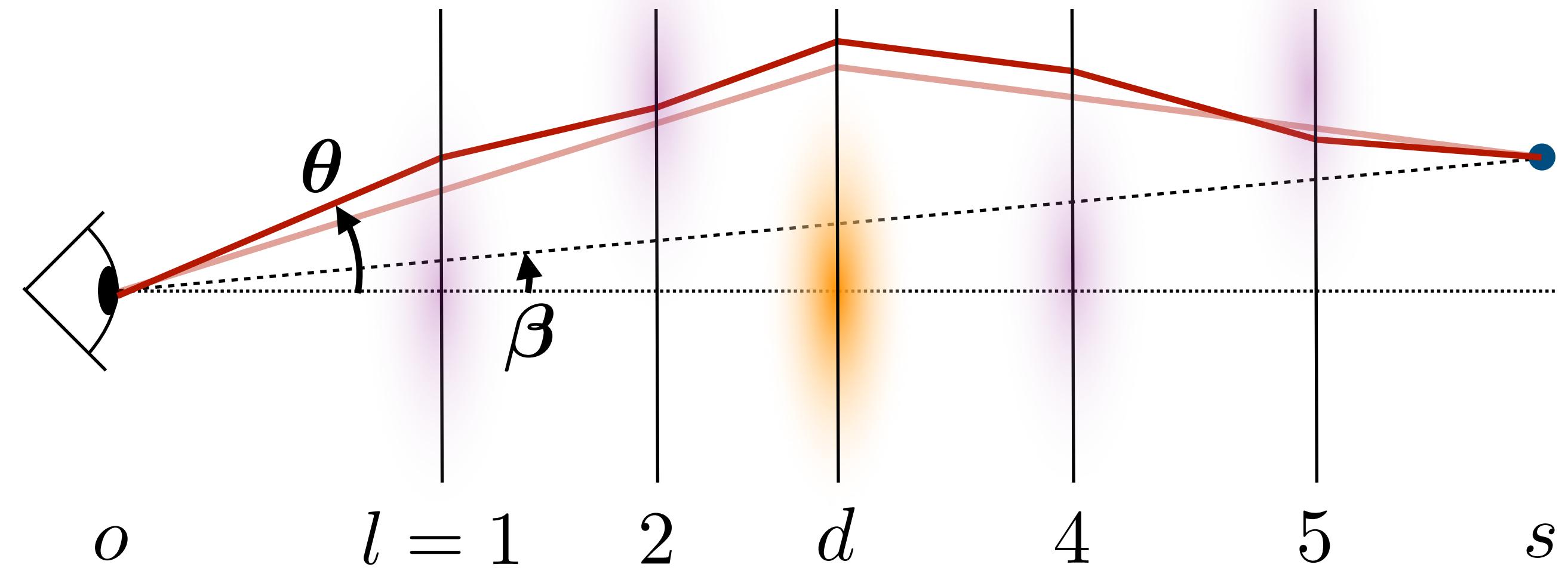
$$\alpha(\theta) \equiv \frac{D_{ds}}{D_{os}} \hat{\alpha}_d(D_{os}\theta)$$

Lens equation

$$\beta = \theta - \alpha(\theta)$$

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

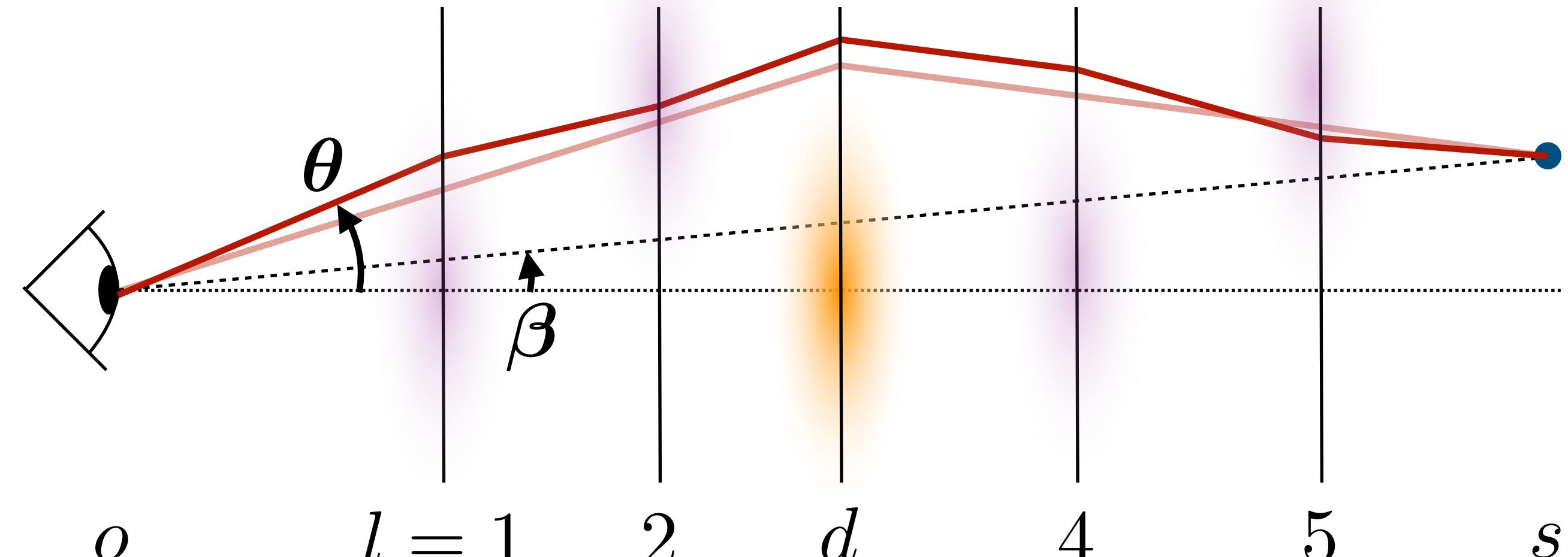
$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

$$\beta = \theta - \alpha(\theta)$$

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

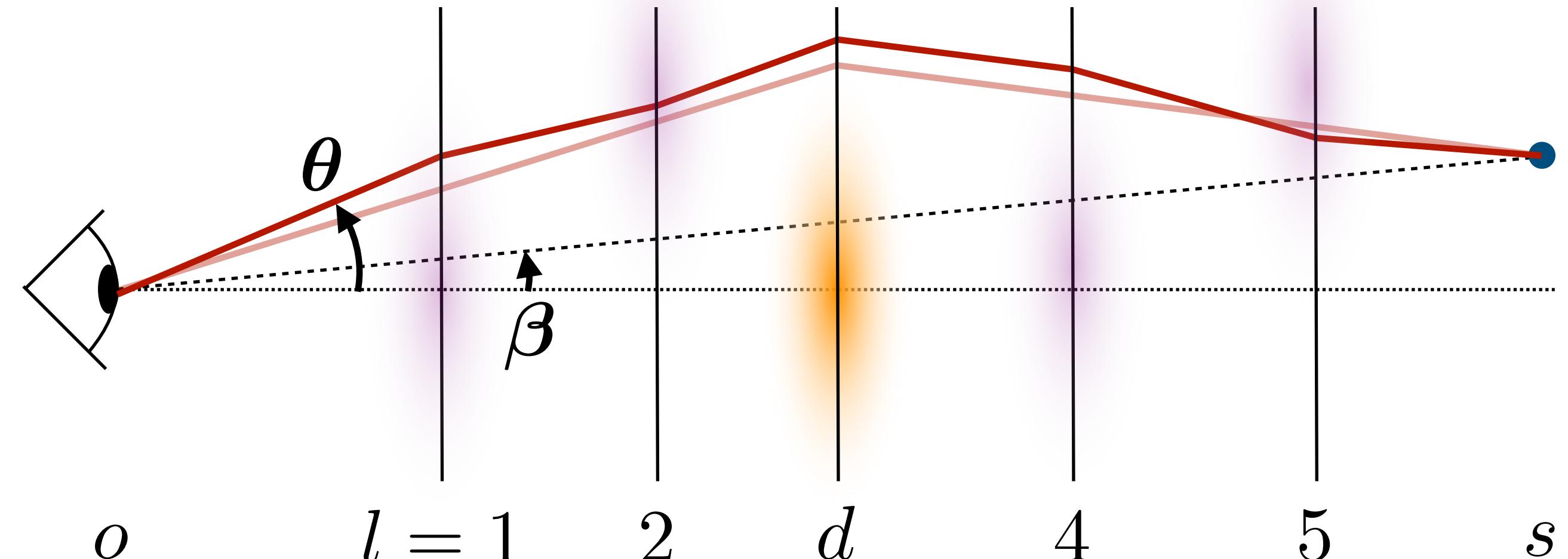
$$\beta = \theta - \alpha(\theta)$$

Dominant-lens approximation

$$\forall l \neq d \quad |\alpha_{ilj}| \ll |\alpha_{idj}|$$

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

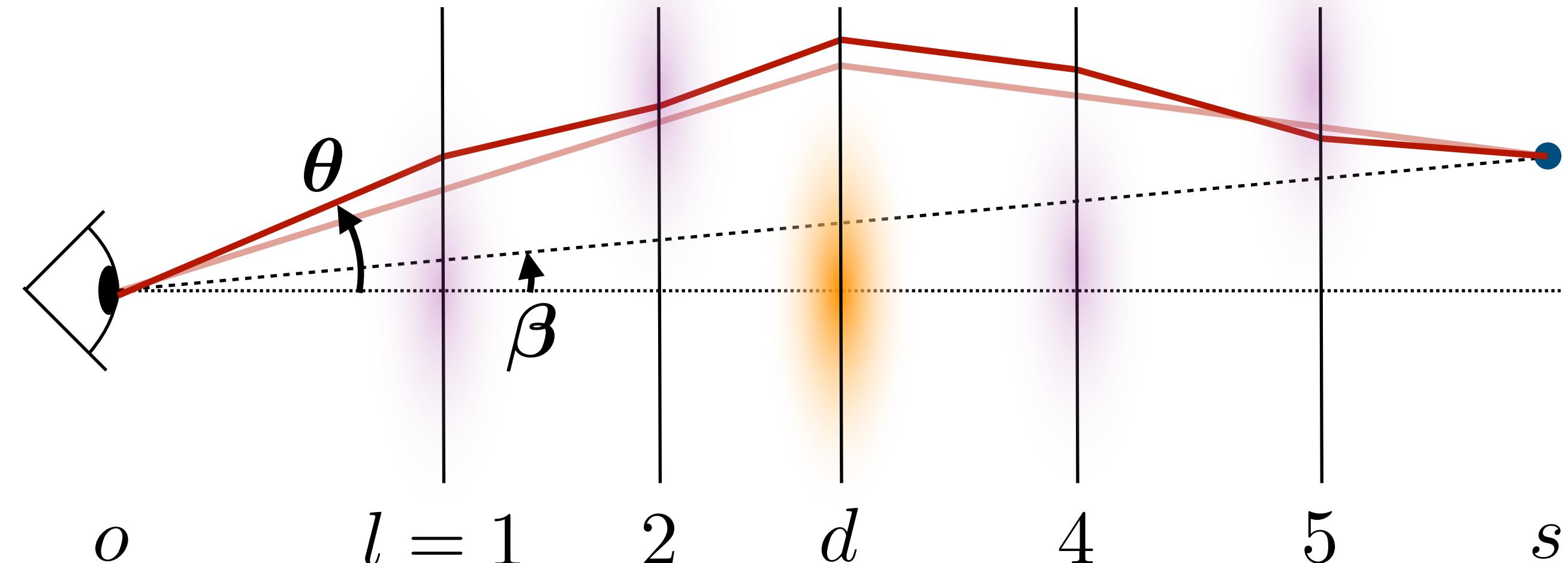
Lens equation

$$\beta = \theta - \alpha(\theta)$$

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

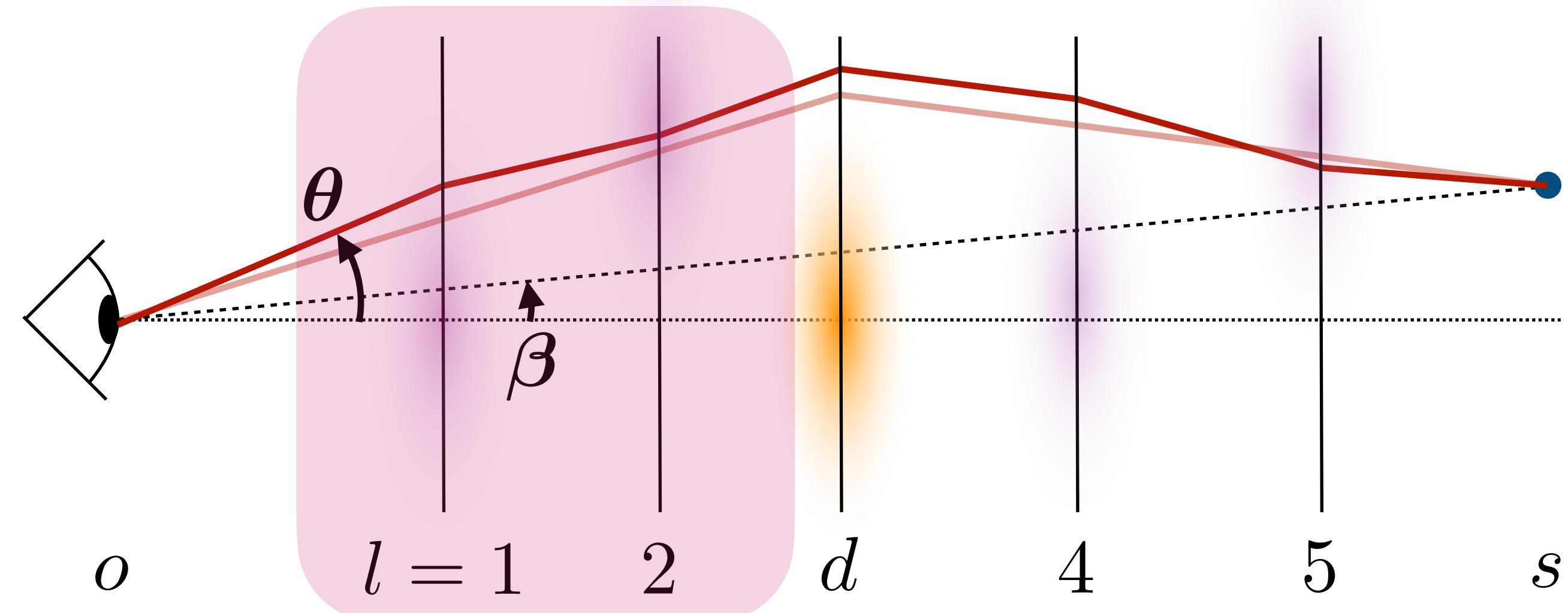
$$\beta = \theta - \alpha(\theta)$$

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

main lens

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

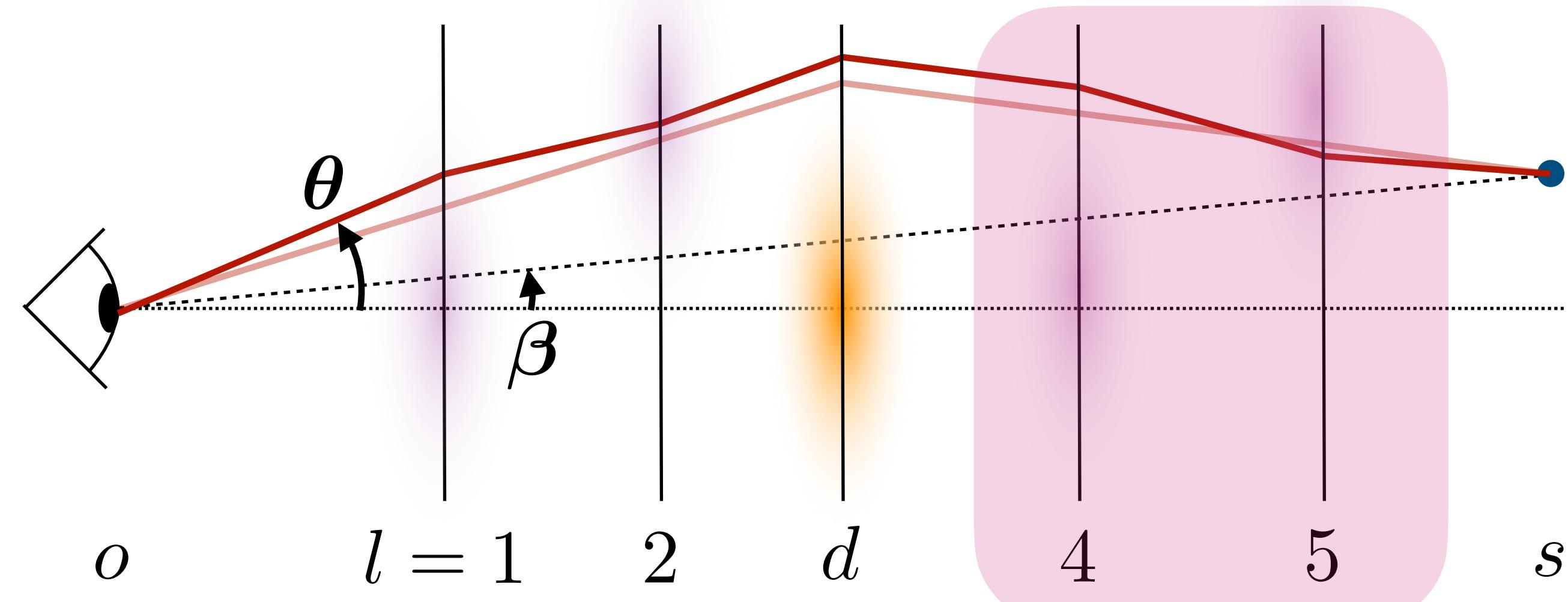
$$\beta = \theta - \alpha(\theta)$$

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

main lens foreground displacement

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

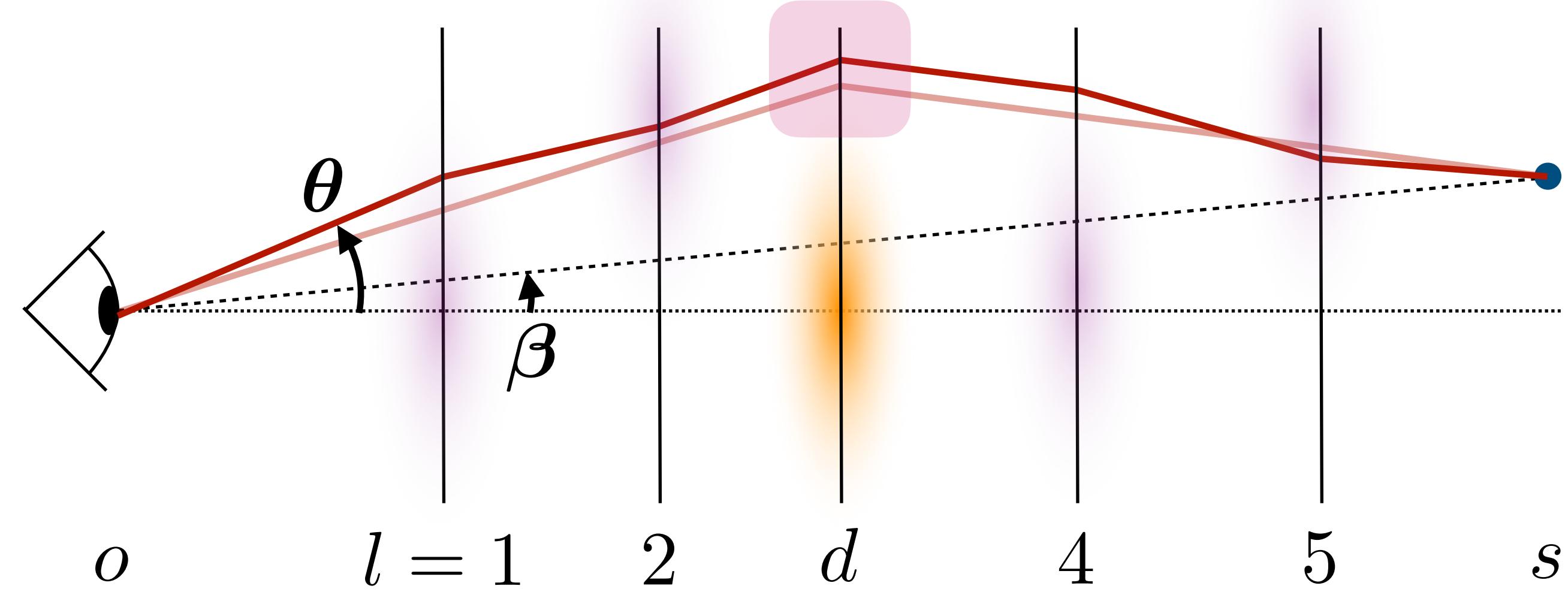
$$\beta = \theta - \alpha(\theta)$$

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

main lens foreground displacement background displacement

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2\mathbf{x} 4G\Sigma_l(\mathbf{x}) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

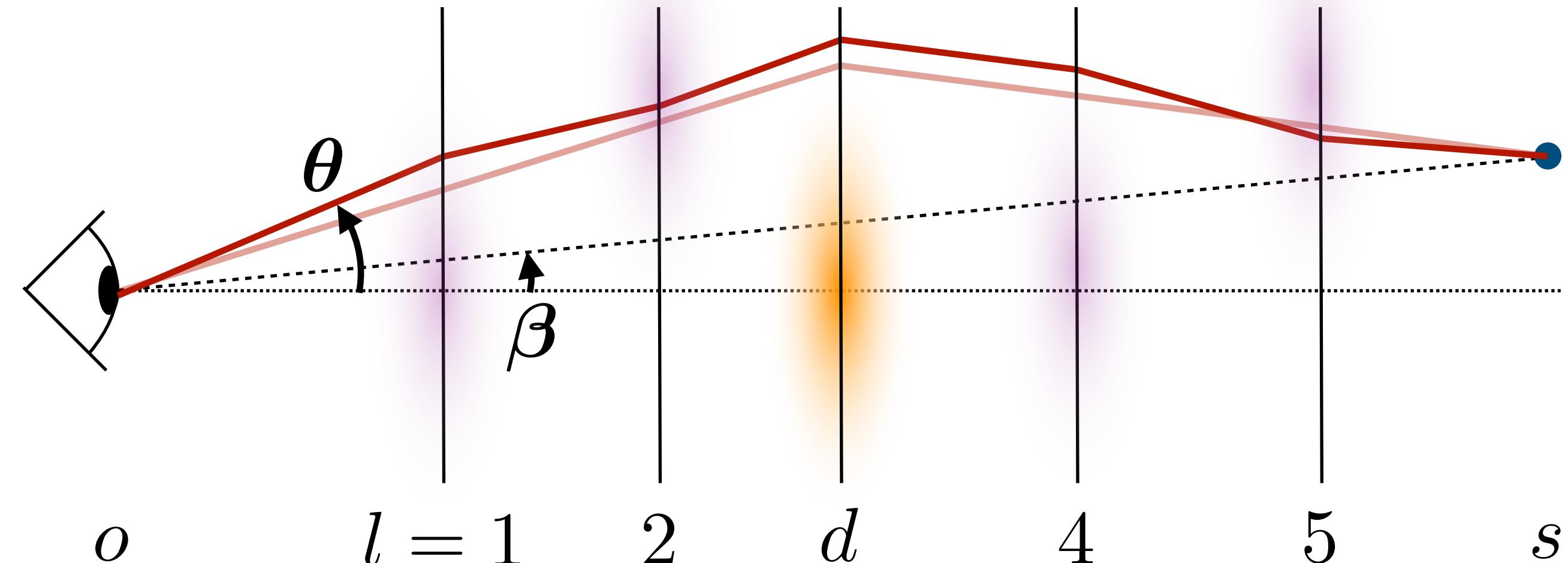
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main lens
 post-Born
 foreground
displacement
 background
displacement

Dominant-lens approximation

NB: differs from Birrer+ (2016)



Deflection angle by lens l

$$\hat{\alpha}_l(\mathbf{x}_l) = \int d^2x \, 4G\Sigma_l(x) \frac{\mathbf{x}_l - \mathbf{x}}{|\mathbf{x}_l - \mathbf{x}|^2}$$

Partial displacement angle

$$\alpha_{ilj}(\theta) \equiv \frac{D_{lj}}{D_{ij}} \hat{\alpha}_l(D_{il}\theta)$$

Lens equation

$$\beta = \theta - \alpha(\theta)$$

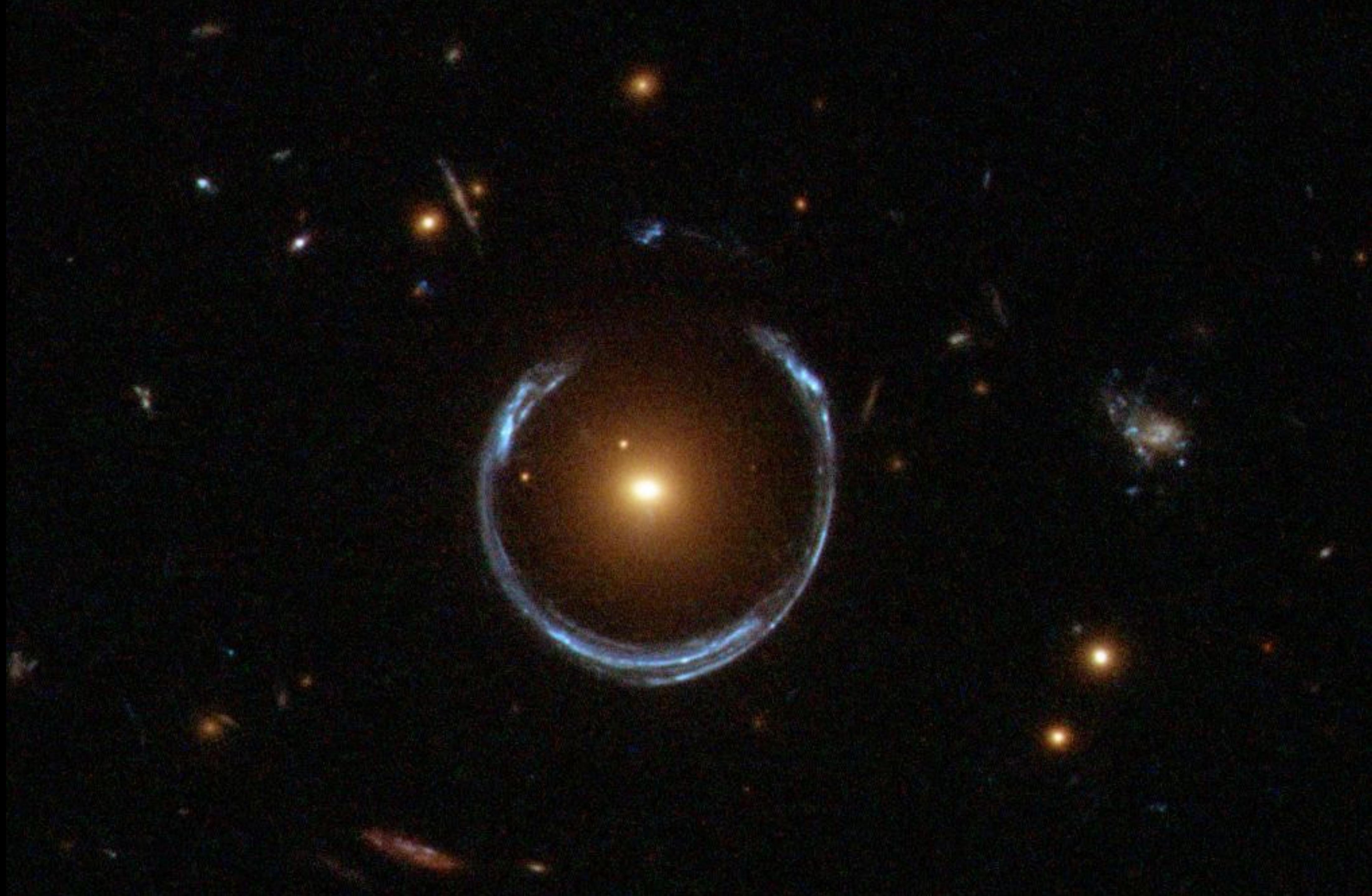
$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

main lens	post-Born	foreground displacement	background displacement
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An application
COSMIC SHEAR
WITH EINSTEIN RINGS

An application COSMIC SHEAR WITH EINSTEIN RINGS

original idea formulated by Birrer et al. (2017)



see also: Kovner (1987)
Bar-Kana (1996)
Schneider (1997)
McCully et al. (2014)

Perturbers in the tidal regime

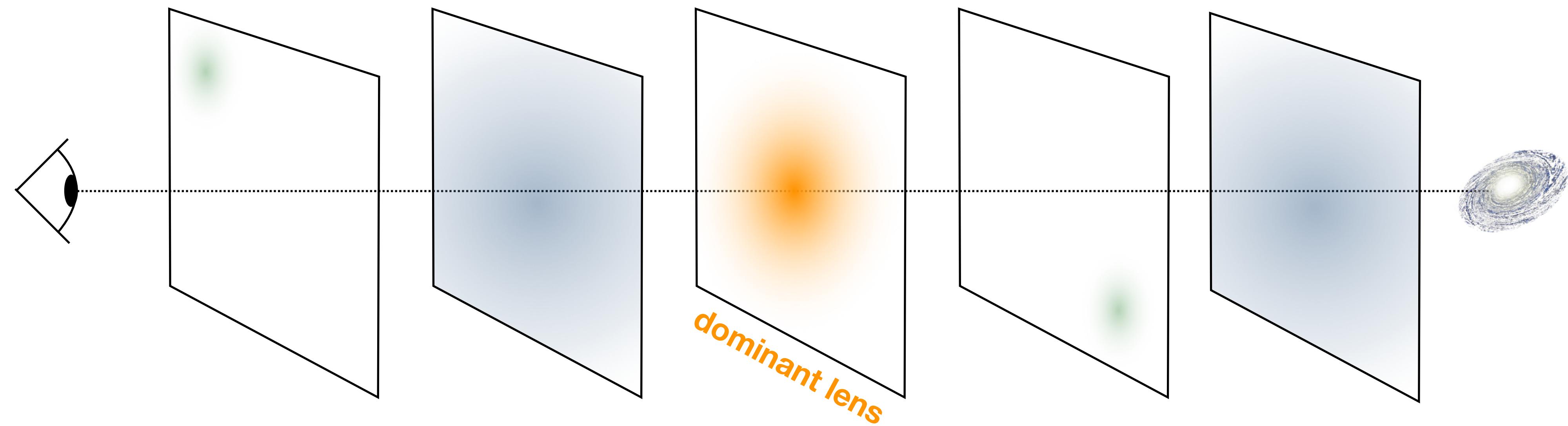
$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols} [\theta - \alpha_{odl}(\theta)]$$

see also: Kovner (1987)
Bar-Kana (1996)
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Perturbers in the tidal regime

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

Tidal regime: $\forall l \neq d \quad \alpha_{ilj}(\theta) = \Gamma_{ilj}\theta$



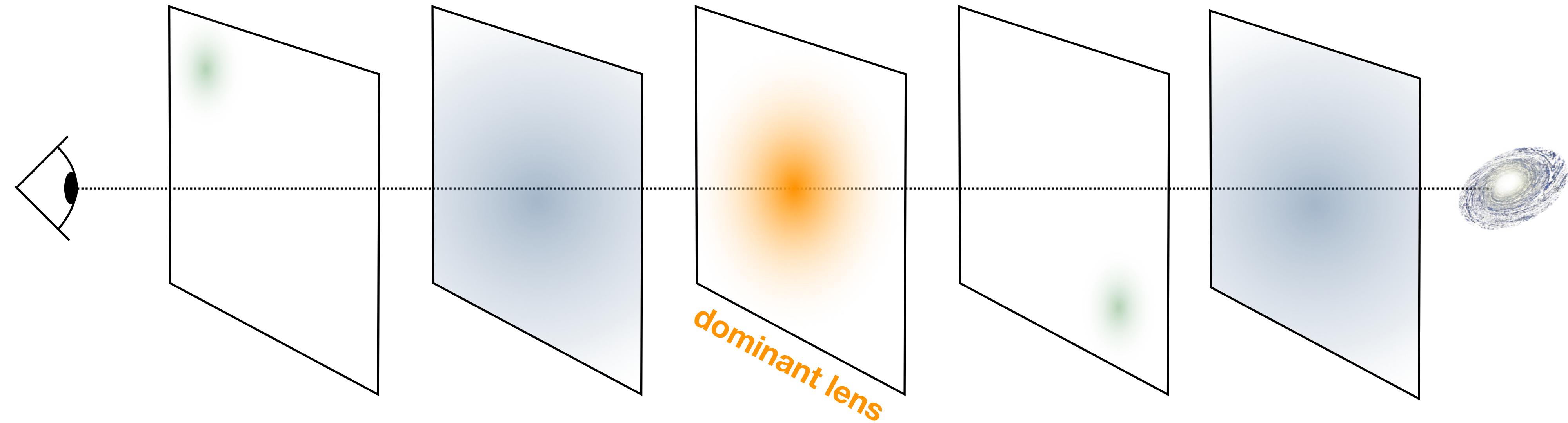
see also: Kovner (1987)
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Perturbers in the tidal regime

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols}[\theta - \alpha_{odl}(\theta)]$$

Tidal regime: $\forall l \neq d \quad \alpha_{ilj}(\theta) = \Gamma_{ilj}\theta$

$$\Gamma_{ilj} = \begin{bmatrix} \kappa_{ilj} + \text{Re}(\gamma_{ilj}) & \text{Im}(\gamma_{ilj}) \\ \text{Im}(\gamma_{ilj}) & \kappa_{ilj} - \text{Re}(\gamma_{ilj}) \end{bmatrix}$$



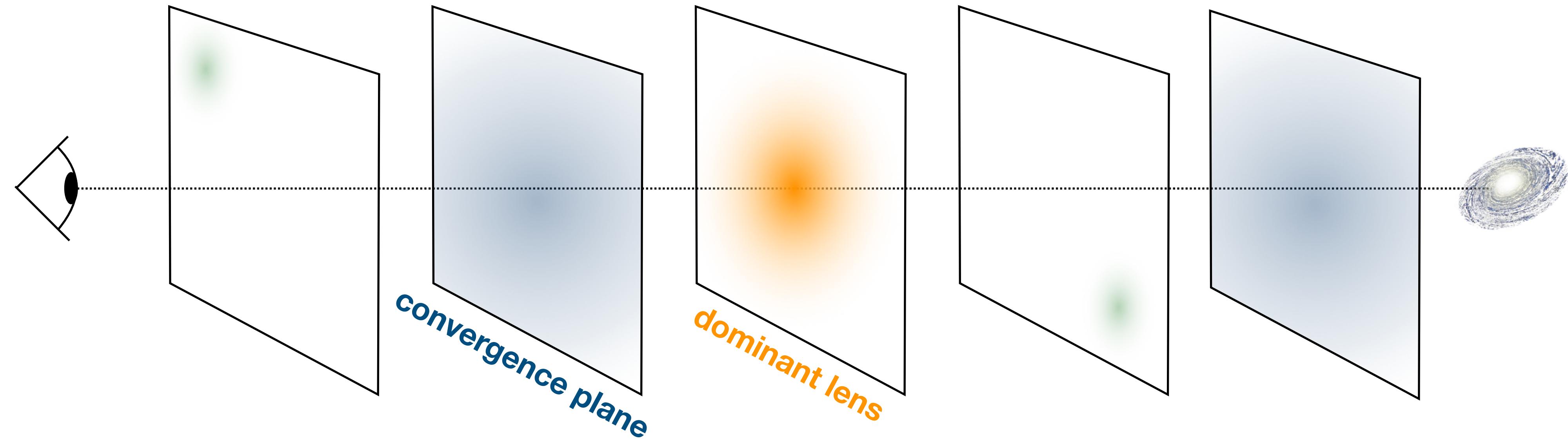
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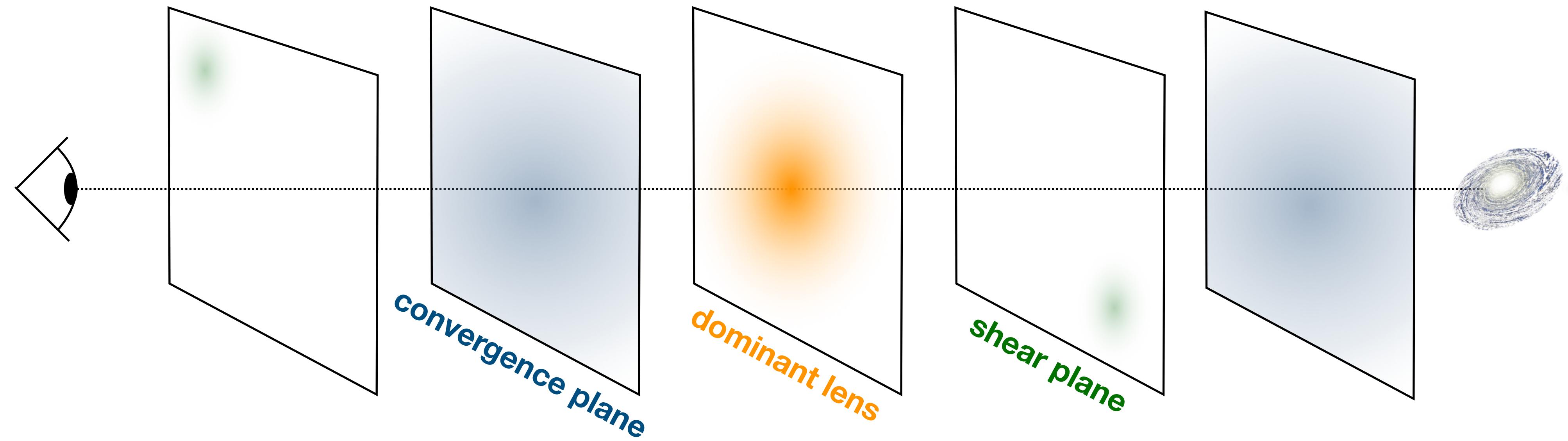
see also: Kovner (1987)
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 Schneider (1997)
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Perturbers in the tidal regime

$$\alpha(\theta) = \alpha_{ods} \left[\theta - \sum_{l < d} \alpha_{old}(\theta) \right] + \sum_{l < d} \alpha_{ols}(\theta) + \sum_{l > d} \alpha_{ols} [\theta - \alpha_{odl}(\theta)]$$

Tidal regime: $\forall l \neq d \quad \alpha_{ilj}(\theta) = \Gamma_{ilj} \theta$

$$\Gamma_{ilj} = \begin{bmatrix} \kappa_{ilj} + \text{Re}(\gamma_{ilj}) & \text{Im}(\gamma_{ilj}) \\ \text{Im}(\gamma_{ilj}) & \kappa_{ilj} - \text{Re}(\gamma_{ilj}) \end{bmatrix}$$



see also: Kovner (1987)
 Bar-Kana (1996)
 Schneider (1997)
 McCully et al. (2014)

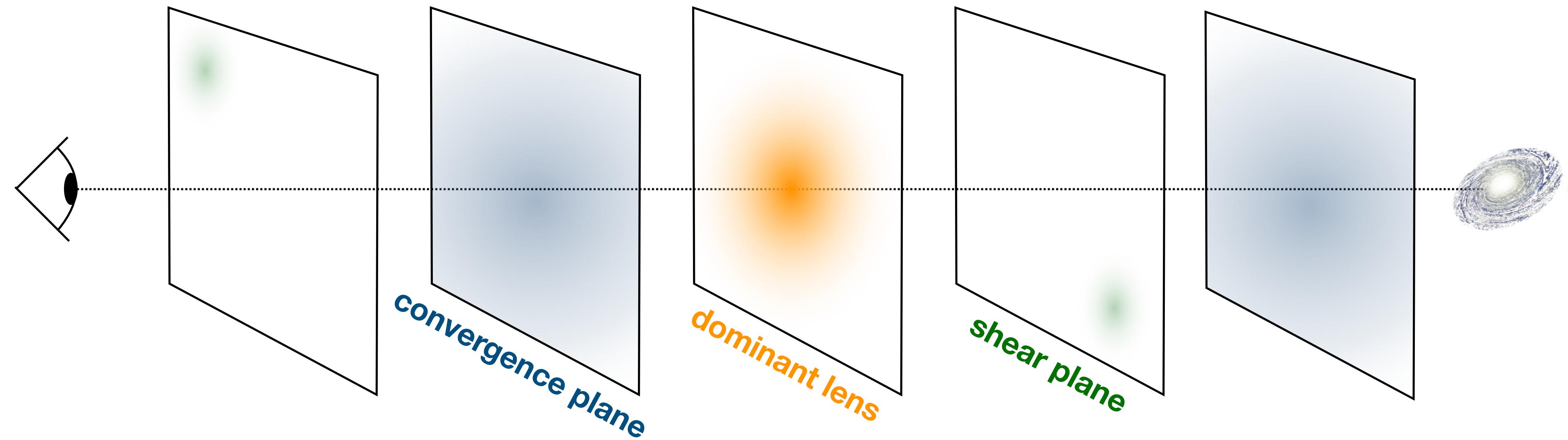
Perturbers in the tidal regime

$$\alpha(\theta) = (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] + \Gamma_{os}\theta$$

$$\Gamma_{ij} = \sum_{i < l < j} \Gamma_{ilj}$$

Tidal regime: $\forall l \neq d \quad \alpha_{ilj}(\theta) = \Gamma_{ilj}\theta$

$$\Gamma_{ilj} = \begin{bmatrix} \kappa_{ilj} + \text{Re}(\gamma_{ilj}) & \text{Im}(\gamma_{ilj}) \\ \text{Im}(\gamma_{ilj}) & \kappa_{ilj} - \text{Re}(\gamma_{ilj}) \end{bmatrix}$$



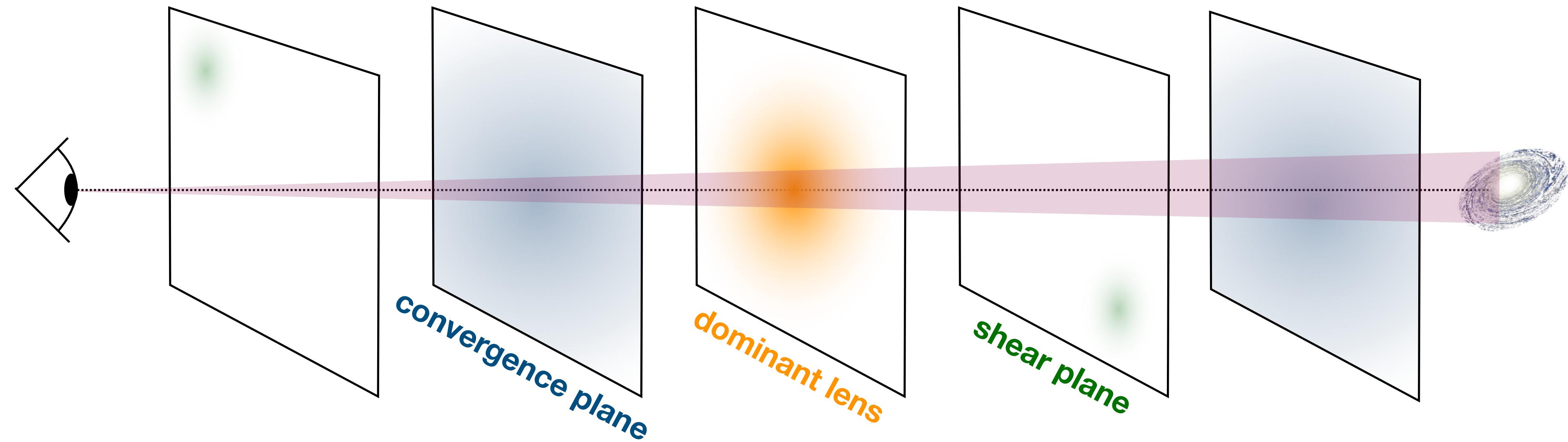
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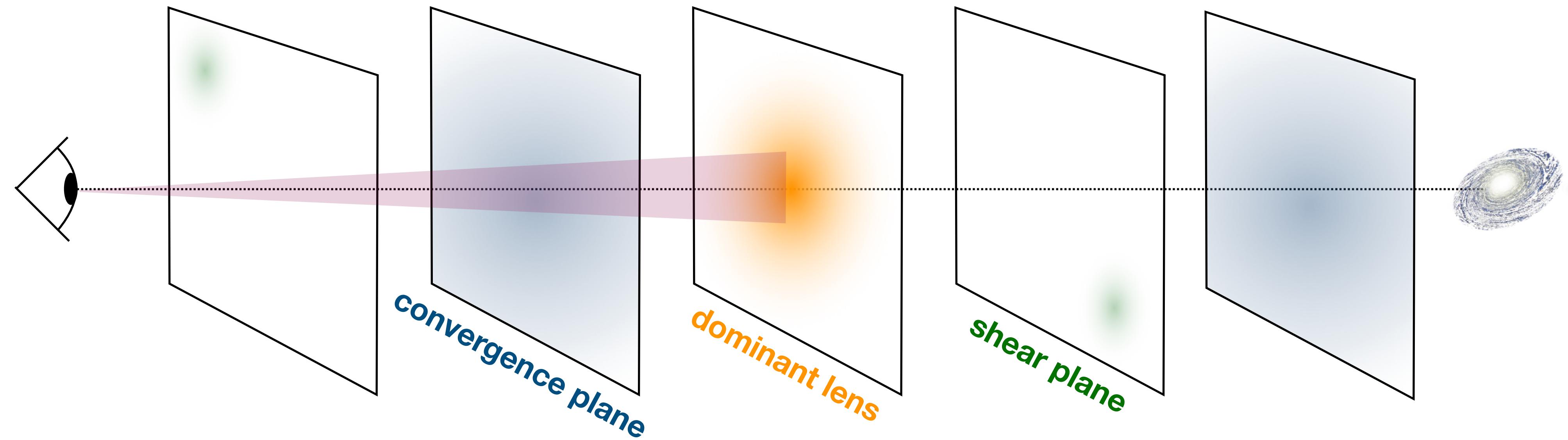
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Perturbers in the tidal regime

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see also: Kovner (1987)
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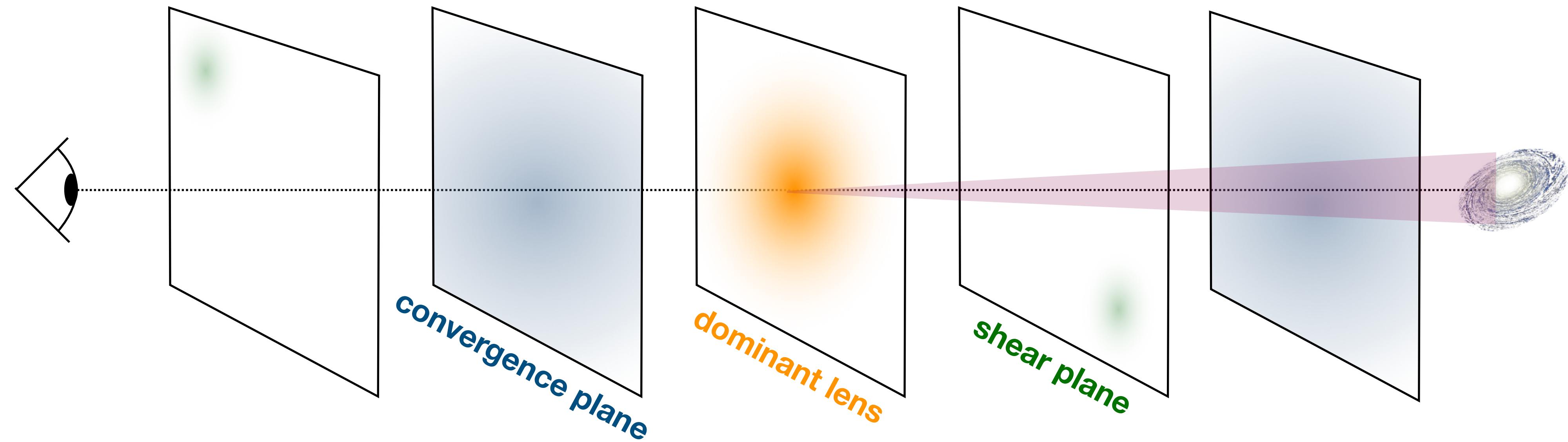
Perturbers in the tidal regime

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Tidal regime: $\forall l \neq d \quad \alpha_{ilj}(\theta) = \Gamma_{ilj}\theta$

$$\Gamma_{ilj} = \begin{bmatrix} \kappa_{ilj} + \text{Re}(\gamma_{ilj}) & \text{Im}(\gamma_{ilj}) \\ \text{Im}(\gamma_{ilj}) & \kappa_{ilj} - \text{Re}(\gamma_{ilj}) \end{bmatrix}$$



Minimal lens model

$$\beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta]$$

Minimal lens model

$$\beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta]$$

unknown

Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

$$\psi_{mod}(\theta) \equiv \psi_{ods}[(1 - \Gamma_{od})\theta]$$

Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

$$\psi_{mod}(\theta) \equiv \psi_{ods}[(1 - \Gamma_{od})\theta] \quad \text{internal degeneracy}$$

Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{\text{LOS}})\theta - \frac{d\psi_{\text{mod}}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

$$\psi_{\text{mod}}(\theta) \equiv \psi_{ods}[(1 - \Gamma_{od})\theta] \quad \text{internal degeneracy}$$

$$\Gamma_{\text{LOS}} \equiv \Gamma_{os} + \Gamma_{od} - \Gamma_{ds}$$

Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

$\psi_{mod}(\theta) \equiv \psi_{ods}[(1 - \Gamma_{od})\theta]$ internal degeneracy

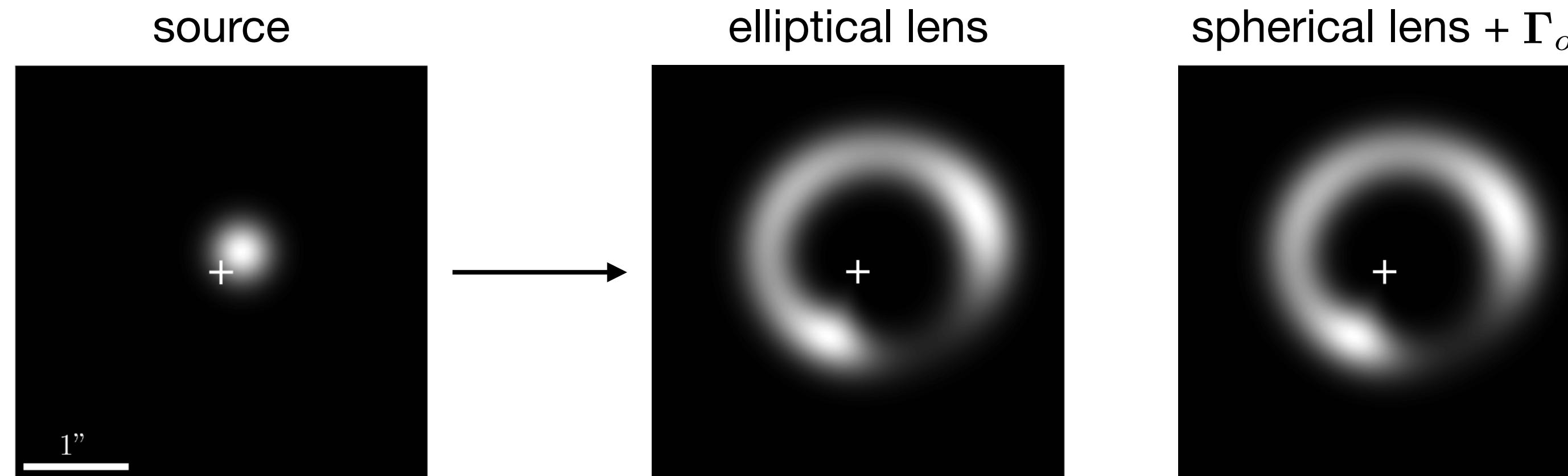
$\Gamma_{LOS} \equiv \Gamma_{os} + \Gamma_{od} - \Gamma_{ds}$ external degeneracy

Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

$\psi_{mod}(\theta) \equiv \psi_{ods}[(1 - \Gamma_{od})\theta]$ internal degeneracy

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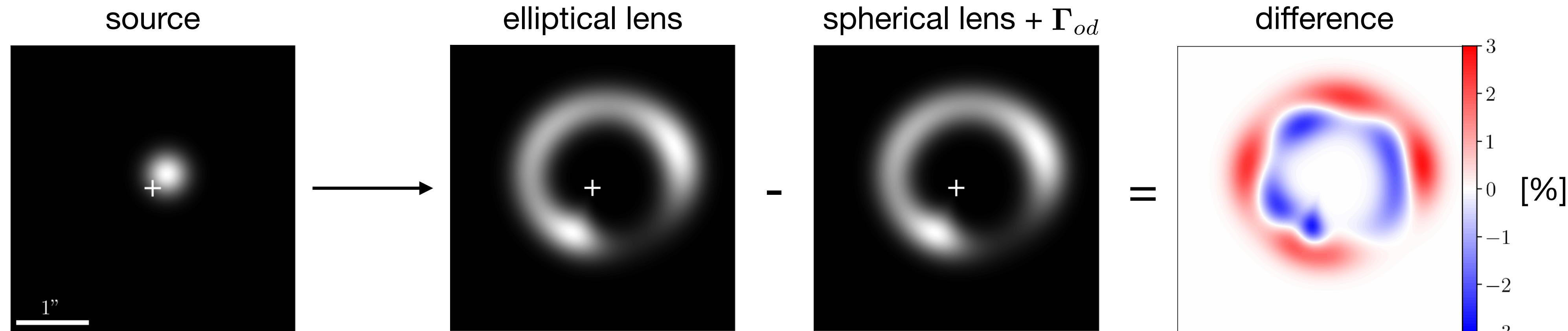


Minimal lens model

$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

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$\Gamma_{LOS} \equiv \Gamma_{os} + \Gamma_{od} - \Gamma_{ds}$ external degeneracy

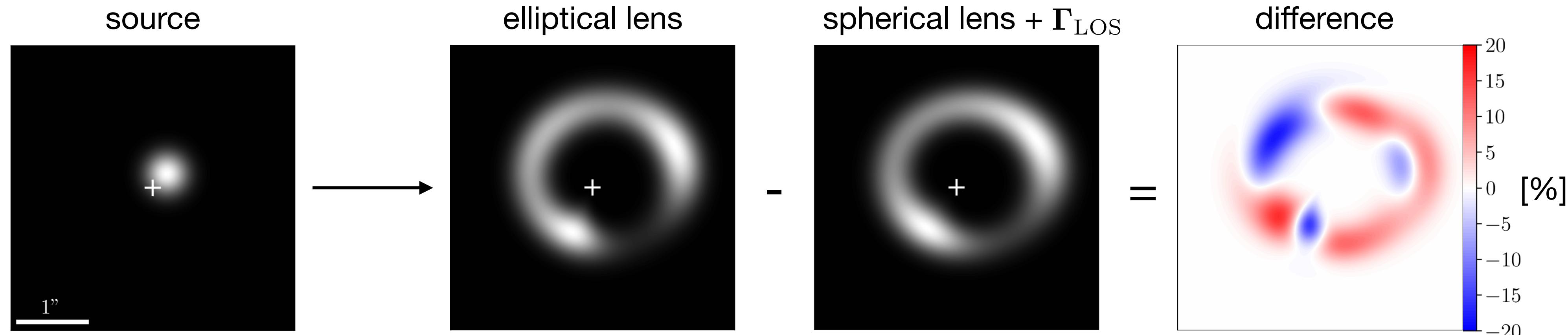


Minimal lens model

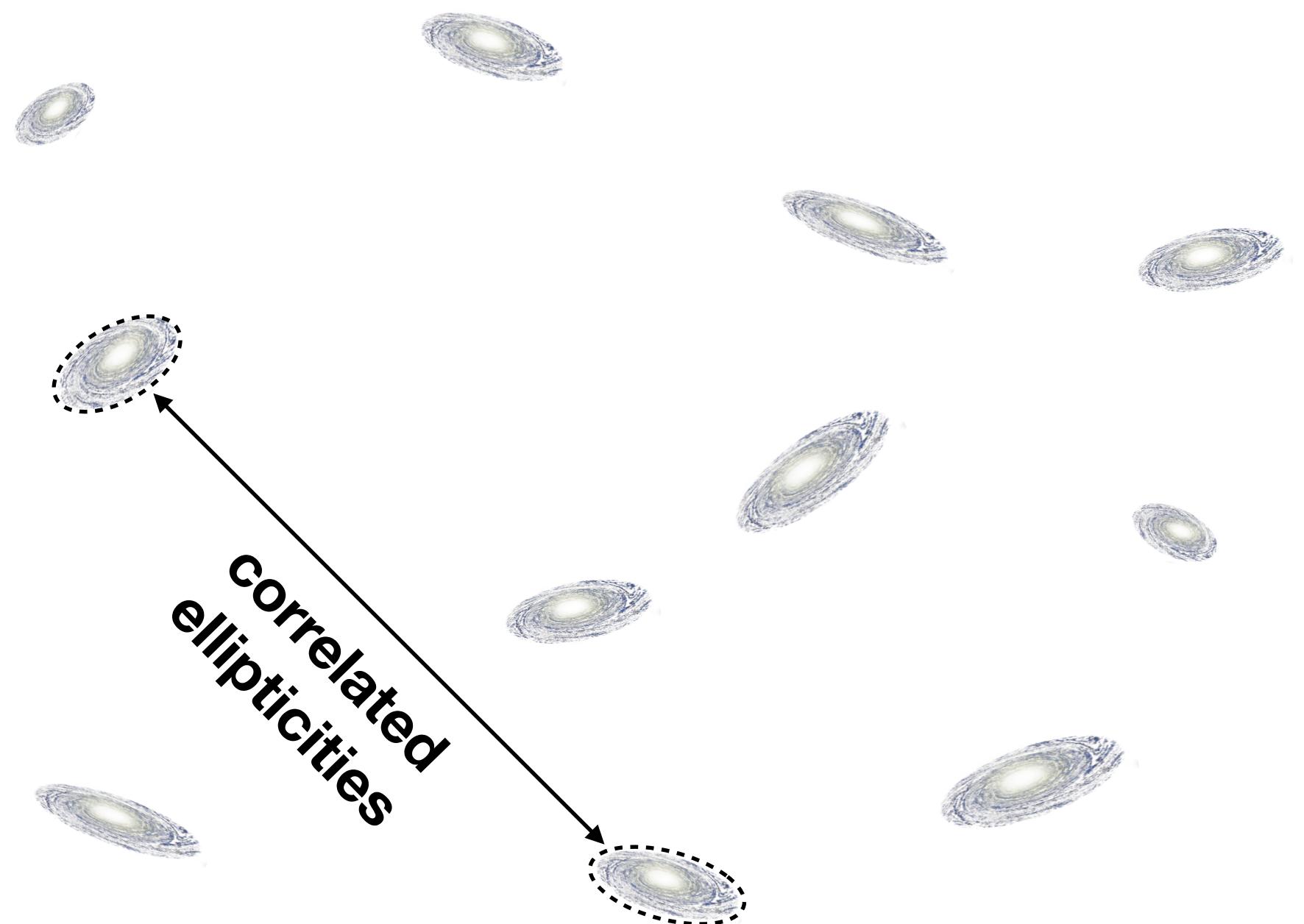
$$(1 - \Gamma_{od} + \Gamma_{ds}) \cdot \begin{cases} \beta = (1 - \Gamma_{os})\theta - (1 - \Gamma_{ds})\alpha_{ods}[(1 - \Gamma_{od})\theta] \\ \tilde{\beta} = (1 - \Gamma_{LOS})\theta - \frac{d\psi_{mod}}{d\theta} \end{cases} \quad [\text{minimal lens model}]$$

$\psi_{mod}(\theta) \equiv \psi_{ods}[(1 - \Gamma_{od})\theta]$ internal degeneracy

$\Gamma_{LOS} \equiv \Gamma_{os} + \Gamma_{od} - \Gamma_{ds}$ external degeneracy



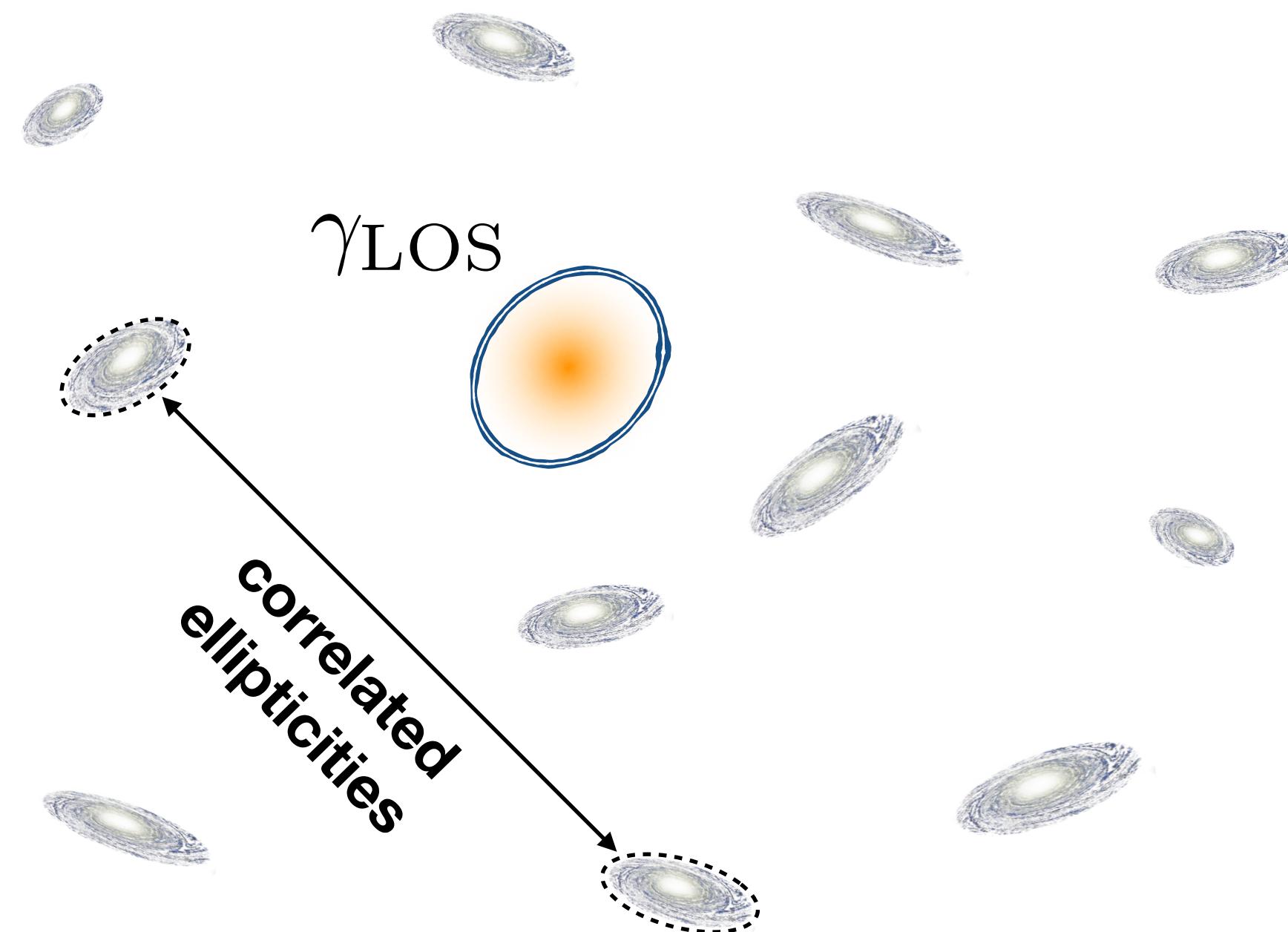
Cosmic shear with Einstein rings



Difficulties of weak-lensing observations

- Shape noise
- Intrinsic alignments

Cosmic shear with Einstein rings

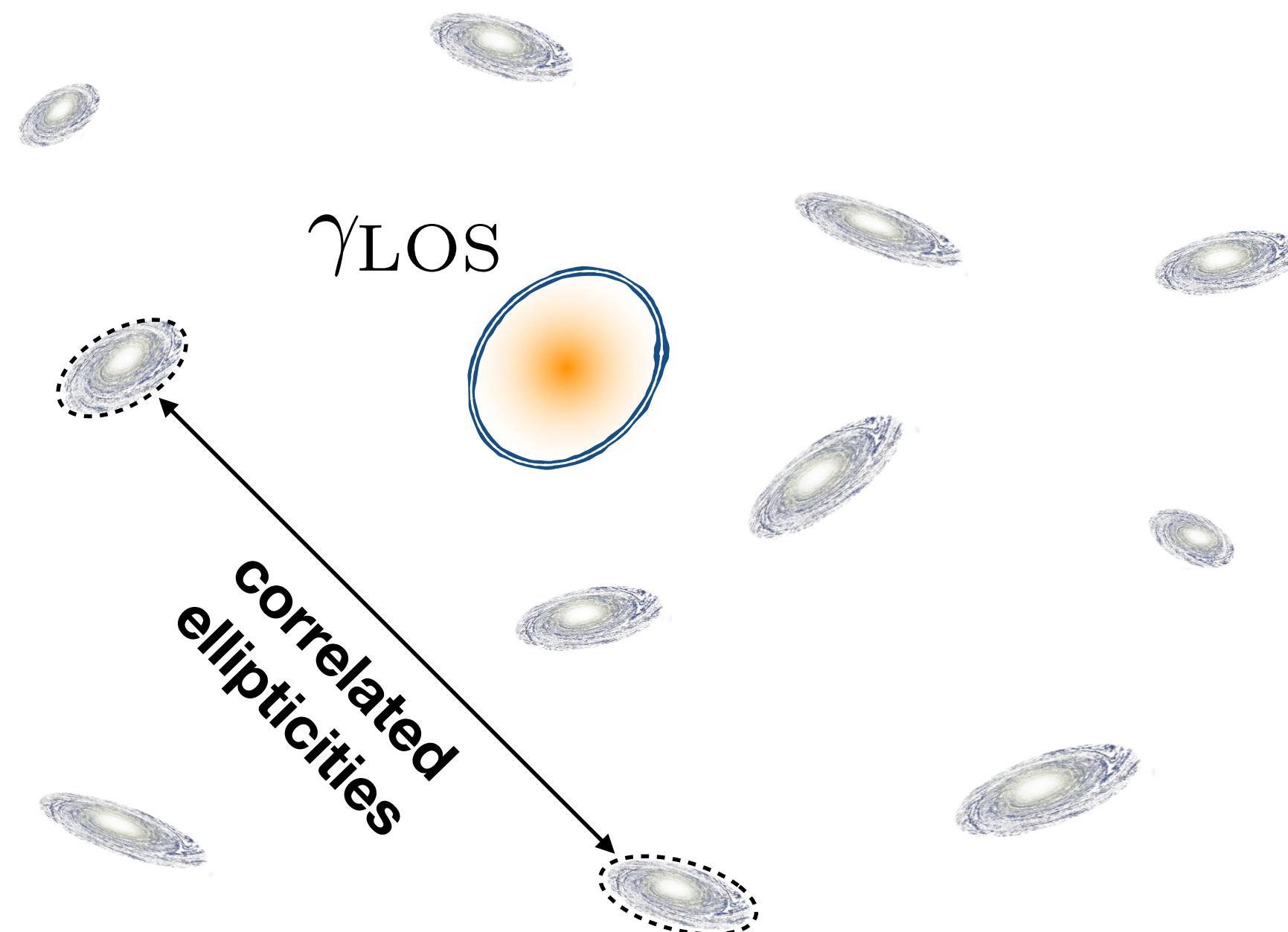


Difficulties of weak-lensing observations

- Shape noise
- Intrinsic alignments

Einstein rings in the field:
Direct and accurate measurement of γ_{LOS}

Cosmic shear with Einstein rings



Difficulties of weak-lensing observations

- Shape noise
- Intrinsic alignments

Einstein rings in the field:
Direct and accurate measurement of γ_{LOS}

Message: synergies between strong and weak lensing must be exploited

Conclusion

Line-of-sight effects = exterior perturbations to strong lenses
“the weak lensing of strong lensing”

Our technical increment

General formalism to model LOS effects with a dominant lens

Main result 1

Identified a measurable line-of-sight shear; should improve cosmic shear

Main result 2 (not discussed here, live session?)

Use distortions of critical curves to break the mass-sheet degeneracy

Credits



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The video made use of the soundtrack of *The Good, the Bad, and the Ugly* by Ennio Morricone.