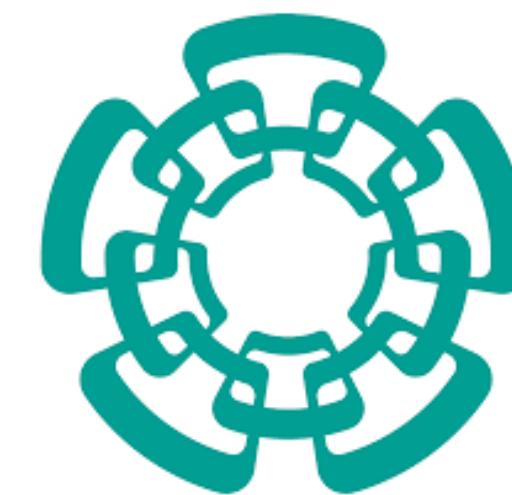


Multi-Scalar Field Dark Matter

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Vázquez.

Cosmology from Home 2021.



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Outline.

- >Introduction.
- > Multi Scalar Field Dark Matter:
 - > Numerical analysis.
 - > Models constraints.
- > Summary.

Λ CDM.

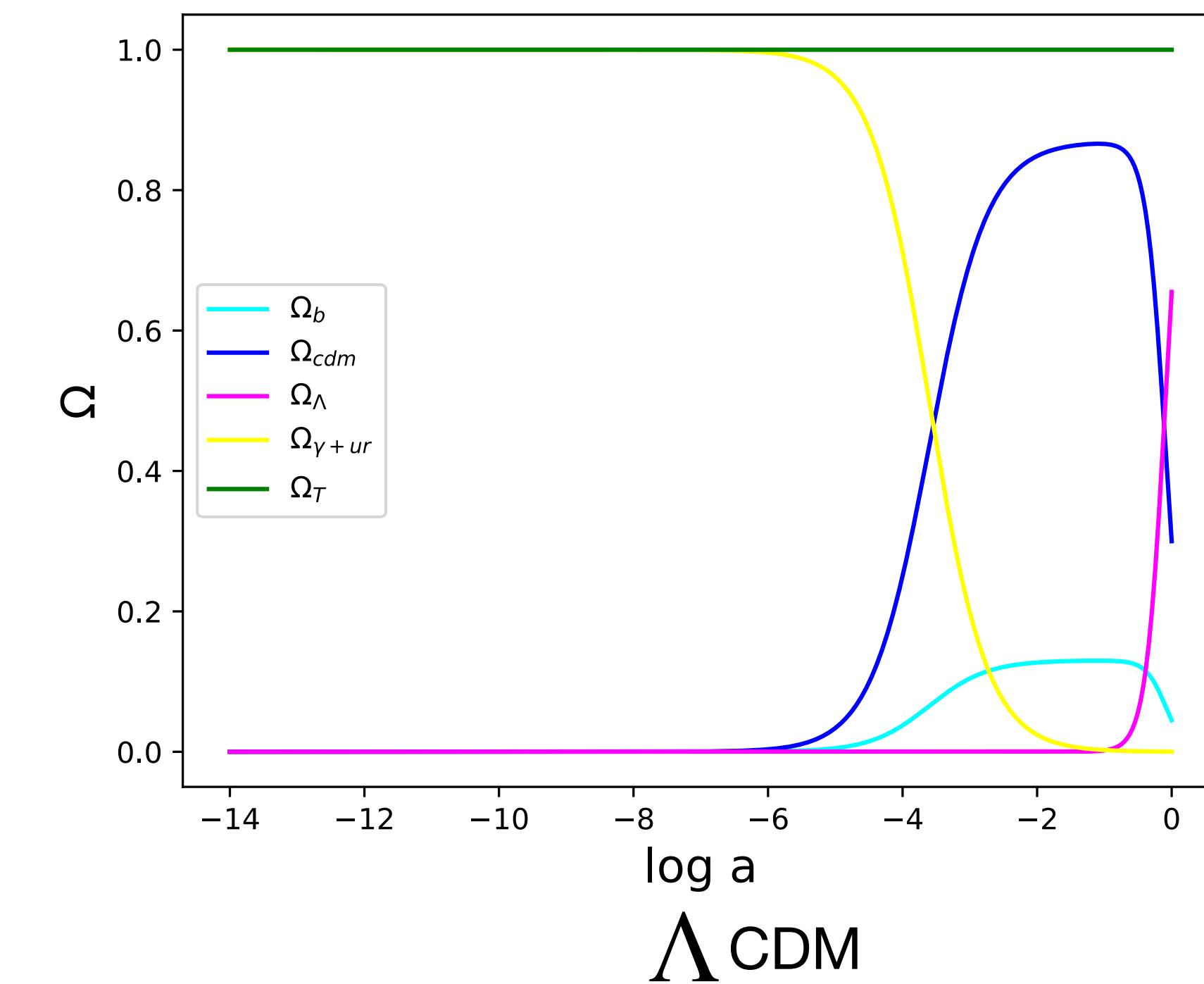
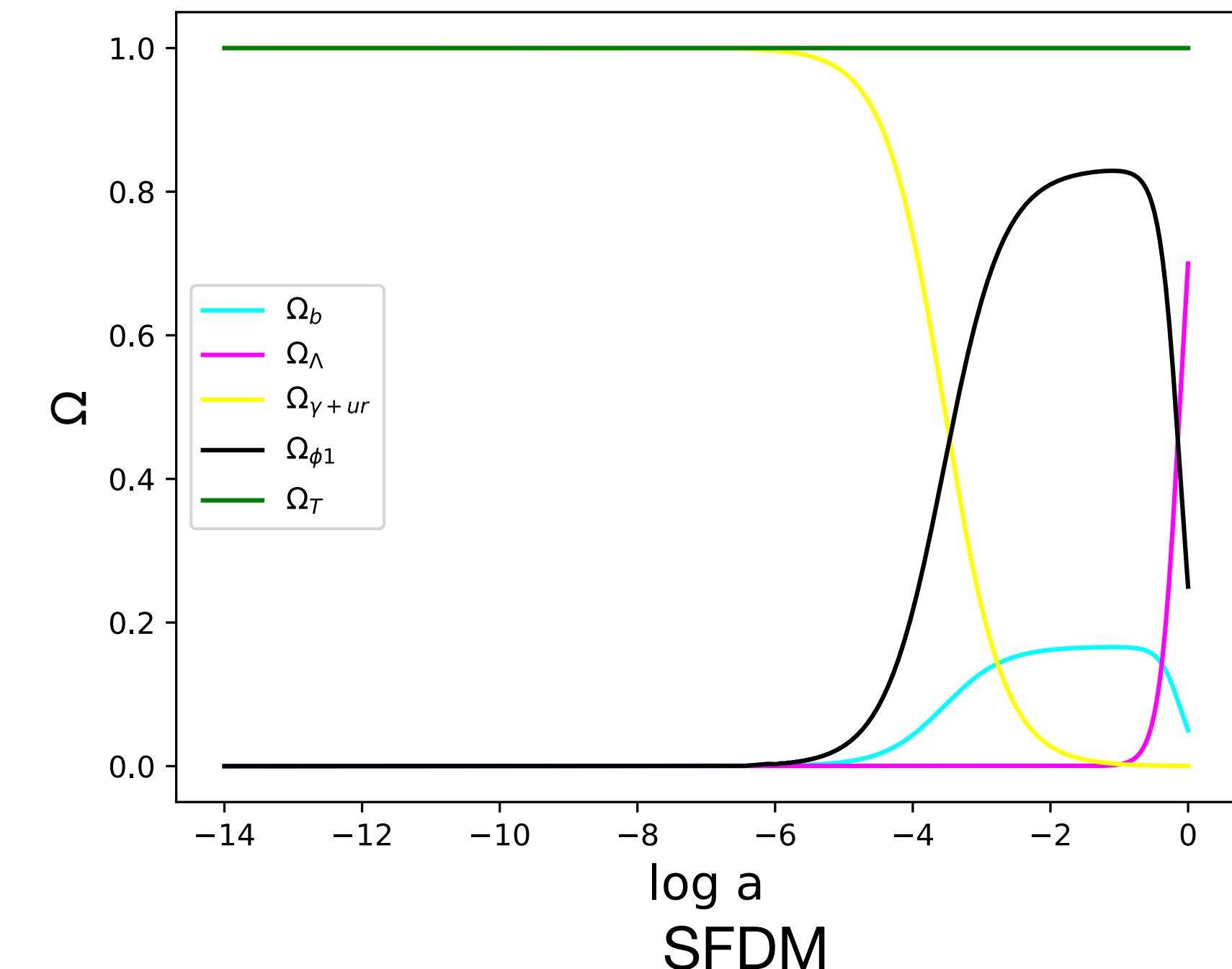
- > Cosmological standard model [4].
- > The Universe is currently expanding at an accelerated rate and has no curvature.
- > Dark Matter is cold, that is, it is composed of non-relativistic particles.
- > It is considered cosmological constant as Dark Energy.

Componente	ω
Matter	0
Radiation	1/3
Cosmological Constant	-1

- > The Universe has matter (dark and baryonic), radiation (photons and neutrinos) and dark energy (cosmological constant).
- > On large scales, Λ CDM and the observations agree but there are problems at smaller scales [5-6].
- > There are alternatives to solve them [6, 50].

SFDM.

- > Scalar Field Dark Matter [19].
- > In this model dark matter is made up of a **scalar field**, with a potential, and galactic halos are formed due to the **Bose-Einstein** condensation of this field [7-15].
- > The mass value of the boson associated with this field is
 $m \sim 10^{-22} eV.$
- > **SFDM** solves some problems of Λ **CDM** [11-15].



SFDM.

-> However, there is still work to be done.

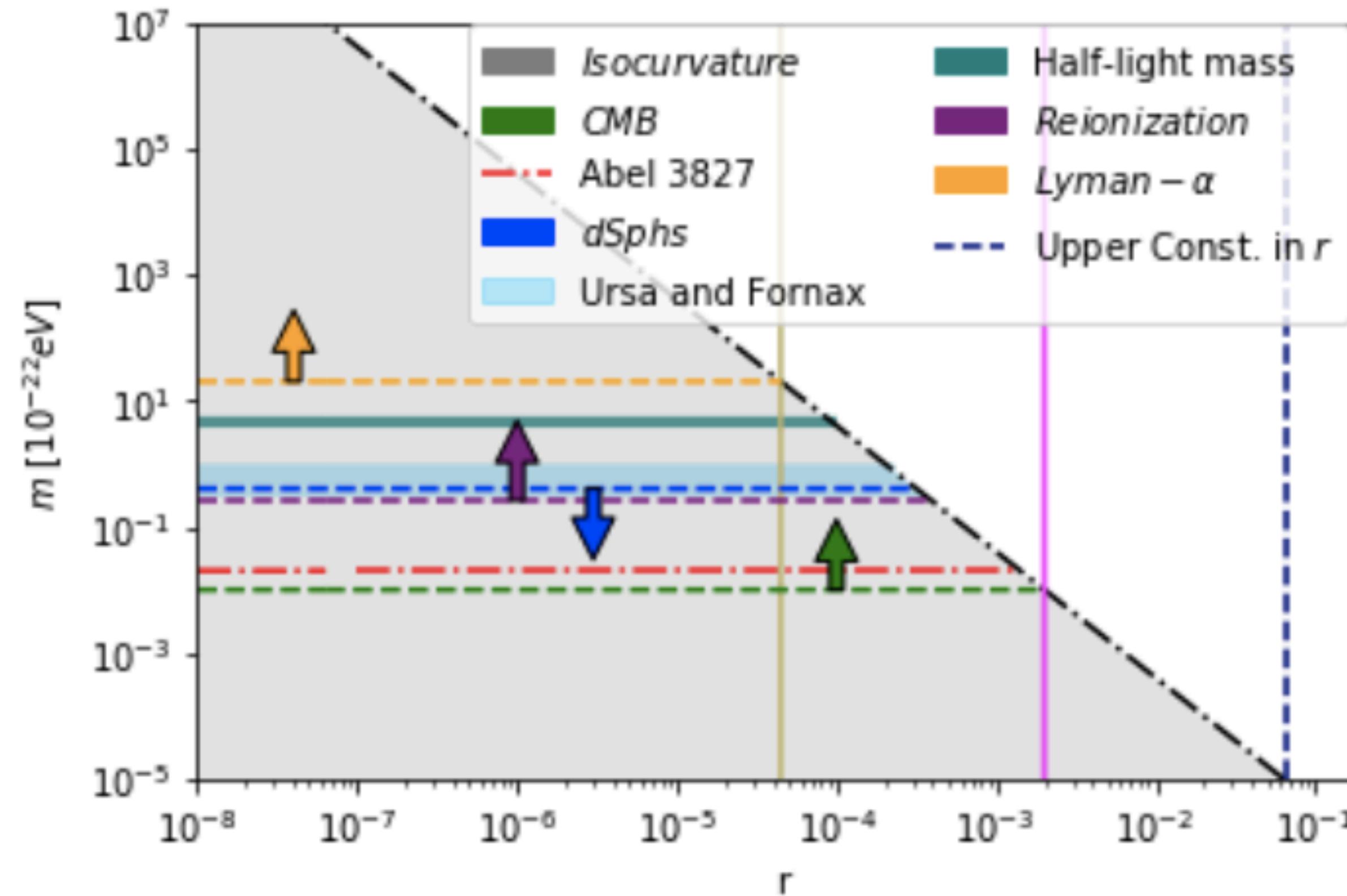


FIG. 1: Isocurvature constraints for the SFDM candidate. [20]

SFDM.

-> However, there is still work to be done.

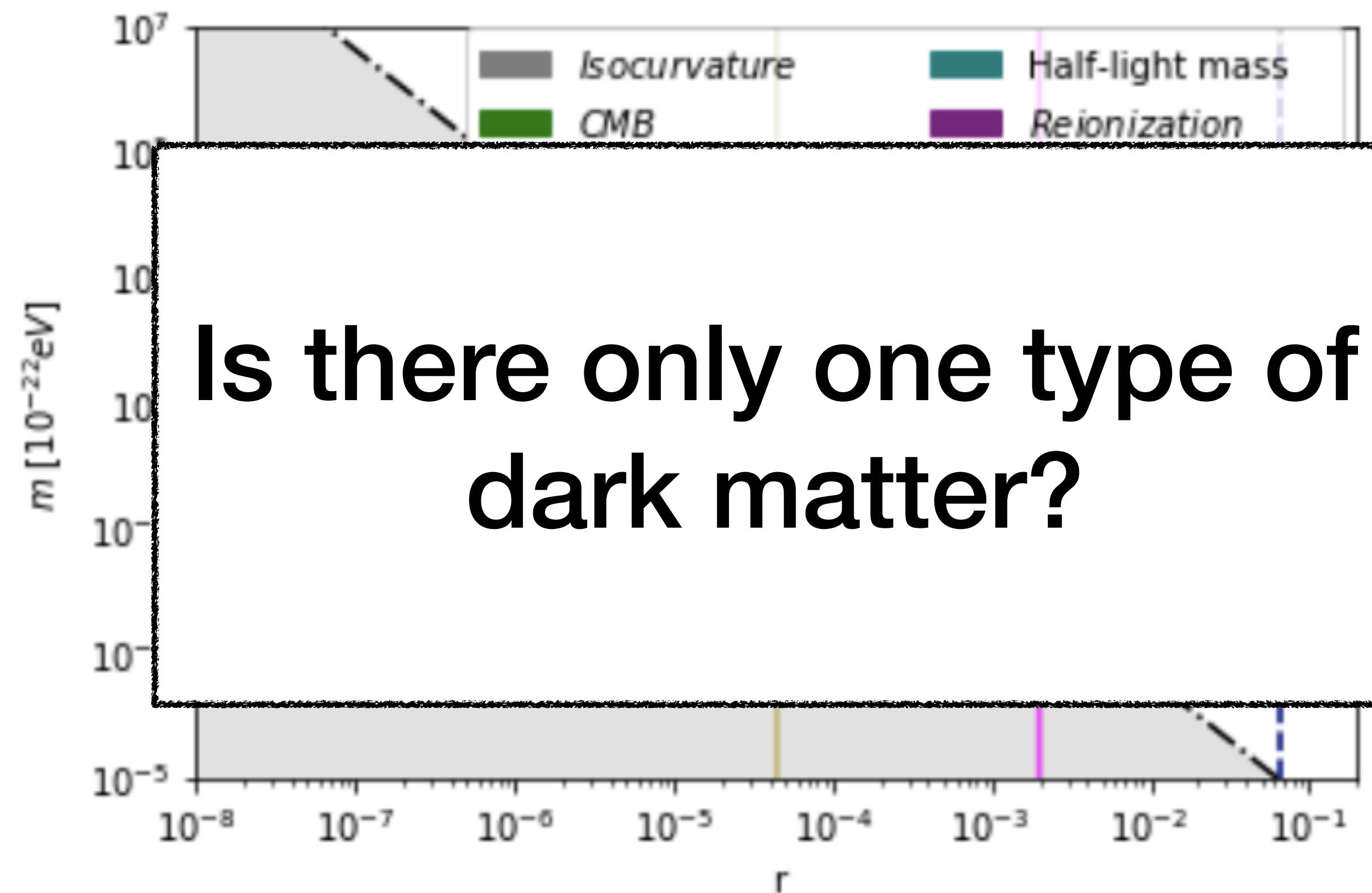


FIG. 1: Isocurvature constraints for the SFDM candidate. [20]

SFDM.

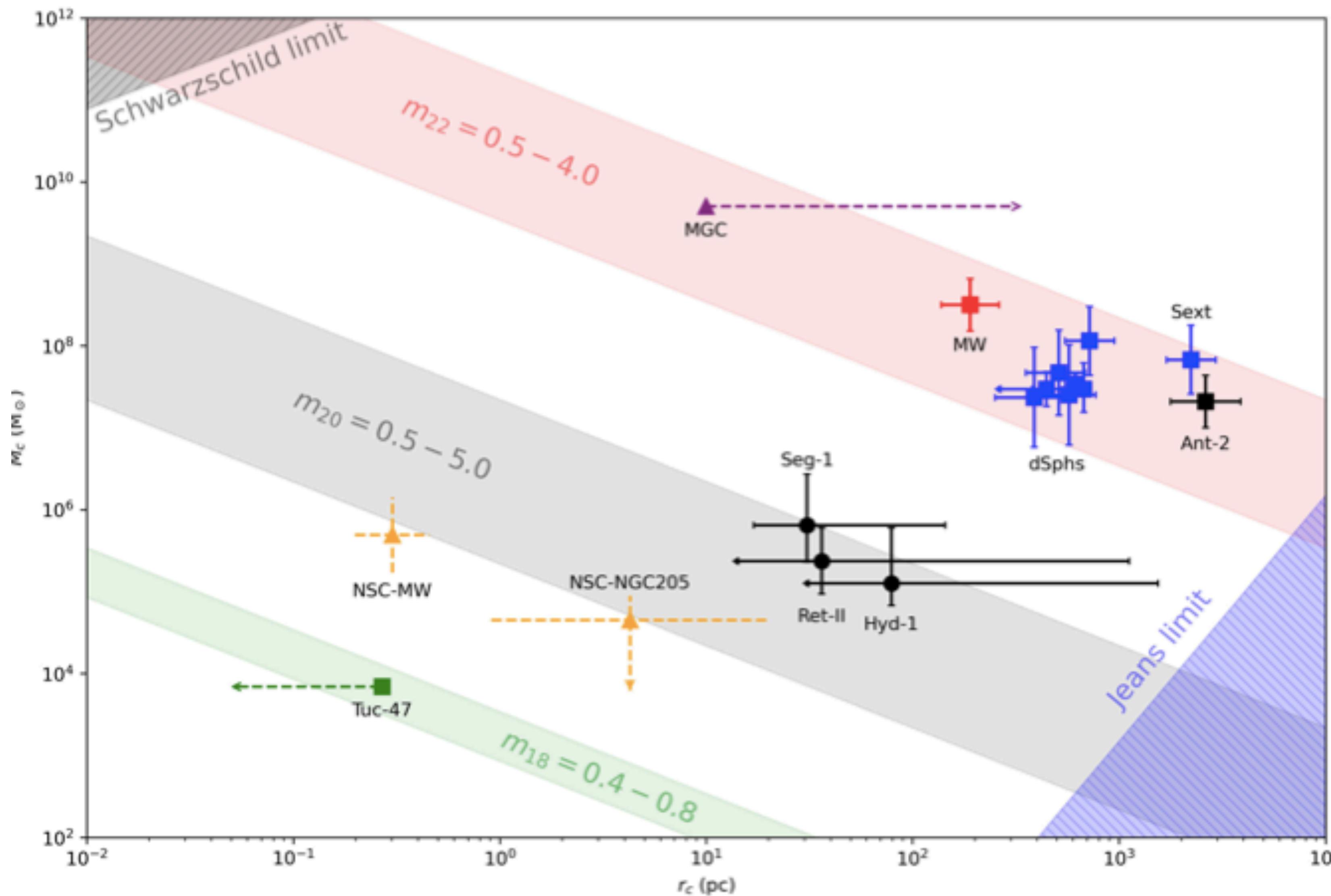


FIG. 1. The Soliton Core Mass-Radius Plot [1811.03771].

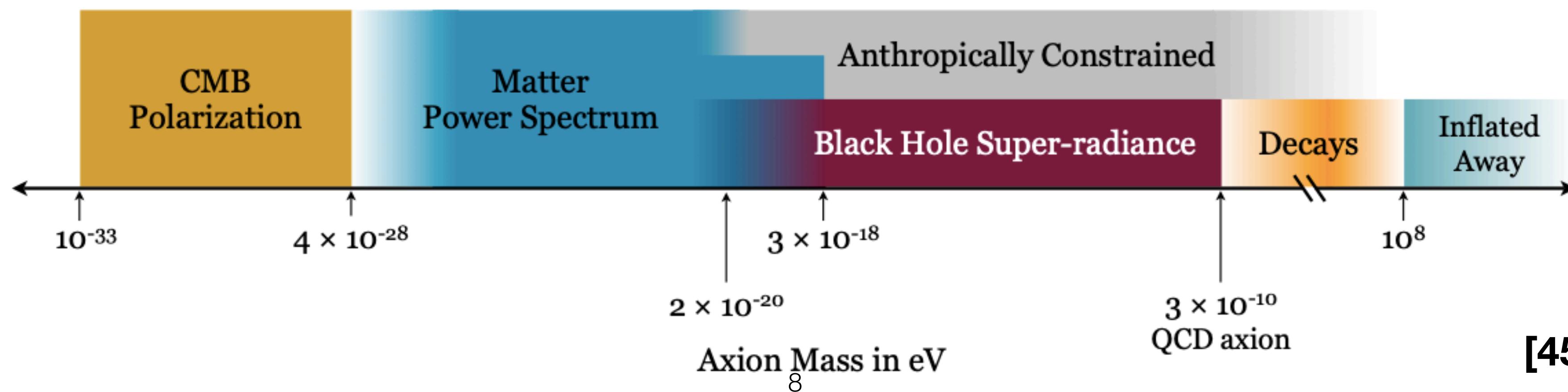
Multi-SFDM.

-> In this model, we propose that dark matter is made up of multiple scalar fields with different masses.

-> As a first approximation, we consider that the scalar fields are **spatially homogeneous**, **real** and that they **do not** interact with each other.

-> We consider that the Universe contains baryons, dark matter, dark energy (cosmological constant), photons and neutrinos.

-> The Universe has no curvature and we describe it with the FLRW metric.



[45]

Multi-SFDM (Background).

Klein-Gordon

$$\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{dV(\phi_i)}{d\phi_i} = 0, \quad i = 1, 2, \dots, n$$

$$H^2 - \frac{\kappa^2}{2} \left(\sum_I \rho_I + \sum_i \left(\frac{1}{2} \dot{\phi}_i + V(\phi_i) \right) \right) = 0,$$

$$\dot{\rho}_I + 3\frac{\dot{a}}{a} (\rho_I + P_I) = 0.$$

Change of variables. [47]

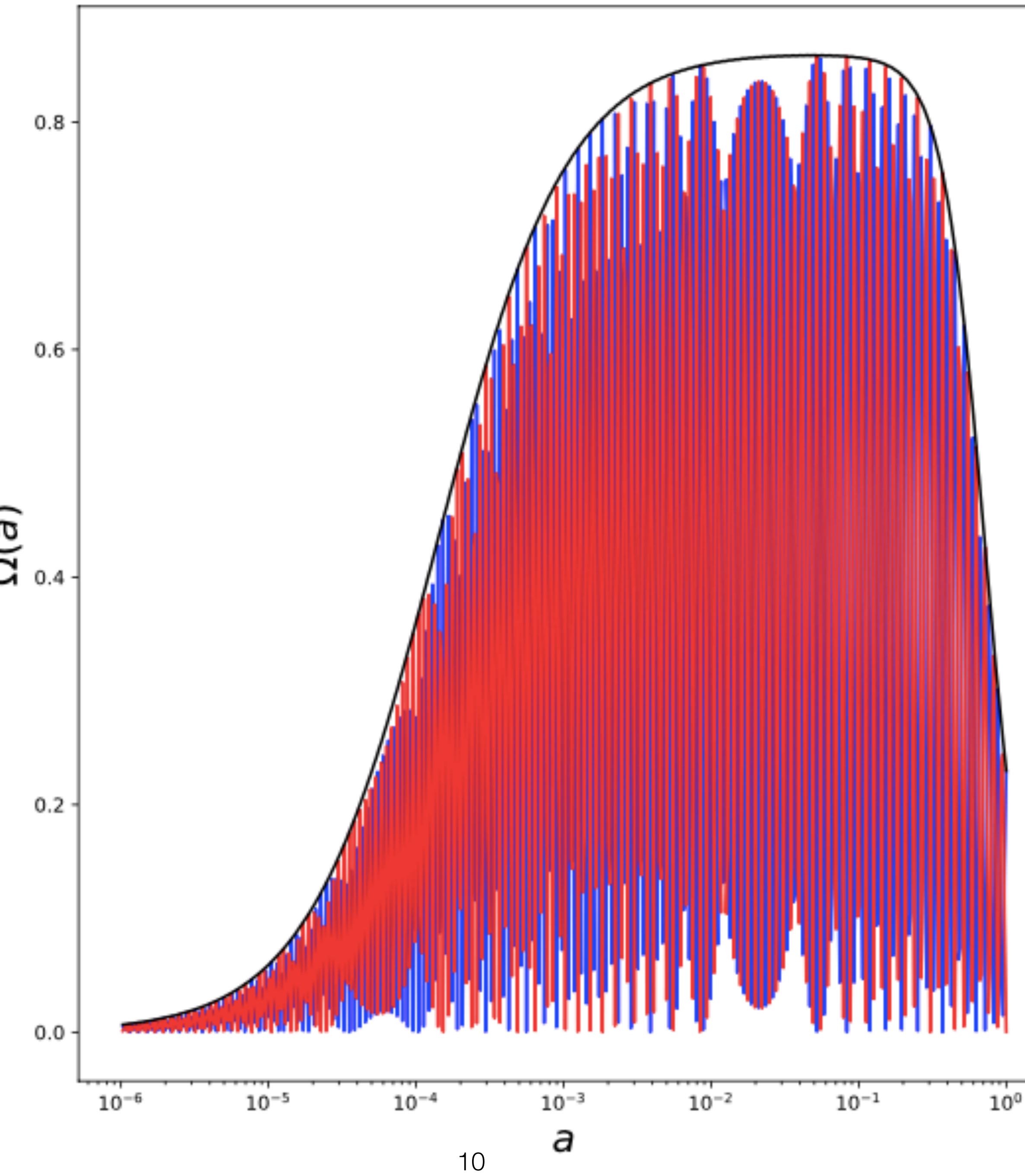
$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \quad y_i \equiv \frac{\kappa V_i^{1/2}}{\sqrt{3}H}, \quad y_{i,j} \equiv -\frac{2\kappa 6^{j/2} \partial_{\phi_i}^j V_i^{1/2}}{\sqrt{3}\kappa^j H}.$$

Multi-SFDM

Change of vari

$$x_i \equiv \frac{\kappa \dot{\phi}_i}{\sqrt{6}H},$$

$$H^2 - \frac{\kappa}{c_s}$$



$$\frac{\kappa 6^{j/2}}{\sqrt{3}\kappa^j} \frac{\partial_{\phi_i}^j V_i^{1/2}}{H}.$$

Multi-SFDM (Background).

$$\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{dV(\phi_i)}{d\phi_i} = 0,$$

$$H^2 - \frac{\kappa^2}{2} \left(\sum_I \rho_I + \sum_i \left(\frac{1}{2}\dot{\phi}_i + V(\phi_i) \right) \right) = 0,$$

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Change of variables. [47]

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \quad y_i \equiv \frac{\kappa V_i^{1/2}}{\sqrt{3}H}, \quad y_{i,j} \equiv -\frac{2\kappa 6^{j/2}}{\sqrt{3}\kappa^j} \frac{\partial_{\phi_i}^j V_i^{1/2}}{H}.$$

$$x_i = \Omega_{\phi_i}^{1/2} \sin \left(\frac{\theta_i}{2} \right), \quad y_i = \Omega_{\phi_i}^{1/2} \cos \left(\frac{\theta_i}{2} \right).$$

Multi-SFDM (Background).

$$\begin{aligned}\theta'_i &= -3 \operatorname{sen}(\theta_i) + y_{i,1}, \\ \Omega'_{\phi_i} &= 3 \left(\omega_{tot} + \cos(\theta_i) \right) \Omega_{\phi_i},\end{aligned}$$

$$y'_{i,j} = -\frac{\dot{H}}{H^2} y_{i,j} + \Omega_{\phi_i}^{1/2} \operatorname{sen}\left(\frac{\theta_i}{2}\right) y_{i,j+1}.$$

Potentials.

$$V(\phi) = 1/2m^2\phi^2$$

$$V(\phi) = m^2f^2 [1 + \cos(\phi/f)]$$

$$V(\phi) = m^2f^2 [\cosh(\phi/f) - 1]$$

To compare
with known
results.

$$y'_{i,1} = \frac{3}{2} (1 + \omega_{tot}) y_{i,1} + \frac{\lambda_i}{2} \Omega_{\phi_i} \sin(\theta_i)$$

$$\lambda_i = 3/\kappa^2 f_i^2$$

Multi-SFDM (Linear perturbations).

$$\ddot{\phi} = -3H\dot{\phi} - \left[\frac{k^2}{a^2} + \frac{d^2V(\phi)}{d\phi^2} \right] \varphi - \frac{1}{2}\dot{h}\dot{\phi}.$$

Klein-Gordon perturbed equation [48]

Change of variables [21, 25].

$$\sqrt{\frac{2}{3}} \frac{\kappa \dot{\phi}_i}{H} = -\Omega_{\phi_i}^{1/2} e^{\alpha_i} \cos\left(\frac{\vartheta_i}{2}\right),$$

$$\delta_{0i} = -e^{\alpha_i} \sin\left(\frac{\theta_i - \vartheta_i}{2}\right),$$

$$\frac{\kappa y_{i,1} \phi_i}{\sqrt{6}} = -\Omega_{\phi_i}^{1/2} e^{\alpha_i} \sin\left(\frac{\vartheta_i}{2}\right).$$

$$\delta_{1i} = -e^{\alpha_i} \cos\left(\frac{\theta_i - \vartheta_i}{2}\right).$$

Multi-SFDM (Linear perturbations).

$$\delta'_{0i} = \left[-3 \sin(\theta_i) - \omega_i (1 - \cos(\theta_i)) \right] \delta_{1i} + \omega_i \sin(\theta_i) \delta_{0i} - \frac{1}{2} h' (1 - \cos(\theta_i)),$$

$$\delta'_{1i} = \left[-3 \cos(\theta_i) - \omega_i \sin(\theta_i) + \frac{y_{i,2}}{y_{i,1}} \Omega_{\phi_i}^{1/2} \sin\left(\frac{\theta_i}{2}\right) \right] \delta_{1i} + \left[\omega_i (1 + \cos(\theta_i)) - \frac{y_{i,2}}{y_{i,1}} \Omega_{\phi_i}^{1/2} \cos\left(\frac{\theta_i}{2}\right) \right] \delta_{0i} - \frac{1}{2} h' \sin(\theta_i).$$

$$\omega_i \equiv \frac{k^2}{a^2 H^2 y_{i,1}},$$

With potentials:

$$\delta'_{1i} = \left[-3 \cos(\theta_i) - \left(\omega_i - \frac{\lambda_i}{2y_{i,1}} \Omega_{\phi_i} \right) \sin(\theta_i) \right] \delta_{1i} + \left(\omega_i - \frac{\lambda_i}{2y_{i,1}} \Omega_{\phi_i} \right) (1 + \cos(\theta_i)) \delta_{0i} - \frac{1}{2} h' \sin(\theta_i),$$

$$\omega'_i = \omega_i \left[\frac{3}{2} \omega_{tot} - \frac{1}{2} - \frac{\lambda_i}{2y_{i,1}} \Omega_{\phi_i} \sin(\theta_i) \right].$$

Multi-SFDM (Perturbaciones lineales).

$$\delta'_{0i} = \left[-3 \sin(\theta_i) - \omega_i (1 - \cos(\theta_i)) \right] \delta_{1i} + \omega_i \sin(\theta_i) \delta_{0i} - \frac{1}{2} h' (1 - \cos(\theta_i)),$$



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the Cosmic Linear Anisotropy Solving System



$$\delta'_{1i} = \left[-3 \cos(\theta_i) - \left(\omega_i - \frac{\lambda_i}{2y_{i,1}} \Omega_{\phi_i} \right) \sin(\theta_i) \right] \delta_{1i} + \left(\omega_i - \frac{\lambda_i}{2y_{i,1}} \Omega_{\phi_i} \right) (1 + \cos(\theta_i)) \delta_{0i} - \frac{1}{2} h' \sin(\theta_i),$$

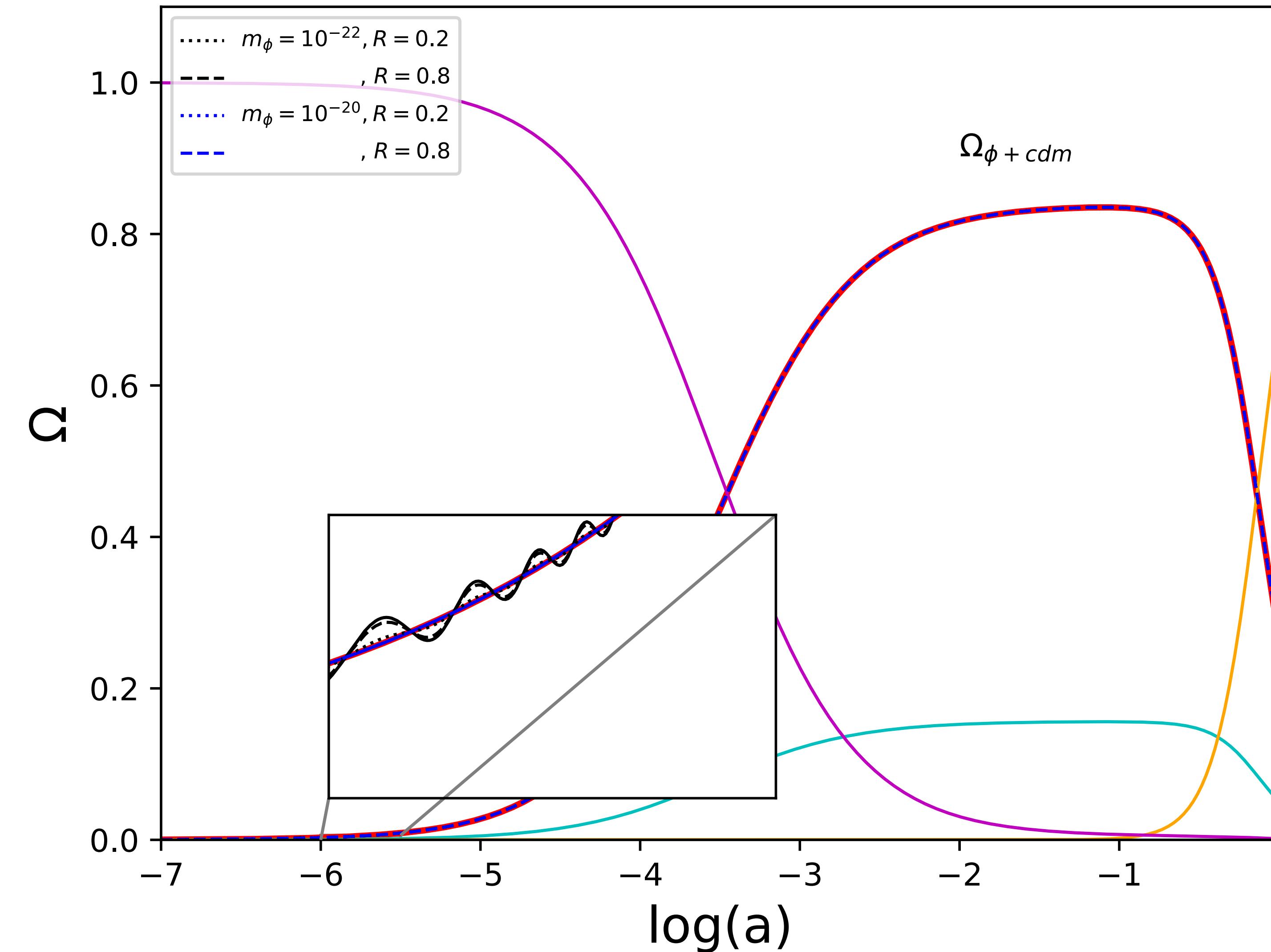
$$\omega'_i = \omega_i \left[\frac{3}{2} \omega_{tot} - \frac{1}{2} - \frac{\lambda_i}{2y_{i,1}} \Omega_{\phi_i} \sin(\theta_i) \right].$$

Numerical Analysis.

- > We are going to solve the particular case in which we consider two components of dark matter. The first is CDM with SFDM and twoSFDM.
- > We define the auxiliar parameter:

$$R = \frac{\Omega_{\phi 1,0}}{\Omega_{\phi 1,0} + \Omega_{\phi 2,0} + \Omega_{CDM,0}}$$

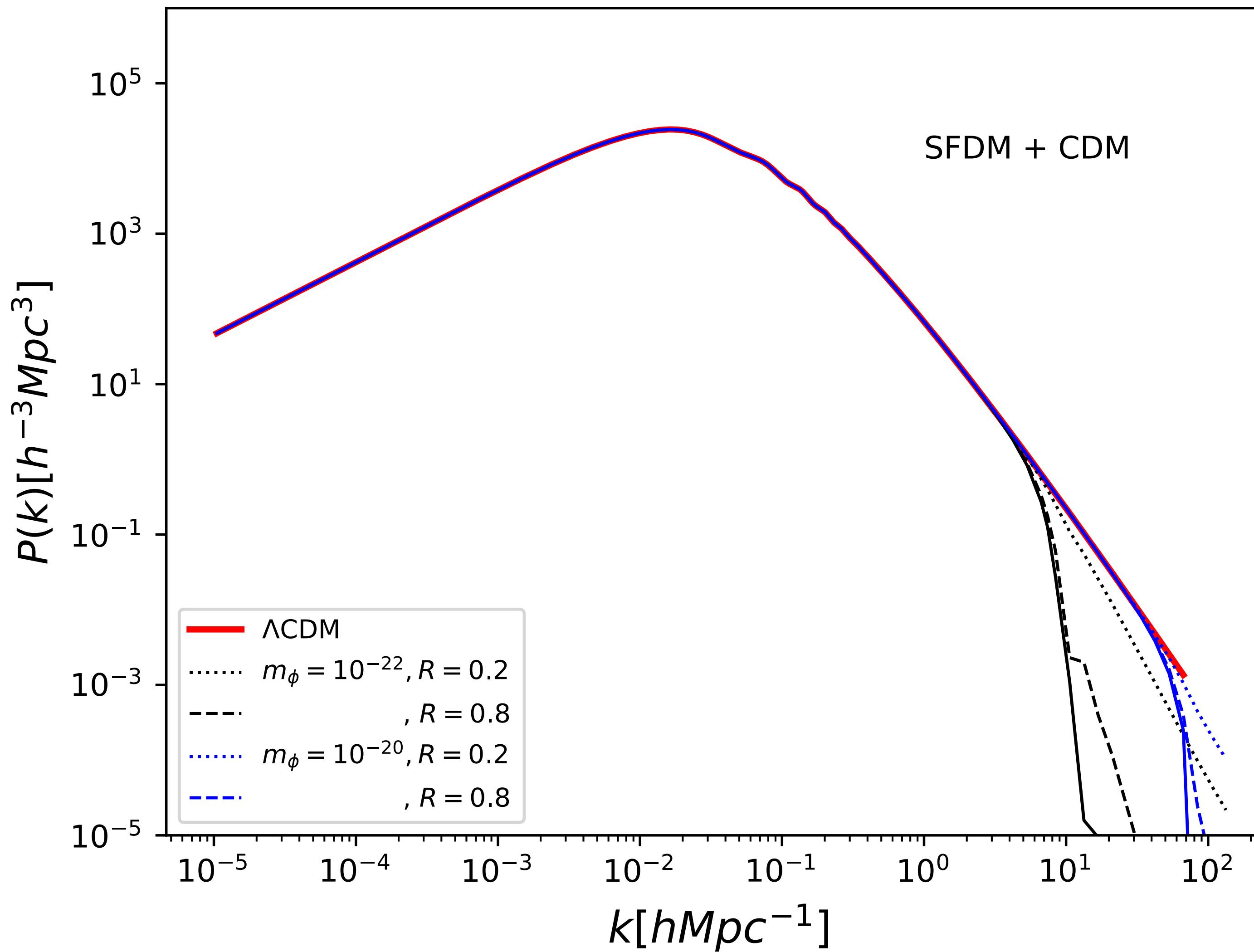
CDM with SFDM.



-> If $R > 5$ the oscillations are close to those of a single scalar field.

-> If $R < 5$ the oscillations are less evident until they disappear when $R = 0$, that is, CDM dominates.

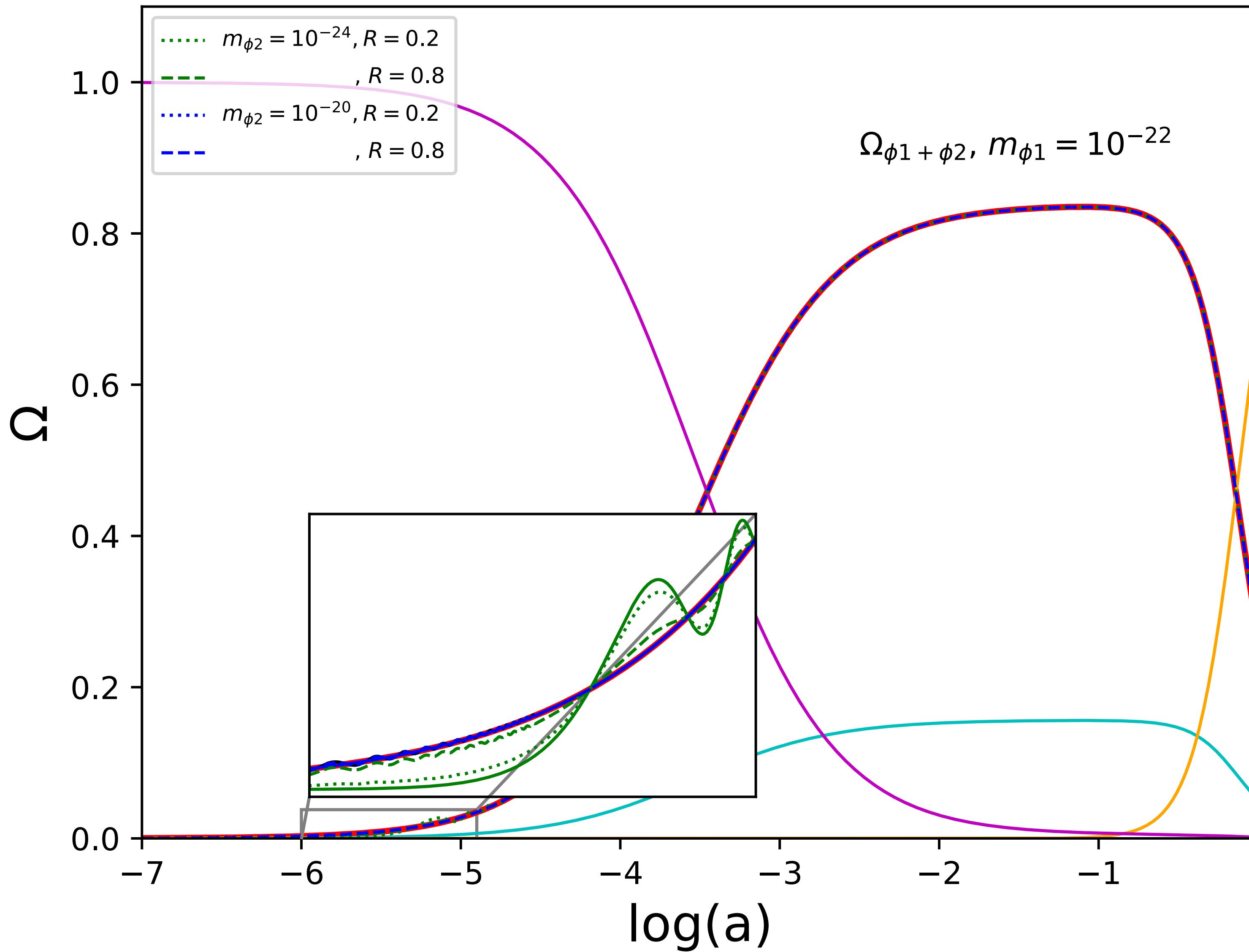
CDM con SFDM.



- > The scalar field characteristic cut-off occurs when $m_{\phi 1} < 10^{-20} eV$.
- > It disappears when $m_{\phi 1} > 10^{-20} eV$.

TwoSFDM.

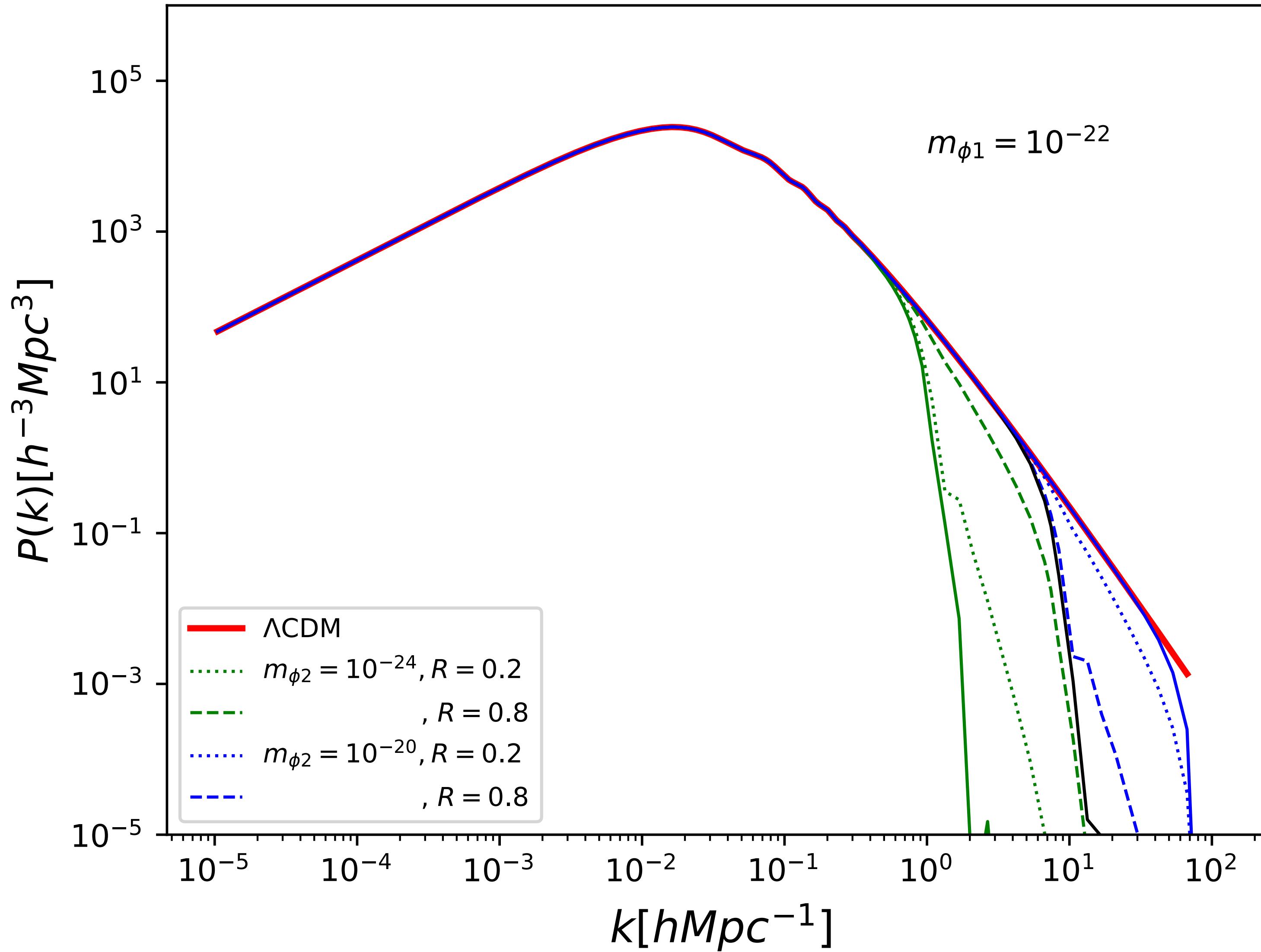
$$V(\phi) = 1/2m^2\phi^2$$



- > If $R > 5$ the oscillations are close to those of the field 1.
- > If $R < 5$ the oscillations are close to those of the field 2.
- > The oscillations are bounded by the single field case (solid lines).

TwoSFDM.

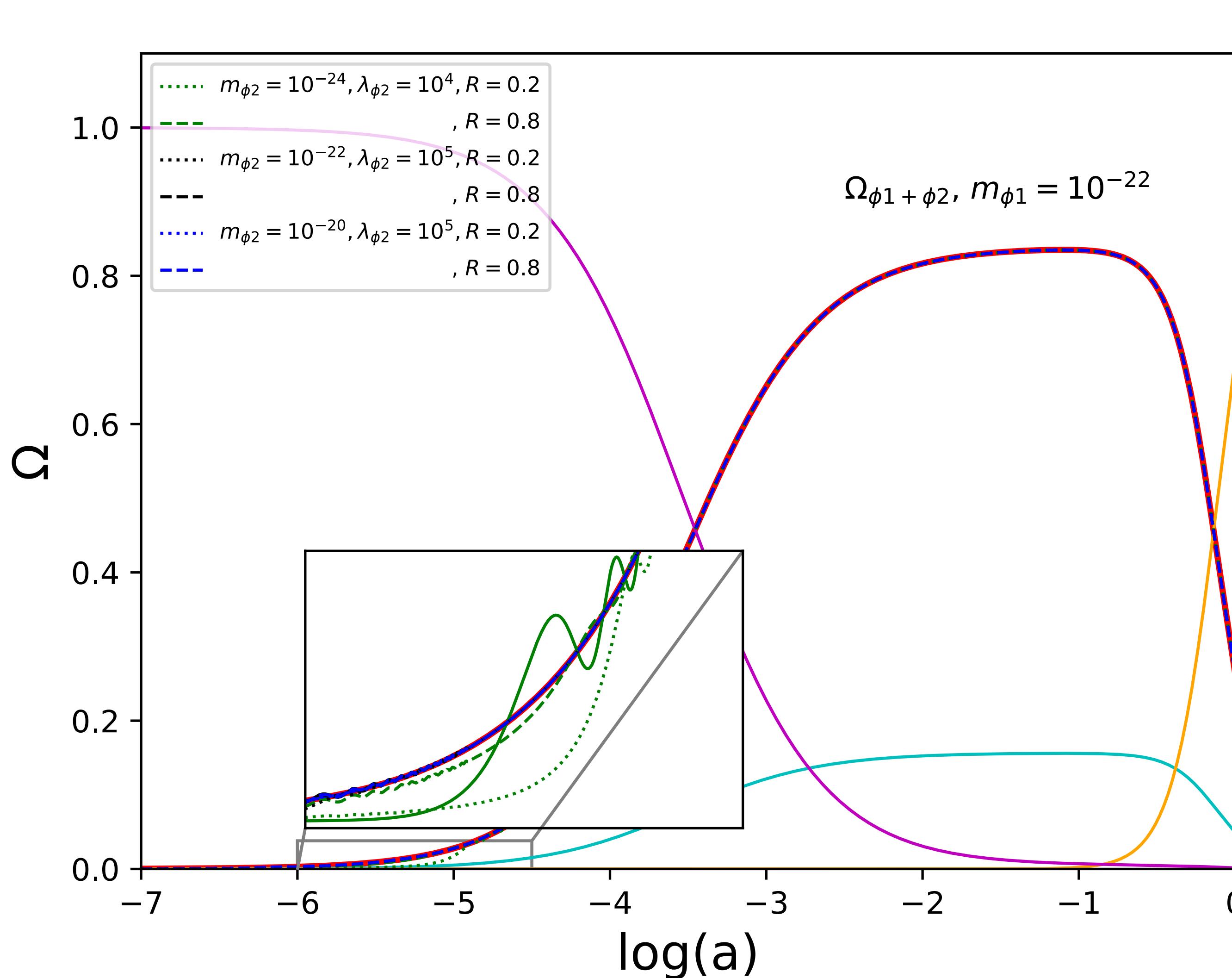
$$V(\phi) = 1/2m^2\phi^2$$



- > If $R > 5$ the MPS approaches the MPS of field 1.
- > If $R < 5$ the MPS approaches the MPS of field 2.

TwoSFDM.

$$V(\phi) = 1/2m^2\phi^2$$



$$V(\phi) = m^2 f^2 [1 + \cos(\phi/f)]$$

-> If $R > 5$ the oscillations are close to those of the field 1.

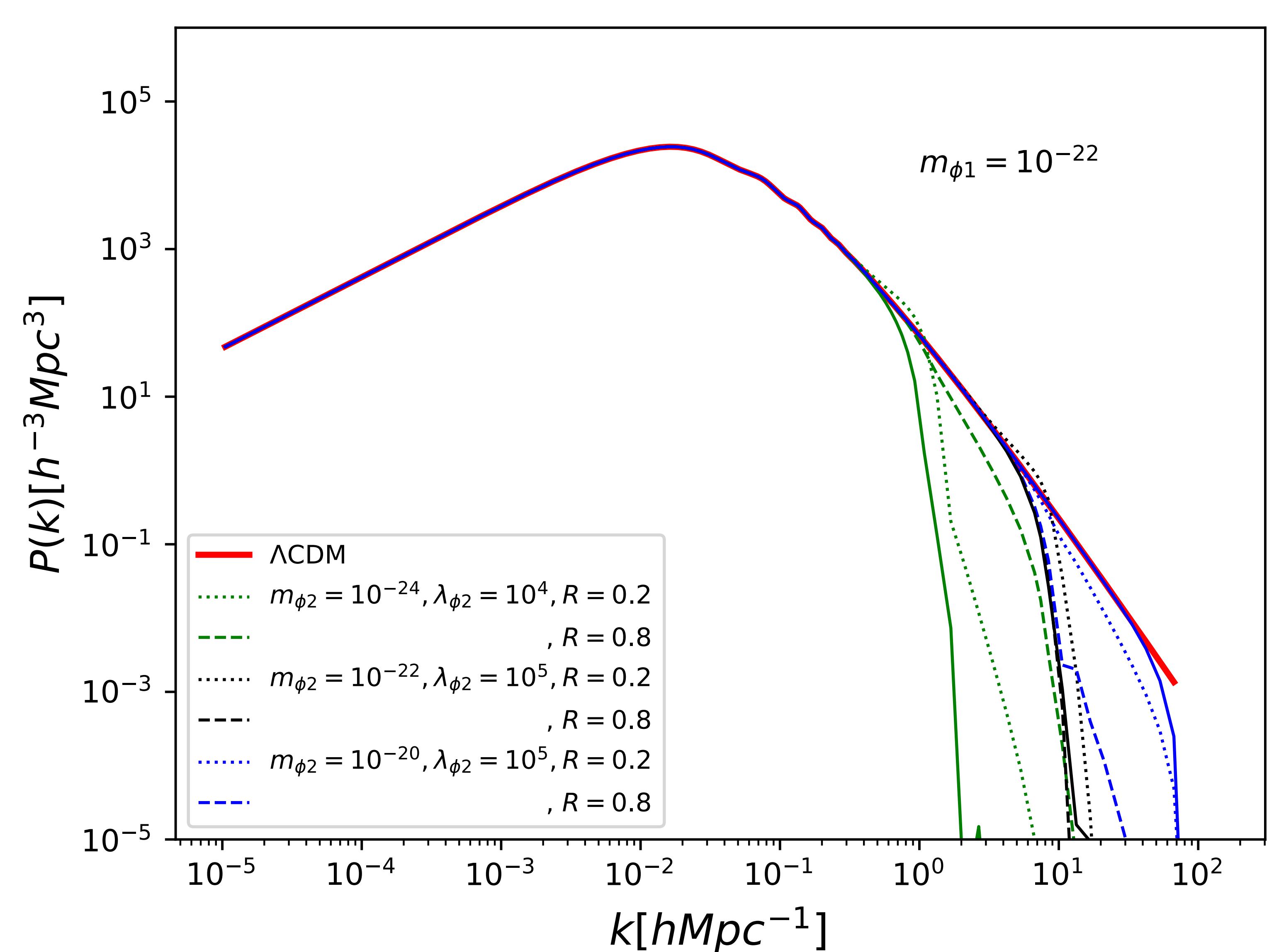
-> If $R < 5$ the oscillations are close to those of the field 2.

-> The oscillations are bounded by the single field case (solid lines)

-> The evolution does not depends on λ_{ϕ_2} .

TwoSFDM.

$$V(\phi) = 1/2m^2\phi^2$$



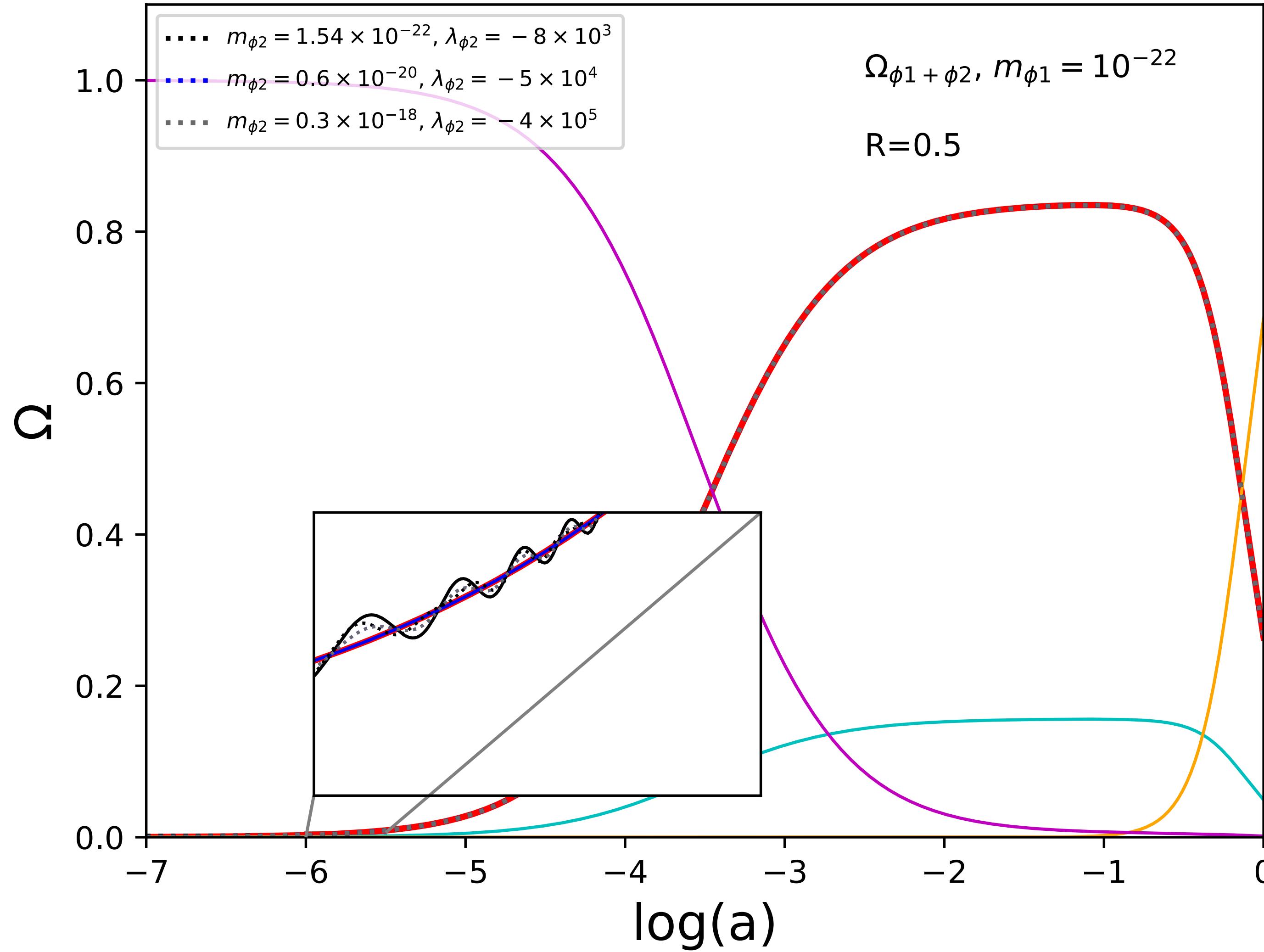
$$V(\phi) = m^2 f^2 [1 + \cos(\phi/f)]$$

- > If $R > 5$ the MPS approaches the MPS of field 1.
- > If $R < 5$ the MPS approaches the MPS of field 2.
- > The characteristic bump of the potential \cos appears when $R < 5$, the mass is light and λ_{ϕ_2} has a large value.

TwoSFDM.

$$V(\phi) = 1/2m^2\phi^2$$

$$V(\phi) = m^2f^2 [\cosh(\phi/f) - 1]$$

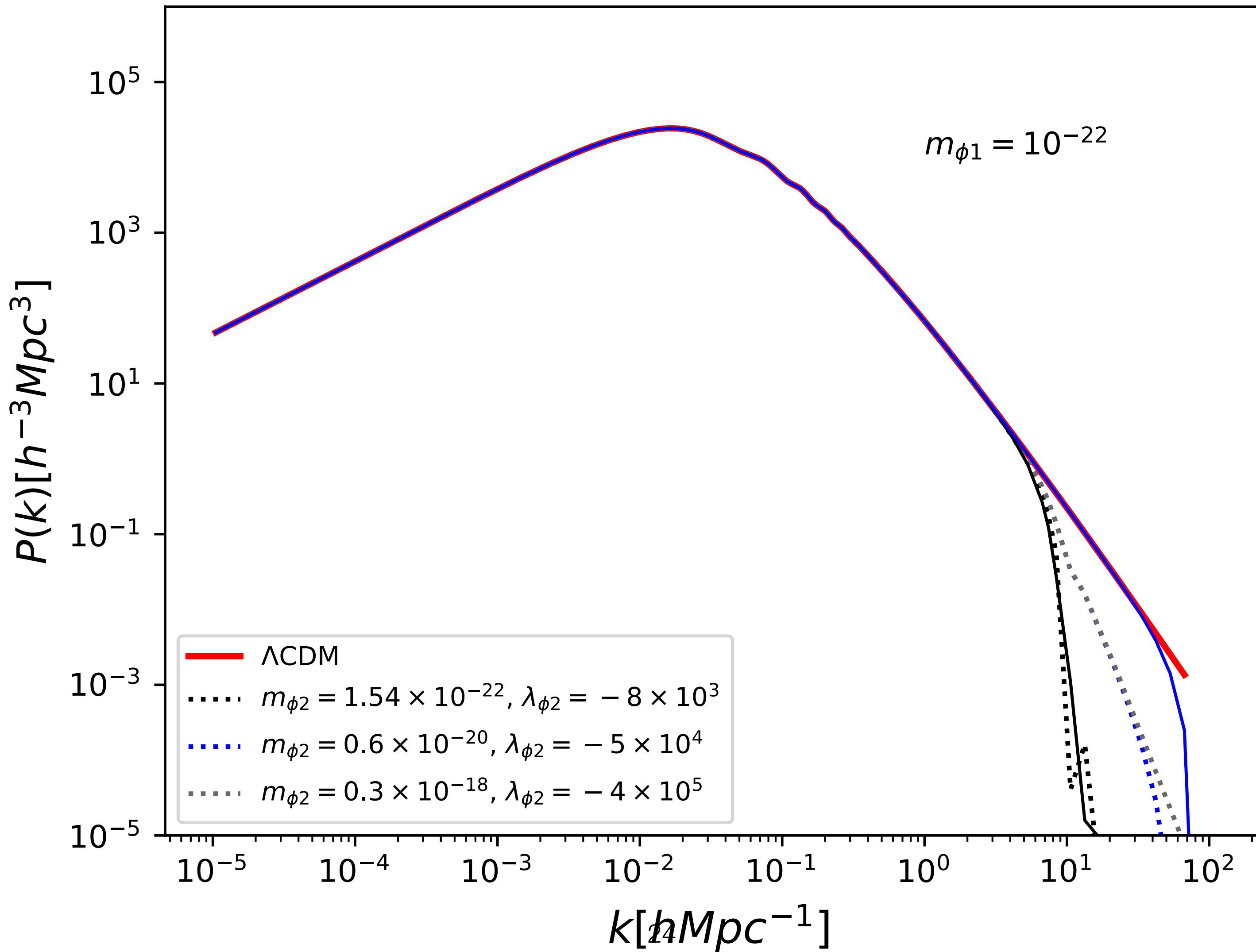


-> The oscillations are bounded by the single field case (solid lines).

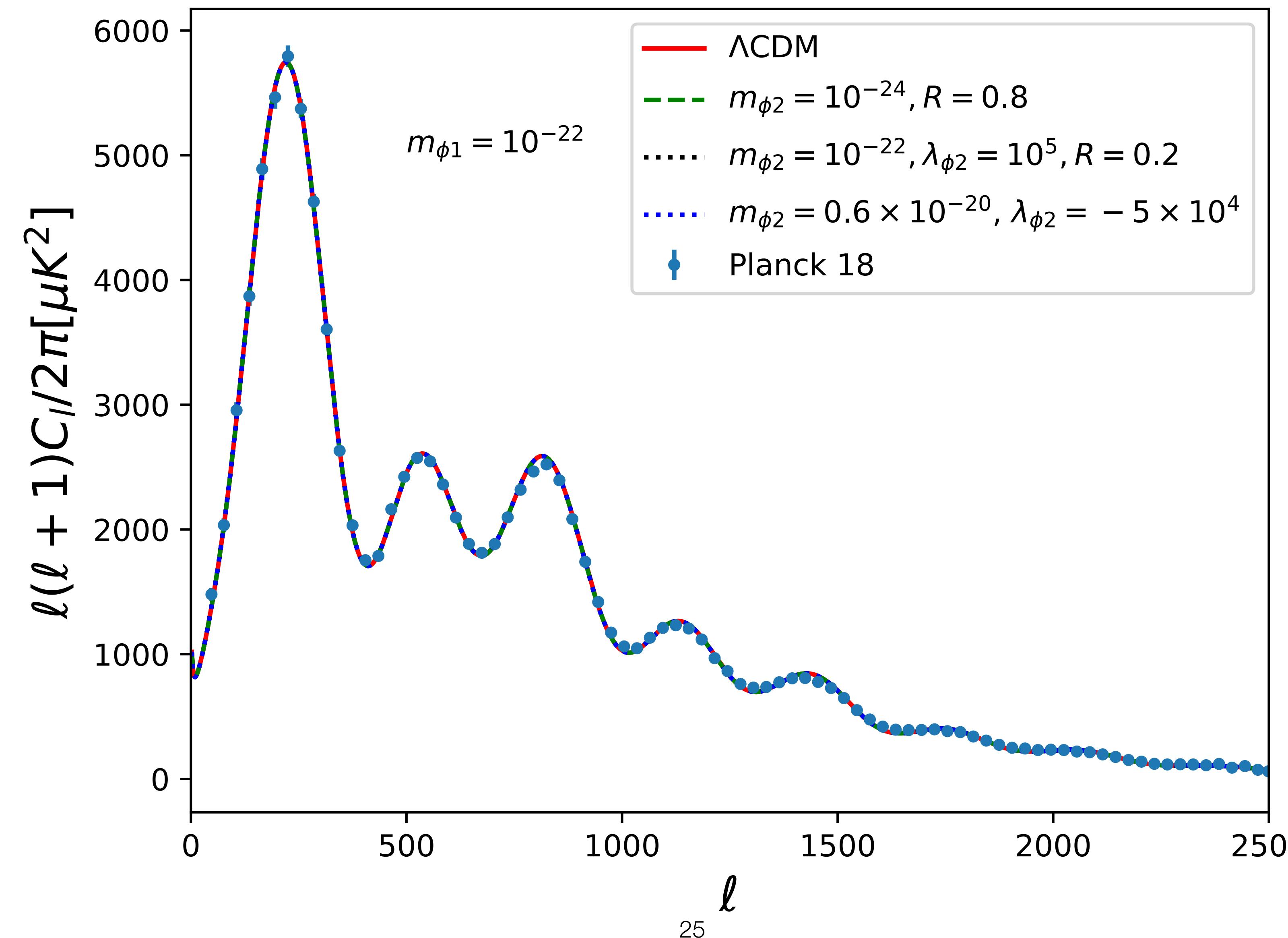
TwoSFDM.

$$V(\phi) = 1/2m^2\phi^2$$

$$V(\phi) = m^2f^2 [\cosh(\phi/f) - 1]$$



TwoSFDM.



There are no differences.

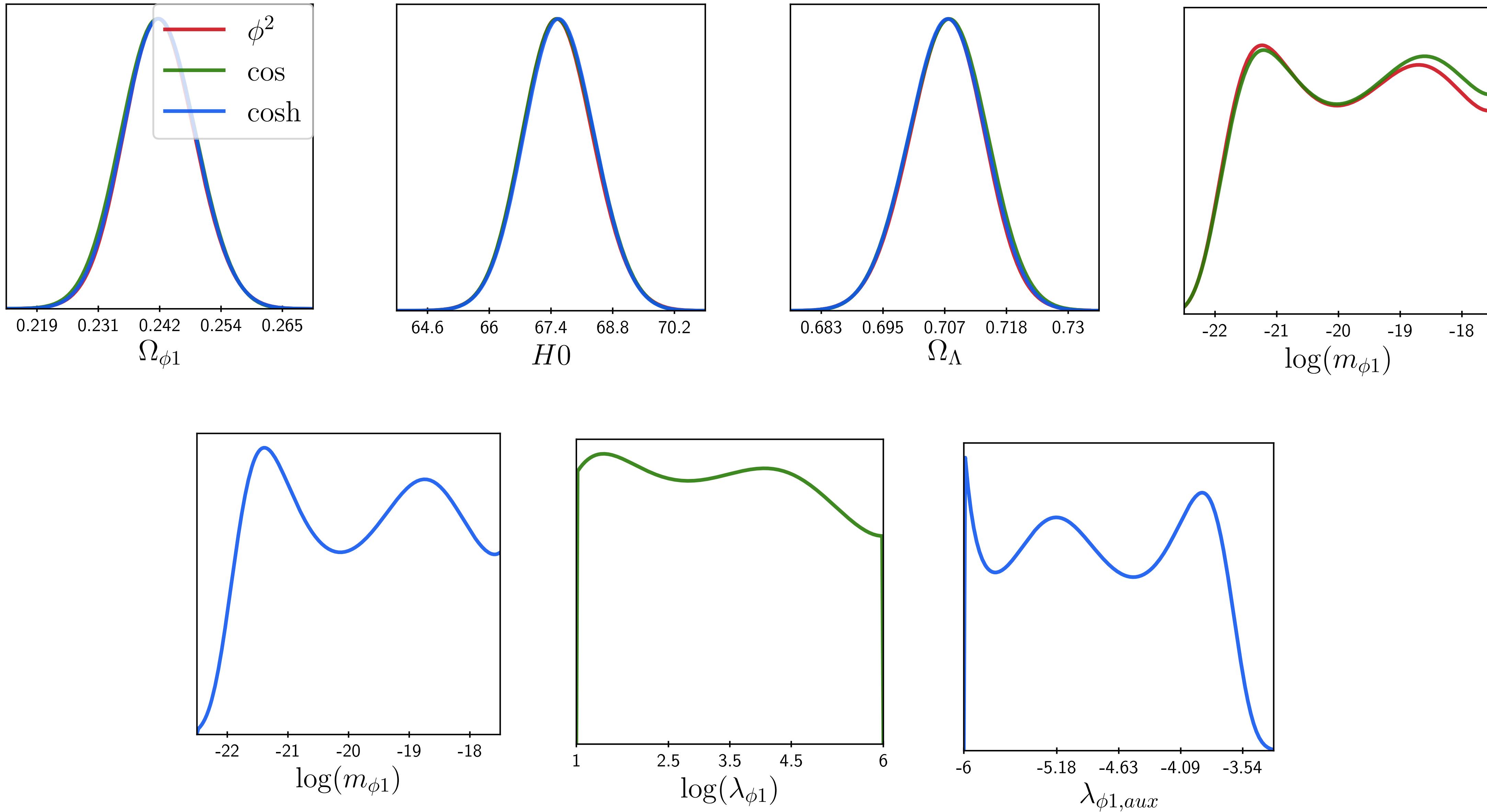
Models Constraints.

- > We use MontePython [51, 52].
- > We use the Lyman-a 3D matter power spectrum data from BOSS and eBOSS collaboration [53]. We use too the Ly-a BAO from eBOSS DR14 [54], the Galaxy BAO from DR12 [55], 6dFGS [56] and SDSS DR7 [57], a gaussian prior on the physical baryon density, $\omega_{b,0} = [0.005,0.1]$, and the Supernovas survey Pantheon [58].

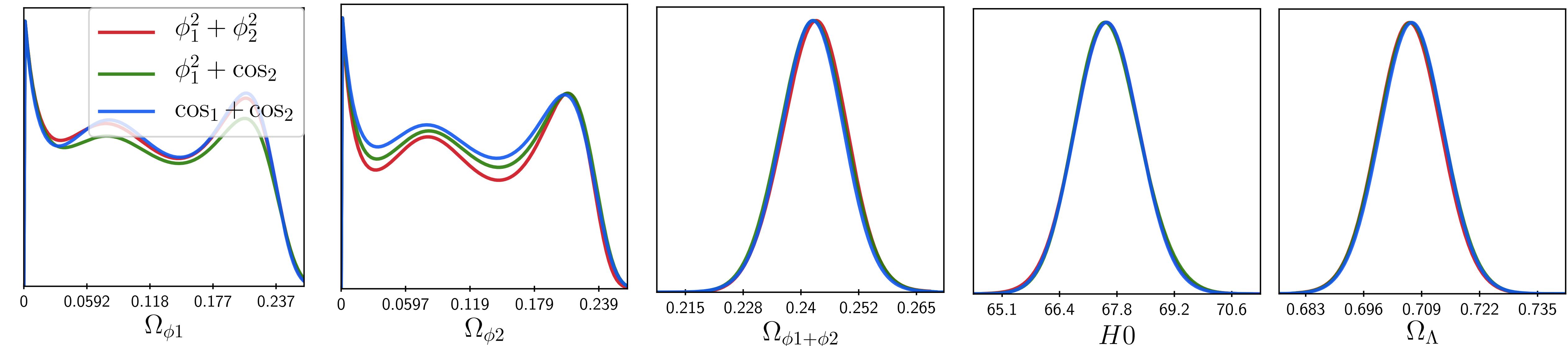
Cosmological parameter inference with Bayesian statistics

Luis E. Padilla,^{1, 2, *} Luis O. Tellez,¹ Luis A. Escamilla,¹ and J. Alberto Vazquez^{3, 1, †}

SFDM



TwoSFDM.



$m_1 \sim 10^{-22} eV$

$m_2 \sim 10^{-20} eV$

1811.03771

Summary.

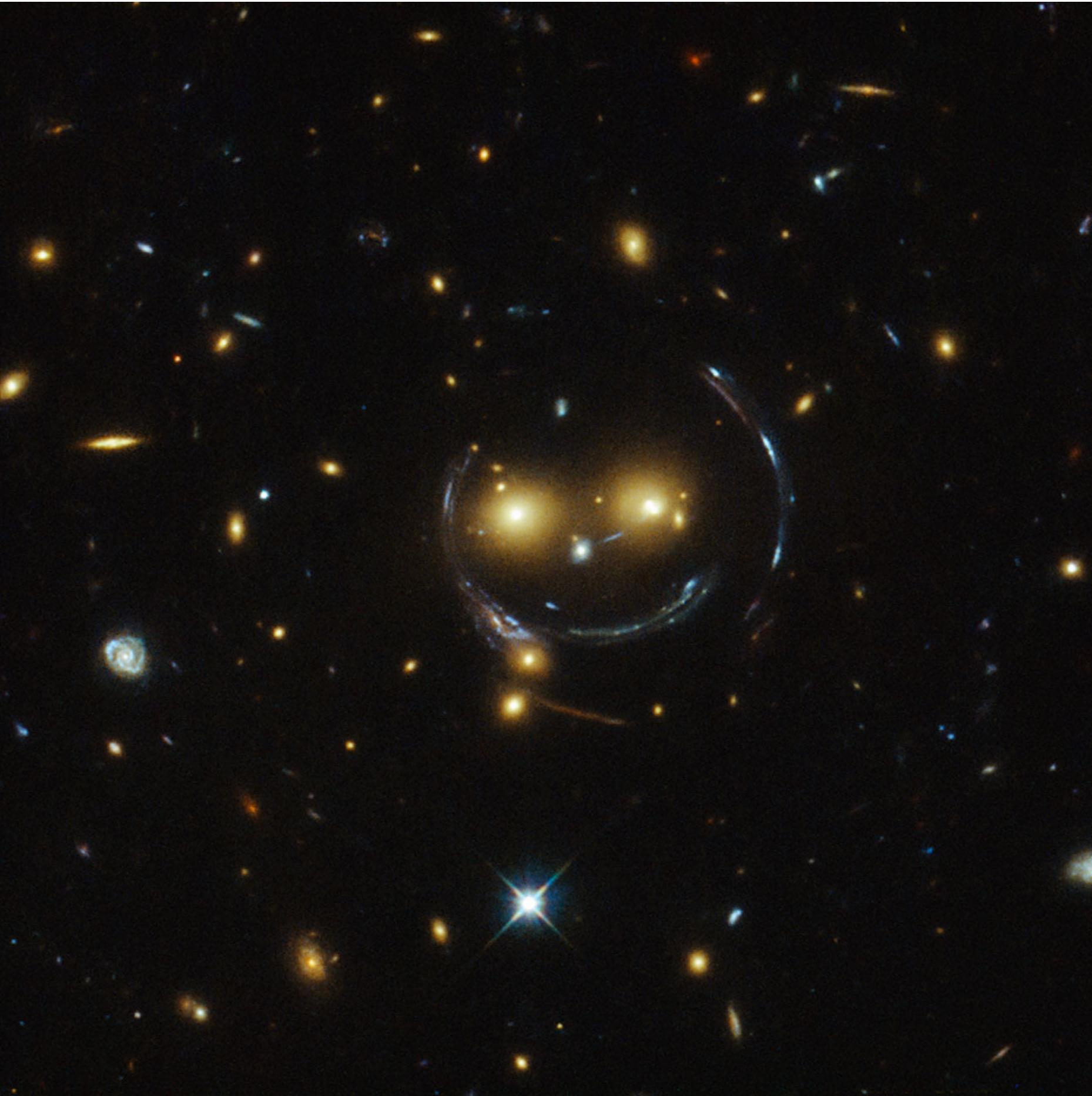
- > The main result of this work (soon in the arXiv) is that adding more scalar fields in order to explain astrophysical phenomena does not affect the known cosmology. So the MSFDM can be an alternative candidate to dark matter that can explain the observations at the cosmological and astrophysical levels. The results presented here can be generalized to a greater number of fields with different potentials.
- > We recover the results presented in 1811.03771. The authors propose two scalar field with masses $m_1 \sim 10^{-22} eV$ and $m_2 \sim 10^{-20} eV$, where the lightest field dominates.

Future work.

- > Study the Bayes factor.
- > Finish the paper :O

	ϕ^2	\cos	\cosh	$\phi_1^2 + \phi_2^2$	$\phi_1^2 + \cos_2$	$\cos_1 + \cos_2$
$ \mathcal{B}_{\Lambda\text{CDM},i} $	1.59	1.69	0.85	2.79	3.74	5.68

TABLE I. Bayes factor



Thank you!

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