

Entropy in the early universe

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Cosmology from Home 2021

Outline

1. Motivation
2. Reversible cosmology
3. Entropic forces in mechanics
4. Entropic forces in General Relativity
5. Non-equilibrium cosmology
6. Conclusions

1. Motivation

- General Relativity is a time-reversible theory.
- Most of the universe expansion history is adiabatic, but there are a few non-equilibrium epochs.
- Irreversible phenomena are not included in General Relativity in a complete and systematic way.

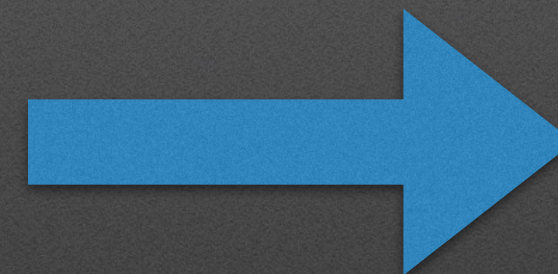
2. Reversible cosmology

Homogeneous and isotropic universe

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right)$$

Filled with a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$$



+

Einstein

Friedmann equations

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho \quad (\text{F1})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \quad (\text{F2})$$

2. Reversible cosmology



What can we say about this perfect fluid? Energy conservation:

$$D_\mu T^{\mu\nu} = 0 \rightarrow \dot{\rho} + 3H(\rho + p) = 0$$

But this also arises from the second law of thermodynamics

$$T \frac{dS}{dt} = \frac{d}{dt} (\rho a^3) + p \frac{d}{dt} (a^3) = 0 \quad \rightarrow$$

Which is true only in
equilibrium

 Internal energy
Change  Work

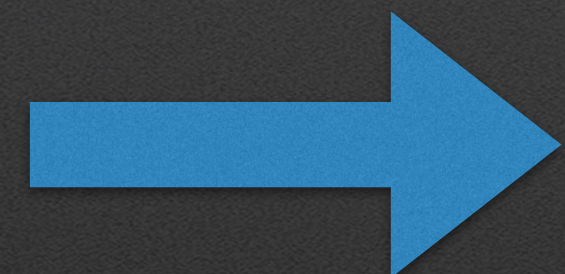
More generally $T \frac{dS}{dt} \geq 0$

2. Reversible cosmology

Should we go beyond adiabatic cosmology?

Continuity equation becomes $\dot{\rho} + 3H(\rho + p) = \frac{T\dot{S}}{a^3}$

+F1



Non-equilibrium F2

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3p - \frac{T\dot{S}}{a^3 H} \right)$$

3. Entropic forces in mechanics

Can we make the previous statement more rigorous?

We will use the variational formulation of non-equilibrium thermodynamics
(Gay-Balmaz & Yoshimura, 2017)

Stationary-action
principle

+

Laws of
thermodynamics



Defines the (physical)
Variational problem



Imposes constraints on



3. Entropic forces in mechanics

Consider a simple mechanical system

$$\mathcal{S} = \int dt L(q, \dot{q}, S) \rightarrow \delta \mathcal{S} = \int dt \left(\left(\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \right) \delta q + \frac{\partial L}{\partial S} \delta S \right)$$

Add the constraint

Constrained equation of motion

$$\frac{\partial L}{\partial S} \delta S = f(q, \dot{q}) \delta q$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = f(q, \dot{q})$$

Friction or
Entropic force

3. Entropic forces in mechanics

But what is $f(q, \dot{q})$?

It needs to be obtained from the phenomenological constraint

$$\frac{\partial L}{\partial S} \dot{S} = f(q, \dot{q}) \dot{q} \quad \rightarrow \quad \text{Usually } \frac{\partial L}{\partial S} = -T \text{ so } f(q, \dot{q}) \dot{q} \leq 0$$

Dynamics is
modified

+

Symmetry under time
reversion is broken

4. Entropic forces in General Relativity

Let us extend this formalism to General Relativity

$$\mathcal{S} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R + \int d^4x \mathcal{L}_m(g_{\mu\nu}, s)$$

$$\delta\mathcal{S} = \int d^4x \left(\frac{1}{2\kappa} \frac{\delta(\sqrt{-g}R)}{\delta g^{\mu\nu}} + \frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} + \int d^4x \frac{\delta\mathcal{L}_m}{\delta s} \delta s$$

Add the constraint

Non-equilibrium Einstein field equation

$$\frac{\delta\mathcal{L}_m}{\delta s} \delta s = \frac{1}{2} \sqrt{-g} f_{\mu\nu} \delta g^{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa \left(T_{\mu\nu} - f_{\mu\nu} \right) \quad \leftarrow \text{Friction or Entropic force}$$

4. Entropic forces in General Relativity

What do we know about this friction tensor $f_{\mu\nu}$?

Bianchi identities $\longrightarrow D^\mu T_{\mu\nu} = D^\mu f_{\mu\nu}$

Not much more so far... we need a proper notion of time evolution.

4. Entropic forces in General Relativity

Arnowitt, Deser and Misner (1959)

Let us work in the ADM formalism  Proper notion of time evolution

(3+1) splitting of space-time into constant time hypersurfaces Σ_t



$$g_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu$$



Induced
metric on Σ_t



Normal
vector to Σ_t

Initial value problem
with (h_{ij}, \dot{h}_{ij}) at t_0

Equivalently

$$ds^2 = - (Ndt)^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

4. Entropic forces in General Relativity

In the ADM formalism

Einstein field
equation



Hamilton evolution
equations

Canonical conjugate momentum $\Pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}}$ and Hamiltonian $\mathcal{H} = \Pi^{ij} \dot{h}_{ij} - \mathcal{L}$

Evolution equations

$$\frac{\partial \mathcal{H}}{\partial \Pi^{ij}} = \dot{h}_{ij}$$

Entropic
term

$$\tilde{f}_{ij} = h^\mu_i h^\nu_j f_{\mu\nu}$$

Constraints

$$\frac{\partial \mathcal{H}}{\partial N} = 0 \quad \frac{\partial \mathcal{H}}{\partial N^i} = 0$$

$$\frac{\delta \mathcal{H}}{\delta h_{ij}} = -\dot{\Pi}^{ij} - 2\kappa \frac{\delta \mathcal{L}_m}{\delta h_{ij}} - \kappa N \sqrt{h} \tilde{f}^{ij}$$

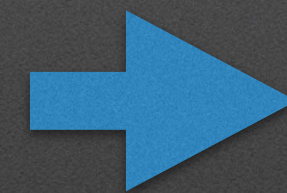


4. Entropic forces in General Relativity

And now what is \tilde{f}_{ij} ?

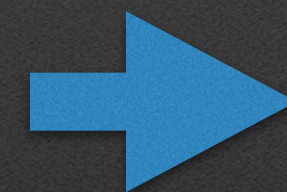
Phenomenological constraints

$$\frac{\partial \mathcal{L}}{\partial s} \mathfrak{L}_n s = \frac{1}{2} N \sqrt{h} \tilde{f}_{ij} \mathfrak{L}_n h^{ij}$$



Internal entropy production

$$\mathfrak{L}_n s = \mathfrak{L}_n s^{tot} - \nabla_i j_s^i$$



Entropy balance equation

4. Entropic forces in General Relativity

Temperature and entropy from the matter content

- Mechanical system

$$L(q, \dot{q}, S) = E_K(q, \dot{q}) - U(q, S) \quad \longrightarrow \quad T = -\frac{\partial L}{\partial S} = \frac{\partial U}{\partial S}$$

- Hydrodynamical matter

$$\mathcal{L} = -\sqrt{-g}\rho(g_{\mu\nu}, s) \quad \longrightarrow \quad T = -\frac{1}{\sqrt{-g}}\frac{\partial \mathcal{L}}{\partial s} = -\frac{\partial \rho}{\partial s}$$

4. Entropic forces in General Relativity

T and S from the gravity sector associated to a horizon H with induced metric h

$$\mathcal{S}_{GHY} = \frac{1}{8\pi G} \int_H d^3y \sqrt{h} K = \frac{1}{8\pi G} \int_H dt \sin \theta d\theta d\varphi \sqrt{h} K$$

- Schwarzschild black hole

$$\mathcal{S}_{GHY} = \frac{1}{2} \int dt M c^2 = - \int dt T_{BH} S_{BH}$$

$$T_{BH} = \frac{\hbar c^3}{8\pi G M}$$

$$S_{BH} = \frac{A c^3}{4G \hbar} = \frac{4\pi G M^2}{\hbar c}$$

\hbar signals an underlying quantum description, but $T_{BH} S_{BH}$ is classical

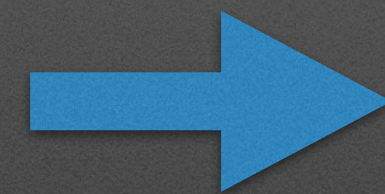
5. Non-equilibrium cosmology

Homogeneous and isotropic universe

$$ds^2 = -N(t)dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2\right)$$

Filled with a fluid with

$$\rho = \rho(a, S) \quad p = p(a, S)$$



Choose
 $N(t) = 1$

Hamilton constraint

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

Hamilton equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\tilde{f}$$

$$\text{with } \tilde{f} = h^{ij}f_{ij}$$

5. Non-equilibrium cosmology

From the phenomenological constraint

$$\tilde{f} = \frac{T\dot{S}}{a^2\dot{a}} \quad \longrightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3} \frac{T\dot{S}}{a^2\dot{a}}$$

Term of entropic origin



Non-equilibrium F2

Entropy production

(P)reheating, phase transitions,
black hole formation...



Cosmic acceleration

6. Conclusions

- The laws of thermodynamics modify the dynamics of any physical system and break time reversibility.
- The Lagrangian and Hamiltonian formulations of General Relativity can be modified to include the second law of thermodynamics.
- In Cosmology, this means the appearance of an additional term of entropic origin in the second Friedmann equation.
- The effect of non-adiabatic phenomena in the expansion history of the universe should be revisited.