Entropy in the early universe

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Outline

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1. Motivation

- General Relativity is a time-reversible theory.
- Most of the universe expansion history is adiabatic, but there are a few non-equilibrium epochs.
- Irreversible phenomena are not included in General Relativity in a complete and systematic way.

2. Reversible cosmology

Homogeneous and isotropic universe

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

Filled with a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

$\longrightarrow$

+ Einstein

Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad \textbf{(F1)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad \textbf{(F2)}$$

2. Reversible cosmology

What can we say about this perfect fluid? Energy conservation:

$$D_\mu T^{\mu\nu} = 0 \rightarrow \dot{\rho} + 3H(\rho + p) = 0$$

But this also arises from the second law of thermodynamics

$$T\frac{dS}{dt} = \underbrace{\frac{d}{dt}\left(\rho a^3\right)}_{\text{Internal energy Change}} + \underbrace{p\frac{d}{dt}\left(a^3\right)}_{\text{Work}} = 0$$

Which is true only in equilibrium

More generally $T\dfrac{dS}{dt} \geq 0$

2. Reversible cosmology

Should we go beyond adiabatic cosmology?

Continuity equation becomes $\dot{\rho} + 3H(\rho + p) = \frac{T\dot{S}}{a^3}$

+F1

Non-equilibrium F2 $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p - \frac{T\dot{S}}{a^3 H}\right)$

3. Entropic forces in mechanics

Can we make the previous statement more rigorous?

We will use the variational formulation of non-equilibrium thermodynamics (Gay-Balmaz & Yoshimura, 2017)

Stationary-action principle + Laws of thermodynamics

Stationary-action principle → Defines the (physical) Variational problem

Laws of thermodynamics → Imposes constraints on → Defines the (physical) Variational problem

3. Entropic forces in mechanics

Consider a simple mechanical system

$$\mathcal{S} = \int dt L(q, \dot{q}, S) \to \delta \mathcal{S} = \int dt \left(\left(\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \right) \delta q + \frac{\partial L}{\partial S} \delta S \right)$$

Add the constraint

Constrained equation of motion

$$\frac{\partial L}{\partial S} \delta S = f(q, \dot{q}) \delta q \longrightarrow \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = f(q, \dot{q})$$

Friction or
Entropic force

3. Entropic forces in mechanics

But what is $f(q, \dot{q})$?

It needs to be obtained from the phenomenological constraint

$$\frac{\partial L}{\partial S}\dot{S} = f(q, \dot{q})\dot{q}$$

Usually $\dfrac{\partial L}{\partial S} = -T$ so $f(q, \dot{q})\dot{q} \leq 0$

Dynamics is modified + Symmetry under time reversion is broken

4. Entropic forces in General Relativity

Let us extend this formalism to General Relativity

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + \int d^4x\mathcal{L}_m(g_{\mu\nu}, s)$$

$$\delta\mathcal{S} = \int d^4x\left(\frac{1}{2\kappa}\frac{\delta(\sqrt{-g}R)}{\delta g^{\mu\nu}} + \frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}\right)\delta g^{\mu\nu} + \int d^4x\frac{\delta\mathcal{L}_m}{\delta s}\delta s$$

Add the constraint

Non-equilibrium Einstein field equation

$$\frac{\delta\mathcal{L}_m}{\delta s}\delta s = \frac{1}{2}\sqrt{-g}f_{\mu\nu}\delta g^{\mu\nu} \longrightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa\left(T_{\mu\nu} - f_{\mu\nu}\right) \longleftarrow$$

Friction or
Entropic force

4. Entropic forces in General Relativity

What do we know about this friction tensor $f_{\mu\nu}$?

Bianchi identities $\longrightarrow$ $D^\mu T_{\mu\nu} = D^\mu f_{\mu\nu}$

Not much more so far… we need a proper notion of time evolution.

4. Entropic forces in General Relativity

Arnowitt, Deser and Misner (1959)

Let us work in the ADM formalism → Proper notion of time evolution

(3+1) splitting of space-time into constant time hypersurfaces Σ_t

Initial value problem with $\left(h_{ij}, \dot{h}_{ij}\right)$ at t_0

$$g_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu$$

Induced metric on Σ_t

Normal vector to Σ_t

Equivalently

$$ds^2 = -(Ndt)^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

4. Entropic forces in General Relativity

In the ADM formalism

Einstein field equation $\longleftrightarrow$ Hamilton evolution equations

Canonical conjugate momentum $\Pi^{ij} = \dfrac{\partial \mathscr{L}}{\partial \dot{h}_{ij}}$ and Hamiltonian $\mathscr{H} = \Pi^{ij}\dot{h}_{ij} - \mathscr{L}$

Evolution equations

$$\frac{\partial \mathscr{H}}{\partial \Pi^{ij}} = \dot{h}_{ij}$$

$$\frac{\delta \mathscr{H}}{\delta h_{ij}} = -\dot{\Pi}^{ij} - 2\kappa \frac{\delta \mathscr{L}_m}{\delta h_{ij}} - \kappa N\sqrt{h}\tilde{f}^{ij}$$

Entropic term

$$\tilde{f}_{ij} = h^{\mu}{}_{i} h^{\nu}{}_{j} f_{\mu\nu}$$

Constraints

$$\frac{\partial \mathscr{H}}{\partial N} = 0 \qquad \frac{\partial \mathscr{H}}{\partial N^i} = 0$$

4. Entropic forces in General Relativity

And now what is $\tilde{f}_{ij}$?

Phenomenological constraints

$$\frac{\partial \mathscr{L}}{\partial s}\pounds_n s = \frac{1}{2}N\sqrt{h}\tilde{f}_{ij}\pounds_n h^{ij}$$ → Internal entropy production

$$\pounds_n s = \pounds_n s^{tot} - \nabla_i j^i_s$$ → Entropy balance equation

4. Entropic forces in General Relativity

Temperature and entropy from the matter content

- Mechanical system

$$L(q, \dot{q}, S) = E_K(q, \dot{q}) - U(q, S) \quad \longrightarrow \quad T = -\frac{\partial L}{\partial S} = \frac{\partial U}{\partial S}$$

- Hydrodynamical matter

$$\mathscr{L} = -\sqrt{-g}\rho(g_{\mu\nu}, s) \quad \longrightarrow \quad T = -\frac{1}{\sqrt{-g}}\frac{\partial \mathscr{L}}{\partial s} = -\frac{\partial \rho}{\partial s}$$

4. Entropic forces in General Relativity

T and S from the gravity sector assciated to a horizon H with induced metric h

$$\mathcal{S}_{GHY} = \frac{1}{8\pi G}\int_H d^3y\sqrt{h}K = \frac{1}{8\pi G}\int_H dt\sin\theta d\theta d\varphi\sqrt{h}K$$

- Schwarzschild black hole

$$\mathcal{S}_{GHY} = \frac{1}{2}\int dt\, Mc^2 = -\int dt\, T_{BH}S_{BH}$$

$$T_{BH} = \frac{\hbar c^3}{8\pi GM}$$

$$S_{BH} = \frac{Ac^3}{4G\hbar} = \frac{4\pi GM^2}{\hbar c}$$

$\hbar$ signals an underlying quantum description, but $T_{BH}S_{BH}$ is classical

5. Non-equilibrium cosmology

Homogeneous and isotropic universe

$$ds^2 = -N(t)dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

Filled with a fluid with

$$\rho = \rho(a, S) \quad p = p(a, S)$$

Choose

$N(t) = 1$

Hamilton constraint

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

Hamilton equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\tilde{f}$$

with $\tilde{f} = h^{ij} f_{ij}$

5. Non-equilibrium cosmology

From the phenomenological constraint

$$\tilde{f} = \frac{T\dot{S}}{a^2\dot{a}}$$

Term of entropic origin

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\frac{T\dot{S}}{a^2\dot{a}}$$

Non-equilibrium F2

Entropy production
(P)reheating, phase transitions, black hole formation... → Cosmic acceleration

6. Conclusions

- The laws of thermodynamics modify the dynamics of any physical system and break time reversibility.
- The Lagrangian and Hamiltonian formulations of General Relativity can be modified to include the second law of thermodynamics.
- In Cosmology, this means the appearance of an additional term of entropic origin in the second Friedmann equation.
- The effect of non-adiabatic phenomena in the expansion history of the universe should be revisited.