Dynamics of f(Q, T) gravity with variable deceleration parameter

Laxmipriya Pati
Department of Mathematics
BITS-Pilani, Hyderabad Campus, India

Cosmology From Home conference-2021

Topic Outlines

- Introduction of $f(Q, T)$ gravity
- Some crucial information which helps to relate $f(Q, T)$ gravity
- Requirement of $f(Q, T)$ gravity in the action
- Mathematical formalism
- Models with hybrid scale factor
- Analysis of the model
- Results and Discussion

Introduction of $f(Q, T)$ gravity

• General relativity is basically a geometric theory, which is formulated in the Riemann metrical space and it has a great role within modified theories of gravity and also it helps to describe the gravitational field[1].

• We propose an extention of the symmetric teleparallel gravity in which the gravitational action L is given by an arbitrary function f, of the non-metricity Q and the trace of the matter-energy momentum tensor T, so that [2] $L = f(Q, T)$.

---

[1] Y. Xu, T. Harko, et al.,*Eur. Phys. J. C*, **80**, 449 (2020).

[2] Y. Xu, G. Li,T. Harko, S. Liang, *Eur. Phys. J. C*, **79**, 708 (2019).

Conts..

• We imposed basically two cosmological models which is functional form of $f(Q,T)$ and the format of these functional forms are taken as $f(Q,T) = aQ + bT$, $f(Q,T) = aQ^{n+1} + bT$.

• One of the simplest possibilities of extending Einstein's gravity is to introduce an arbitrary function f of the Ricci scalar R into the gravitational action, Which thus becomes,

$$S = \int (\frac{R}{2\kappa^2} + L_m)\sqrt{-g}d^4x$$

.

• A second approach to extend the Hilbert-Einstein action is to assume the existence of a nonminimal coupling between geometry and matter.

Some crucial information which helps to relate $f(Q, T)$ gravity

- [3] Aldrovandi and Pereira have presented a systematic discussions on the teleprallel gravity.

- [4] Ferraro and Fiorini have solved the particle horizon problem in flat FRW space-time.

- [5] Linder has shown the result for cosmic acceleration with the behaviour of the effective equation of state parameter with respect to the function, which describes the exponential dependency on torsion scalar.

---

[3] R. Aldrovandi, J.G Pereira, *Teleparallel Gravity, Fundamental Theories of physics*, **173**, Spinger, Heidelberg, (2013).

[4] R.Ferrarol, Fiorini, *Phys. Rev. D*, **75**, 084031 (2007).

[5] E.V Linder, *Phys. Rev. D*, **81**, 127301 (2010).

Conts..

• [6] Harko et al. have obtained the initial inflationary phase, the matter-dominated expansion, and then the late-time accelerating phase in f(T, τ) gravity.

• [7]Capozziello et al. have used the observational data of Big bang Nucleosynthesis to constrain the f(T) gravity model for the power law, exponential law and square root exponential law.

• [8]Otalora and Reboucas have presented the Godel type geometries and its solution in f(T) gravity.

---

[6]T. Harko, F.S.N Lobo, G.Otalora, E.N Saridakis, *JCAP*, **12**, 021 (2014).

[7]S. Capazziello, G. Lambiase, E.N Saridakis *Eur. Phys. J. C,*, **77**, 576 (2017).

[8]G. Otalora, M. j. Reboucas, *Eur. Phys. J. C,*, **77**, 799 (2017).

Requirement of $f(Q, T)$ gravity in the action

- At the classical level, several approaches are presented to address the observational results concerning the structure, formation and dynamics of the Universe, but the search for a satisfactory gravity theory is still to be established.

- $f(Q, T)$ leads to a non-conservation of energy-momentum tensor which may be responsible to achieve a late time cosmic acceleration in the models.

Mathematical Formalism

The gravitational action is

$$S = \int \left[\frac{1}{16\pi} f(Q, T) + L_M \right] \sqrt{-g} d^4x$$

By varying the above gravitational action we get the general field equation of f(Q,T) gravity which is represented below.

$$-\frac{2}{\sqrt{-g}} \nabla_\alpha (f_Q \sqrt{-g} P^\alpha{}_{\mu\nu}) - \frac{1}{2} f g_{\mu\nu} + f_T (T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q (P_{\mu\alpha\beta} Q_\nu{}^{\alpha\beta} - 2 Q^{\alpha\beta}{}_\mu P_{\alpha\beta\nu}) = 8\pi T_{\mu\nu} \quad (1)$$

we define the trace of the non-metricity tensor is

$$Q_\alpha = Q_\alpha{}^\mu{}_\mu \qquad \tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu} \quad (2)$$

Where,

$$p^{\alpha}_{\mu\nu} = -\frac{1}{2}L^{\alpha}_{\mu\nu} + \frac{1}{4}(Q^{\alpha} - \tilde{Q}^{\alpha})g_{\mu\nu} - \frac{1}{4}\delta^{\alpha}_{(\mu}Q_{\nu)} \tag{3}$$

$$L^{\alpha}_{\beta\gamma} = -\frac{1}{2}g^{\alpha\lambda}(\bigtriangledown_{\gamma}g_{\beta\lambda} + \bigtriangledown_{\beta}g_{\lambda\gamma} - \bigtriangledown_{\lambda}g_{\beta\gamma}) \tag{4}$$

$$L^{\alpha}_{\beta\gamma} = -\frac{1}{2}g^{\alpha\lambda}(Q_{\gamma\beta\lambda} + Q_{\beta\lambda\gamma} - Q_{\lambda\beta\gamma}) \tag{5}$$

Also given

$$Q_{\lambda\mu\nu} = -\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + g_{\nu\sigma}\hat{\Gamma}^{\sigma}_{\mu\lambda} + g_{\sigma\mu}\hat{\sigma}^{\sigma}_{\nu\lambda}$$

• The symmetric teleparallel gravity is a geometrical description of gravity, which is fully equivalent to general relativity. This equivalent can be easily proved in the so called coincident gauge[9], for which

$$\hat{\Gamma}^{\lambda}_{\mu\nu} \equiv 0$$

The disformation tensor can be expressed as,

$$\Gamma^{\lambda}_{\mu\nu} = -L^{\lambda}_{\mu\nu} \tag{6}$$

$$Q \equiv -g^{\mu\nu}(L^{\alpha}_{\beta\mu}L^{\beta}_{\nu\alpha} - L^{\alpha}_{\beta\alpha}L^{\beta}_{\mu\nu}) \tag{7}$$

[9] S. Nojiri and S. D. Odintsov, *Phys. Rept.*, **505**, 59, (2011).

Cosmological evolution of the FLRW Universe in $f(Q, T)$ gravity

$$ds^2 = a^2(t)(dx^2 + dy^2 + dz^2) - N^2(t)dt^2 \tag{8}$$

$$H = \frac{\dot{a}}{a}, \qquad \tilde{T} \equiv \frac{\dot{N}}{N} \tag{9}$$

By solving equation(7) we get

$$Q = 6\frac{H^2}{N^2}$$

The energy momentum tensor is given by

$$T^{\mu}_{\nu} = diag(-\rho, p, p, p)$$

Also,

$$\Theta^{\mu}_{\nu} = diag(2\rho + p, -p, -p, -p)$$

To simplify the mathematical formalism we introduce the notations

$$F \equiv f_Q \qquad and \quad 8\pi\tilde{G} \equiv f_T$$

By using FLRW metric from the field equation we can easily find

$$\frac{f}{2} - 6F\frac{H^2}{N^2} = 8\pi\tilde{G}(\rho + p) \tag{10}$$

$$\frac{f}{2} - \frac{2}{N^2}\left[\left(\dot{F} - F\tilde{T}\right)H + F\left(\dot{H} + 3H^2\right)\right] = -8\pi p \tag{11}$$

Next we consider the standard case when $N = 1$ which is the case of standard FLRW geometry.Thus we get

$$Q = 6H^2 \tag{12}$$

and the generalized Friedmann equations reduces to

$$\rho = \frac{1}{8\pi}\left[\frac{f}{2} - 6FH^2 - 2\frac{\tilde{G}}{1+\tilde{G}}(\dot{F}H + F\dot{H})\right] \tag{13}$$

$$p = -\frac{1}{8\pi}\left[\frac{f}{2} + 6FH^2 + 2(\dot{F}H + F\dot{H})\right] \tag{14}$$

A. Case-I

Consider the functional form $f(Q,T) = aQ + bT$ such that $F = \frac{\partial f}{\partial Q} = a$ and $\tilde{G} = \frac{\partial f}{\partial T} = b$. So the pressure and energy density can be obtained as,

$$p = \frac{-12a\left(\alpha + \frac{\beta}{t}\right)^2 (b + 8\pi) + 4a(3b + 16\pi)\left(\frac{\beta}{t^2}\right)}{b^2 - (3b + 16\pi)2} \quad (15)$$

$$\rho = \frac{12a\left(\alpha + \frac{\beta}{t}\right)^2 (b + 8\pi) + 4ab\left(\frac{\beta}{t^2}\right)}{b^2 - (3b + 16\pi)2} \quad (16)$$

$$\omega = \frac{-3\left(\alpha + \frac{\beta}{t}\right)^2 (b + 8\pi) + (3b + 16\pi)\left(\frac{\beta}{t^2}\right)}{3(b + 8\pi)\left(\alpha + \frac{\beta}{t}\right)^2 + b\left(\frac{\beta}{t^2}\right)} \quad (17)$$

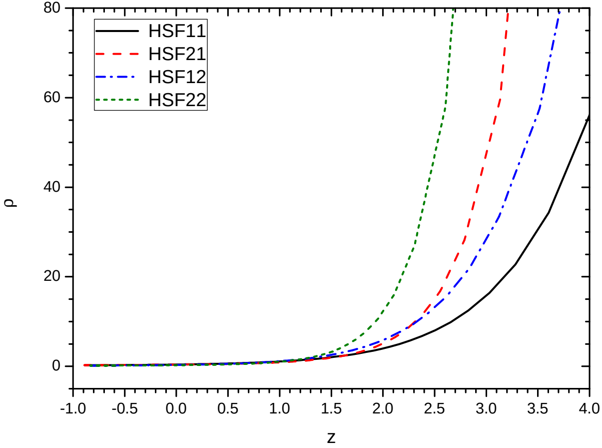

Figure 1: The evolutionary aspect of the energy density for the HSF models. Here we have used the parameter space a = -4.4, b = 0.01.

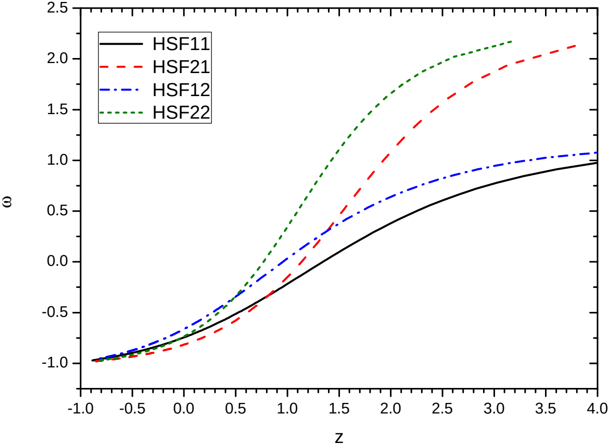

Figure 2: The equation of state parameter in $f(Q, T)$ gravity. The parameter space used for the figure is a = -4.4, b = 0.01.

B. Case-II

Here we will take the functional, $f(Q,T) = aQ^{n+1} + bT$

$$p = \frac{a\left(6H^2\right)^n \left[(12bn + 6b - 12n(3b+16\pi) - 6(3b+16\pi))H^2 - 4(3b+16\pi)(n+1)\dot{H} - \frac{2(3b+16\pi)n(n+1)}{3H}\right]}{b^2 - (3b+16\pi)^2} \tag{18}$$

$$\rho = \frac{a\left(6H^2\right)^n \left[(12n(3b+16\pi) + 6(3b+16\pi) - 12bn - 6b)H^2 - 4b(n+1)\dot{H} - \frac{2bn(n+1)}{3H}\right]}{b^2 - (3b+16\pi)^2} \tag{19}$$

$$\omega = \frac{\left[(12bn + 6b - 12n(3b+16\pi) - 6(3b+16\pi))H^2 - 4(3b+16\pi)(n+1)\dot{H} - \frac{2(3b+16\pi)n(n+1)}{3H}\right]}{\left[(12n(3b+16\pi) + 6(3b+16\pi) - 12bn - 6b)H^2 - 4b(n+1)\dot{H} - \frac{2bn(n+1)}{3H}\right]} \tag{20}$$

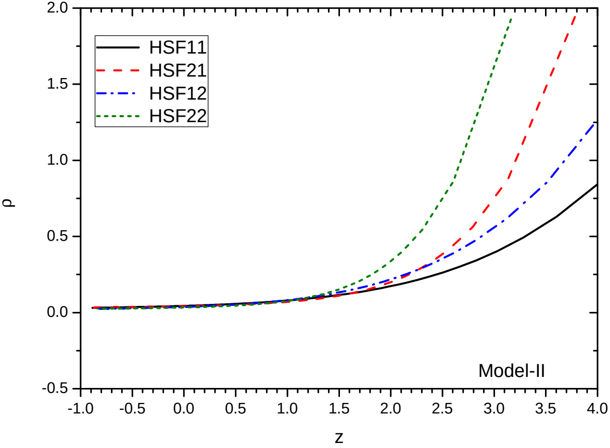

Figure 3: The evolutionary aspect of the energy density for the HSF models. Here we have used the parameter space a = -4.4, b = 0.01, n = -0.4.

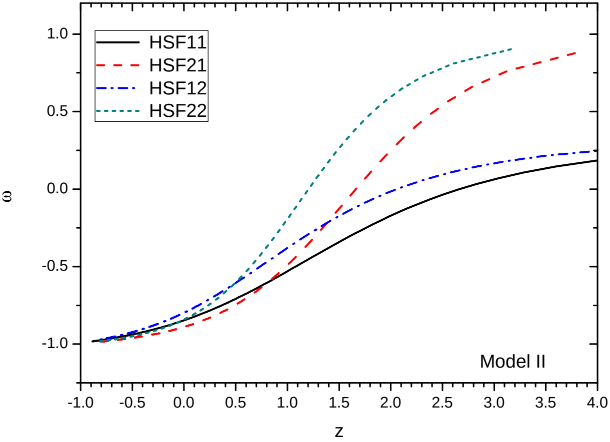

Figure 4: The equation of state parameter in $f(Q, T)$ gravity. The parameter space used for the figure is a = -4.4, b = 0.01, n = -0.4.

Results and Discussion

• The behaviour of energy density and EoS parameter mostly depends on the value of the model parameters a and b, and that of scale factor parameters α and β. It is worthy to mention here that the late time cosmic acceleration behaviour can be assessed in dark energy [a] and $f(R, T)$ gravity [b] cosmological models by constraining the parameter of the scale factor, $\alpha > 0$ and $0 < \beta < \frac{1}{3}$.

[a] B.Mishra, S.K Tripathy, *Mod. Phys. Lett. A*, **30**, 1550175 (2015).
[b] B.Mishra, S.K Tripathy, S. Tarai, *Mod. Phys. Lett. A*, **33**, 180052 (2018).

• This is essential in getting accelerating cosmological models without invoking any dark energy components to the matter field.

• It is found from our investigations that, the model parameter a does not affect the evolutionary aspect of the EoS parameter. However, the parameter b affects marginally the behaviour of ω.

•Another parameter n that appears in the second $f(Q, T)$ model has a substantial effect on the EoS parameter in the sense that, with an increase in n, the value of ω at the present epoch increases.

• Besides the above, we also have used some recent constructed hybrid scale factors to investigate the cosmic dynamics.

• The HSF models provide a signature flipping behaviour of the deceleration parameter with early deceleration and late time acceleration.

•HSF models coincide with the concordant ΛCDM value $\omega =$ -1 at late times of cosmic evolution which is inconformity with recent observations.

• The HSF models predict reasonable values of the deceleration parameter at the present epoch as well as the values of transit redshift.

• The model parameters are chosen suitably so as to provide a physically acceptable energy density.

•In view of this, these models may be useful in the context of the search for geometrical alternatives to dark energy.

In Collaboration With

Prof. Bivudutta Mishra
Birla Institute of Technology and Science-Pilani, Hyderabad Campus, India

S.K Tripathy
Indira Gandhi Institute of Technology, Sarang, Dhenkanal, India


Thank you