Primordial black holes in an early matter era and stochastic inflation

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This talk is based on [1912.01638], [2001.08220] and [2006.14597] [1,2,3]. Full list of references at the end!

Overview and Introduction

- Primordial Black Holes are the oldest dark matter candidates. They are interesting because they do not require physics beyond inflation.

- A large window of masses remains viable (much smaller than LIGO observations, assuming peaked distribution) [4,5,6,8]

$$10^{-16}\,M_{\odot} \lesssim M_{\mathrm{PBH}} \lesssim 10^{-11} M_{\odot}.$$

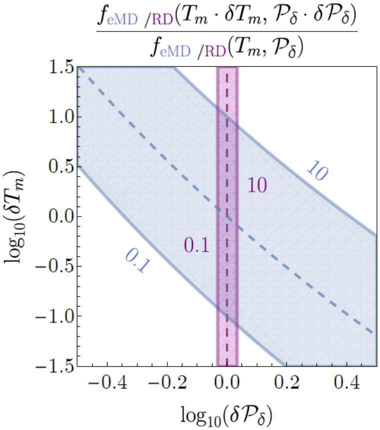
- Their astrophysical signatures (gravitational waves, lensing, etc.) could be probed within the next decade [7].

- We wish to determine the effects on the PBH abundance of
  1. The equation of state of the Universe at the time of their formation.
  2. The backreaction of quantum fluctuations on the classical trajectory.

- We explore these aspects in the context of a numerical inflationary model, and an analytical one.

PBHs from Inflation

PBHs are black holes formed in the early universe by mechanisms different to stellar collapse. For PBHs to form, we need large density fluctuations $\delta = \delta\rho/\rho$, produced during inflation [9].



$$\frac{f_{\mathrm{eMD}}\ /_{\mathrm{RD}}(T_m \cdot \delta T_m, \mathcal{P}_\delta \cdot \delta \mathcal{P}_\delta)}{f_{\mathrm{eMD}}\ /_{\mathrm{RD}}(T_m, \mathcal{P}_\delta)}$$

One can show that (T_m is the transition temperature between eMD and RD, $\mathcal{P}_\delta(k)$ encodes how fluctuations are distributed)

$$f_{\mathrm{PBH}}(T_m, \mathcal{P}_\delta) = \Omega_{\mathrm{PBH}}/\Omega_{\mathrm{DM}} \propto \beta_{\mathrm{MD/RD}},$$

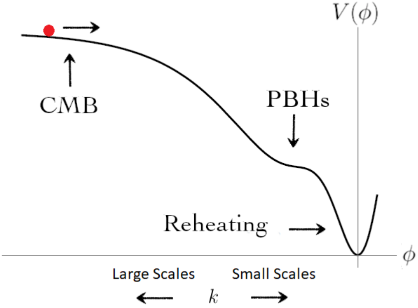
with [14,15]

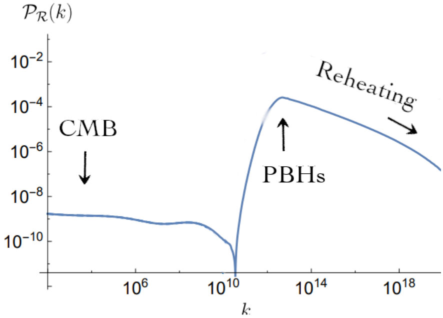
$$\beta_{\mathrm{RD}}(k) \propto \frac{1}{\sqrt{\mathcal{P}_\delta}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\mathcal{P}_\delta}\right) d\delta,$$

$$\beta_{\mathrm{MD}}(k) \propto \mathcal{P}_\delta \exp\left[-\alpha \left(\frac{\mathcal{I}^4}{\mathcal{P}_\delta}\right)^{1/3}\right].$$

The latter takes into account non-sphericity and angular momentum.

The power spectrum $\mathcal{P}_\delta(\boldsymbol{k})$ can be computed, and is already measured. The PBH masses are $M_{\mathrm{RD}} \propto \boldsymbol{k}^{-2}$ and $M_{\mathrm{MD}} \propto \boldsymbol{k}^{-3}$ and the power spectrum is $\mathcal{P}_\delta(\boldsymbol{k}) \sim H^4/\dot{\phi}^2$ (slow-roll). Figures from [3,10].





Collapse during matter-domination has two big advantages,

1. The abundance is **much** less sensitive to small changes in $\mathcal{P}_\delta$.
2. The power spectrum required to get a significant PBH abundance is much smaller than in RD ($\mathcal{P}_{\mathrm{RD}} \sim 10^{-2}$ vs $\mathcal{P}_{\mathrm{MD}} \sim 10^{-4}$).

The Simplest Model

Consider a scalar field coupled to gravity in the Jordan frame [10]

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(M_p^2+\xi\phi^2)R + \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\right].$$

We can redefine the fields as $\Omega^2 \equiv 1+\xi\phi^2/M_p^2$ and $g_{\mu\nu} \to \Omega^2[\phi]\, g_{\mu\nu}$,

$$\Omega^2\frac{dh}{d\phi} = \left[\Omega^2 + \frac{3}{2}M_p^2\left(\frac{d\Omega^2}{d\phi}\right)^2\right]^{1/2},$$

where h is obtained by solving this equation and is such that the kinetic term is canonically normalized,

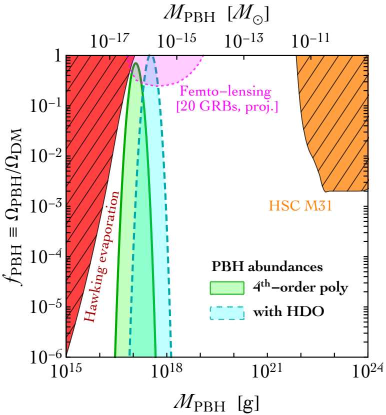
$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_p^2R + \frac{1}{2}g_{\mu\nu}\partial^\mu h\partial^\nu h - V(\phi(h))/\Omega^4\right].$$

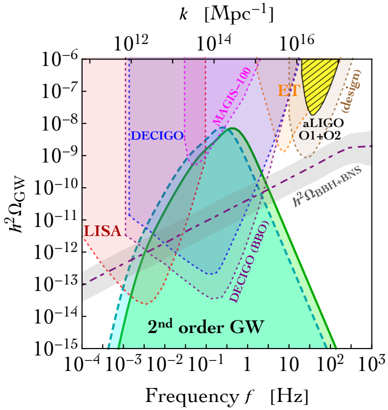
Arguably the simplest potential is a polynomial ($\phi(h)$ is monotonic)

$$U(h) \equiv \frac{V}{\Omega^4} = \left.\frac{a_2\phi^2+a_3\phi^3+a_4\phi^4}{(1+\xi\phi^2/M_p^2)^2}\right|_{\phi=\phi(h)}$$

The main issue is adjusting the spectral index, which is in tension with evaporation bounds, $n_s^{\rm pred} \simeq 0.949$ but $n_s^{\Lambda\rm CDM} = 0.9649 \pm 0.0042$.

1. Extend ΛCDM, since $n_s^{\Lambda\mathrm{CDM}+N_{\mathit{eff}}+dn_s/d\log(k)} = 0.950 \pm 0.011$ [11]
2. Add higher-dimensional operators $c_n \phi^n / \Lambda^{n-4}$ (expected anyway)





The Stochastic Formalism

What is Stochastic Inflation?

In stochastic inflation, quantum fluctuations backreact on the classical trajectory of the inflaton, modifying its background evolution [12,13,16],

$$\frac{d\bar{\phi}}{dN} = -\frac{\partial_\phi V}{3H^2} + \frac{H}{2\pi}\xi_\phi \quad \rightarrow \quad \mathcal{P}_\delta \ll 1 \ \text{(slow roll)}$$

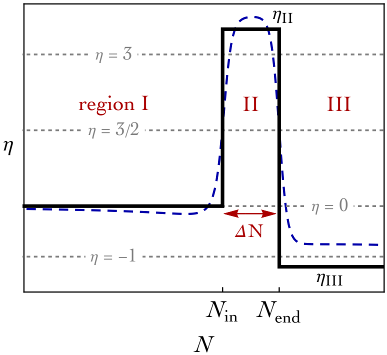
The fields are split into a coarse-grained part and a perturbation by introducing a cutoff σ. The fields satisfy the Langevin equations,

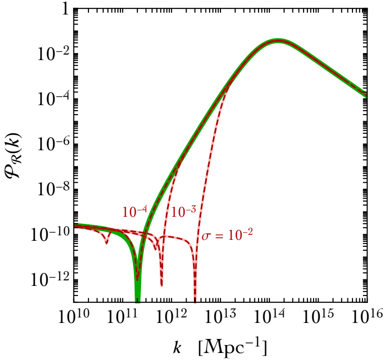
$$\frac{d\bar{\phi}}{dt} = \frac{\bar{\pi}}{a^3} + \xi_\phi, \qquad \text{and} \qquad \frac{d\bar{\pi}}{dt} = -a^3\frac{dV}{d\phi}\bigg|_{\bar{\phi}} + \xi_\pi,$$

where ξ_i are noise operators. These are classical stochastic variables, since $[\xi_\phi(t,\boldsymbol{x}'),\xi_\pi(t,\boldsymbol{x})] \rightarrow 0$ on small scales. With an analytical approach we can find explicit expressions for the noise.

Analytical Model

The enhancement of the power spectrum in any inflection point potential can be understood by considering a three-region model, with $\eta \sim \ddot{\phi}/(H\dot{\phi})$,



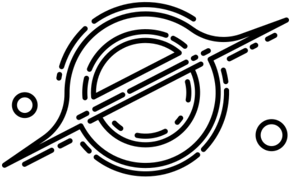


The power spectrum is, in terms of classical stochastic variables $\delta\phi_{\rm st}$, $\delta\pi_{\rm st}$

$$\mathcal{P}_\mathcal{R} = \frac{1}{2\epsilon_{\rm cl}}\Big[D_{\phi\phi} + \underbrace{2\langle\delta\phi_{\rm st}\delta\pi_{\rm st}\rangle - 2(\epsilon_{\rm cl}-\eta_{\rm cl})\langle\delta\phi_{\rm st}^2\rangle}_{0}\Big].$$

Conclusions

- The simplest potential that can produce PBHs is viable, provided ΛCDM is extended, or higher-dimensional operators are considered.
- If dark matter is in the form of PBHs, the corresponding GW signal should be observable by LISA and DECIGO if they form during RD.
- We have shown that, at leading order, stochastic inflation does not affect the power spectrum, even in the presence of a USR phase (the result might change with a full calculation [16]).
- PBH formation in an early matter-dominated era has significant advantages, namely, that a smaller enhancement of the power spectrum is required, and the potential parameters are less tuned.



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