THE UNIVERSITY of EDINBURGH

Cosmology from Home 2021

Universal signature of primordial entanglement

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Why open QFTs in cosmology?

- Wilsonian EFTs describe physics below a particular energy scale by integrating out high-energy dof.

- For inflationary perturbations, the sub-Hubble modes would be integrated out.

Caution: In Wilsonian case, energy conservation ensures that high-energy dof are not part of the system. For inflation, such modes are not excluded by any conservation law. *(Burgess et al., 1408.5002, 1512.00169; Shandera et al., 1708.00493; …)*

Inflation is an example of the interplay between microscopic and macroscopic scales.

- Thus, one cannot use standard EFTs to describe the evolution of inflationary perturbations. We need the technology of **open EFTs**.

- In principle, open QFT techniques are also required to study the **decoherence** of cosmological perturbations. *(See e.g. Nelson, 1601.03734; Gong & Seo, 1903.12295)*

- Open QFT techniques allow to describe the **non-unitary** evolution of a **pure state** into a **mixed state**, capturing the effects of entanglement into a *reduced density matrix.*

- Notice that physics at different scales are able to influence each other through quantum entanglement.

- Entanglement is a non-local effect, so whatever effect it has on observable quantities, it is a manifestation of the quantum nature of primordial fluctuations.

Plan of action

- We will compute the effects of entanglement on observable magnitudes like the power spectrum.

- Similar programs have been performed for several inflationary scenarios, typically with multiple fields. *(Boyanovsky, 1506.07395, 1804.07967)*

- Here, we will consider single-field inflation with an interaction provided by gravitational nonlinearities arising from the Einstein-Hilbert action. In this sense, we present a **universal lower bound** on the observable consequences of primordial quantum entanglement.

- No UV-complete theory of gravity is assumed, although we consider GR to be applicable during inflation.

Free theory

$$\mathcal{S}^{(2)} = \int d^4x \; a^3\epsilon \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial\zeta)^2\right]$$

Mukhanov-Sasaki variable: $\chi = z(\tau)\zeta$ $\qquad z^2 = 2\epsilon a^2 M_{\mathrm{P}}^2$

$$\mathcal{S}^{(2)} = \int d\tau d^3x \; \left[(\chi')^2 - (\partial\chi)^2 + \frac{z''}{z}\chi^2\right]$$

$$\chi_k'' + \left(k^2 - \frac{2}{\tau^2}\right)\chi_k = 0 \qquad \chi_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

$$\hat{H}^{(2)} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left(\underbrace{k\left[\hat{c}_{\mathbf{k}}\hat{c}^{\dagger}_{\mathbf{k}} + \hat{c}_{-\mathbf{k}}\hat{c}^{\dagger}_{-\mathbf{k}}\right]}_{\text{Massless scalar field in flat space}} \underbrace{-i\frac{z'}{z}\left[\hat{c}_{\mathbf{k}}\hat{c}_{-\mathbf{k}} - \hat{c}^{\dagger}_{\mathbf{k}}\hat{c}^{\dagger}_{-\mathbf{k}}\right]}_{\text{Squeezing interaction}}\right)$$

$$|n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle = \left[\frac{1}{n!}\left(\hat{c}^{\dagger}_{\mathbf{k}}\hat{c}^{\dagger}_{-\mathbf{k}}\right)^n\right]|0_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle$$

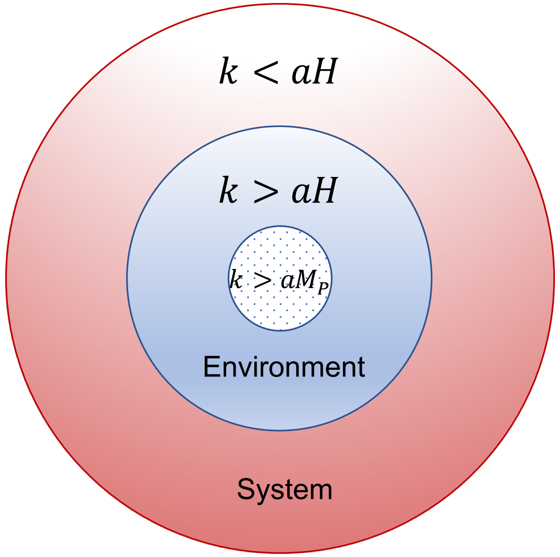
$$|SQ(k,\tau)\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{-2in\phi_k}\tanh^n r_k\,|n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

System & environment dof

Mode functions:

$$\chi_k^{\mathcal{E}}(\tau) = \chi_k(\tau)\theta(k - aH)$$

$$\chi_k^{\mathcal{S}}(\tau) = \chi_k(\tau)\theta(aH - k)$$



Wavefunction: $$|\Psi(\tau)\rangle = \boxed{|SQ(\tau)\rangle_{k<aH}} \otimes \boxed{|0\rangle_{k>aH}}$$

Squeezed state

Bunch-Davies state

Gravitational non-linearities

$$\mathcal{L}^{(3)} = \int d^3x \; a\epsilon^2\zeta(\partial\zeta)^2$$

$$\hat{H}_{\rm int} = -\int d^3x \; \mathcal{L}^{(3)} = \lambda(\tau)\int d^3x \; \hat{\chi}(\mathbf{x})(\partial\hat{\chi}(\mathbf{x}))^2$$

$$\lambda \equiv -a^2\epsilon^2/z^3$$

In the interaction picture:

$$\hat{\mathcal{O}}_I = \hat{U}_0^\dagger\hat{\mathcal{O}}\hat{U}_0$$

$$\hat{H}_{\rm I}(\tau) = \lambda(\tau)\int d^3x \, \hat{\chi}(\tau,\mathbf{x})(\partial\hat{\chi}(\tau,\mathbf{x}))^{\mathbf{2}}$$

Now, it's time for some approximations…

- Consider 2 environment modes and 1 system mode.

$$
\begin{aligned}
\hat{H}_I(\tau) &\simeq \lambda(\tau) \int d^3x \; \hat{\chi}^{\mathcal{S}}(\tau, \mathbf{x}) \left(\partial \hat{\chi}^{\mathcal{E}}(\tau, \mathbf{x})\right)^2 \\
&= -\lambda(\tau) \int_{\Delta_k} (\mathbf{k_2} \cdot \mathbf{k_3}) \hat{\chi}^{\mathcal{S}}_{\mathbf{k_1}}(\tau) \hat{\chi}^{\mathcal{E}}_{\mathbf{k_2}}(\tau) \hat{\chi}^{\mathcal{E}}_{\mathbf{k_3}}(\tau)
\end{aligned}
$$

$$
\int_{\Delta_k} \equiv \int d^3k_1 \int d^3k_2 \int d^3k_3 \; (2\pi)^{-6} \delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})
$$

Entanglement entropy may lead to interesting consequences for inflation, not only in terms of observables. *(Brahma, Alaryani and Brandenberger, 2005.09688)*

$$s_{\rm ent} \lesssim \ln(\lambda^2)\epsilon_H H^2 M_{\rm Pl} a^2 \qquad s_{\rm th} = \frac{4\pi^2}{45} g_* T_R^3 \simeq \frac{4\pi^2}{45} g_* H^{3/2} M_{\rm Pl}^{3/2}$$

$$s_{\rm th} > s_{\rm ent}$$

Second law of thermodynamics

$$N < \frac{1}{4}\ln\left(\frac{M_{\rm Pl}}{H}\right) + \frac{1}{2}\ln\epsilon_H^{-1} \qquad N < \frac{5}{4}\ln\left(\frac{M_{\rm Pl}}{H}\right) - \frac{9}{2}\ln 10$$

$\sim$

TCC!

$$N < \ln\left(\frac{M_{\rm Pl}}{H}\right)$$

- Initial factorization

$$|\Psi(\tau_0)\rangle = |\mathcal{S}_0\rangle \otimes |\mathcal{E}_0\rangle$$

$$\rho_I(\tau_0) = \rho_{\mathcal{S}}(\tau_0) \otimes \rho_{\mathcal{E}}(\tau_0)$$

- Weak coupling and Born approximation

$$\rho_{\mathcal{E}}(\tau) \approx \rho_{\mathcal{E}}(\tau_0)$$

$$\rho_I(\tau) = \rho_{\mathcal{S}}(\tau) \otimes \rho_{\mathcal{E}}(\tau_0)$$

Setting up the master equation

$$\frac{d\rho_I}{d\tau} = -i\left[\hat{H}_I(\tau), \rho_I(\tau)\right]$$

Von Neumann equation

$$= -i[\hat{H}_I(\tau), \rho_I(\tau_0)] - \int_{\tau_0}^{\tau} d\tau' \Big\{\hat{H}_I(\tau)\hat{H}_I(\tau')\rho_I(\tau') - \hat{H}_I(\tau)\rho_I(\tau')\hat{H}_I(\tau')$$
$$-\hat{H}_I(\tau')\rho_I(\tau')\hat{H}_I(\tau) + \rho_I(\tau')\hat{H}_I(\tau')\hat{H}_I(\tau)\Big\}$$

$$\rho_r(\tau) = \mathrm{Tr}_{\mathcal{E}}\left[\rho_I(\tau)\right]$$

Reduced density matrix

Master equation

$$\rho_r'(\tau) = \int \frac{d^3p}{(2\pi)^3}\ \lambda(\tau) \int_{\tau_0}^{\tau} d\tau'\ \lambda(\tau') \left\{ \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau)\hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau')\rho_r(\tau')K_p(\tau,\tau') - \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau)\rho_r(\tau')\hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau')K_p^*(\tau,\tau') \right.$$
$$\left. -\ \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau')\rho_r(\tau')\hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau)K_p(\tau,\tau') + \rho_r(\tau')\hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau')\hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau)K_p^*(\tau,\tau') \right\}$$

$$K_{p_1}(\tau,\tau') = -2\int \frac{d^3p_2}{(2\pi)^3}\,(\mathbf{p_2}\cdot\mathbf{p_3})^2\,\chi^{\varepsilon}_{p_2}(\tau)\chi^{\varepsilon}_{p_2}(\tau')^*\chi^{\varepsilon}_{p_3}(\tau)\chi^{\varepsilon}_{p_3}(\tau')^*$$

Kernel

$$\mathbf{p_3} = -(\mathbf{p_1}+\mathbf{p_2})$$

$$K_{p_1}(\tau,\tau') \approx -\frac{e^{2i(\tau-\tau')/\tau}\left[1-e^{-ip_1(\tau-\tau')}\right]\left[\tau-(1-i)\tau'\right]^2}{8\pi^2 p_1\tau^4(\tau')^2(\tau-\tau')^2}$$

Perturbative solution

$$
\begin{aligned}
\rho_r(\tau) \approx &\rho_r(\tau_0) + \sum_{\mathbf{p}} \int_{\tau_0}^{\tau} d\tau' \lambda(\tau') \int_{\tau_0}^{\tau'} d\tau'' \lambda(\tau'') \Big\{ \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau') \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau'') \rho_r(\tau_0) K_p(\tau', \tau'') \\
&- \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau') \rho_r(\tau_0) \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau'') K_p^*(\tau', \tau'') - \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau'') \rho_r(\tau_0) \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau') K_p(\tau', \tau'') \\
&+ \rho_r(\tau_0) \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau'') \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau') K_p^*(\tau', \tau'') \Big\}
\end{aligned}
$$

$$
\left\langle \hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{q}}(\tau) \right\rangle = \mathrm{Tr}\left[\hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{q}}(\tau) \rho_r(\tau) \right]
$$

A sanity check…

$$\Delta_\zeta^2(q) = \frac{q^3}{2\pi^2 z^2} \left\langle \hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{q}}(\tau) \right\rangle$$

$$= \frac{q^3}{2\pi^2 z^2} \mathrm{Tr}\left[\hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{q}}(\tau) \rho_r(\tau_0) \right] \approx \frac{1}{2\epsilon M_{\mathrm{Pl}}^2} \left(\frac{H}{2\pi} \right)^2$$

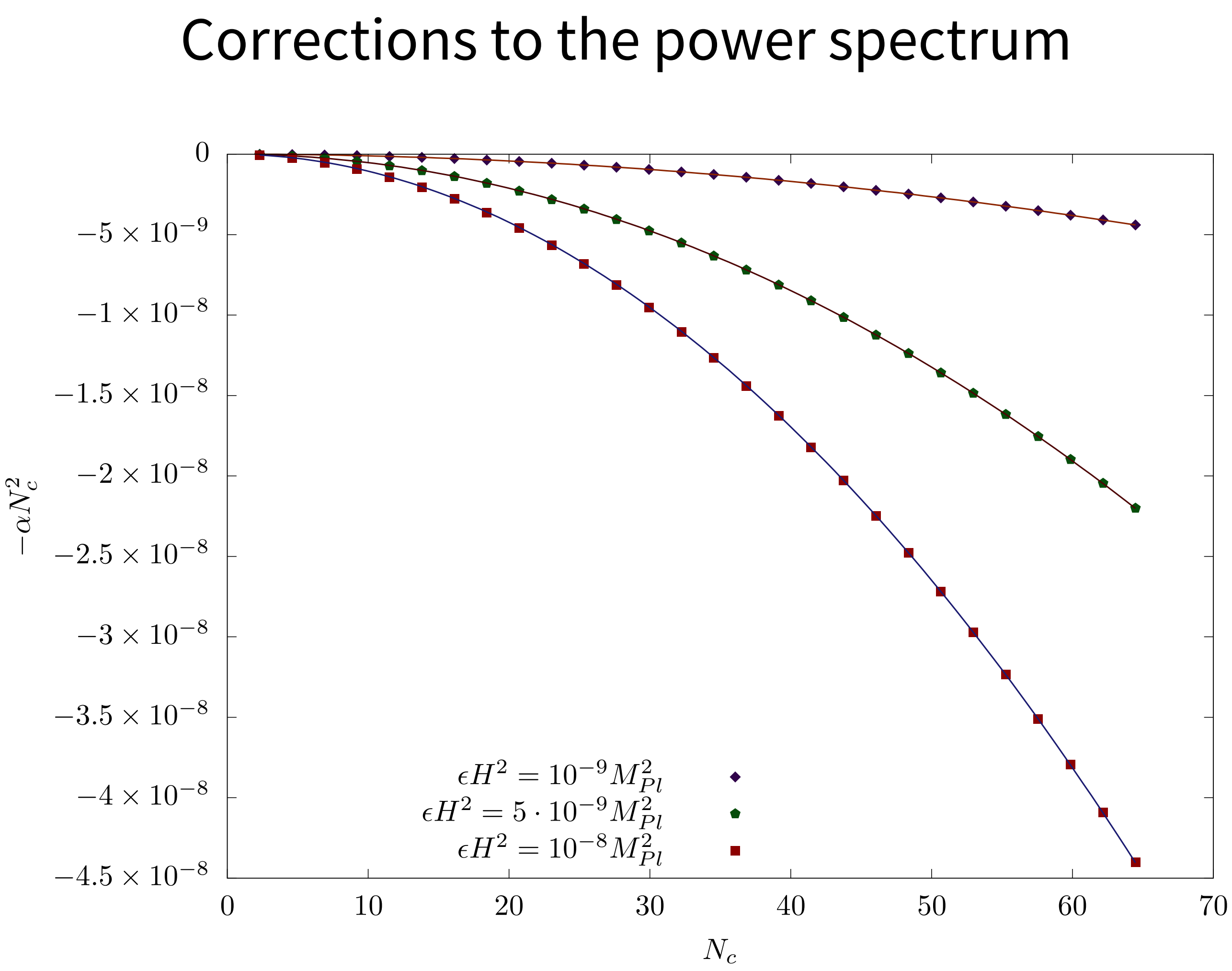
$$|0\rangle\langle 0|$$

And now including entanglement effects:

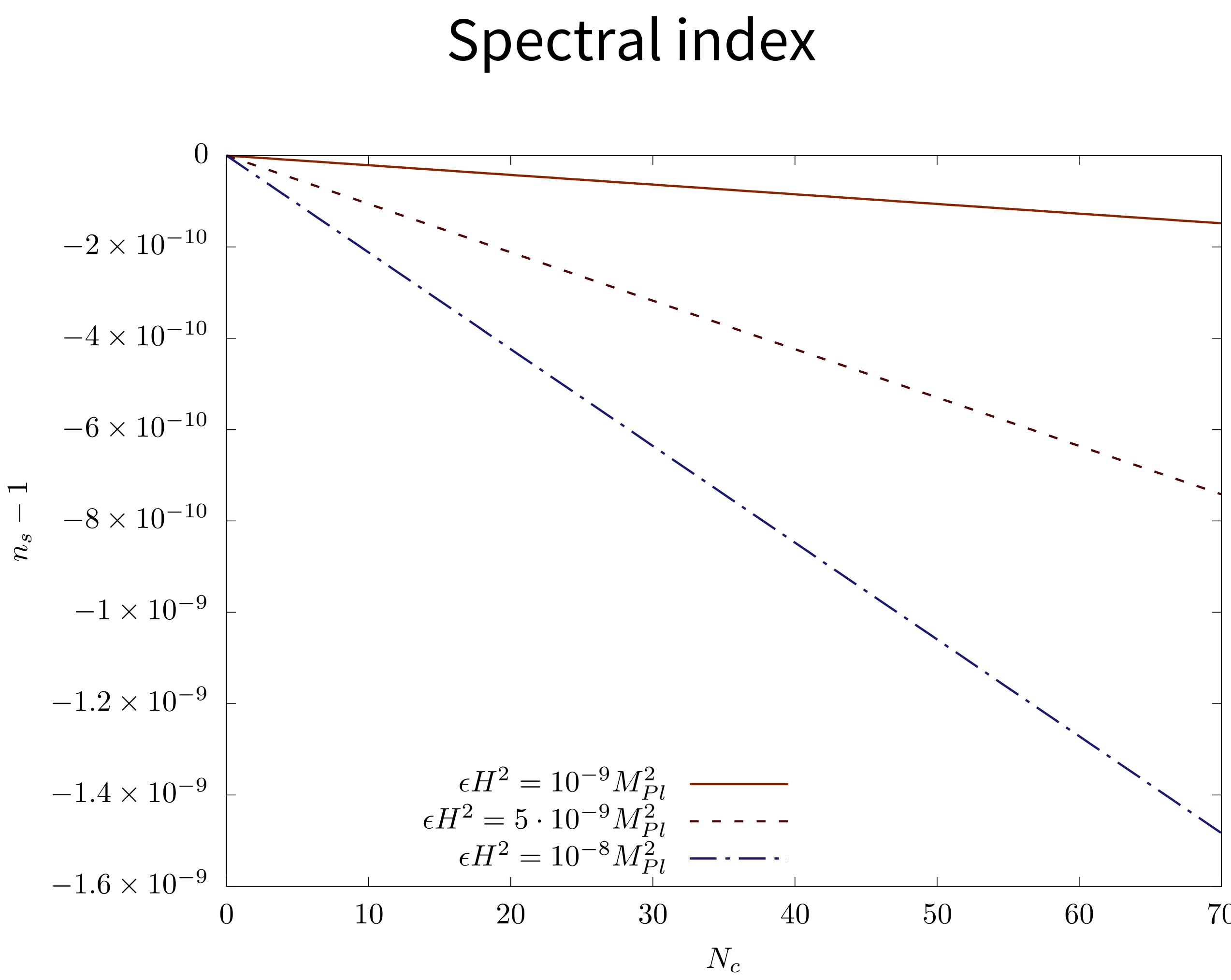
$$\left\langle \hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau)\hat{\chi}^{\mathcal{S}}_{-\mathbf{q}}(\tau)\right\rangle = \frac{1}{2q}\left(1+\frac{1}{(q\tau)^2}\right) + 2\int_{-1/q}^{\tau} d\tau'\lambda(\tau')\int_{-1/q}^{\tau'} d\tau''\lambda(\tau'') \left\{K_q(\tau',\tau'')\left[(\chi_q(\tau))^2\chi_q^*(\tau')\chi_q^*(\tau'') - |\chi_q(\tau)|^2\chi_q(\tau')\chi_q^*(\tau'')\right] + \text{c.c.}\right\}$$

$$\boxed{\Delta_\zeta^2 = \frac{1}{2\epsilon M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2(1-\alpha N_c^2)}$$

$$\alpha \approx 0.00211886\frac{\epsilon H^2}{2M_{\rm Pl}^2} \qquad N_c \equiv -\ln(-q\tau)$$



Corrections to the power spectrum
$-\alpha N_c^2$
0
-5×10^{-9}
-1×10^{-8}
-1.5×10^{-8}
-2×10^{-8}
-2.5×10^{-8}
-3×10^{-8}
-3.5×10^{-8}
-4×10^{-8}
-4.5×10^{-8}
0 10 20 30 40 50 60 70
N_c
$\epsilon H^2 = 10^{-9} M_{Pl}^2$
$\epsilon H^2 = 5\cdot10^{-9} M_{Pl}^2$
$\epsilon H^2 = 10^{-8} M_{Pl}^2$



Spectral index
$n_s - 1$
0
-2×10^{-10}
-4×10^{-10}
-6×10^{-10}
-8×10^{-10}
-1×10^{-9}
-1.2×10^{-9}
-1.4×10^{-9}
-1.6×10^{-9}
0 10 20 30 40 50 60 70
N_c
$\epsilon H^2 = 10^{-9} M_{Pl}^2$
$\epsilon H^2 = 5\cdot10^{-9} M_{Pl}^2$
$\epsilon H^2 = 10^{-8} M_{Pl}^2$

Remarks

- These are the *smallest* effects expected due to entanglement.
- It is fair to assume that inflationary models predicting non-vanishing non-gaussianities would present bigger corrections.
- Other interactions can also lead to an enhanced effect. One can play the same game with every interaction and observable, including those associated to tensor perturbations, where the coupling constant is not suppressed by slow-roll parameters.
- The use of the open QFT approach is crucial to correctly capture the physics of the problem.

Non-perturbative resummation *(Boyanovsky, 1506.07395)*

$$\hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau) = \chi^+_q(\tau)\hat{P}_{\mathbf{q}} - \chi^-_q(\tau)\hat{X}_{\mathbf{q}}$$

$$\chi^+_q(\tau) = \sqrt{\frac{-\pi\tau}{2}}Y_{3/2}(|q\tau|)\,, \qquad \chi^-_q(\tau) = \sqrt{\frac{-\pi\tau}{2}}J_{3/2}(|q\tau|)$$

$$\hat{P}_{\mathbf{q}} = \frac{i}{\sqrt{2}}\left(\hat{c}^\dagger_{-\mathbf{q}} - \hat{c}_{\mathbf{q}}\right)\,, \qquad \hat{X}_{\mathbf{q}} = \frac{1}{\sqrt{2}}\left(\hat{c}_{\mathbf{q}} + \hat{c}^\dagger_{-\mathbf{q}}\right)$$

$$\left\langle \hat{\chi}^{\mathcal{S}}_{\mathbf{q}}(\tau)\hat{\chi}^{\mathcal{S}}_{-\mathbf{q}}(\tau)\right\rangle \approx \left[\chi^+_q(\tau)\right]^2\left\langle \hat{P}_{\mathbf{q}}\hat{P}_{-\mathbf{q}}\right\rangle \approx \frac{1}{2q^2\tau^3}$$

Now, let us take the **Markovian approximation**:

$$\rho_r(\tau') \to \rho_r(\tau)$$

$$\rho_r'(\tau) = \sum_{\mathbf{p}} \lambda(\tau) \int_{\tau_0}^{\tau} d\tau' \; \lambda(\tau') \left\{ \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau') \rho_r(\tau) K_p(\tau, \tau') - \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau) \rho_r(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau') K_p^*(\tau, \tau') \right.$$
$$\left. - \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau') \rho_r(\tau) \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau) K_p(\tau, \tau') + \rho_r(\tau) \hat{\chi}^{\mathcal{S}}_{-\mathbf{p}}(\tau') \hat{\chi}^{\mathcal{S}}_{\mathbf{p}}(\tau) K_p^*(\tau, \tau') \right\}$$

$$\frac{d}{d\tau} \left\langle \hat{P}_{\mathbf{q}} \hat{P}_{-\mathbf{q}} \right\rangle = -\frac{\epsilon H^2}{48\pi^2 M_{\mathrm{Pl}}^2} \frac{\ln(|q\tau|)}{\tau} \left\langle \hat{P}_{\mathbf{q}} \hat{P}_{-\mathbf{q}} \right\rangle$$ (Valid in the long-time limit)

$$\left\langle \hat{P}_{\mathbf{q}} \hat{P}_{-\mathbf{q}} \right\rangle = f(\tau) \left\langle \hat{P}_{\mathbf{q}} \hat{P}_{-\mathbf{q}} \right\rangle (\tau_*)$$

$$f(\tau) = \exp\left[-\frac{\epsilon H^2}{48\pi^2 M_{\rm Pl}^2}\int_{-1/q}^{\tau} d\tau' \frac{\ln(-q\tau)}{\tau}\right] = \exp\left[-\frac{\epsilon H^2}{96\pi^2 M_{\rm Pl}^2}\ln^2(|q\tau|)\right]$$

$$\boxed{\Delta_\zeta^2 = \frac{1}{2\epsilon M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2 e^{-\alpha N_c^2}}$$

$$\alpha = 0.00211086\frac{\epsilon H^2}{2M_{\rm Pl}^2}$$

Previously, we had estimated numerically:

$$\alpha \approx 0.00211886\frac{\epsilon H^2}{2M_{\rm Pl}^2}$$

Concluding remarks

- A complete description of the dynamics of inflationary perturbations requires the technology of open QFTs.
- Even if sub and super-horizon perturbations are initially uncorrelated, the interaction between them, as predicted by GR, evolves the state from a pure one to a mixed one, generating entanglement.
- The quantum entanglement of cosmological perturbations leaves fingerprints on observable quantities like the power spectrum, and can be considered a signal of their quantum nature.
- We found a good agreement in the results obtained using a perturbative approach and the Markovian approximation. However, if the latter is not valid (for some region of parameter space), a resummation may not be possible and the duration of inflation could be constrained under the argument of maintaining perturbative control over the expansion.