

Hubble's law in general space-times

– a numerical relativity study of anisotropic signatures in luminosity distance cosmography

Hayley Macpherson and Asta Heinesen



CENTRE DE RECHERCHE ASTROPHYSIQUE DE LYON

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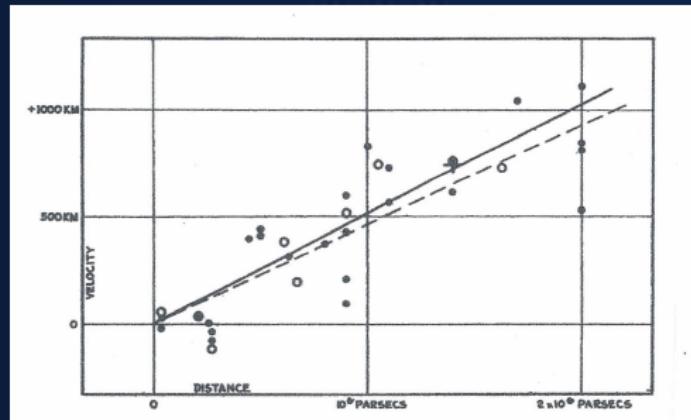
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Introduction

Typically distance–redshift data is analysed isotropically.

Discovery of the Hubble-Lemaître law $d_L \approx z/H_0$: G. Lemaître (1927), V. M. Slipher (1917), E. P. Hubble (1929).

Today: large cosmological data sets. Modern supernovae catalogues consist of 10^3 SN1a. $\rightarrow 10^5$ SN1a within this decade (LSST, WFIRST, ...).



E. P. Hubble (1929): Velocity-Distance Relation among Extra-Galactic Nebulae. Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. ©US National Academy of Sciences.

The luminosity distance Hubble law FLRW geometry

The luminosity distance Hubble law, M. Visser (2004):

$$d_L = \frac{1}{H_0} z + \frac{1-q_0}{2H_0} z^2 + \frac{-1+3q_0^2+q_0-j_0+\Omega_{k0}}{6H_0} z^3 + \mathcal{O}(z^4)$$

$$\begin{aligned} H &\equiv \frac{\dot{a}}{a}, & q &\equiv -\frac{\ddot{a}}{aH^2}, & j &\equiv \frac{\dot{\dot{a}}}{aH^3}, & \Omega_k &\equiv \frac{-k}{a^2 H^2} \\ \cdot &\equiv \frac{d}{dt}, & k &\in \{-1, 0, 1\} \end{aligned}$$

Purely geometrical result. Relies only on the FLRW metric assumption, but not on the field equations governing the scale factor: “FLRW cosmography”

Data from sources in the $\mathcal{O}(z^n)$ vicinity of the observer is described by a *finite* number of parameters

The luminosity distance Hubble law general geometry

A. Heinesen, JCAP05(2021)008 [arXiv:2010.06534]

Isotropic FLRW series expansion → generic anisotropic expansion.

Builds on work by, e.g., J. Kristian and R. K. Sachs (1966), S. Seitz, P. Schneider and J. Ehlers (1994), C. Clarkson and O. Umeh (2011), C. Clarkson, G. F. R. Ellis, A. Faltenbacher, R. Maartens, O. Umeh and J. P. Uzan (2012)

The luminosity distance Hubble law:

$$d_L = \frac{1}{\mathfrak{H}_o} z + \frac{1-\mathfrak{Q}_o}{2\mathfrak{H}_o} z^2 + \frac{-1+3\mathfrak{Q}_o^2+\mathfrak{Q}_o-\mathfrak{J}_o+\mathfrak{R}_o}{6\mathfrak{H}_o} z^3 + \mathcal{O}(z^4)$$

$$H \rightarrow \mathfrak{H}, \quad q \rightarrow \mathfrak{Q}, \quad j \rightarrow \mathfrak{J}, \quad \Omega_k \rightarrow \mathfrak{R}$$

The functions $\mathfrak{H}, \mathfrak{Q}, \mathfrak{J}, \mathfrak{R}$ in general vary with the point of observation, and the direction of sight.

The luminosity distance Hubble law general geometry

A. Heinesen, JCAP05(2021)008 [arXiv:2010.06534]

The luminosity distance Hubble law for a general congruence of observers and emitters in a general space-time:

$$d_L = \frac{1}{\mathfrak{H}_0} z + \frac{1-\mathfrak{Q}_0}{2\mathfrak{H}_0} z^2 + \frac{-1+3\mathfrak{Q}_0^2+\mathfrak{Q}_0-\mathfrak{J}_0+\mathfrak{R}_0}{6\mathfrak{H}_0} z^3 + \mathcal{O}(z^4)$$

$$\mathfrak{H} = -\frac{\frac{dE}{d\lambda}}{E^2}, \quad \mathfrak{Q} = -1 - \frac{1}{E} \frac{\frac{d\mathfrak{H}}{d\lambda}}{\mathfrak{H}^2},$$

$$\mathfrak{R} = 1 + \mathfrak{Q} - \frac{1}{2E^2} \frac{k^\mu k^\nu R_{\mu\nu}}{\mathfrak{H}^2}, \quad \mathfrak{J} = \frac{1}{E^2} \frac{\frac{d^2\mathfrak{H}}{d\lambda^2}}{\mathfrak{H}^3} - 4\mathfrak{Q} - 3$$

E : Photon energy as measured by emitters/observers

$\frac{d}{d\lambda} \equiv k^\mu \nabla_\mu$: Derivative along photon 4-momentum, k^μ

$R_{\mu\nu}$: Ricci curvature of the space-time

The luminosity distance Hubble law general geometry

A. Heinesen, JCAP05(2021)008 [arXiv:2010.06534]

Simple multipole expansions in direction of incoming light, e^μ , seen by the observer

Physically interpretable multipole coefficients

$$H \rightarrow \mathfrak{H} = \frac{1}{3}\theta - e^\mu a_\mu + e^\mu e^\nu \sigma_{\mu\nu}, \quad 1 \text{ dof} \rightarrow 9 \text{ dof}$$

$$9 \text{ dof} \left\{ \begin{array}{l} \theta : \text{expansion of observer congruence} \\ a^\mu : 4\text{-acceleration of observer congruence} \\ \sigma_{\mu\nu} : \text{shear of observer congruence} \end{array} \right.$$

$$q \rightarrow \mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^\mu \overset{1}{\mathfrak{q}}_\mu + e^\mu e^\nu \overset{2}{\mathfrak{q}}_{\mu\nu} + e^\mu e^\nu e^\rho \overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^\mu e^\nu e^\rho e^\kappa \overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\mathbf{e})}, \quad 1 \text{ dof} \rightarrow 16 \text{ independent dof}$$

$$j_0 - \Omega_{k0} \rightarrow \mathfrak{J} - \mathfrak{R} = 1 + \frac{\overset{0}{\mathfrak{t}} + e \cdot \overset{1}{\mathfrak{t}} + ee \cdot \overset{2}{\mathfrak{t}} + eee \cdot \overset{3}{\mathfrak{t}} + eeee \cdot \overset{4}{\mathfrak{t}} + eeeee \cdot \overset{5}{\mathfrak{t}} + eeeeeee \cdot \overset{6}{\mathfrak{t}}}{\mathfrak{H}^3(\mathbf{e})}, \quad 1 \text{ dof} \rightarrow 36 \text{ independent dof}$$

The general luminosity distance Hubble law

– some remarks

- For a *general* cosmological space-time, the luminosity distance series expansion in redshift is given 9, 25, 61 degrees of freedom in the $\mathcal{O}(z)$, $\mathcal{O}(z^2)$, $\mathcal{O}(z^3)$ vicinity of the observer.
- This opens the door for model-independent analysis. Test of the FLRW ansatz, model independent constraints on kinematic variables and space-time curvature.
- No need for correction of data before cosmological fit (peculiar velocity corrections in Λ CDM analysis of supernovae).
- Motivated search for anisotropies.
- Until fully model-independent data analysis is possible: Simplifying constraints to investigate datasets.
- Investigate luminosity distance of artificial observers within realistic numerical simulations.

The general luminosity distance Hubble law

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The general luminosity distance Hubble law

Luminosity distance and anisotropic sky-sampling at low redshifts: a numerical relativity study

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(Dated: March 22, 2021)

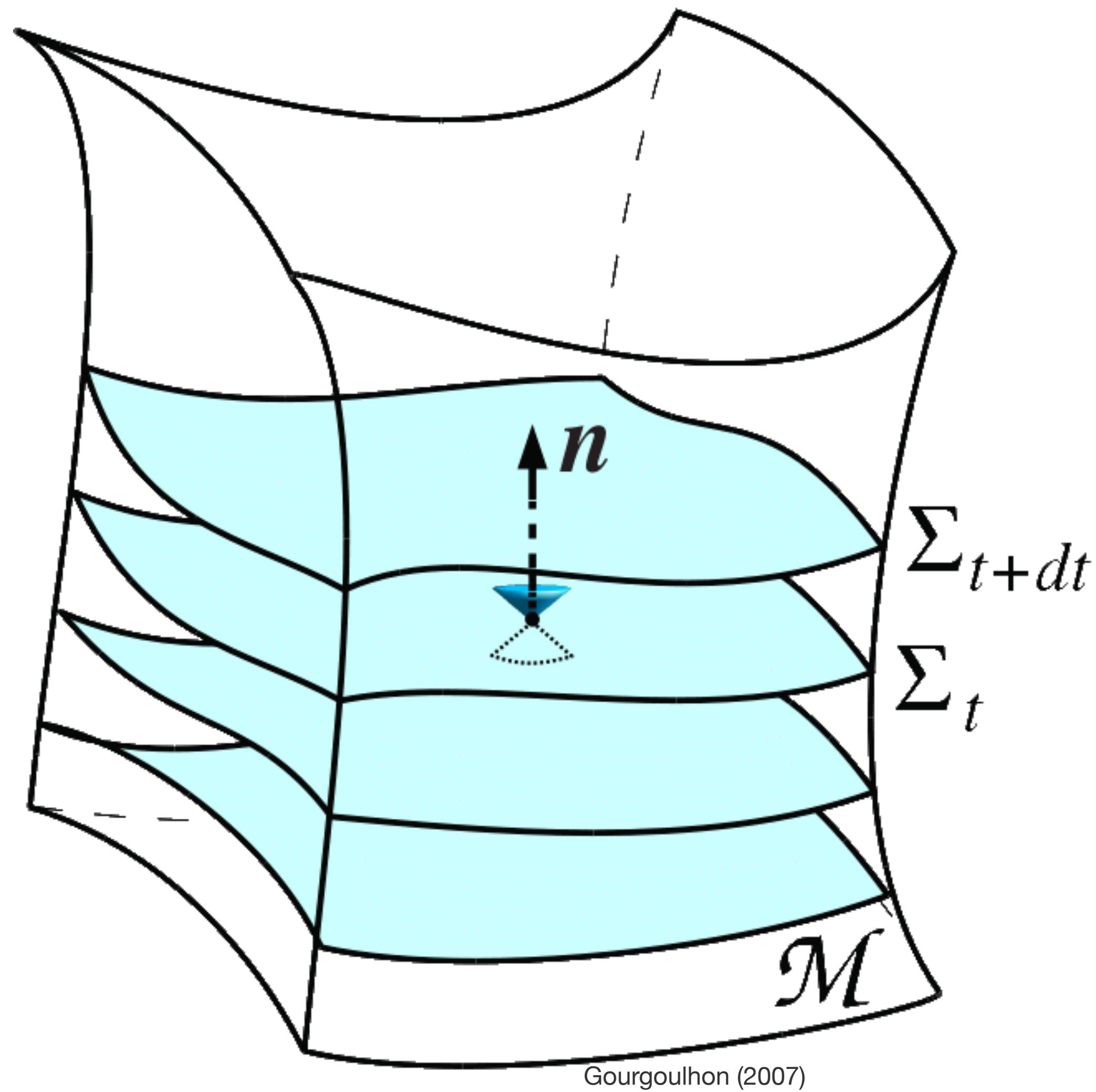
Most cosmological data analysis today relies on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, providing the basis of the current standard cosmological model. Within this framework, interesting tensions between our increasingly precise data and theoretical predictions are coming to light. It is therefore reasonable to explore the potential for cosmological analysis outside of the exact FLRW cosmological framework. In this work we adopt the general luminosity-distance series expansion in redshift with *no assumptions* of homogeneity or isotropy. This framework will allow for a full model-independent analysis of near-future low-redshift cosmological surveys. We calculate the effective observational ‘Hubble’, ‘deceleration’, ‘curvature’ and ‘jerk’ parameters of the luminosity-distance series expansion in numerical relativity simulations of realistic structure formation, for observers located in different environments and with different levels of sky-coverage. With a ‘fairly-sampled’ sky, we find 2% and 15% cosmic variance in the ‘Hubble’ and ‘deceleration’ parameters, respectively, compared to their analogies in the FLRW model. On top of this, we find that typical observers measure maximal sky-variance of 7% and 550% in the same parameters. Our work suggests the inclusion of low-redshift anisotropy in cosmological analysis could be important for drawing correct conclusions about our Universe.

arXiv:2103.11918

We used numerical relativity simulations

since the formalism contains no assumptions on the metric tensor, we wanted our simulations to reflect this generality as close as possible

- Investigate luminosity distance of artificial observers within realistic numerical simulations.



Gourgoulhon (2007)

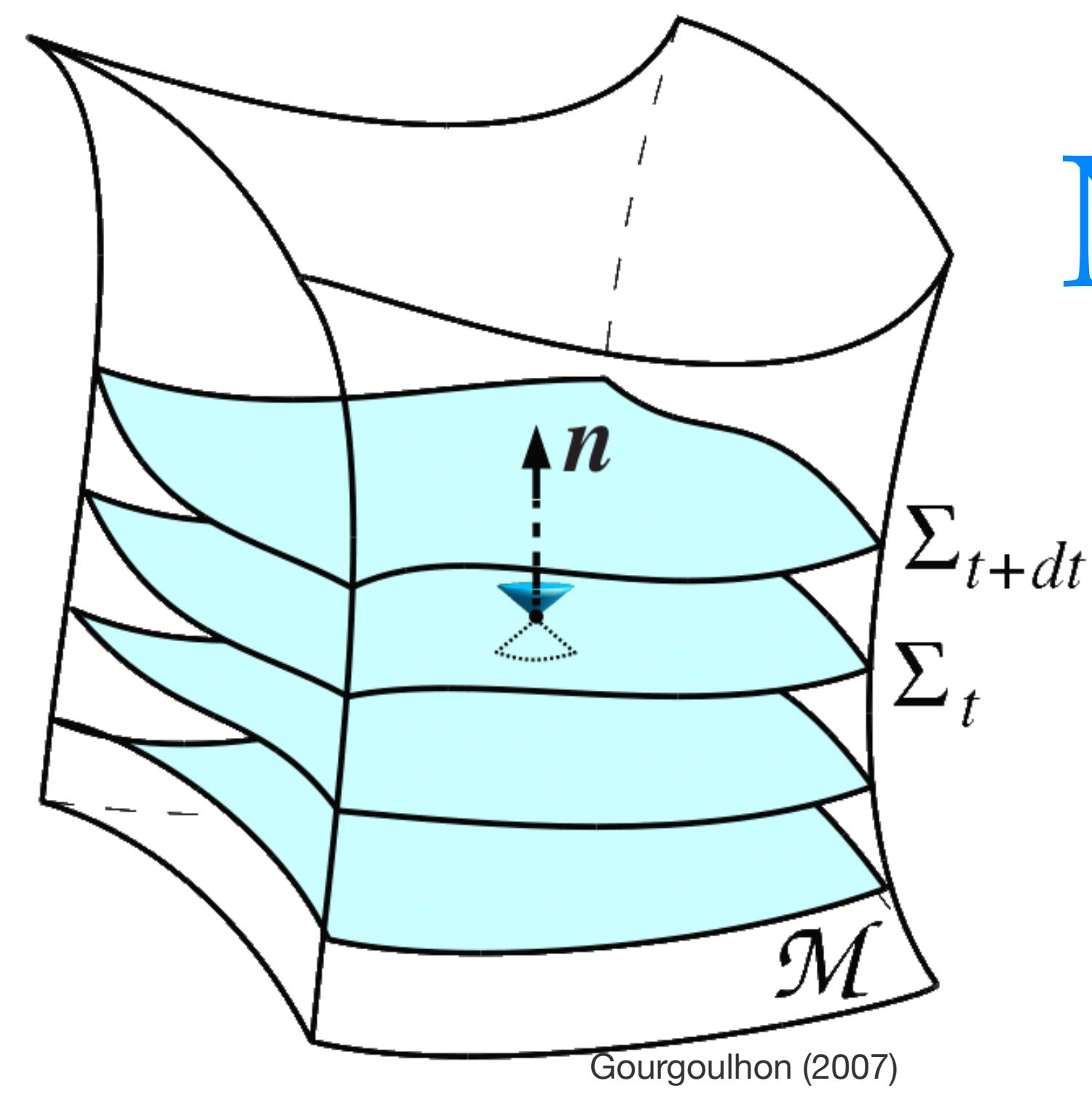
Numerical relativity

Allows us to solve the field equations with no simplifying assumptions to gravity (i.e., not weak) or geometry (i.e., not flat locally or globally)

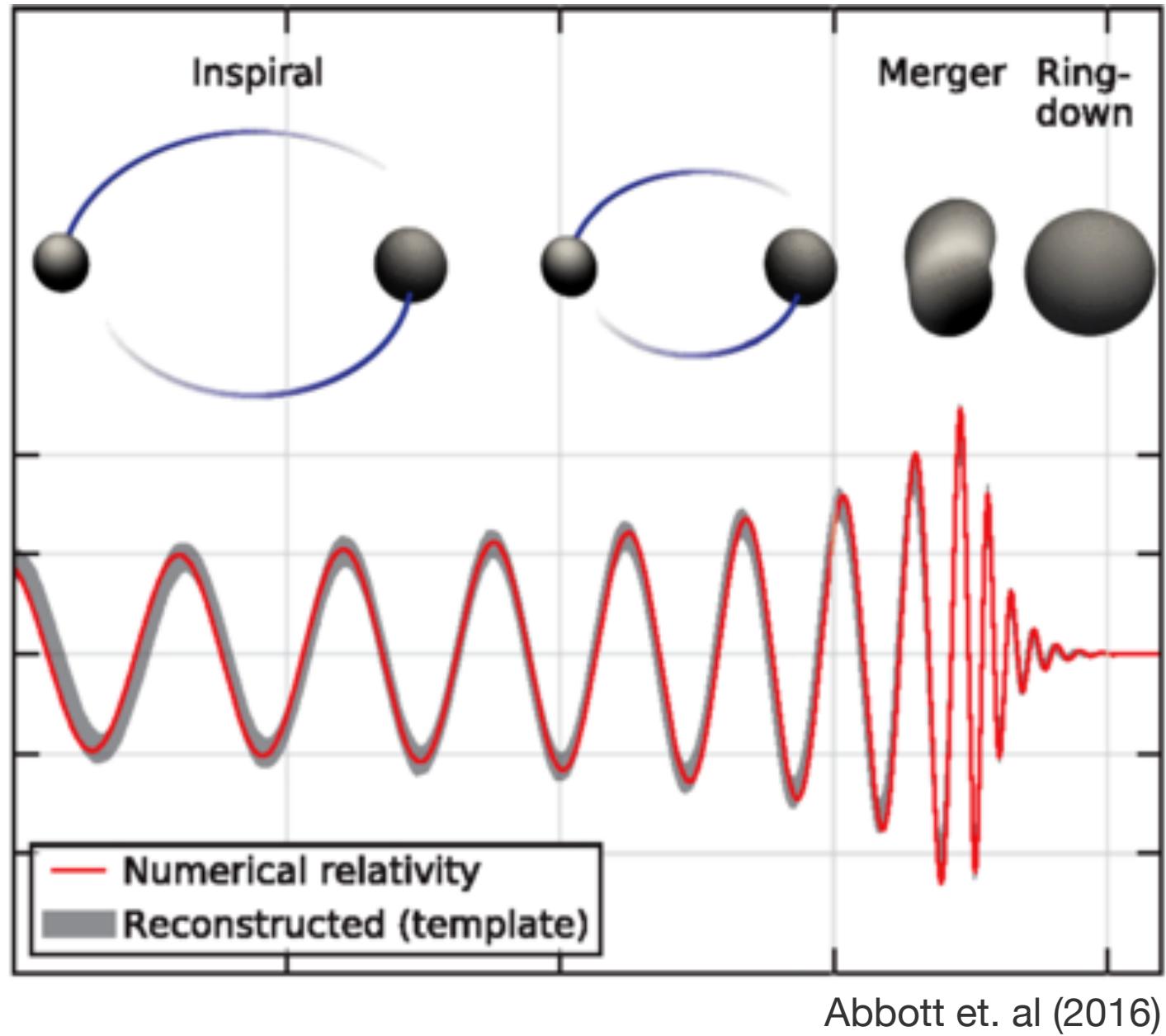
*Based on a 3+1 split of
spacetime \rightarrow space + time*

*This means we can write Einstein's equations as a set of evolution equations for these spatial slices
And evolve them in all generality!*

Numerical relativity



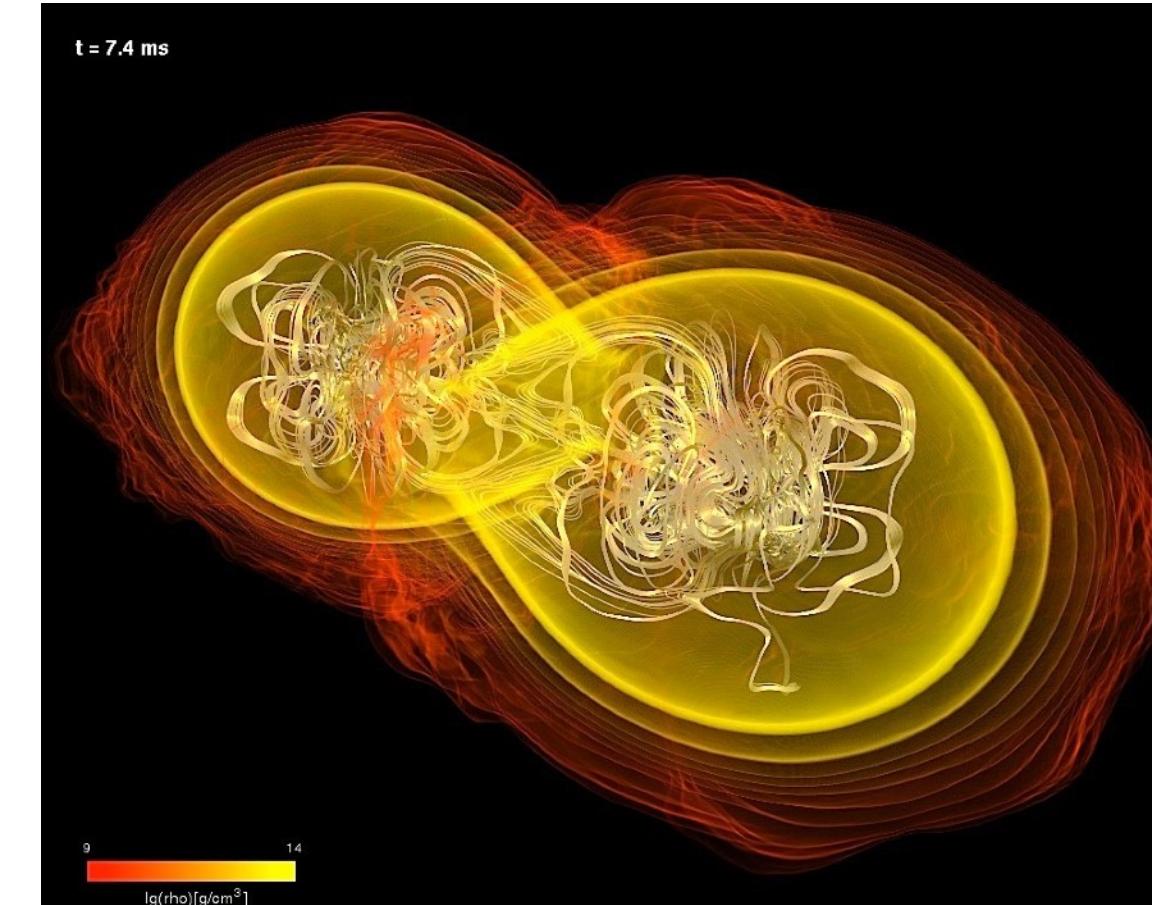
Gourgoulhon (2007)



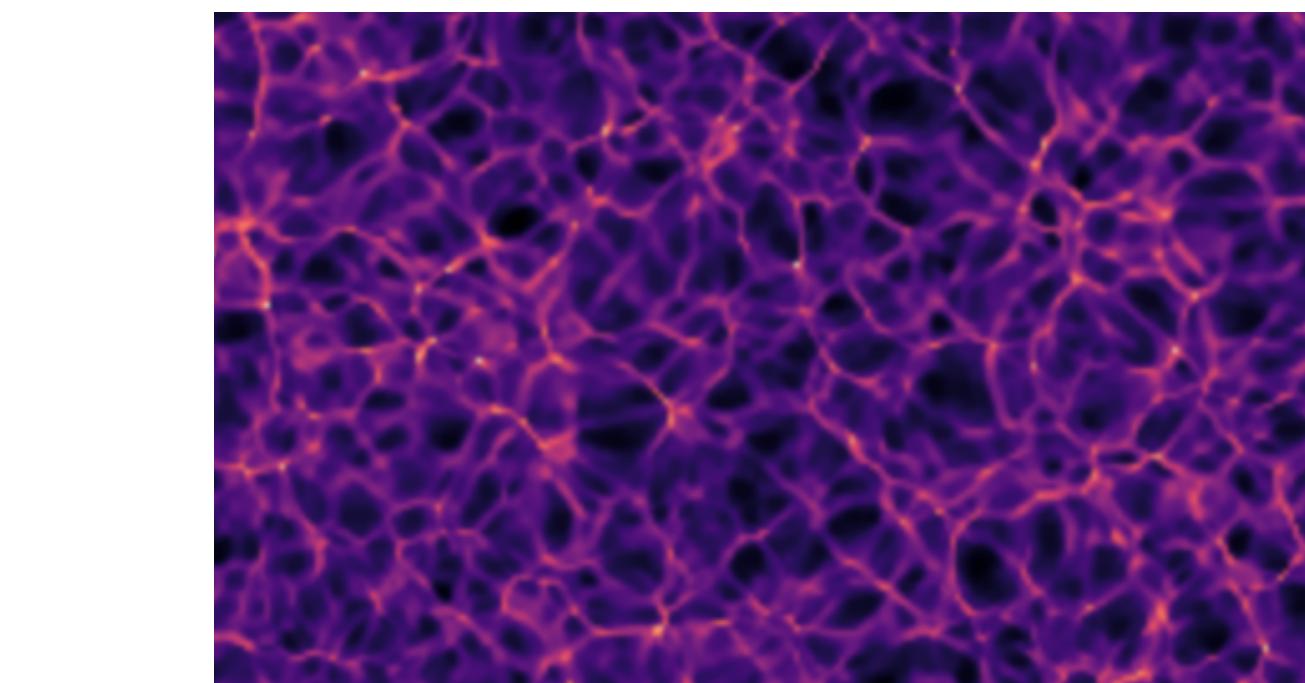
Abbott et. al (2016)



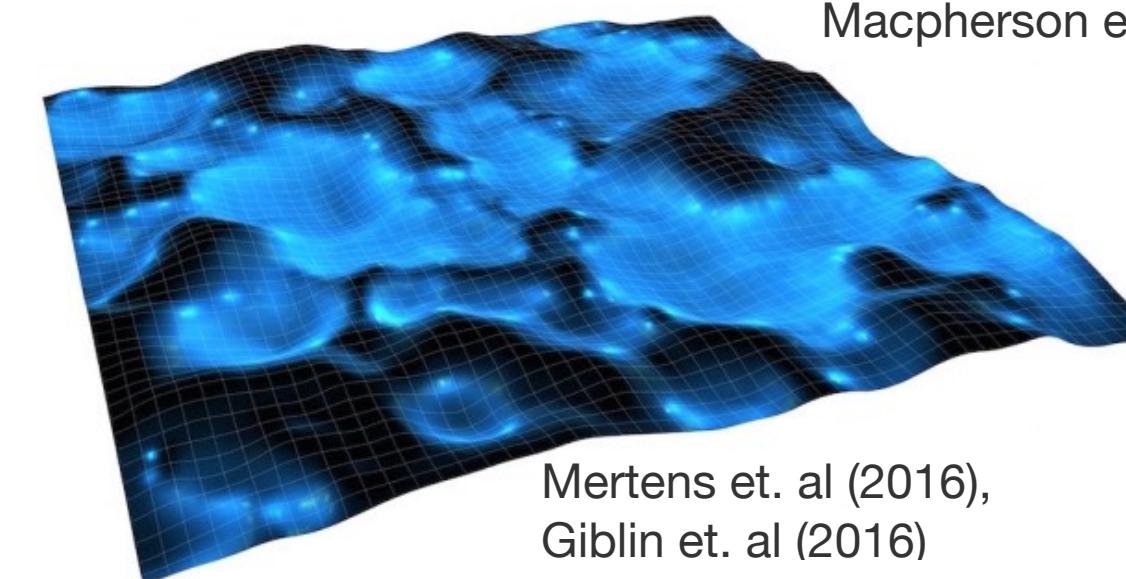
Liska et. al (2018)



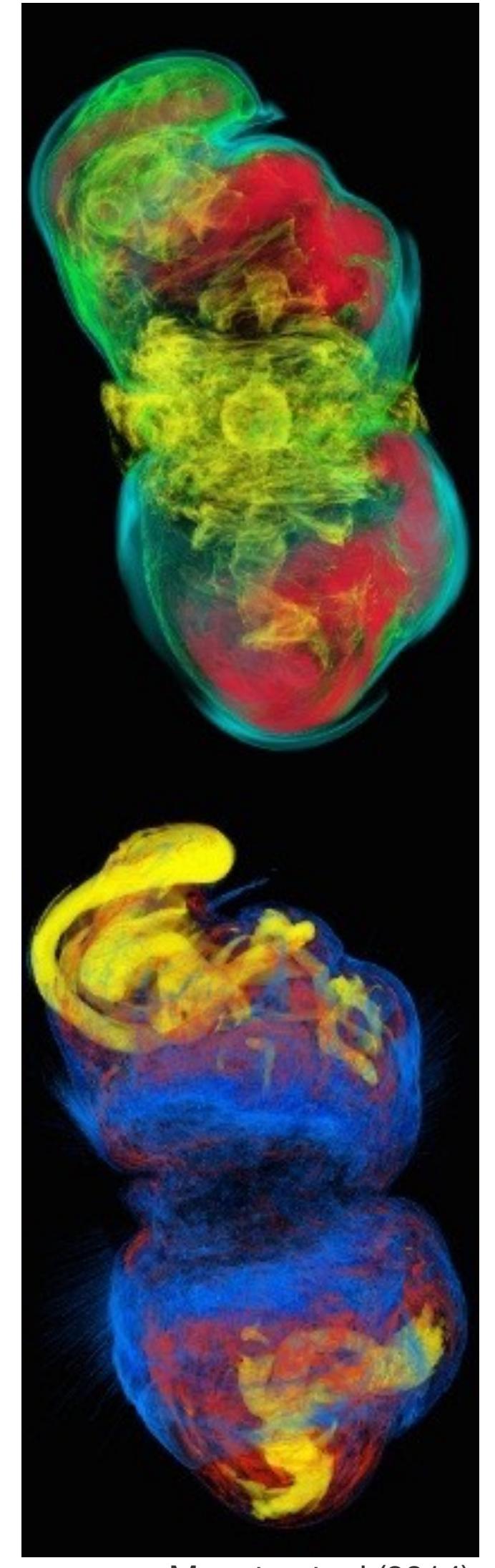
Giacomazzo et. al (2011)



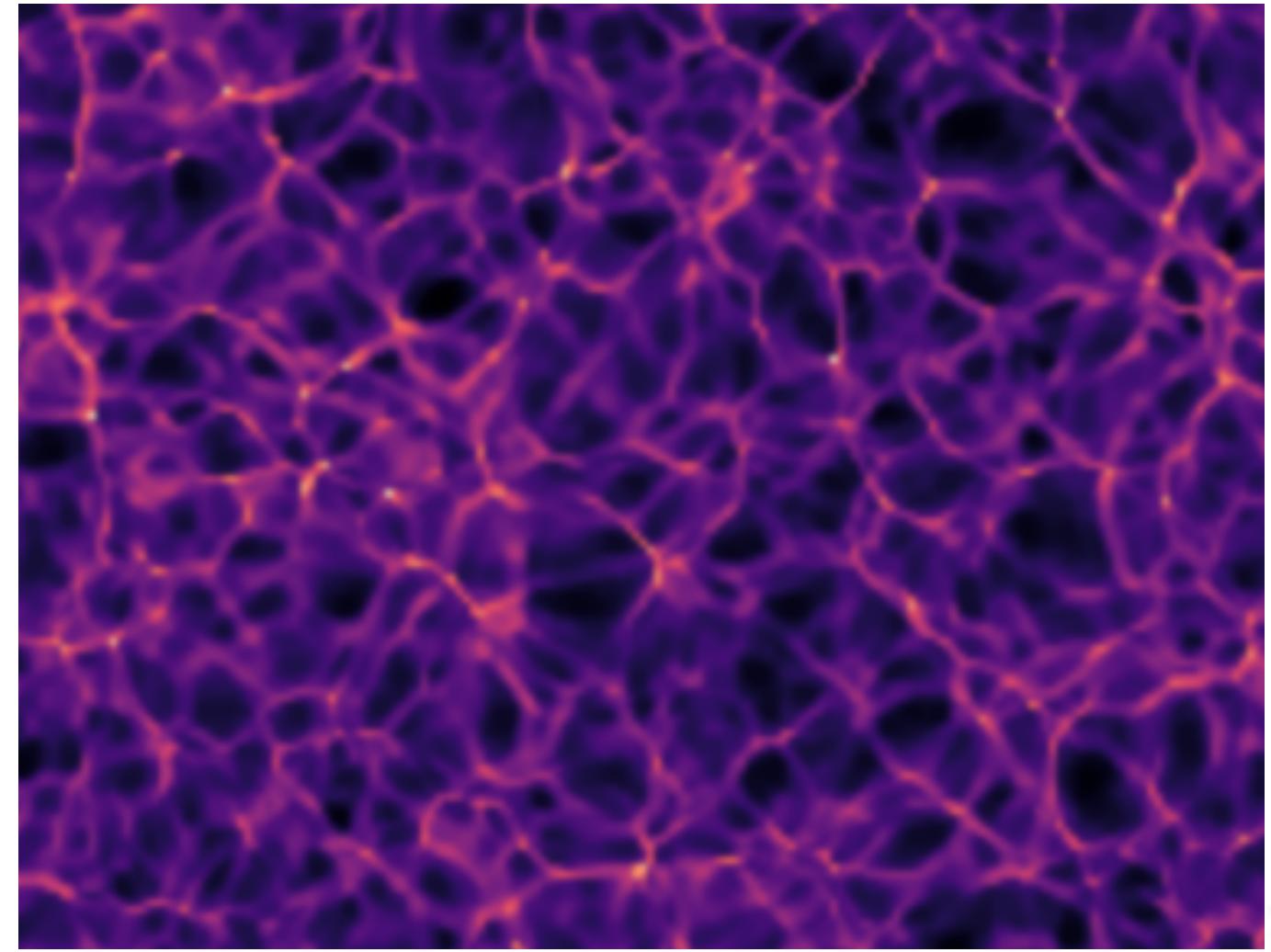
Macpherson et. al (2019)



Mertens et. al (2016),
Giblin et. al (2016)

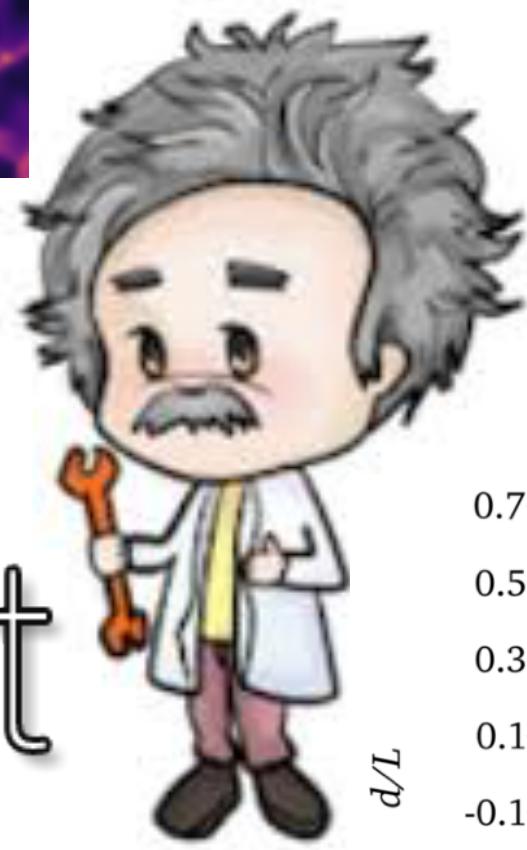


Moesta et. al (2014)

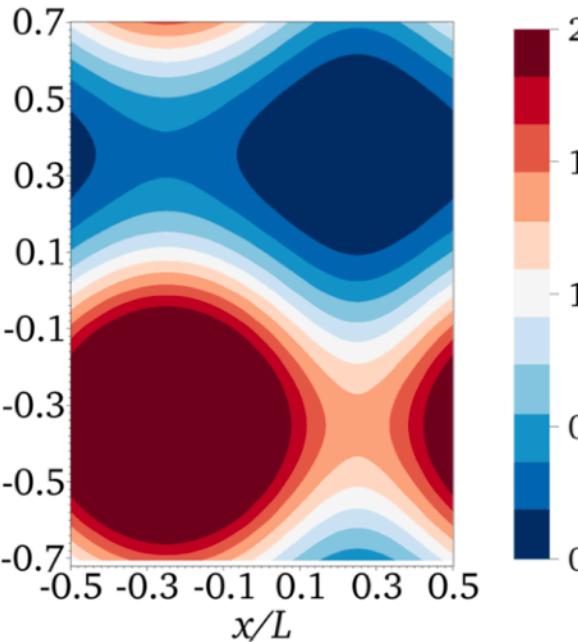


Macpherson et. al (2017,2018,2019)

einstein
toolkit

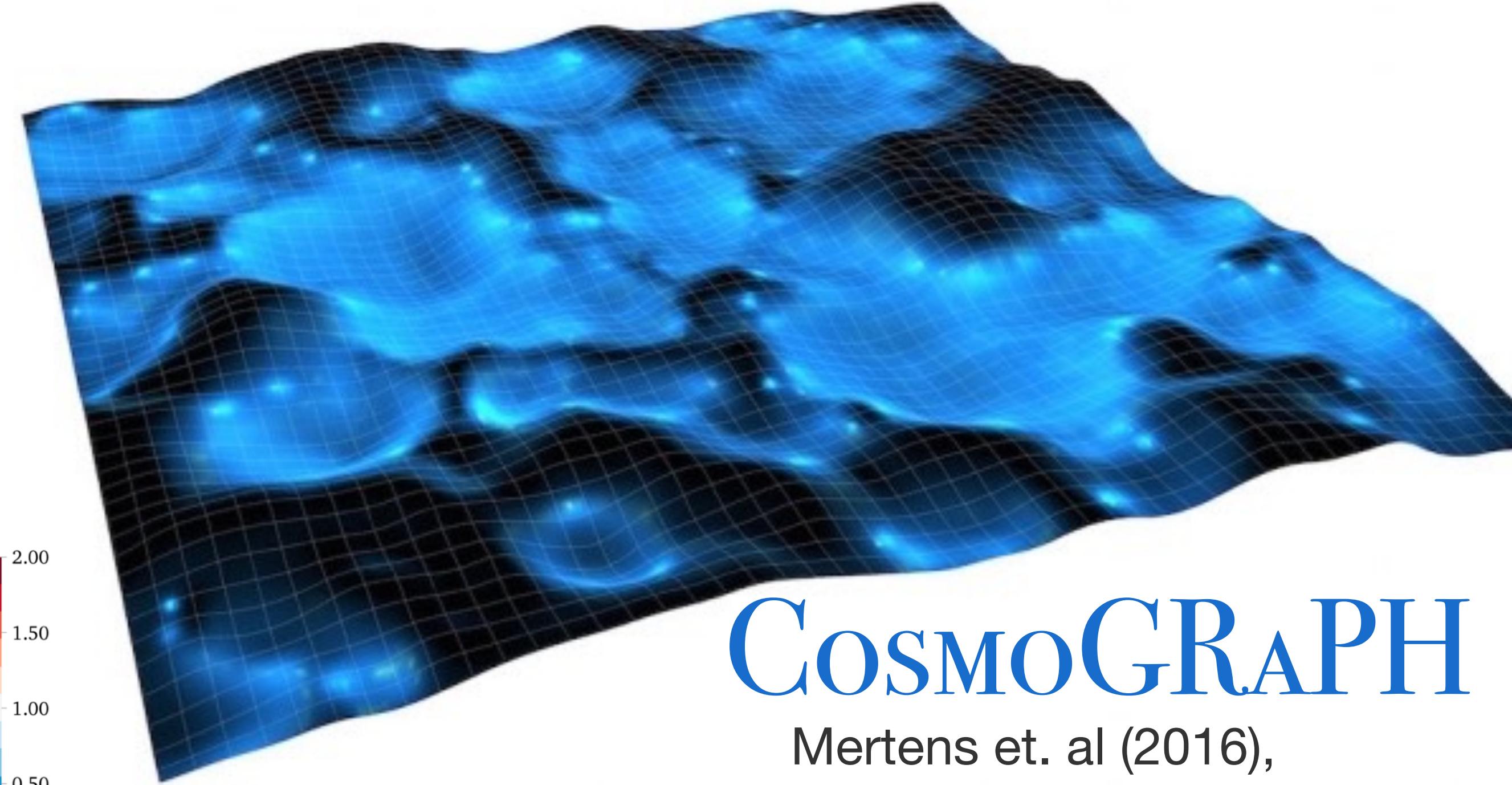


Bentivegna & Bruni (2016)
Bentivegna (2016)



Numerical relativity

for inhomogeneous & anisotropic cosmology



CosmoGRAPH

Mertens et. al (2016),
Giblin et. al (e.g., 2016-2019)

(some approximations to GR)

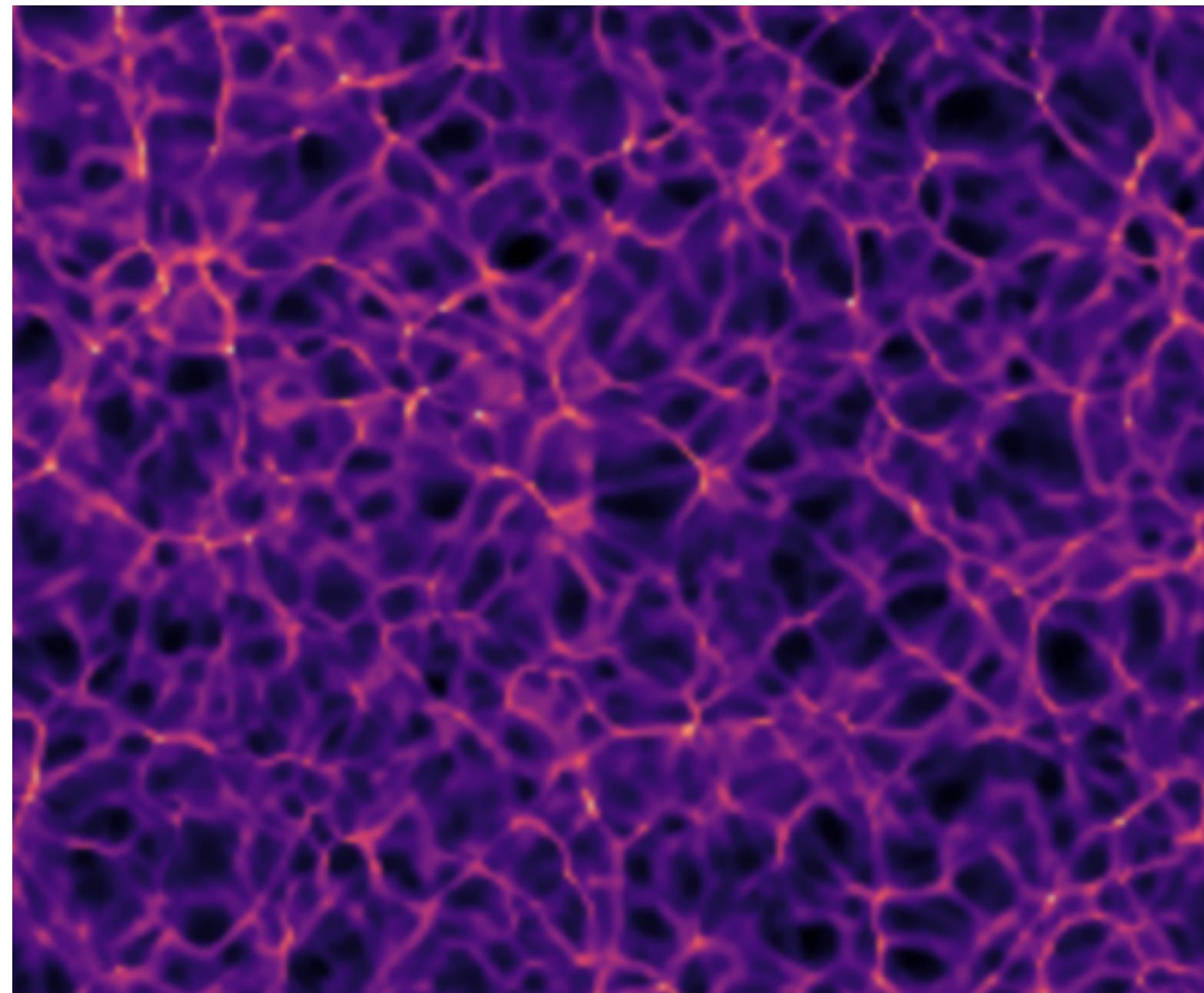
and... Daverio et. al (2017,2019), East et. al (2018), Adamek et. al (e.g., 2013-2019), Barrera-Hinojosa & Li (2020a,b)

gevolution

GRAMSES

Numerical relativity

for inhomogeneous & anisotropic cosmology



Macpherson et. al (2017,2018,2019)

einstein
toolkit



Cactus / Einstein Toolkit

- Widely used
- Free and open-source

FLRWsolver

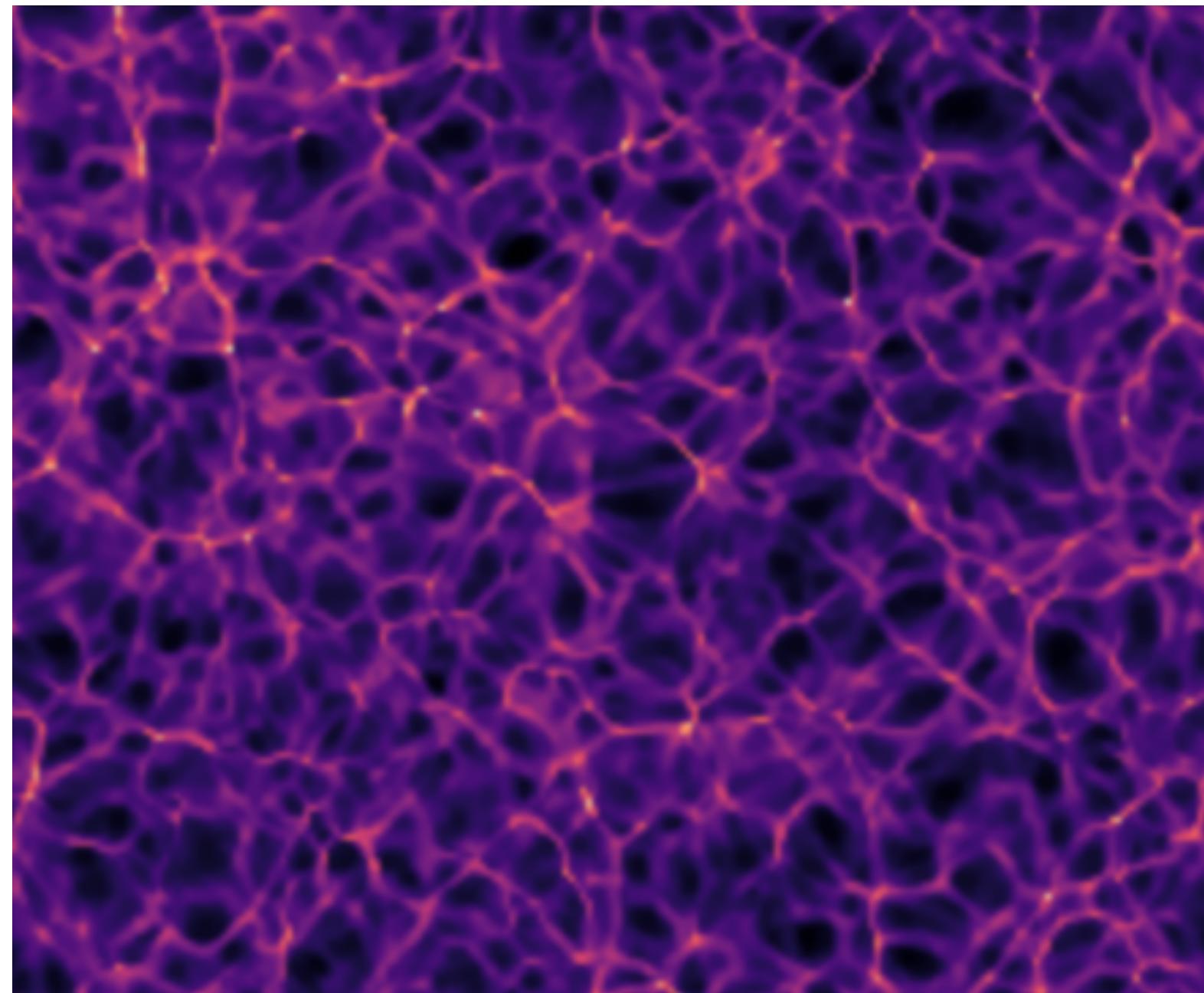
- A module to initialise realistic cosmological spacetimes
- tested in [arxiv:1611.05447](https://arxiv.org/abs/1611.05447)

Available at:

github.com/hayleyjm/FLRWsolver_public

Numerical relativity

for inhomogeneous & anisotropic cosmology



Macpherson et. al (2017,2018,2019)

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Initial conditions

- Linear perturbations around an EdS background
- Gaussian random perturbations following the matter power spectrum at the CMB

Simulations

- Matter dominated, i.e., no dark energy
- Fluid approx. for hydrodynamics

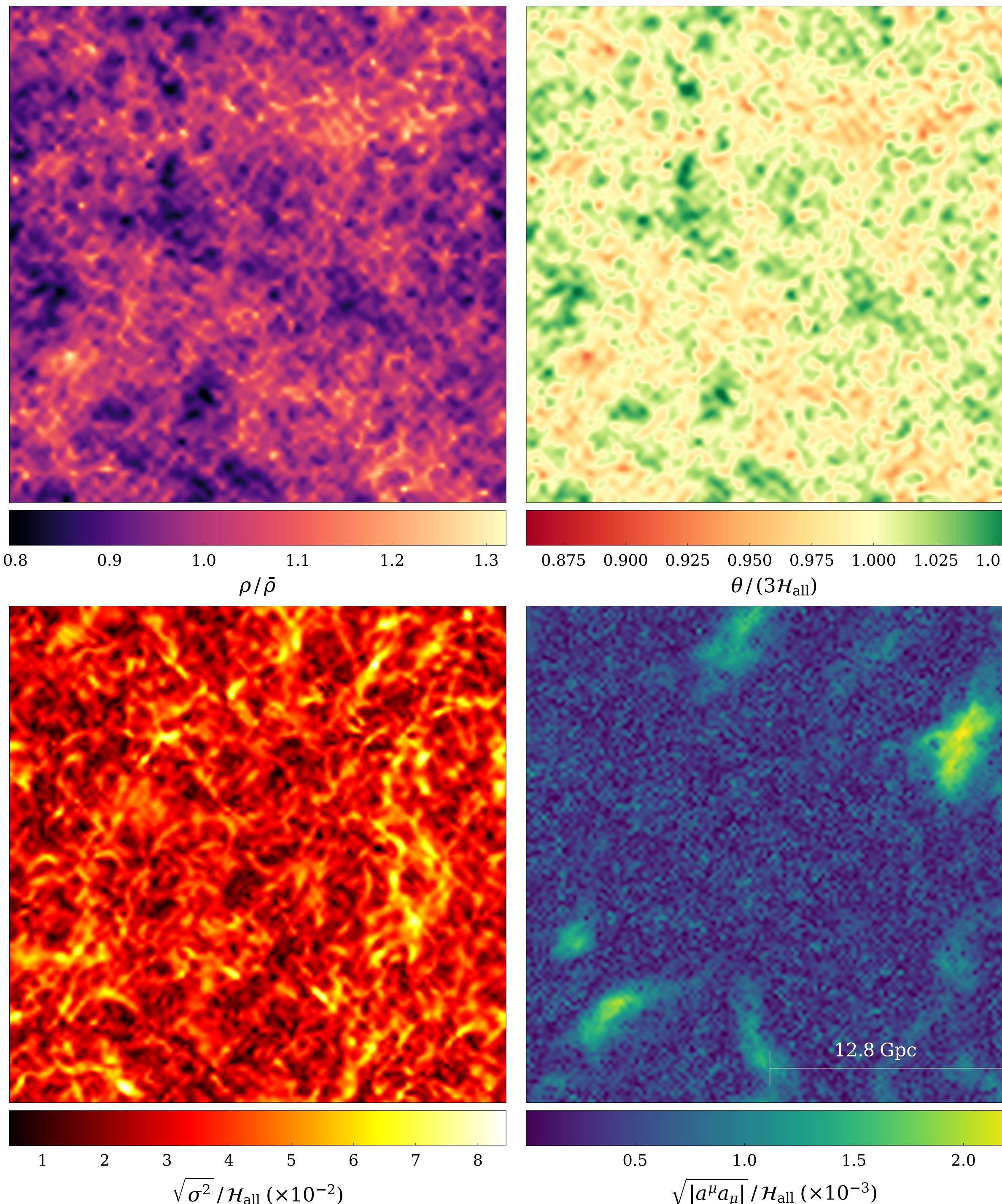
A note about smoothing

- 💡 The general series expansion has *limited assumptions* (see Heinesen, 2020, for details)...
- 💡 Effective Hubble needs to be **differentiable** and **positive everywhere** we apply the series
 - 💡 i.e., no collapse happening
 - 💡 This means we need to define a *smoothing scale*
 - 💡 We choose conservative scales above that at which our Universe is assumed to be described as FLRW
 - 💡 *Interesting to see if we recover the appropriate EdS parameters in this case*
 - 💡 100 Mpc and 200 Mpc grid cells to ensure *no smaller-scale structure can form*
 - 💡 Means our box sizes are 12.8 to 25.6 Gpc in size

Density field

Note: no dark energy
In terms of global averages, we find these sims are EdS to within numerical errors

Shear field



Expansion rate

Acceleration

Step 1: place observers randomly within the simulation domain

Step 2: each observer gets assigned a certain set of lines of sight

Step 3: for each line of sight, calculate each of the effective parameters

Effective Hubble

$$\mathfrak{H} = \frac{1}{3}\theta - e^\mu a_\mu + e^\mu e^\nu \sigma_{\mu\nu}$$

Effective deceleration

$$\mathfrak{Q} \equiv -1 - \frac{1}{E} \frac{\frac{d\mathfrak{H}}{d\lambda}}{\mathfrak{H}^2}$$

Derivative along the incoming null ray

$$\frac{d}{d\lambda} = k^\mu \nabla_\mu$$

Effective curvature

$$\mathfrak{R} \equiv 1 + \mathfrak{Q} - \frac{1}{2E^2} \frac{k^\mu k^\nu R_{\mu\nu}}{\mathfrak{H}^2}$$

Ricci focussing term

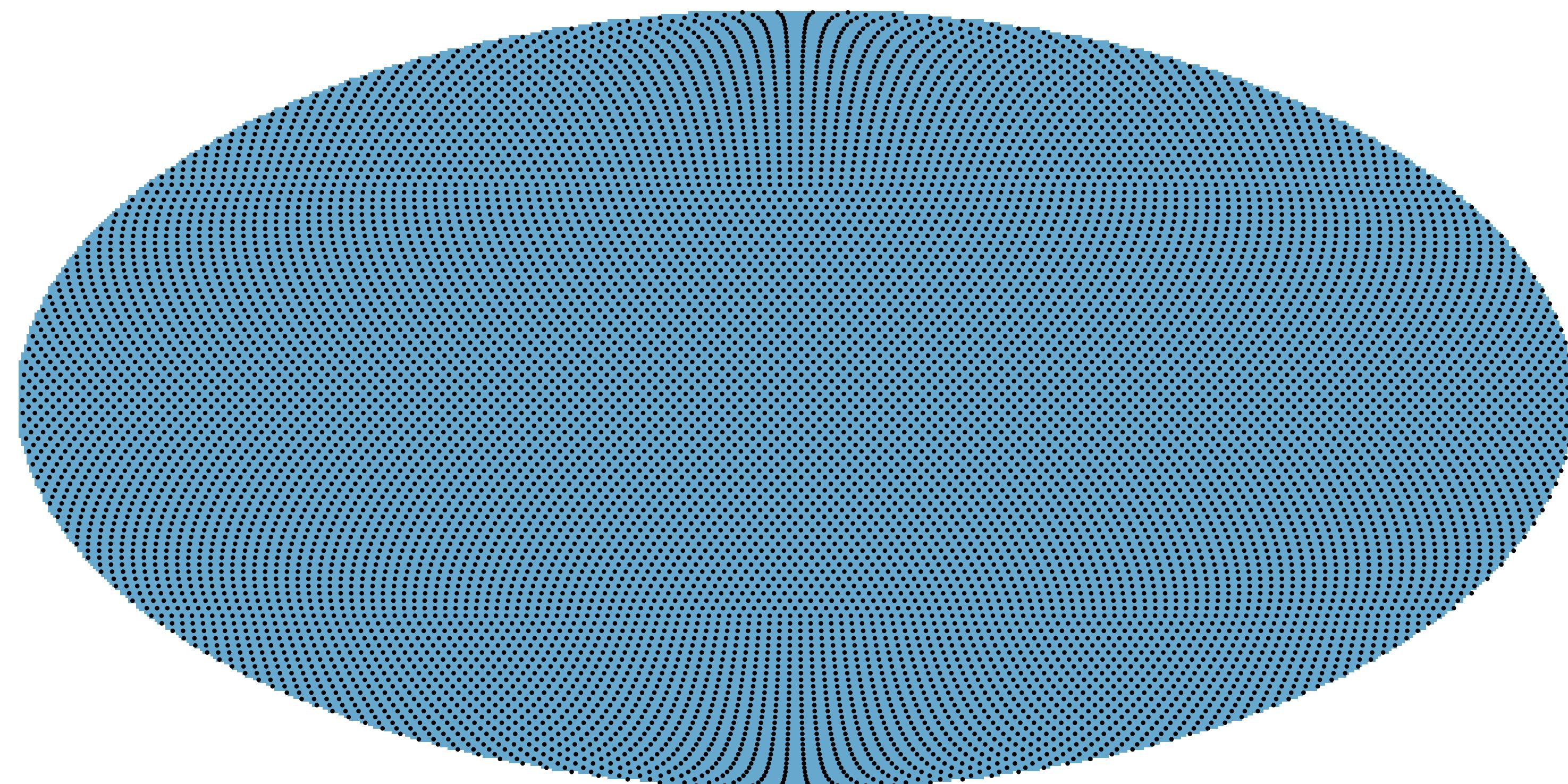
$$\mathfrak{J} \equiv \frac{1}{E^2} \frac{\frac{d^2 \mathfrak{H}}{d\lambda^2}}{\mathfrak{H}^3} - 4\mathfrak{Q} - 3$$

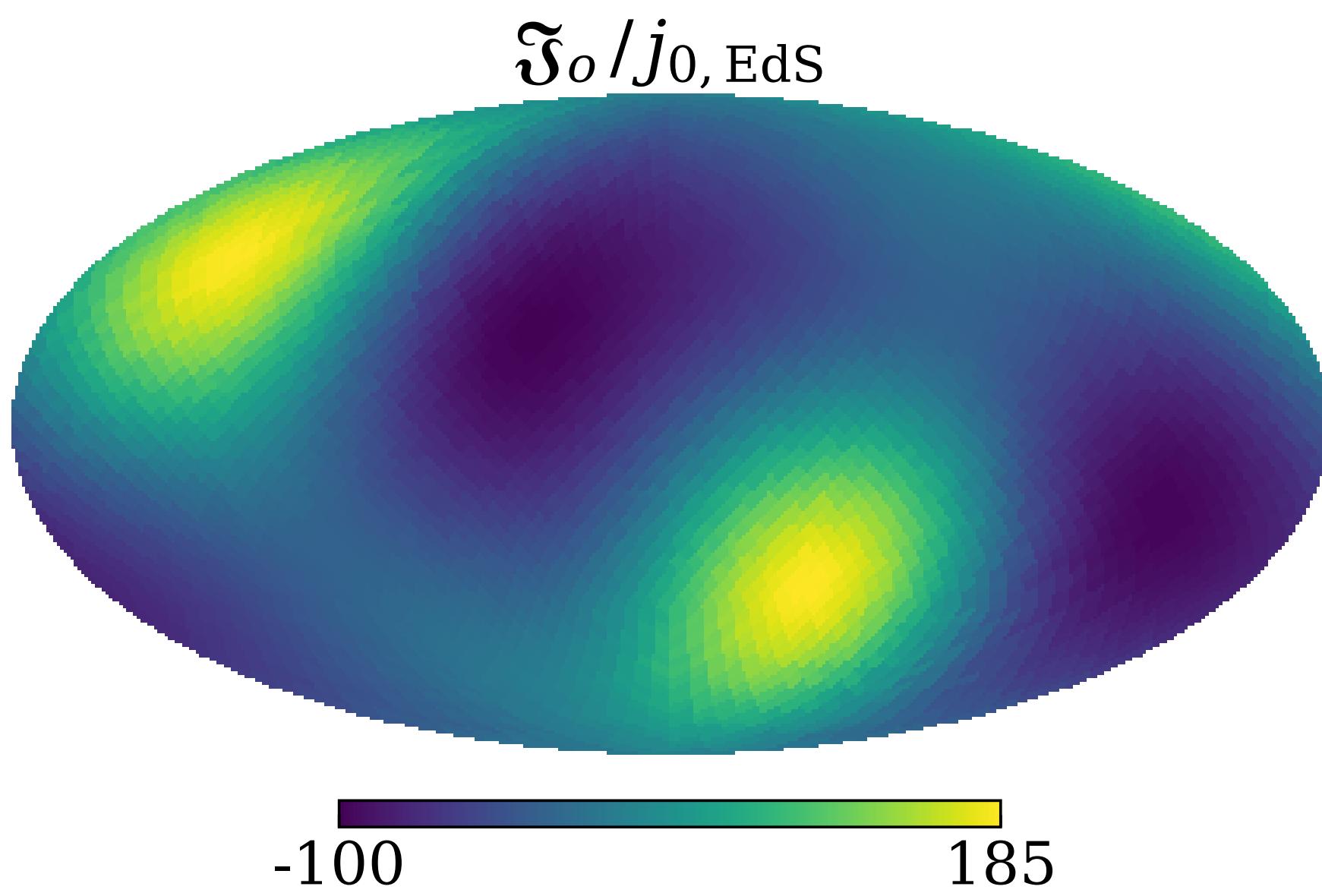
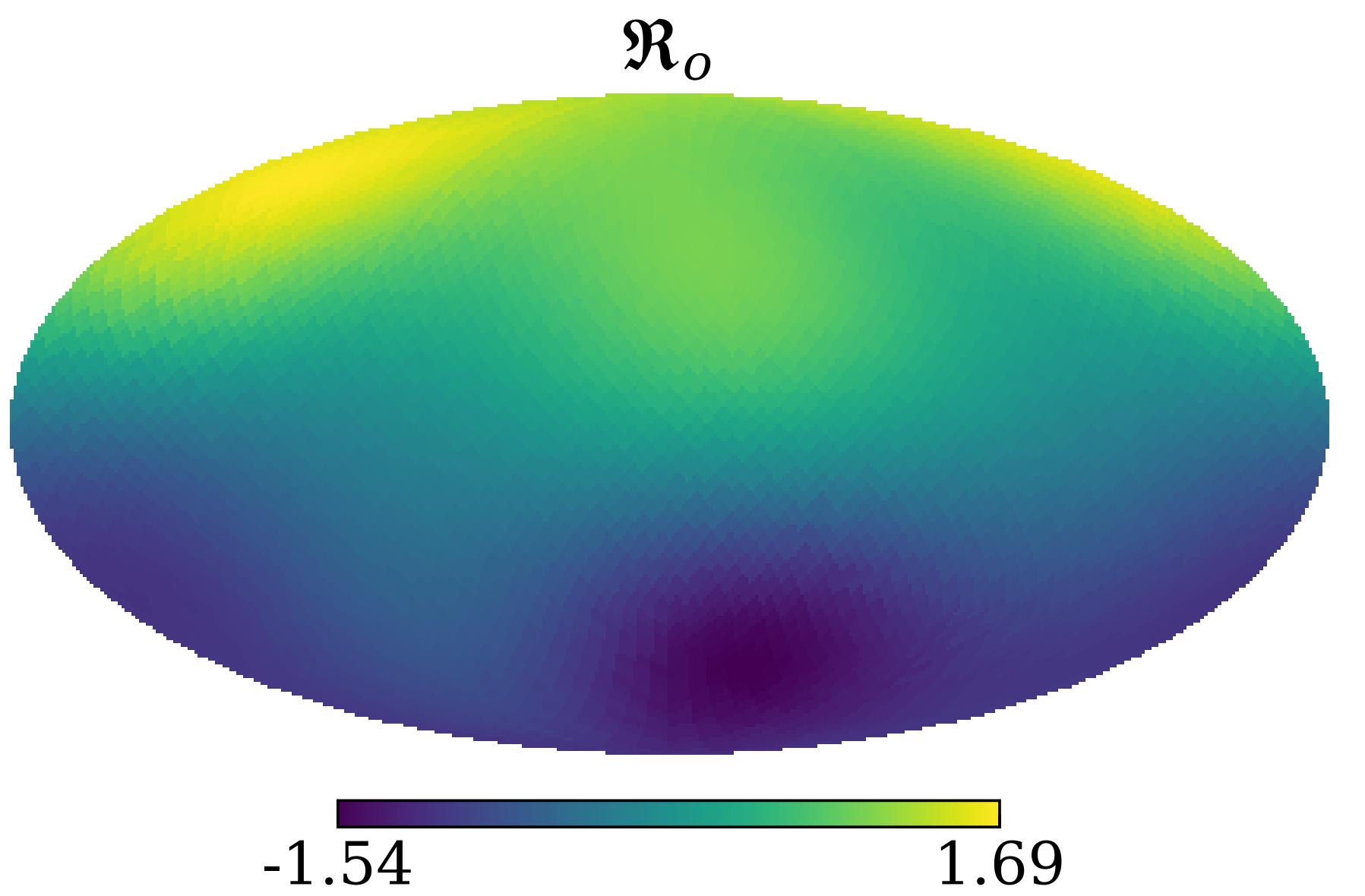
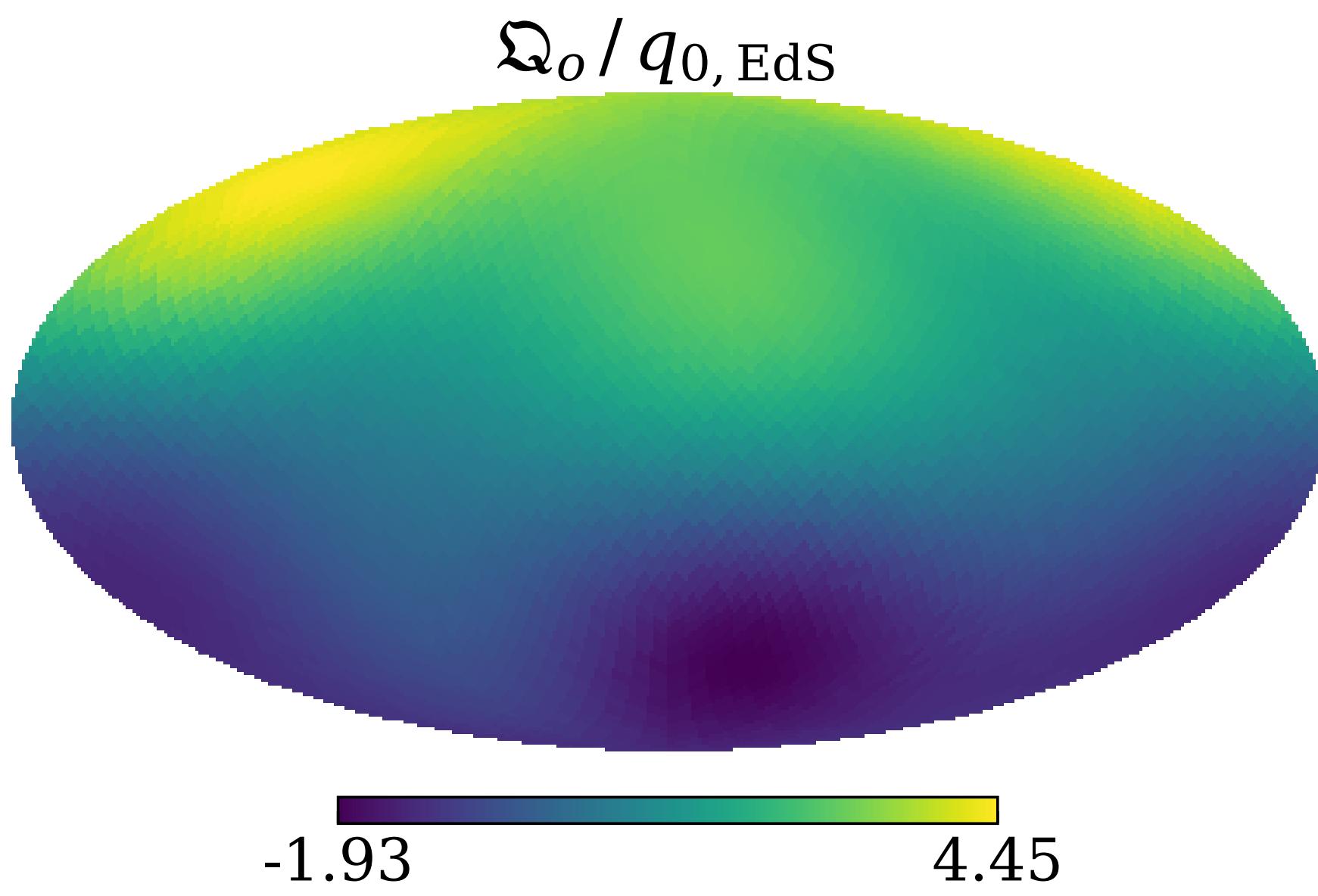
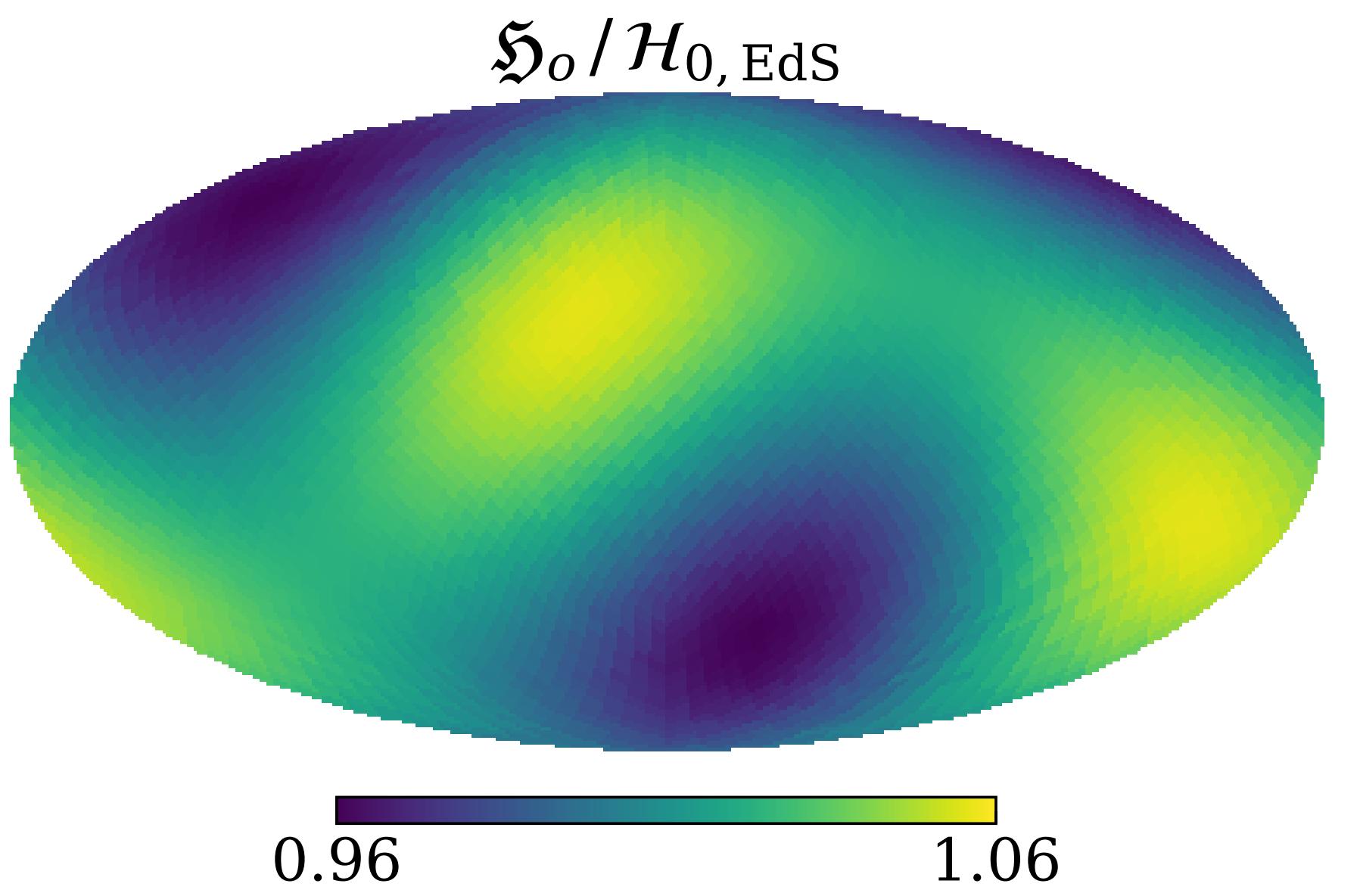
First, consider some observers with **very** well-sampled skies...

e^μ is drawn from HEALPix directions with:

$$12 \times N_{\text{side}}^2 \quad N_{\text{side}} = 32$$

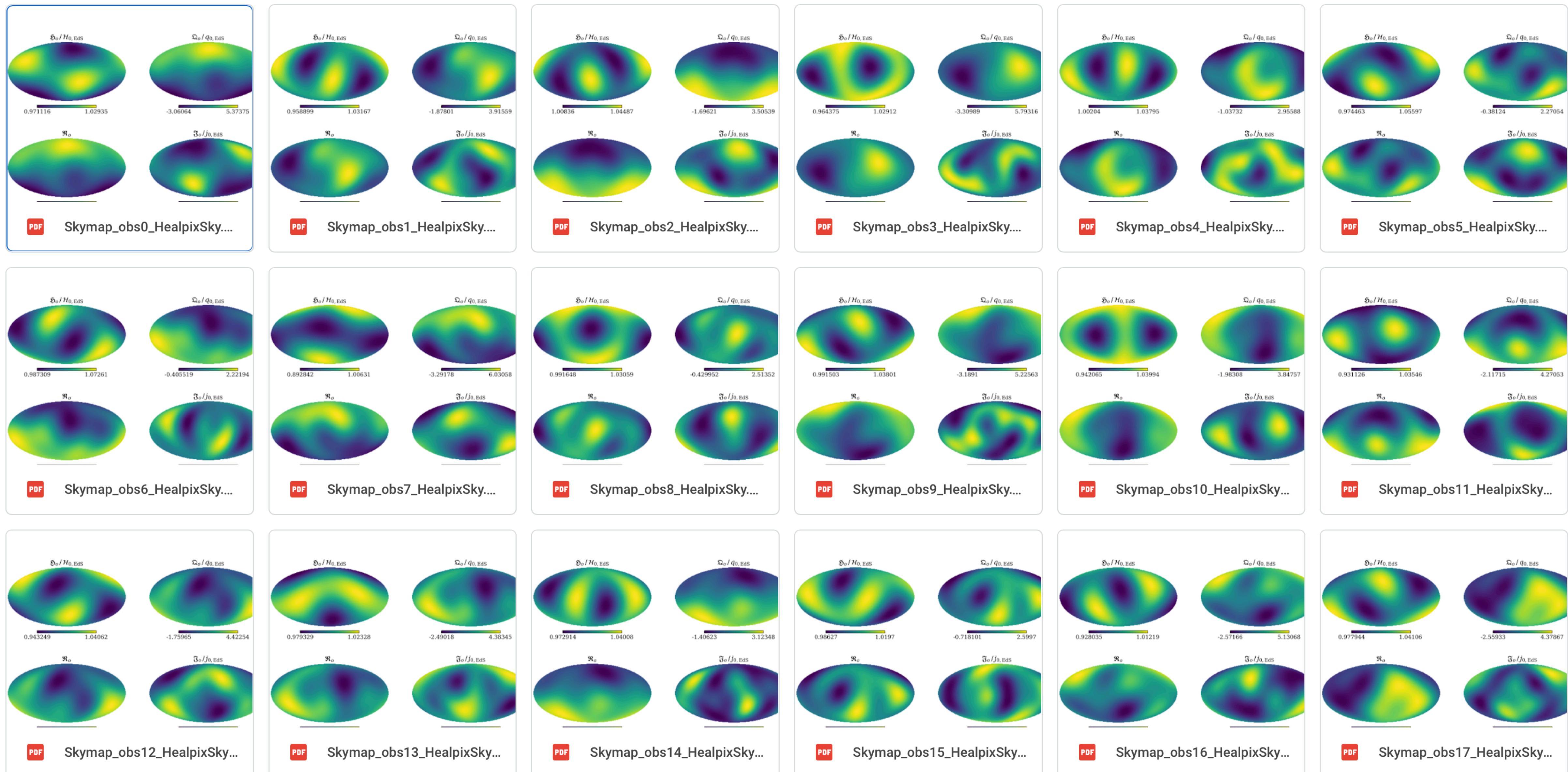
12,288 isotropic lines of sight... calculate effective parameters for each one

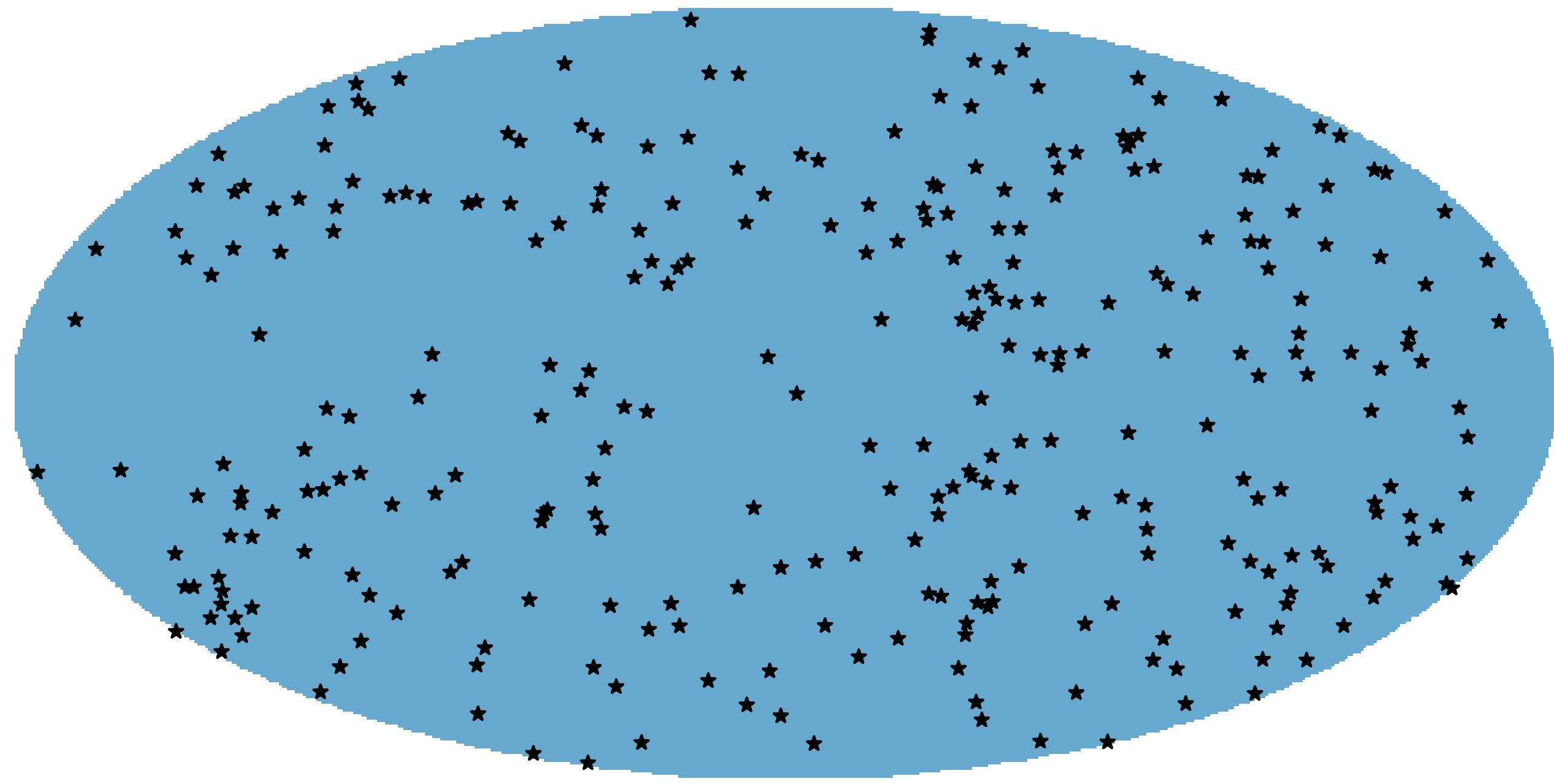




Files

Name ↑





Unfortunately real observations don't sample the sky that well...

Instead consider a 'fairly-sampled' sky with 300 random LOS

Calculate parameters for each LOS and average the result

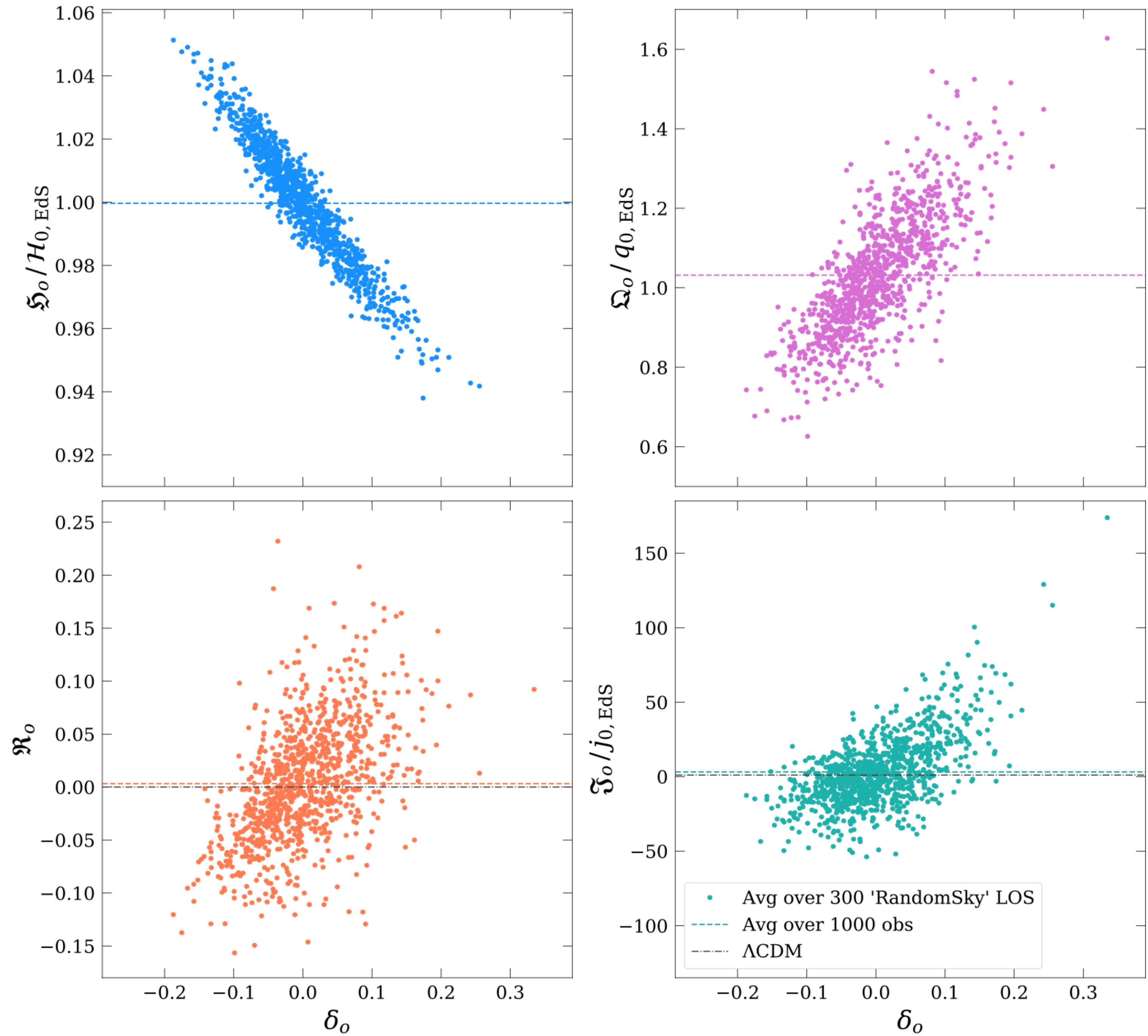
Isotropic contributions

Should be good approx. for monopole contribution

200 Mpc/h smoothing length

Still significant variance w.r.t EdS

“Cosmic variance” in Hubble agrees with, e.g., N-body simulation estimates

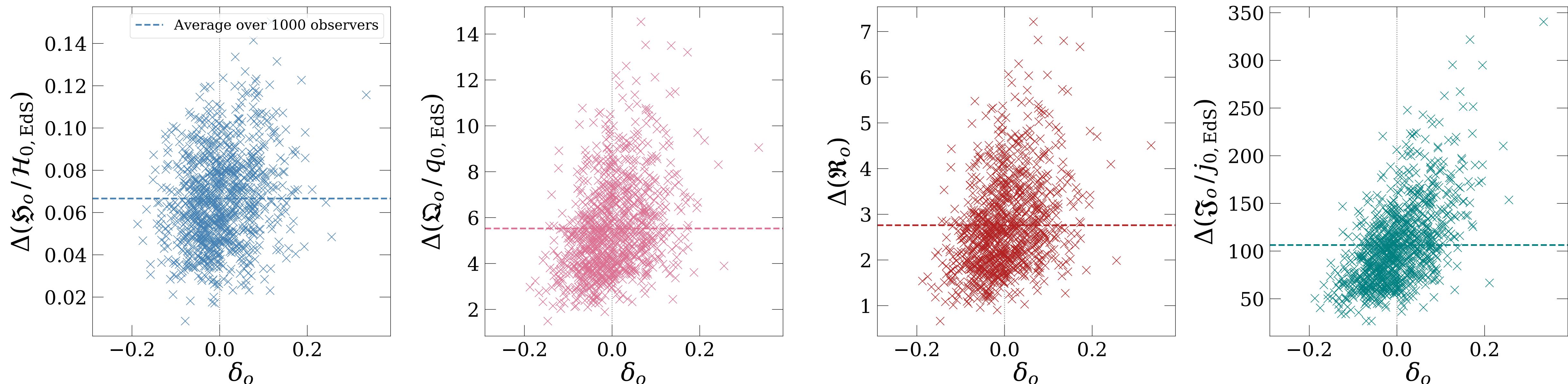


Anisotropic contributions

On top of the monopole, each observer will also measure a variance across their sky

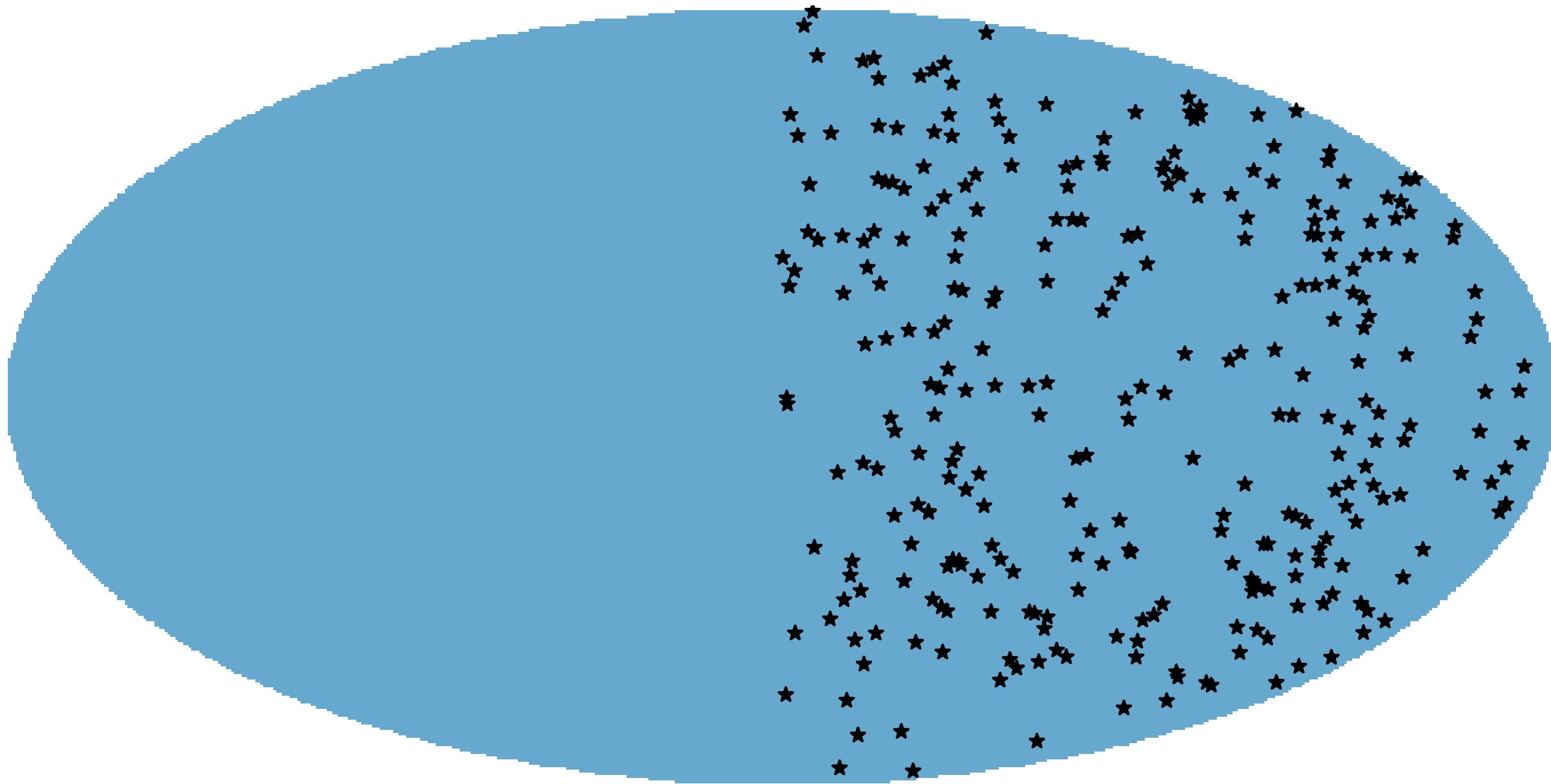
Quantify this with “maximal sky-deviation”

$$\Delta(\mathfrak{H}_o / \mathcal{H}_{0,\text{EdS}}) \equiv \frac{\mathfrak{H}_{o,\text{max}} - \mathfrak{H}_{o,\text{min}}}{\mathcal{H}_{0,\text{EdS}}}$$



Anisotropy does not depend on the local density at the observer

Although, the magnitude of these effects will be smaller in a model universe with typically lower density contrasts



Now consider an exaggerated case of an “unfairly-sampled” sky

300 lines of sight chosen randomly across one half of the sky

Calculate parameters for each LOS and average the result

Gives us an idea of the potential effects due to under-sampling one’s sky

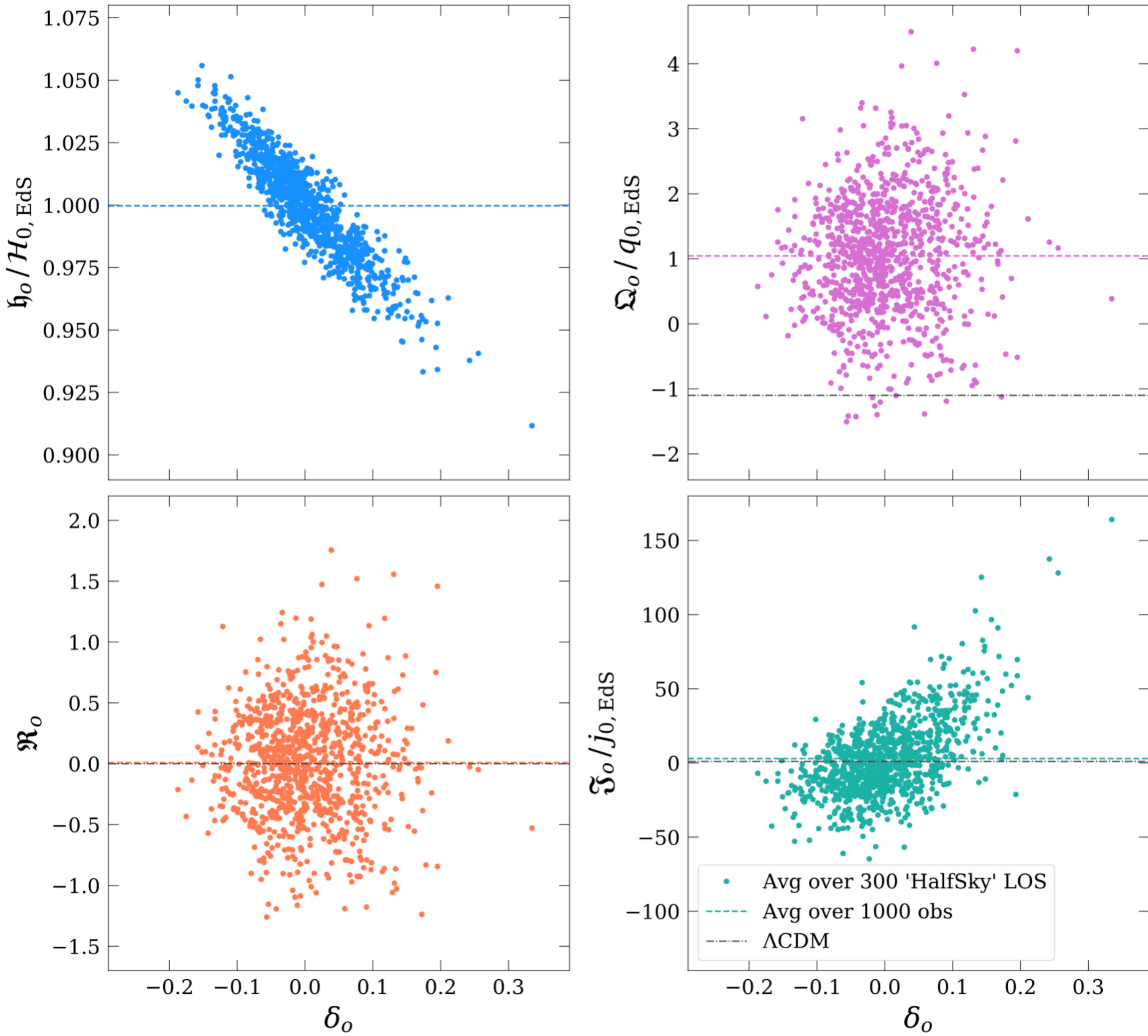
“Unfairly-sampled sky”

Hubble, jerk don't change much (most likely a consequence of the specific half of the sky we chose)

Variance in dipole-dominant parameters increases significantly

Some observers measure acceleration in a decelerating universe (EdS)

This was a drastic unfair sampling, results will be different for different survey geometries



MAIN TAKE-AWAYS

- The coefficients of the $dL(z)$ relation generalise in nontrivial ways in the presence of local inhomogeneity and anisotropy
- We find differences from FLRW even in the monopole limit
 - *In simulations with conservative smoothing and that follow EdS globally*
 - On top of this - parameters vary significantly across individual observer's skies
 - *This could impact, e.g., surveys with incomplete sky coverage*
 - Studies like this can help us determine what constitutes a “reasonable” assumption in simplifying this formalism to potentially apply to data
 - *e.g., which multipoles are dominant in each parameter?*