Could PBHs and secondary GWs have originated from squeezed initial states?

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Cosmology from home
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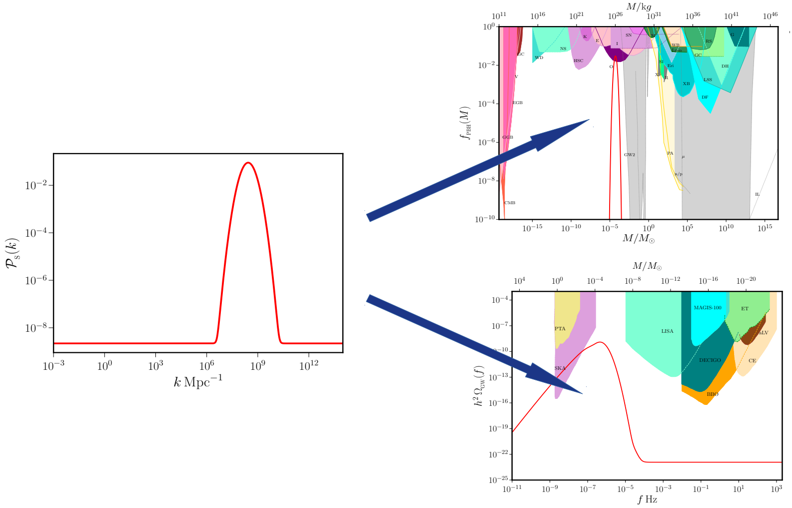
Overview

- Introduction
- Squeezed initial states
- Scalar non-Gaussianity
- Correction to the power spectrum
- PBHs and secondary GWs
- Backreaction
- Conclusion

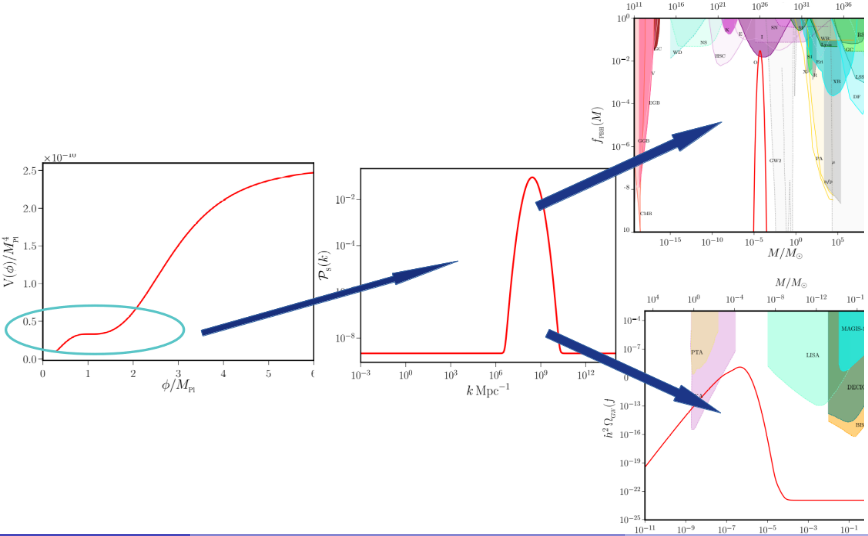
Introduction

Primordial black holes (PBHs) and secondary gravitational waves (GWs) are produced due to enhanced primordial scalar power.



Tweaking the potential

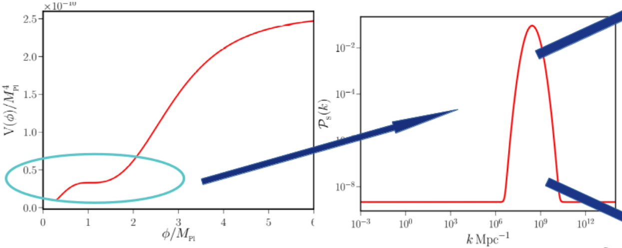
Introducing certain features, like an inflection point, in the potential driving inflation may lead to such enhancement in power.

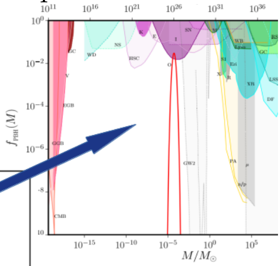


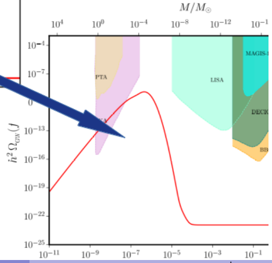
Tweaking the potential

Introducing certain features, like an inflection point, in the potential driving inflation may lead to such enhancement in power.

However, these potentials have to be fine-tuned to achieve the required shape and location of the enhancement.







An alternative method

But there is an appealing alternative.

[1]See, for instance, *R. H. Brandenberger and J. Martin, Int. J. Mod. Phys. A* **17**, *3663 (2002);*
P. D. Meerburg, J. P. van der Schaar and P. .S. Corasaniti, JCAP **05**, *018 (2009);*
I. Agullo and L. Parker, Phys. Rev. D **83**, *063526 (2011);*
S. Kundu, JCAP **02**, *005 (2012);*

An alternative method

But there is an appealing alternative.
What if scalar perturbations are evolved from excited initial states, instead of the standard Bunch-Davies vacuum?[1]

[1] See, for instance, *R. H. Brandenberger and J. Martin, Int. J. Mod. Phys. A* **17**, *3663 (2002);*
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S. Kundu, JCAP **02**, *005 (2012);*

Squeezed initial states

If the scalar perturbations are evolved from non-vacuum initial states - particularly squeezed initial states - one can generate desired features in the power spectrum[2].

$$\mathcal{R}_k(\eta) = \frac{i\,H}{2\,M_{\mathrm{Pl}}\,\sqrt{k^3\,\epsilon_1}}\,\left[\alpha(k)\,\left(1+i\,k\,\eta\right)\,\mathrm{e}^{-i\,k\,\eta} - \beta(k)\,\left(1-i\,k\,\eta\right)\,\mathrm{e}^{i\,k\,\eta}\right]$$

In this scenario, the scalar power spectrum is given by

$$\mathcal{P}_{_{\mathrm{S}}}(k) = \mathcal{P}_{_{\mathrm{S}}}^{0}(k)\,\vert\alpha(k)-\beta(k)\vert^2, \quad \text{where} \quad \mathcal{P}_{_{\mathrm{S}}}^{0}(k) = \frac{H^2}{8\,\pi^2\,M_{\mathrm{Pl}}^2\,\epsilon_1}\,.$$

The coefficients $\alpha(k)$ and $\beta(k)$ are constrained by the condition

$$\vert\alpha(k)\vert^2 - \vert\beta(k)\vert^2 = 1$$

$\alpha(k) = 1$ and $\beta(k) = 0$ corresponds to the Bunch-Davies vacuum.

[2] See, for instance, *R. H. Brandenberger, and J. Martin, Int. J. Mod. Phys. A* **17**, *3663 (2002)*

Modeling the power spectrum

We desire a spectrum with lognormal feature, such as,

$$\mathcal{P}_{\mathrm{s}}(k) = \mathcal{P}_{\mathrm{s}}^{0}(k)\ [1 + g(k)] \quad \text{where} \quad g(k) = \frac{\gamma}{\sqrt{2\,\pi\,\Delta_k^2}} \exp\left[-\frac{\ln^2(k/k_{\mathrm{f}})}{2\,\Delta_k^2}\right].$$

To obtain this spectrum, we model the coefficients as follows.

$$\alpha(k) = \frac{2 + g(k)}{2\sqrt{1 + g(k)}} \quad \text{and} \quad \beta(k) = \frac{-g(k)}{2\sqrt{1 + g(k)}}$$

Since $\beta(k) = 0$ corresponds to Bunch-Davies vacuum,
$\beta(k) \sim g(k) \sim \gamma$ denotes the extent of deviation from the Bunch-Davies vacuum.

Scalar non-Gaussianity

The non-Gaussianity in the scalar perturbation is introduced as[3]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}^{\mathrm{G}}(\eta, \boldsymbol{x}) - \frac{3}{5} f_{_{\mathrm{NL}}} \left[\mathcal{R}^{\mathrm{G}}(\eta, \boldsymbol{x})\right]^2 .$$

From this definition, the scalar non-Gaussianity parameter $f_{_{\mathrm{NL}}}$ can be calculated to be

$$\begin{aligned} f_{_{\mathrm{NL}}} \quad &= \quad -\frac{10}{3} \frac{1}{(2\,\pi)^4}\, k_1^3\, k_2^3\, k_3^3\, G(k_1, k_2, k_3) \\ &\quad \times \left[k_1^3\, \mathcal{P}_{_{\mathrm{S}}}(k_2)\, \mathcal{P}_{_{\mathrm{S}}}(k_3) + \text{two permutations}\right]^{-1} , \end{aligned}$$

where, $\langle \mathcal{R}_{k_1} \mathcal{R}_{k_2} \mathcal{R}_{k_3} \rangle = (2\pi)^{3/2}\, G(k_1, k_2, k_3)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$[4] .

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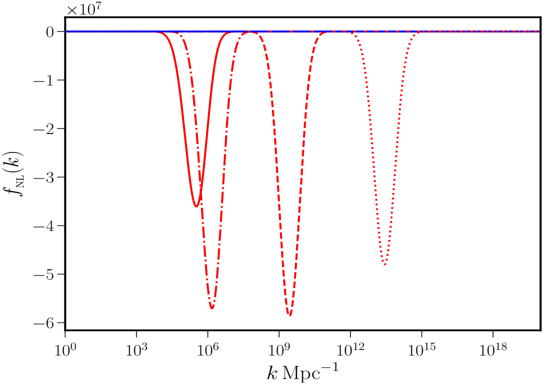
[3] See, for instance, *J. Maldacena, JHEP* **05**, *013 (2003).*

[4] See, for instance, *J. Martin, and L. Sriramkumar, JCAP* **1201**, *008 (2012).*

Computing $f_{\rm NL}$

$$f_{\rm NL} = -\frac{5}{3}\frac{k_f}{k_1}\frac{\gamma\sqrt{2\pi\Delta_k^2}}{\left(\gamma+\sqrt{2\pi\Delta_k^2}\right)}\left(\frac{1}{\gamma+\sqrt{2\pi\Delta_k^2}}+\frac{\gamma+2\sqrt{2\pi\Delta_k^2}}{4(\gamma+\sqrt{2\pi\Delta_k^2})^2}\right).$$



$f_{\rm NL}(k)$ in the squeezed limit (in red) against the expected value from the consistency relation (in blue).

Correction to power spectrum

Using the $f_{_{\rm NL}}$ introduced as

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}^{\rm G}(\eta, \boldsymbol{x}) - \frac{3}{5}\, f_{_{\rm NL}}\, \left[\mathcal{R}^{\rm G}(\eta, \boldsymbol{x})\right]^2 ,$$

we compute the correction to the scalar power spectrum[5].

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}}\, \hat{\mathcal{R}}_{\boldsymbol{k}'} \rangle = \frac{2\,\pi^2}{k^3}\, \delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}') \left[\mathcal{P}_{_{\rm S}}(k) + \left(\frac{3}{5}\right)^2 \frac{k^3}{2\,\pi}\, f_{_{\rm NL}}^2 \int {\rm d}^3\boldsymbol{p}\, \frac{\mathcal{P}_{_{\rm S}}(p)}{p^3}\, \frac{\mathcal{P}_{_{\rm S}}\left(|\boldsymbol{k}-\boldsymbol{p}|\right)}{|\boldsymbol{k}-\boldsymbol{p}|^3}\right]$$

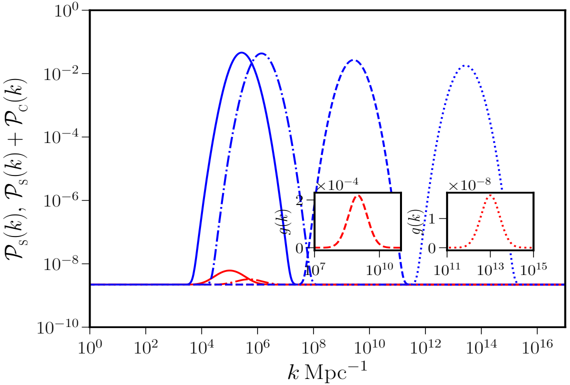
Thus the correction to the spectrum $\mathcal{P}_{_{\rm C}}(k)$ is

$$\mathcal{P}_{_{\rm C}}(k) = \left(\frac{3}{5}\right)^2 f_{_{\rm NL}}^2 \int_0^\infty {\rm d}v \int_{|1-v|}^{1+v} \frac{{\rm d}u}{v^2\, u^2}\, \mathcal{P}_{_{\rm S}}(k\, v)\, \mathcal{P}_{_{\rm S}}(k\, u)\, .$$

[5] See, for example, *R.-g. Cai, S. Pi, and M. Sasaki, Phys. Rev. Lett.* **122**, *201101 (2019) ; C. Unal, Phys. Rev.* **D 99**, *041301 (2019)* .

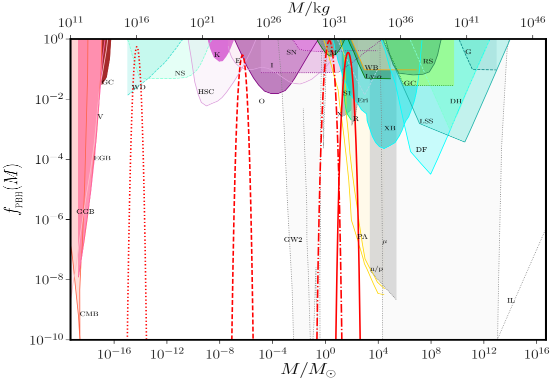
Correction dominates over the original spectrum

$$\mathcal{P}_{\rm C}(k) = \left(\frac{3}{5}\right)^2 \, f_{_{\rm NL}}^2(k) \int_0^\infty {\rm d}v \int_{|1-v|}^{1+v} \frac{{\rm d}u}{v^2\,u^2}\,\mathcal{P}_{\rm S}(k\,v)\,\mathcal{P}_{\rm S}(k\,u)\,.$$



The corrected power spectra (in blue) against the original spectra (in red).

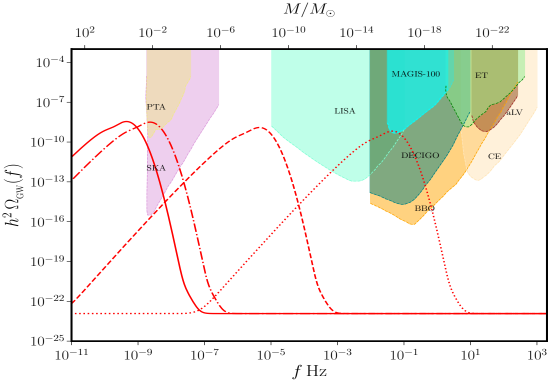
PBHs from squeezed initial states



The mass distribution of $f_{\rm PBH}$ obtained from squeezed initial states[6]

[6] For the calculation of $f_{\rm PBH}$, see, *H. V. Ragavendra, L. Sriramkumar and J. Silk, JCAP* **05**, *010 (2021)*. For constraints, see, *B. Carr, F. Kuhnel, and M. Sandstad, Phys. Rev.* **D 94**, *083504 (2016)*.

Secondary GWs from squeezed initial states



The spectra of $\Omega_{\rm GW}$ obtained from squeezed initial states[7]

[7] For the calculation of $\Omega_{\rm GW}$, see, *H. V. Ragavendra, L. Sriramkumar and J. Silk, JCAP* **05**, *010 (2021)*. For constraints, see, *C. Moore, R. H. Cole, and C. P. L. Berry, Class. Quant. Grav.* **32**, *015014 (2015)*.

Backreaction

However, in this scenario, the perturbations may have energy density larger than that of the inflaton field when they are deep inside the Hubble radius. Such backreaction may disrupt the background evolution[8].
The energy density of the perturbations in our case is calculated to be

$$\rho_{\mathcal{R}} = \frac{1}{a^4} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\,\pi)^3}\, k\, |\beta(k)|^2 = \frac{\gamma^2\, \mathrm{e}^{4\,\Delta_k^2}}{16\,\pi^{5/2}\,\Delta_k} \left(\frac{k_{\mathrm{f}}}{a}\right)^4 .$$

To avoid backreaction, we demand

$$\rho_{\mathcal{R}} \ll \rho_{\mathrm{I}}, \quad \text{where} \quad \rho_{\mathrm{I}} = 3\,H^2\,M_{\mathrm{Pl}}^2 .$$

This gives us the condition on the parameter γ to be[9]

$$\gamma \ll \frac{10^{9/2}}{\sqrt{r}} \left(\frac{k_{\mathrm{min}}}{k_{\mathrm{f}}}\right)^2 .$$

[8] See, for instance, *R. Holman, JCAP* **05**, *001 (2008); S. Kundu, JCAP* **02**, *005 (2012).*
[9] *H. V. Ragavendra, L. Sriramkumar, and J. Silk, JCAP* **05**, *010 (2021)*

Summary

- The mechanism of evolving perturbations from non-vacuum initial states seemed promising in enhancing the scalar power.
- The non-Gaussianity parameter $f_{_{\rm NL}}$ was obtained to be extremely large in this scenario leading to significant correction to the power spectrum.
- Hence we expected PBHs and secondary GWs to originate from squeezed initial states and also possibly constrain the states against observations.
- But the issue of **backreaction** strongly constrains any deviation of the initial state from the standard Bunch-Davies vacuum, thereby severely limiting the production of PBHs and secondary GWs.

Outlook

- The non-Gaussian correction we have calculated is sufficient to examine the imprints on Ω_{GW}. However, the f_{NL} shall also modify the probability distribution of matter density that form PBHs. Hence, such non-Gaussian effects due to squeezed initial states on f_{PBH} should be analyzed separately.

Outlook

- The non-Gaussian correction we have calculated is sufficient to examine the imprints on Ω_{GW}. However, the f_{NL} shall also modify the probability distribution of matter density that form PBHs. Hence, such non-Gaussian effects due to squeezed initial states on f_{PBH} should be analyzed separately.
- It would be interesting to calculate higher order moments, such as the trispectrum, to see if they lead to stronger enhancement to the power spectrum within the limits of backreaction.

Conclusion

Thank you for your attention.

Reference:

H. V. Ragavendra, *L. Sriramkumar and J. Silk, JCAP* **05**, *010 (2021) [arXiv:2011.09938 [astro-ph.CO]].*