Cosmology from Home - 2021

α'-Cosmological Tale: String Cosmology Backgrounds from Classical String Geometry

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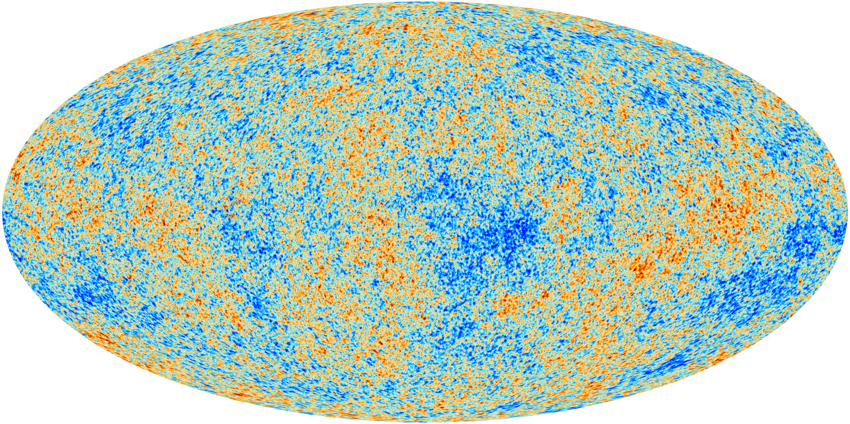
Observations



Black Body Radiation with average $T \sim 2.7$ K and fluctuations of order $\Delta T/T \sim 10^{-5}$. (Planck Collaboration, '13)

ΛCDM Model

With only 6 parameters it explains all the current cosmological data. Among these parameters, two are related to the initial fluctuations that gave rise to the CMB

$$P(k) = \Delta_R^2 k^{n_s - 1},$$

where $\Delta_R^2 \sim 2.5 \times 10^{-9}$ and $n_s \sim 0.9667$[1]. The small value of $n_s - 1$ encodes the almost scale-invariance of the power spectrum.

[1] (Planck Collaboration, '13)

How does one obtain such a spectrum? The fact it's almost scale-invariant and thermalized is puzzling and would seem to imply acausal physics.

1970: Peebles and Yu showed that in order to recover an almost scale-invariant power spectrum there should be a way to produce standing-wave (constant amplitude) fluctuations on super-Hubble scales (H^{-1}) before the CMB was emitted.

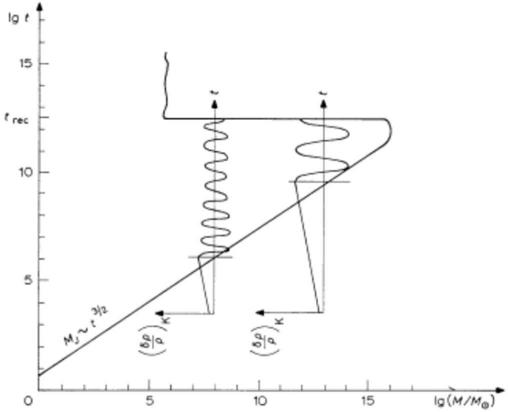
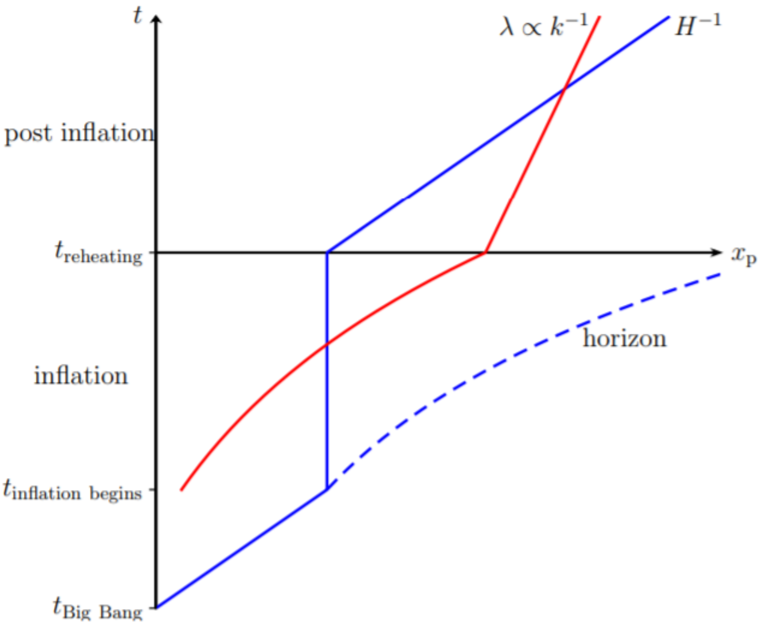


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

(Sunyaev and Zeldovich, '70)

Inflation

First causal scenario that yields such physics by invoking **a phase of quasi-de Sitter expansion**. It predicts an almost scale-invariant power spectrum for the fluctuations with a red tilt: $n_s - 1 < 0$. (Mukhanov and Chibisov, '81)

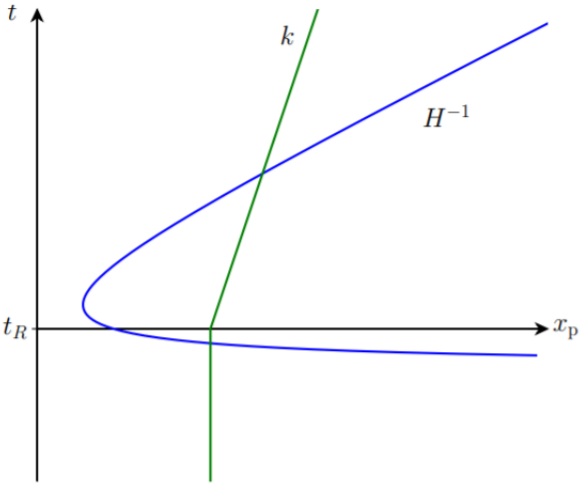


t
$\lambda \propto k^{-1}$
H^{-1}
post inflation
$t_{\text{reheating}}$
x_{p}
horizon
inflation
$t_{\text{inflation begins}}$
$t_{\text{Big Bang}}$

Problems with Inflation

- Non-fundamental scalar field
- Eternal inflation and multiverse (Brandenberger, Costa, GF, '15)
- Trans-planckian problem (Martin and Brandenberger, '01)
- **Singularity** (Borde and Vilenkin '94)

Alternatives? Yes!



Emergent Scenario together with holographic scaling.

Where are we at?

- Standard Cosmological Model remains incomplete
- Singularities are classical, thus finding a good QG theory should do it: **string theory**[2]
- Strings allow for new degrees of freedom and introduce new symmetries/dualities

[2] No convincing embedding of inflation in string theory so far, e.g. swampland conjectures (Vafa et al., '18) and TCC (Bedroya et al., '19).

Talking of strings...

- Perturbative structure of the theory is well known
  - Expansion in g_s: dimensionless string coupling (*QFTish*)
  - $\boxed{\text{Expansion in } \sqrt{\alpha'}}$: string length (*stringish*)
- Low energy EFT (supergravities, GR limit) valid for small curvature: $\sqrt{\alpha'}/R_c \ll 1$
- Singularities happen for $\mathcal{R} \to \infty$, so not only α' corrections but non-perturbative regime is fundamental (Wang, Wu, Yang and Ying, '19)

Backdrop Story

- B-supergravity action + cosmological ansatz: global $O(d,d)$ (Meissner and Veneziano, '91) generalizing scale-factor duality (Veneziano, '91)
- $O(d,d)$ Covariant String Cosmology developed but lowest order in α' (Gasperini and Veneziano, '92)
- Impressively, $O(d,d)$ symmetry present to all orders in α' as long as fields are spatially independent (Sen, '92)
- $O(d,d)$ transf. receive α' corrections, **but**:
  - one can redefine the fields to keep their form at least in 1st order in α' (Meissner, '97)
  - assuming this is the case to ALL orders (which certainly happens in conventional string field theory variables (Kugo and Zwiebach, '92)), one can classify all $O(d,d)$ invariant α'-corrections (Hohm and Zwiebach, '19)
- No matter sources considered though!

Objectives

1. To construct a full α'-corrected manifestly $O(d,d)$ covariant formulation of String Cosmology in the presence of matter
2. To build a non-singular very early universe model including all stringy corrections

α'-complete Cosmology

(Bernardo, Brandenbeger, GF - hep-th/1911.00088)

The action is

$$S_T^{(\alpha')} = \frac{1}{2\kappa^2} \int d^d x dt n e^{-\Phi} \left[-(\mathcal{D}\Phi)^2 + \sum_{k=1}^{\infty} \alpha'^{k-1} c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k} \right] + S_m[\Phi, n, \mathcal{S}, \chi]$$

The EOM are

$$2\mathcal{D}^2\Phi - (\mathcal{D}\Phi)^2 - \sum_{k=1}^{\infty} \alpha'^{k-1} c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k} = \kappa^2 e^{\Phi} \bar{\sigma}$$

$$(\mathcal{D}\Phi)^2 - \sum_{k=1}^{\infty} \alpha'^{k-1} (2k-1) c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k} = 2\kappa^2 \bar{\rho} e^{\Phi}$$

$$\mathcal{D}\left(e^{-\Phi} \sum_{k=1}^{\infty} \alpha'^{k-1} 4 k c_k \mathcal{S} (\mathcal{D}\mathcal{S})^{2k-1} \right) = -\kappa^2 \eta \bar{\mathcal{T}}$$

FLRW α'-complete Cosmology

Friedmann-like equations are

$$\begin{aligned} \dot{\Phi}^2 + HF'(H) - F(H) &= 2\kappa^2 e^{\Phi}\bar{\rho} \\ \dot{H}F''(H) - \dot{\Phi}F'(H) &= -2\kappa^2 e^{\Phi} d\bar{p} \\ 2\ddot{\Phi} - \dot{\Phi}^2 + F(H) &= \kappa^2 e^{\Phi}\bar{\sigma} \end{aligned}$$

$$\dot{\bar{\rho}} + dH\bar{p} = \frac{1}{2}\dot{\Phi}\bar{\sigma}$$

Invariant under the scale factor duality $a \to 1/a$, which implies

$$H \to -H, \quad \Phi \to \Phi, \quad \bar{\rho} \to \bar{\rho}, \quad \bar{p} \to -\bar{p}, \quad \bar{\sigma} \to \bar{\sigma}$$

assuming matter action is duality invariant. This is a remnant of the $O(d,d)$ transformation for the FLRW background.

De Sitter Solutions

- Asymptotically appealing to early- and late-time cosmology
- Imposing $H = H_0$ into the equations leads to two conditions $(\bar{\sigma} = 0)$:

$$\frac{\dot{w}}{w} - dH_0 w = -\frac{F'(H_0)}{2dw}$$

$$F(H_0) = \frac{H_0}{2} F'(H_0), \quad w \neq 0$$

while the shifted dilaton evolves as

$$\dot{\Phi} = dH_0 w - \frac{\dot{w}}{w} \pm \sqrt{\left(\frac{\dot{w}}{w} - dH_0 w\right)^2 - F(H_0)}$$

- Constant equation of state: $\dot{\Phi} = 0$ or $\dot{\Phi} = dH_0 w_0$

Einstein Frame

Let's write the Hubble parameter in the Einstein frame, where we typically do cosmology:

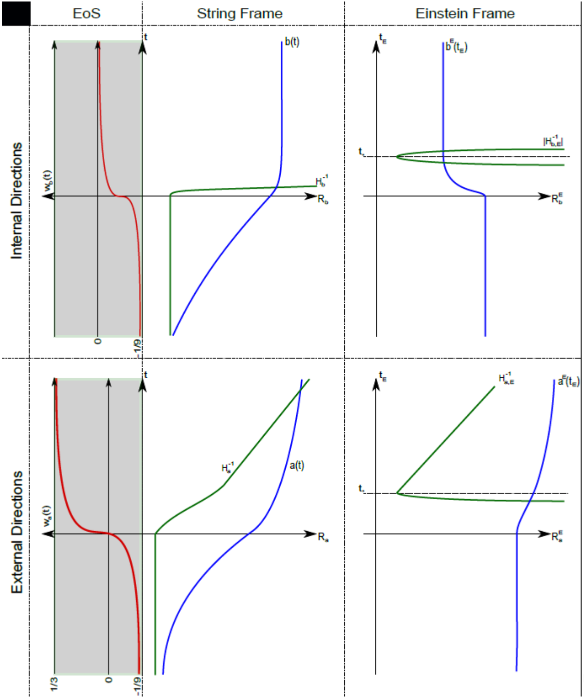
$$H_E(t_E) \sim -a(t)^{\frac{d}{d-1}} e^{\frac{\Phi}{d-1}} (\dot{\Phi} + H)$$
$$\frac{dH_E(t_E)}{dt_E} \sim \ddot{\Phi} + \dot{H} + \frac{1}{d-1}(dH + \dot{\Phi})(H + \dot{\Phi})$$

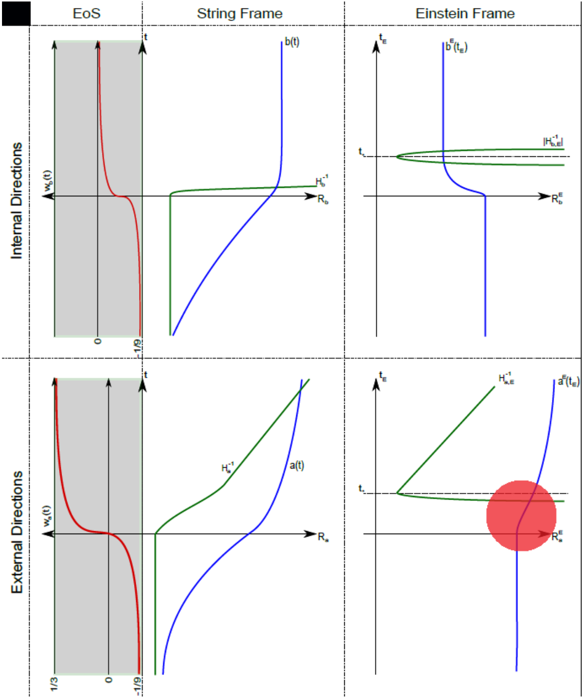
We immeditally see that if $\dot{\Phi} = -H_0$ we have dS in the String frame and a static universe in Einstein frame. But we have just seen that $\dot{\Phi} = dH_0 w_0$, so that would imply precisely a **winding equation of state**: $w = -1/d$.

Rose-Tinted Model of the Early Universe

(Bernardo, Brandenbeger, GF - hep-th/2005.08324 and hep-th/2007.14096)

- First we need to improve the formalism to tackle a class of anisotropic models, since critical strings demand higher dimensional spaces with internal directions that should be small (and static) in order to make contact with our $4D$ universe
- Given we cannot treat a continuous evolution of the EoS, we model the dynamics by different stages with a constant EoS: winding, pressureless, radiation
- We start off with a compactified dS_{10} solution in the String frame with a winding EoS (static in Einstein frame) with all directions expanding until their physical radius become of the string size


EoS
String Frame
Einstein Frame
Internal Directions
External Directions
t
b(t)
H_b^-1
R_b
w_b(t)
0
-1/9
t_E
b^E(t_E)
|H_b,E^-1|
t_*
R_b^E
H_a^-1
a(t)
R_a
w_a(t)
1/3
H_a,E^-1
a^E(t_E)
R_a^E



EoS
String Frame
Einstein Frame
Internal Directions
External Directions
$w_b(t)$
0
-1/9
t
b(t)
H_b^{-1}
R_b
t_E
$b^E(t_E)$
$|H_{b,E}^{-1}|$
t_*
R_b^E
$w_a(t)$
1/3
0
-1/9
t
H_a^{-1}
a(t)
R_a
t_E
$H_{a,E}^{-1}$
$a^E(t_E)$
t_*
R_a^E

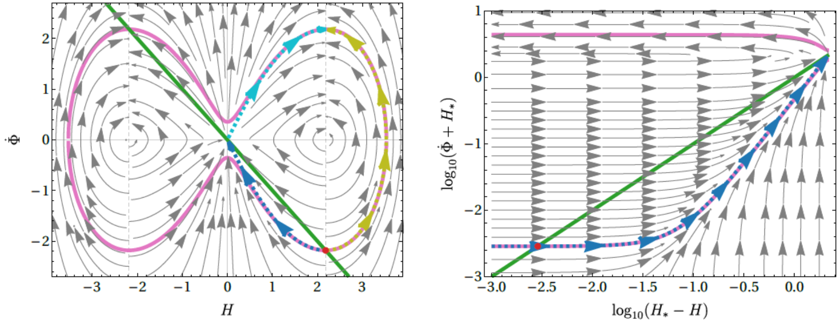
Energy Conditions

(Quintin, Bernardo, GF - hep-th/2105.01083)

- SEC violation in the Einstein frame can be achieved with a positive pressure thanks to the tower of α' corrections: accelerated expansion easily obtainable
- For NEC violations, hints that it is possible in the high-curvature regime with $\alpha' H^2 \sim \mathcal{O}(1)$, where the corrections become crucial

Non-singular bouncing cosmologies

(Quintin, Bernardo, GF - hep-th/2105.01083)



FRG ansatz:

$$F(H) = -dH^2 - 1 - \frac{23}{12}H^2\left(\frac{3}{2}+H^2\right) + \frac{\sqrt{2}}{|H|}(1+H^2)^{3/2}\arctan\left(\frac{|H|}{\sqrt{2(1+H^2)}}\right)$$

Kalb-Ramond fields

(Bernardo, Chouha, GF - hep-th/2104.15131)

- We extend the cosmological ansatz to include also the flux associated with the B-field
- The single-trace EOMs have their symmetry enhanced: $O(2)$ symmetry group, allowing rotations between the metric and the B-field
- Considering the structure of the lowest-order equations in the presence of matter, we are able to add matter to the framework
- Among the solutions found, we have shown that despite the metric being isotropic, the formalism allows to find non-perturbative solutions in α' with an anisotropic B-field
- We also argue that the presence of the B-field is needed to modulate the transition between winding and momentum modes, crucial to a cosmological model based on a gas of strings

In 5 years at a dinner...

- We have equations to talk about string cosmology in non-perturbative regimes **including** the string scale
- They depend on an infinite number of unknown coefficients... **BUT**
  - They are useful to show that likely most perturbative results break down closer to the string scale
  - We found no obstructions to build controversial solutions like de Sitter non-perturbatively (both in Einstein and String frames) and the emergent scenario may naturally be realized
- The framework allows many different cosmological solutions, including the construction of a stringy early universe model with 4 large dimensions that may stand as an alternative to inflation
- Non-singular bounces might be realizable

Beyond the horizon...

- To build other realistic cosmological scenarios (de Sitter with exit mechanism, etc)
- Extend the formalism such that full $O(d,d)$ symmetry is implemented
- Other compactifications scheme? We made progress only for asymptotically static internal dimensions
- To consider cosmological perturbations in the model

The end.

Thank you!