Scalar-tensor mixing from icosahedral inflation

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Mainly based on Alberto Nicolis & GS - arXiv: 2011.04687

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Outline

- Introduction
  - Solid inflation and Icosahedral inflation
- Non-minimal coupling and anisotropic power spectrum
- Mixed tensor – scalar power spectrum
- Observational implications
- Conclusion

Introduction – Solid Inflation

Endlich, Nicolis & Wang, 2012

- Solid inflation is described by coupling a triplet of scalar fields to gravity, with the triplet satisfying the following symmetries,

$$\phi^I \to \phi^I + a^I\,, \quad \phi^I \to O^I{}_J \phi^J$$

  where $O^I{}_J$ is a rotation and a^I is a constant.
- The building block of a solid EFT, to the lowest order, is

$$B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$$

- The total action, upon coupling to gravity, is

$$S_0 = \int d^4x \sqrt{-g}\left[\tfrac{1}{2} M_P^2 R + F\big(B^{IJ}\big)\right]$$

- Such an action admits FRW solutions for the metric.

Introduction – Solid Inflation

- Such an action admits FRW solutions for the metric, with the background values of scalar fields being

$$\langle \phi^I \rangle = x^I$$

- The background values of the metric and the scalar fields are invariant under a combination of spatial translation/rotation and internal translation/rotation.
- This is the “unbroken subgroup”, under which the action for the perturbations,

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{FRW}} + h_{\mu\nu} \ , \qquad \phi^I = \langle \phi^I \rangle + \pi^I \ ,$$

is manifestly invariant.

Introduction – Solid Inflation

- The action up to two derivatives in perturbations is then given by

$$S_{(2)} = S_\gamma + S_L + S_T \ ,$$

where

$$S_\gamma = \tfrac{1}{4} M_{\rm Pl}^2 \int dt\, d^3x\, a^3 \left[\tfrac{1}{2}\dot\gamma_{ij}^2 - \tfrac{1}{2a^2} \big(\partial_m \gamma_{ij}\big)^2 + 2\dot H c_T^2\, \gamma_{ij}^2 \right]$$

$$S_T = M_{\rm Pl}^2 \int dt \int_{\vec k} a^3 \left[\frac{k^2/4}{1 - k^2/4a^2 \dot H} \left|\dot\pi_T^i\right|^2 + \dot H c_T^2\, k^2 \left|\pi_T^i\right|^2 \right]$$

$$S_L = M_{\rm Pl}^2 \int dt \int_{\vec k} a^3 \left[\frac{k^2/3}{1 - k^2/3a^2 \dot H} \left|\dot\pi_L - (\dot H/H)\pi_L\right|^2 + \dot H c_L^2\, k^2 \left|\pi_L\right|^2 \right] .$$

with

$$h_{ij} = a(t)^2 \exp\left(\gamma_{ij}\right) , \qquad \partial_i \gamma_{ij} = \gamma_{ii} = 0$$

$$\pi_j = \frac{\partial_j}{\sqrt{-\nabla^2}} \pi_L + \pi_T^j \ , \qquad \vec\nabla \cdot \vec\pi_T = 0$$

Introduction – Icosahedral Inflation

Kang & Nicolis, 2015

- Specialize from a generic SO(3) element to a discrete subgroup,

$$\phi^I = D^I{}_J \phi^J$$

- Can we make the system intrinsically anisotropic, while giving isotropic background and scalar spectrum for perturbations?
- Quick answer: Yes!
- More elaborated: it is possible to construct a six-index tensor that is anisotropic, and invariant under the icosahedral group, given by

$$T_{aniso}^{ijklmn} = 2(\gamma+2)\delta^{ijklmn} + (\gamma+1)(\delta^{ijkl}\delta^{mn}\delta^{m\ i+1} + \cdots) + (\delta^{ijkl}\delta^{mn}\delta^{m\ i-1} + \cdots)$$

- At the same time, any two-index or four-index tensors invariant under the icosahedral group are isotropic.
- A closer look reveals that for $S^{ijklmn} \equiv \left(\delta^{ij}\delta^{kl}\delta^{mn} + 14 \text{ other permutations}\right)$ we can write

$$T_{\text{aniso}}^{ijkkmm} \equiv \frac{(\gamma+2)}{7} S^{ijklmn} + T_6^{ijklmn}$$

Non-minimal Coupling

- If we were to investigate only minimal coupling, the six-index anisotropic tensor only affects the three-point functions, leaving the power spectrum isotropic.
- If we allow non-minimal coupling, we can include an interaction term for the tensor modes,

$$S_{\rm int} = -\frac{M_{\rm p}^2}{8}\int d^4x\, a^3\, Z\,\frac{\Delta c_\gamma^2}{a^2} T_6^{ijklmn}\partial_i\gamma_{jk}\partial_l\gamma_{mn}$$

- Here Z and Δc_γ^2 are constant at zeroth order in slow roll, with Δc_γ^2 being a small parameter, and we can set Z equal to 1.
- Such an interaction term can come from

$$\Delta S \sim \frac{1}{M^2}\int d\tau d^3x\, a^4\,(B^{IJ})^{-6}\, T_6\cdot(R^{\mu\nu\rho\sigma}\partial_\mu\phi^I\partial_\nu\phi^J\partial_\rho\phi^K\partial_\sigma\phi^L)^3$$

Non-minimal Coupling

- Start our calculation of power spectrum from (0th order in slow-roll)

$$S_\gamma = \frac{M_\mathrm{p}^2}{8}\int \frac{d\tau d^3k}{(2\pi)^3}\, a^2\Big[\gamma'_{ij}(\vec{k},\tau)\gamma'_{ij}(-\vec{k},\tau) - k^2\gamma_{ij}(\vec{k},\tau)\gamma_{ij}(-\vec{k},\tau) - \Delta c_\gamma^2\, k_i k_l\, T_6^{ijklmn}\gamma_{jk}(\vec{k},\tau)\gamma_{mn}(-\vec{k},\tau)\Big]\,.$$

- With the usual polarization decomposition,

$$\gamma_{ij}(\vec{k},\tau) = \sum_{s=\pm}\gamma^s(\vec{k},\tau)\,\epsilon^s_{ij}(\vec{k})$$

and defining $\gamma_i^s \equiv \gamma^s(\vec{k}_i,\tau)$, after some algebra,

$$M^{ss'}(\vec{k}) \equiv T_6^{ijklmn}\,\hat{k}_i\hat{k}_l\,\epsilon^s_{jk}(-\vec{k})\epsilon^{s'}_{mn}(\vec{k})\ ,\quad \langle\gamma_1\gamma_2\rangle_0 \equiv (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\frac{H^2}{M_\mathrm{p}^2}\frac{1}{k_1^3}$$

$$\langle\gamma_1^s\gamma_2^{s'}\rangle = \left[\delta^{ss'} - \tfrac{3}{4}\Delta c_\gamma^2\, M^{ss'}(\vec{k}_1)\right]\langle\gamma_1\gamma_2\rangle_0$$

Mixed Tensor-Scalar Spectrum

- The same term ΔS that gives rise to anisotropic spectrum of tensor-tensor modes also contains a mixed tensor-scalar term,

$$S_{\text{mix}} = -M_{\text{Pl}}^2 \int dt d^3x \, a \, \Delta c_{\gamma\zeta}^2 \, T_6^{ijklmn} \partial_i \pi_j \partial_k \partial_l \gamma_{mn}$$

- After some tedious algebra, and defining $\zeta = -k\,\pi_L/3$,

$$\langle \zeta \gamma^s \rangle' = \frac{3}{2} \frac{2c_L^3 + 4c_L^2 + 6c_L + 3}{(1+c_L)^2} \cdot T_6^{ijklmn} \, \hat{k}_i \hat{k}_j \hat{k}_k \hat{k}_l \, \epsilon_{mn}^s(\vec{k}) \cdot \frac{\Delta c_{\zeta\gamma}^2}{\epsilon c_L^5} \frac{H^2}{M_{\text{Pl}}^2 k^3}$$

- Compared to the tensor-tensor modes,

$$\langle \zeta \gamma \rangle' \sim \frac{\Delta c_{\zeta\gamma}^2}{\epsilon c_L^5} \frac{H^2}{M_{\text{Pl}}^2 k^3} \sim \frac{\Delta c_{\zeta\gamma}^2}{\epsilon c_L^5} \langle \gamma\gamma \rangle' \, ,$$

  which indicates that the mixed tensor-scalar amplitude is larger than the tensor spectrum for $\Delta c_{\zeta\gamma}^2 \gg r$ where $r \sim \epsilon c_L^5$ for solid inflation.

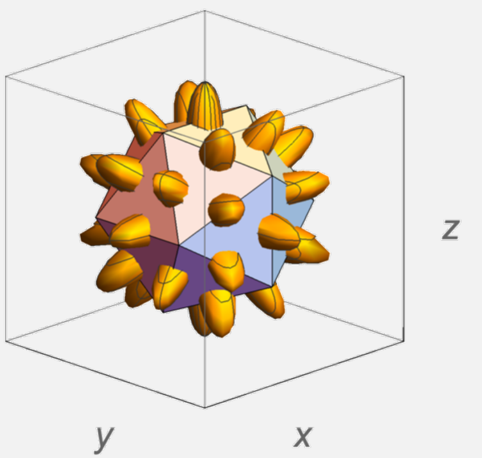
Mixed Tensor-Scalar Spectrum

- Result:

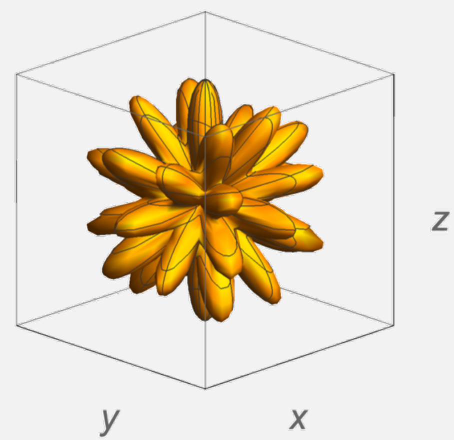
$$\langle \zeta \gamma^s \rangle' = \frac{3}{2} \frac{2c_L^3 + 4c_L^2 + 6c_L + 3}{(1+c_L)^2} \cdot T_6^{ijklmn} \, \hat{k}_i \hat{k}_j \hat{k}_k \hat{k}_l \, \epsilon^s_{mn}(\vec{k}) \cdot \frac{\Delta c^2_{\zeta\gamma}}{\epsilon c_L^5} \frac{H^2}{M_{\rm Pl}^2 k^3}$$

- Visualization:

$$M^{\zeta s}(\vec{k}) \equiv T_6^{ijklmn} \, \hat{k}_i \hat{k}_j \hat{k}_k \hat{k}_l \, \epsilon^s_{mn}(\vec{k})$$



(a) $|M_{\zeta\gamma}|$ overlapping with an icosahedron



(b) $|M_{\zeta\gamma}|$ standing alone

Mixed Tensor-Scalar Spectrum

- Comparison with Anisotropic Inflation,

	Icosahedral Inflation	**Anisotropic Inflation**
Background metric	Isotropic	Anisotropic
Inflaton field	Triplet of scalar fields	One scalar field
Scalar power spectrum	Isotropic	Anisotropic
Scalar-tensor to tensor-tensor ratio	Can be >>1, unrelated to scalar spectrum	<<1, suppressed by small anisotropy

Watanabe et. al. 2009, 2011
Gumrukcuoglu et. al. 2010

Observational Implications

- Our theory is expected to give rise to non-zero T-B and E-B correlations for CMB anisotropies, by breaking full rotational symmetry, while preserving parity.
- $\langle a^{(s)*}_{T/E,lm} a^{(t)}_{B,l'm'} \rangle$ is non-zero if $l = l' \pm n$ for odd n.
- Using $\mathcal{T}^{(s/t)}_{T/E/B,l}(k)$ to represent the transfer functions, we have

$$\langle a^{(s)*}_{T/E,lm} a^{(t)}_{B,l'm'} \rangle = (4\pi)^2 i^{l-l'} (-1)^{l'} \times C \times A_{(l,m),(l',m')} \times \int \frac{dk}{k} \mathcal{T}^{(s)}_{T/E,l}(k) \mathcal{T}^{(t)}_{B,l'}(k)$$

with

$$C = \frac{3}{2} \frac{2c_L^3 + 4c_L^2 + 6c_L + 3}{(1+c_L)^2} \frac{\Delta c^2_{\zeta\gamma} H^2}{\epsilon c_L^5 M^2_{\mathrm{Pl}}} ,$$

$$A_{(l,m),(l',m')} \equiv \int d\Omega_{\hat{k}} \, Y_{lm}(\hat{k}) \left[M^{\zeta+} {}_{-2}Y^*_{l'm'}(\hat{k}) - M^{\zeta-} {}_{2}Y^*_{l'\,m'}(\hat{k}) \right] .$$

Observational Implications

- Both tensor-tensor and scalar-tensor anisotropies can source T-B or E-B correlations, but the scalar-tensor one dominates, if we have $\Delta c^2_{\zeta\gamma} \gg r$.
- Recently there has been discussion about T-B and E-B correlations coming from cosmic birefringence.
- Work in progress to use observational data to constrain our model, as well as comparison with cosmic birefringence…

Conclusion

- We can construct an inflationary model by coupling a triplet of scalar fields, which satisfies internal icosahedral rotational and translational symmetry, to gravity.
- Further allowing for non-minimal coupling, such a theory naturally preserves isotropy for scalar-scalar spectrum, while introducing anisotropies to tensor-tensor spectrum and the bispectrum, as well as non-vanishing scalar-tensor spectrum.
- Scalar-tensor amplitude can dominate over the tensor-tensor amplitude, and feed non-zero T-B and E-B correlations for CMB anisotropies.
- Work in progress to relate to data…