COVARIANT DECOMPOSITION OF THE NONLINEAR GALAXY NUMBER COUNTS AND THEIR MONOPOLE

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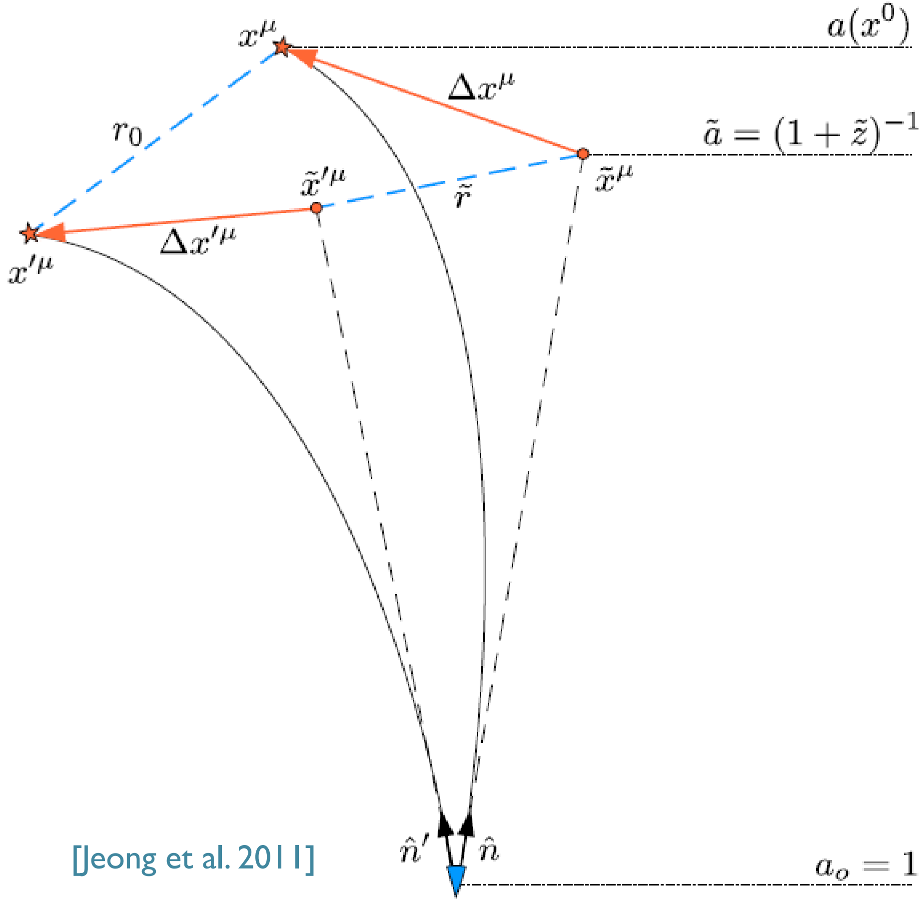
COSMOLOGY FROM HOME 2021

BASED ON

GINAT, DESJACQUES, JEONG & SCHMIDT(2021, FORTHCOMING).

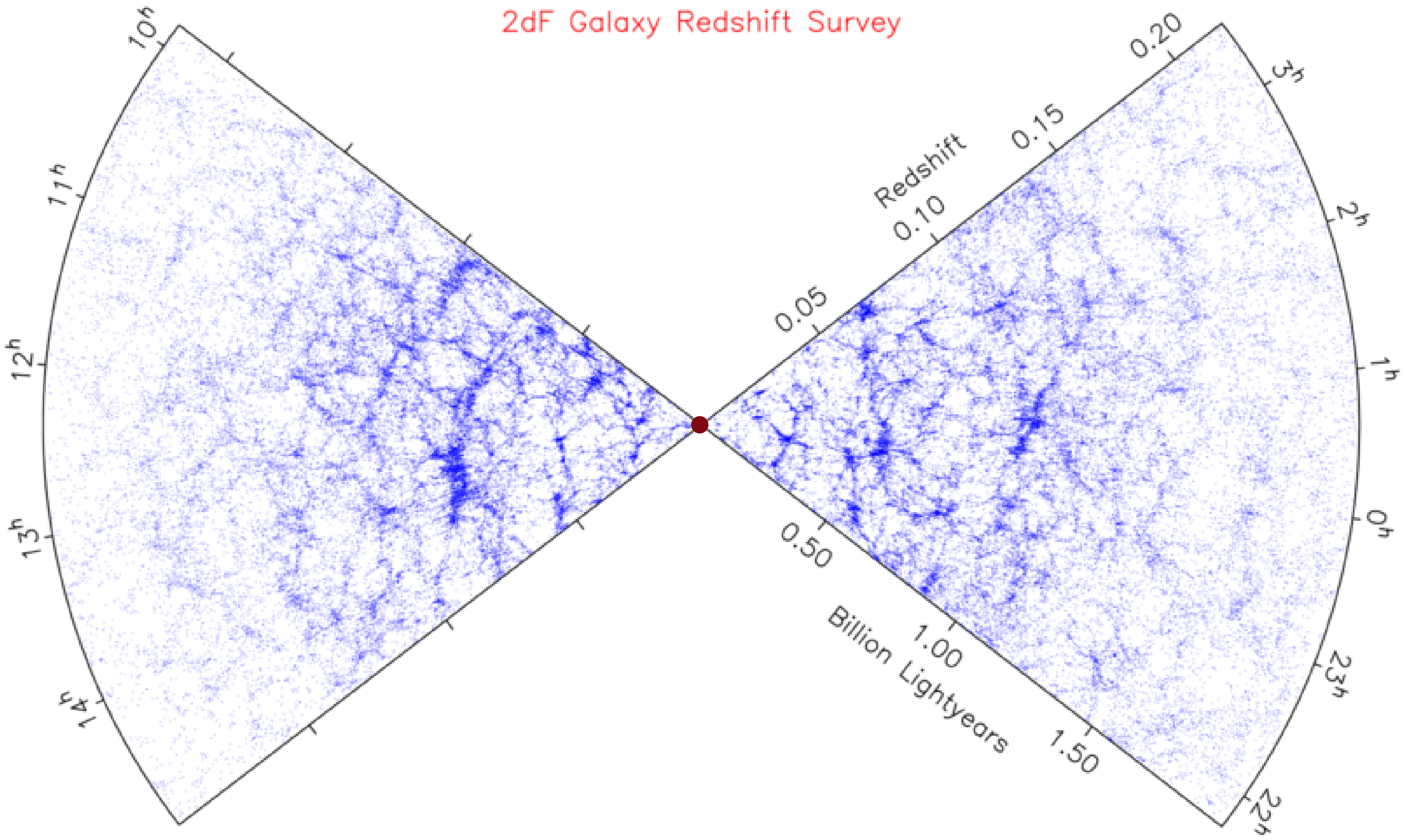
THE UNIVERSE IS ALMOST HOMOGENEOUS AND ISOTROPIC

- The Universe is well-described by an FLRW metric, which is homogeneous and isotropic.
- Corrections enter in the form of perturbations to this metric.
- These imply that the galaxy distribution on the sky is not exactly isotropic.
- Also: light does not travel in straight lines, so the inferred spatio-temporal position of a galaxy is not quite the same as its actual position (projection effects).
- The task at hand: compute the observed galaxy distribution, including all relativistic corrections, to a given order in the cosmological perturbations.
- Bear in mind: the result is an observable quantity, and hence must be gauge-invariant.

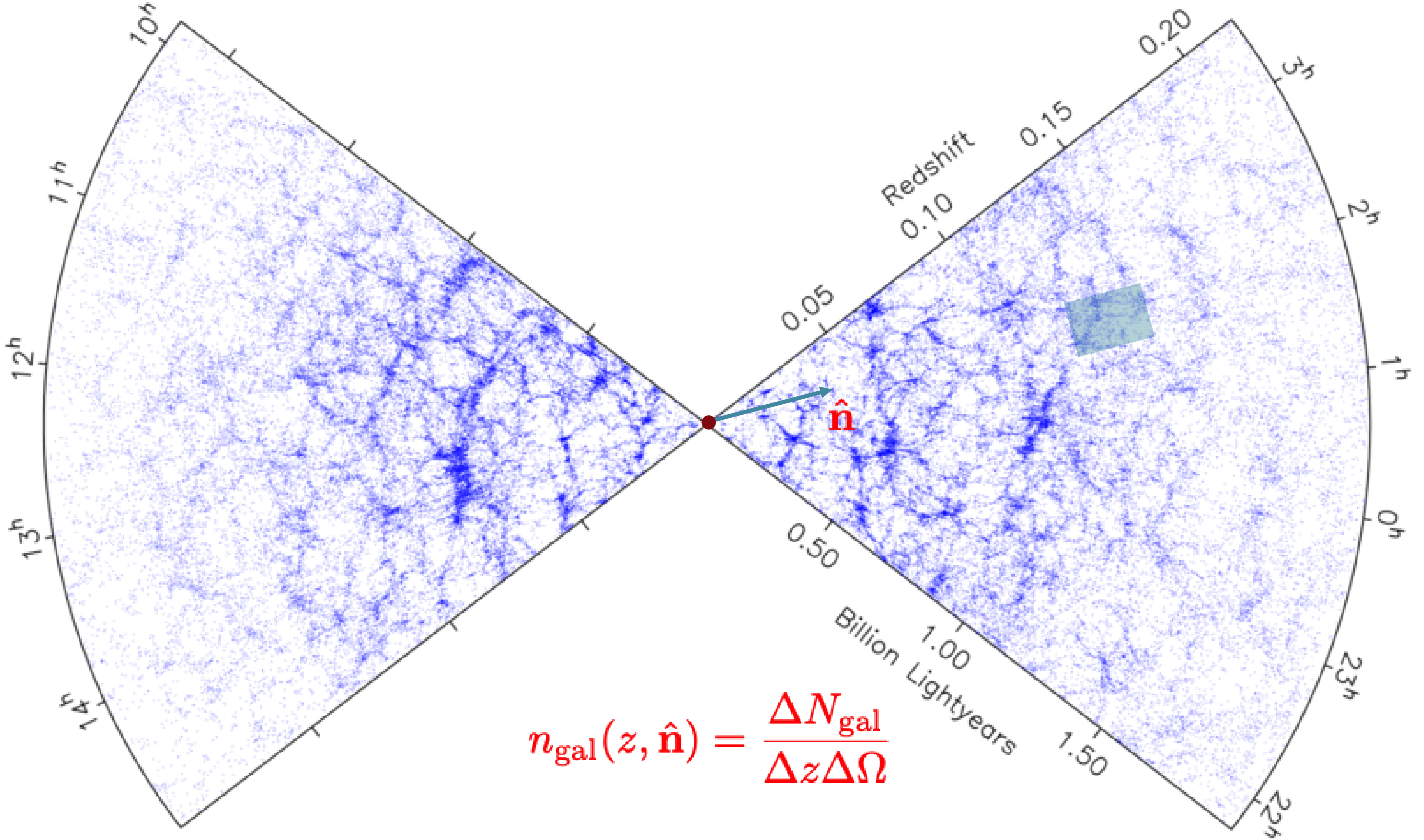


[Jeong et al. 2011]

WHAT WE MEASURE

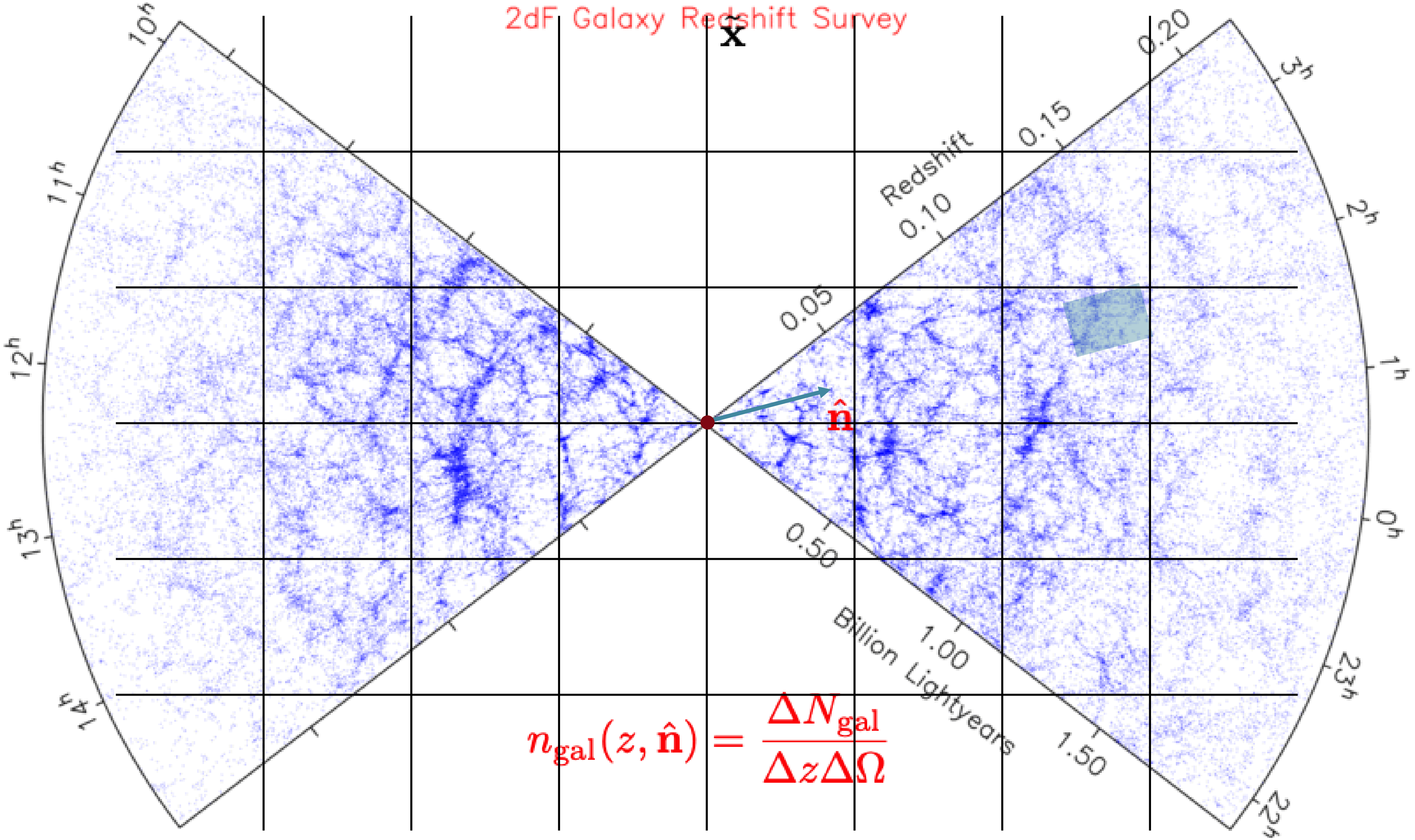


[Peacock et al. 2001]



$$n_{\rm gal}(z, \hat{\mathbf{n}}) = \frac{\Delta N_{\rm gal}}{\Delta z \Delta \Omega}$$

[Peacock et al. 2001]



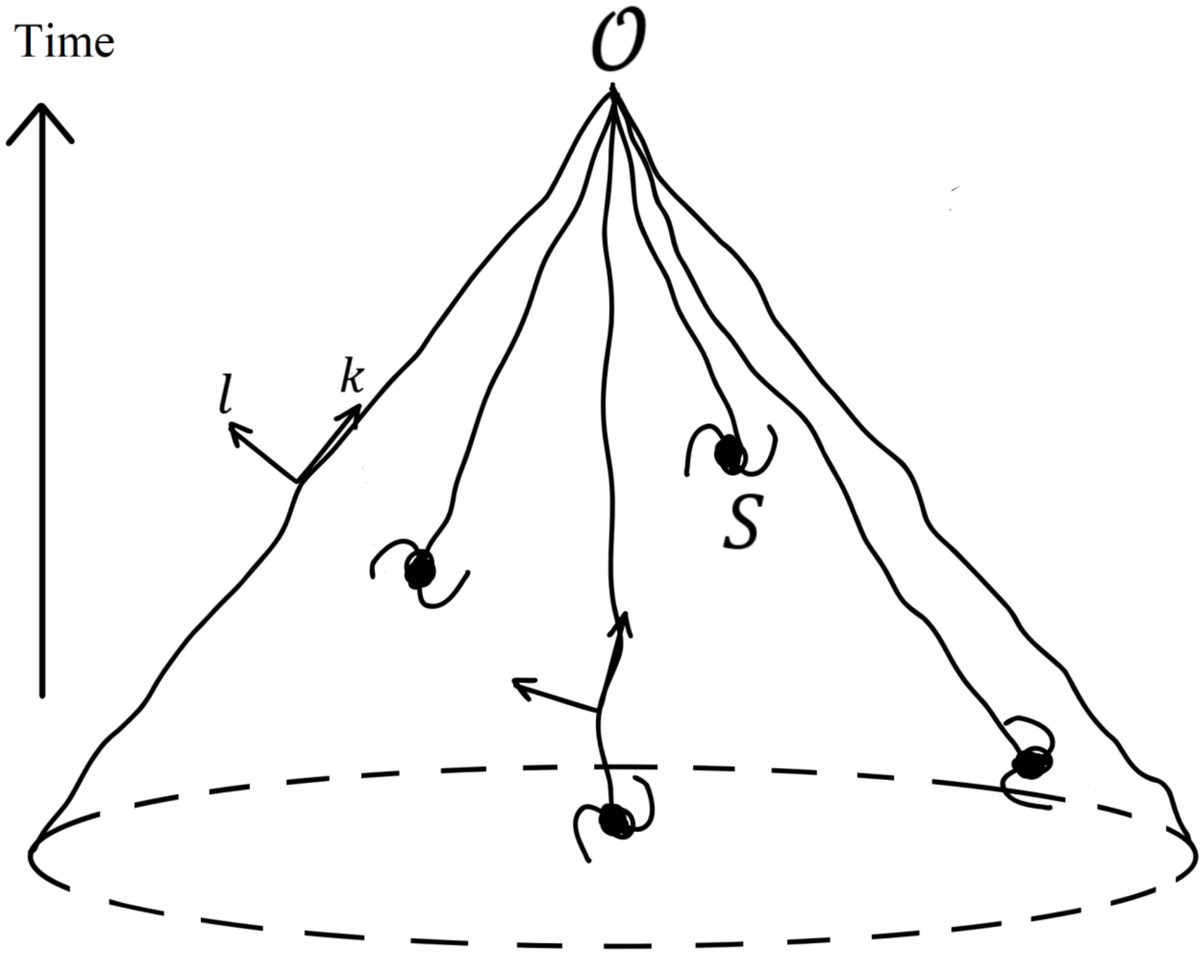
[Peacock et al. 2001]

COSMOLOGICAL OBSERVABLE DEPEND ON THE ENTIRE LINE OF SIGHT

- Cosmological observables, like δ, depend on the entire line of sight from the galaxy to the observer:
- The trajectory of the light used to infer δ is influenced by the cumulative effect of the gravitational field along it (e.g. Yoo et al. 2009, Bonvin & Durrer 2011, Challinor & Lewis 2011, Castorina & Di Dio 2021), including:
  - At the source and observer (boundary/initial conditions)
  - Along the line of sight (e.g. in gravitational redshift).

PAST LIGHT-CONE

- Everything we see is on our past light-cone Σ
- Σ is a function of the proper age of the observer, τ_0.



WHICH CO-ORDINATE SYSTEM SHOULD ONE USE?

- One wishes to characterise the observed galaxy distribution.
- Computing it involves integration over the past light-cone, and the evaluation of tensors on it.
- Hence, it is reasonable to work with a tetrad that is purposely-designed for it.
- Consider a galaxy (S) and the ray of light from it to the observer (O):
  - The ray is a null geodesic, with tangent vector k, directed from S to O.
  - Perpendicular to k, there are two null directions inside Σ; denote their corresponding vectors by $m, \bar{m}$.
  - The additional null direction is denoted by l.
- Together the tetrad $k, l, m, \bar{m}$ form a null tetrad at S (cf., e.g., Penrose & Rindler 1984).
- If $\mathrm{d}U, \mathrm{d}V, \mathrm{d}\zeta, \mathrm{d}\bar{\zeta}$ are their dual 1-forms, the (full, exact) metric is given by

$$\mathrm{d}s^2 = -2\mathrm{d}U\mathrm{d}V - 2\mathrm{d}\zeta\mathrm{d}\bar{\zeta}$$

NULL TETRAD IN MINKOWSKI

$$\mathrm{d}U = \frac{1}{\beta\sqrt{2}}(\mathrm{d}t - \mathrm{d}r)$$

$$\mathrm{d}V = \frac{\beta}{\sqrt{2}}(\mathrm{d}t + \mathrm{d}r)$$

$$\mathrm{d}\zeta = \frac{r}{\sqrt{2}}(\mathrm{d}\theta - \mathrm{i}\sin\theta\,\mathrm{d}\varphi)$$

$$\mathrm{d}\bar{\zeta} = \frac{r}{\sqrt{2}}(\mathrm{d}\theta + \mathrm{i}\sin\theta\,\mathrm{d}\varphi)$$

INTEGRATION ON Σ

- The total number of galaxies in Σ is given by the integrating the galaxy density over Σ.
- The beauty of the above-mentioned null tetrad is that the invariant volume form of Σ is simply

$$\omega_{\tau_0} = -\mathrm{i}\,\mathrm{d}U \wedge \mathrm{d}\zeta \wedge \mathrm{d}\bar{\zeta}$$

- If, in addition, we normalise k such that $k^a = \frac{\mathrm{d}x^a}{\mathrm{d}\lambda}$, along the light-ray, then $\mathrm{d}U = \mathrm{d}\lambda$.
- The galaxy current is $j_a = n_g u_a$, where u_a is the 4-velocity of S, and n_g is the galaxy density. Thus,

$$N(\Sigma) = \int_\Sigma \star j = \int_\Sigma (n_g k^a u_a)\omega_{\tau_0}$$

RELATING THE TETRAD TO INFERRED CO-ORDINATES

- Projection effects are caused by the difference between the inferred 4-position of S and the actual 4-position of S.
- O can construct an inferred (co-moving) co-ordinate system $(\tilde{\eta}, \tilde{\boldsymbol{x}})$, which assign to S the co-ordinates it would have had, given its observed redshift $\tilde{z}$ and sky position $\tilde{n}$, *had the Universe been an FLRW* universe.
- What we need to find to calculate the observed galaxy distribution is therefore to express ω_{τ_0} in the inferred co-ordinates.
- Another advantage of the null tetrad: is has very simple expressions in a locally flat frame of reference at S, $(t_L, \boldsymbol{x}_L)$.
  - Suggests to try to convert directly from $(t_L, \boldsymbol{x}_L)$ to $(\tilde{\eta}, \tilde{\boldsymbol{x}})$.
  - As Σ is 3-dimensional, and ω_{τ_0} is a 3-form, what we're looking for is just a Jacobian!
  - Can be done with the use of cosmic rulers and cosmic clocks (Schmidt & Jeong 2012, Jeong & Schmidt 2014, etc.).

COSMIC CLOCK

- In an FLRW universe, constant observed redshift implies constant proper age; in a perturbed space-time, they are not the same.
- Thus, one can define the cosmic clock (Jeong & Schmidt 2014)

$$T = \ln\left(\frac{a(\tau)}{a(z)}\right)$$

- τ is the proper age of the source, and z is its observed redshift.
- The relations $a(\tau)$ and $a(z)$ are the background relations.
  - Possibly an Alcock-Paczynski effect here.
- As a difference between two observables, T itself is an observable.

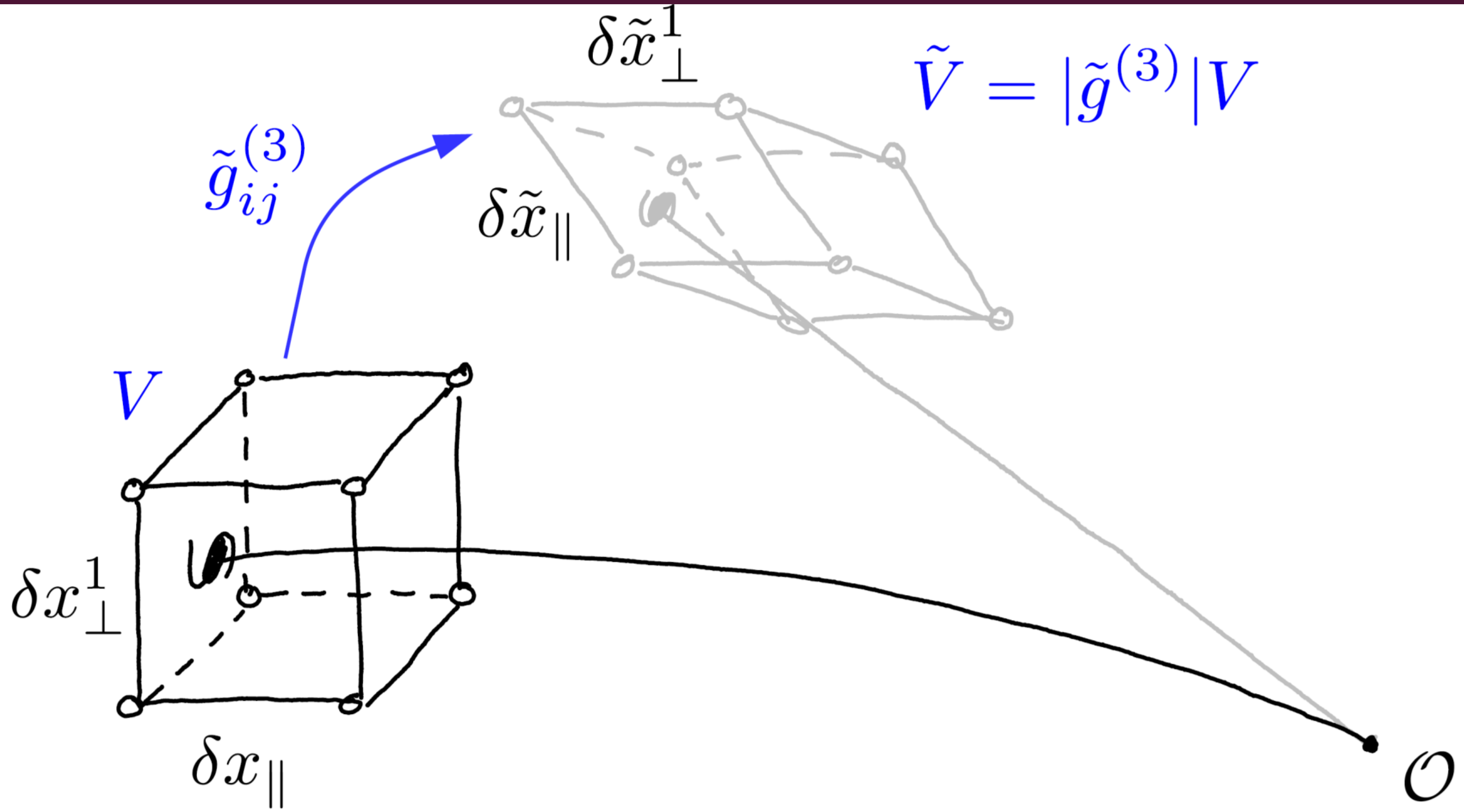
COSMIC RULERS

- Consider now a freely-falling, infinitesimal ruler of intrinsic size $r_0(\tau)$ situated at S.
- In a perturbed space-time, its observed length $\tilde{r}$ is not the same as r_0, and has both longitudinal and transverse (to the line of sight) deformations.
- Define $\tilde{r}_c = \frac{\tilde{r}}{\tilde{a}}$. $\tilde{r}$ can be expanded in r_0 as (Schmidt & Jeong 2012, Jeong & Schmidt 2015):

$$\left(\frac{\tilde{r} - r_0}{\tilde{r}}\right) = \tilde{a}^2 \left(C \frac{\left(\delta \tilde{x}_{\parallel}\right)^2}{\tilde{r}^2} + B_i \frac{\delta \tilde{x}_{\parallel} \delta \tilde{x}_{\perp}^i}{\tilde{r}^2} + A_{ij} \frac{\delta \tilde{x}_{\perp}^i \delta \tilde{x}_{\perp}^j}{\tilde{r}^2} \right)$$

- C, B_i, A_{ij} are cosmic rulers.
- As coefficients in this local metric $\tilde{g}_{ij}^{(3)}$, they, too, are observables, and hence gauge-invariant.

JACOBIANS, AT LAST



JACOBIANS AGAIN

- Let the FLRW volume form in the inferred co-ordinates be $\tilde{\omega}$.
- In the inferred co-ordinates

$$i^*\left[\left(n_g k^a u_a\right)\omega_{\tau_0}\right] = J_T \det g^{(3)}\ \tilde{\omega}.$$

- $\det g^{(3)}$ accounts for the difference between the actual volume element and the inferred one, at constant proper age.
- J_T accounts for the fact that for constant observed redshift (i.e., constant inferred line-of-sight distance), the proper age τ_s is not, in fact, constant.
  - This difference is precisely what the cosmic clock T measures.
- Expressions for them – in the next slides.

VOLUME DISTORTION

$$\det\left(\tilde{g}^{(3)}\right) = (1-\mathcal{C})(1-\mathcal{A}_{mm})(1-\mathcal{A}_{\bar{m}\bar{m}}) - 2\mathcal{B}_m\mathcal{A}_{m\bar{m}}\mathcal{B}_{\bar{m}} \\ - (\mathcal{B}_{\bar{m}}^2 + \mathcal{B}_m^2) + \mathcal{B}_{\bar{m}}^2\mathcal{A}_{mm} + \mathcal{B}_m^2\mathcal{A}_{\bar{m}\bar{m}} - (\mathcal{A}_{m\bar{m}})^2(1-\mathcal{C})$$

AGE DISTORTION

- The size of the ruler at age τ_s is related to its size at age $\tau(\tilde{a})$, where $\tau(a)$ is the *background* time to scale-factor relation, via

$$V_0(\tau_s) = V_0(\tau(\tilde{a})) + \mathcal{T}\frac{\mathrm{d}V_0}{\mathrm{d}\ln\tilde{a}}(\tilde{a}) + \frac{\mathcal{T}^2}{2}\frac{\mathrm{d}^2V_0}{\mathrm{d}(\ln\tilde{a})^2}(\tilde{a}) + \dots$$
$$= \left[\exp\left(s\frac{\mathrm{d}}{\mathrm{d}\ln\tilde{a}}\right)V_0(\tau(\tilde{a}))\right]_{s=\mathcal{T}} .$$

- Whence the Jacobian J_T is simply

$$J_T = \frac{V_0(\tau_s)}{V_0(\tau(\tilde{a}))}$$

DELTA IN TERMS OF RULER QUANTITIES

- Define $\delta_g^{or}(z)$ as the galaxy over-density at a specific observed redshift $\tilde{z}$. Then, combining all of the above:

$$\frac{1+\delta_g^{\rm obs}(\tilde{\boldsymbol{x}})}{1+\delta_g^{\rm or}(\tilde{\boldsymbol{x}})} = J_{\mathcal{T}}(1-\mathcal{C})(1-\mathcal{A}_{mm})(1-\mathcal{A}_{\bar{m}\bar{m}}) - J_{\mathcal{T}}\Big[2\mathcal{B}_m\mathcal{A}_{m\bar{m}}\mathcal{B}_{\bar{m}} + (\mathcal{B}_{\bar{m}}^2+\mathcal{B}_m^2) - \mathcal{B}_{\bar{m}}^2\mathcal{A}_{mm} - \mathcal{B}_m^2\mathcal{A}_{\bar{m}\bar{m}} + (\mathcal{A}_{m\bar{m}})^2(1-\mathcal{C})\Big]$$

- The decomposition expresses δ_{obs} as a combination of gauge-invariant quantities, each of which has a clear physical meaning.
- Requires: evaluating T, C, B_i, A_{ij} to the desired order in the perturbations.
  - Much more manageable than all of δ_g^{obs} directly evaluated to higher order.
- Derivation and more details in Ginat et al. (2021).

AT THE LINEAR LEVEL

- To linear order in the perturbations, we have:
  - $J_T = 1 + 3T$
  - $\det g^{(3)} = 1 - C - M$ (where $M = \mathrm{tr}\, A_{ij}$ is the magnification).

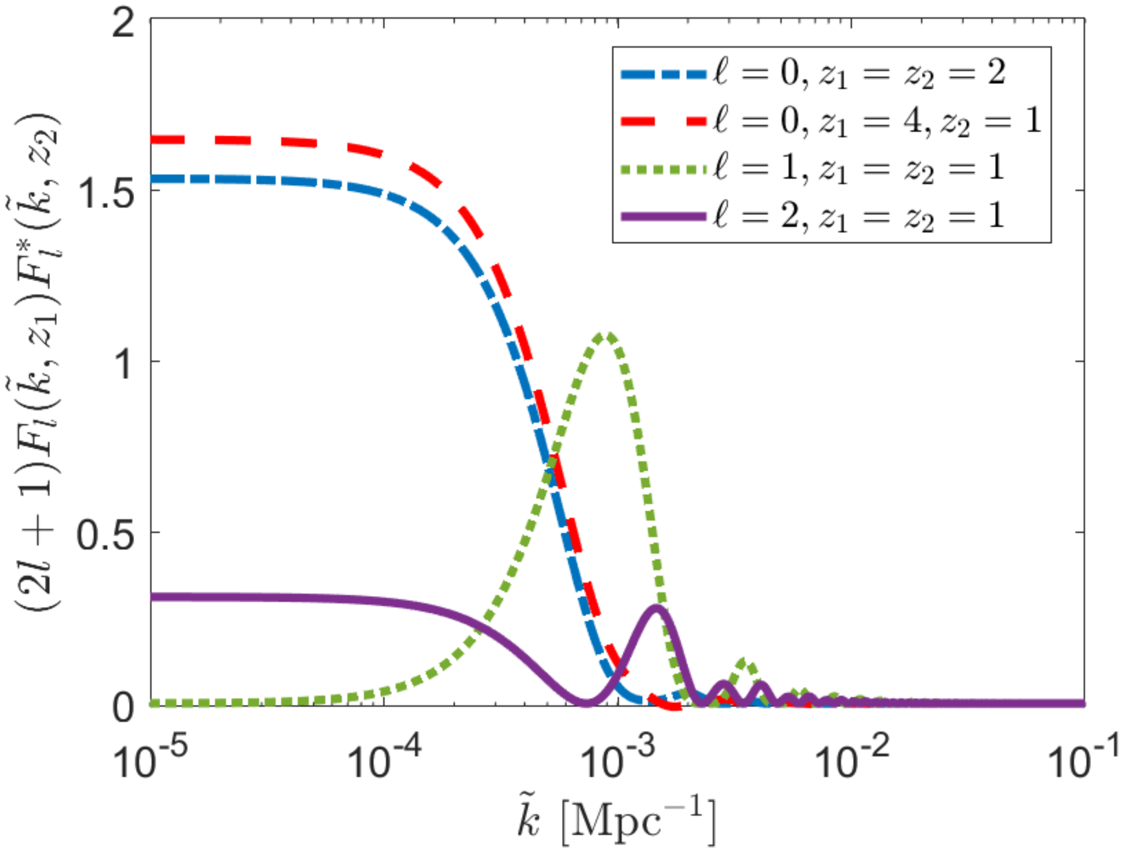
$$\delta_g^{\mathrm{obs}}(\tilde{\boldsymbol{x}}) = -\mathcal{C} - \mathcal{M} + 3\mathcal{T} + \delta_g^{\mathrm{or}}(\tilde{\boldsymbol{x}})$$

- Transform to constant proper time gauge

$$\delta_g^{\mathrm{or}} = \delta_g^{\mathrm{pt}} + \frac{\mathrm{d}\ln \bar{n}_g}{\mathrm{d}\ln \tilde{a}}(\tilde{a})\, \mathcal{T}$$

$\langle (\delta_g)_{\ell m}(z_1)(\delta_g)^*_{\ell' m'}(z_2)\rangle = C_\ell(z_1,z_2)\delta_{\ell\ell'}\delta_{mm'}$

$$C_\ell(z_1,z_2) = \frac{2}{\pi}\int k^2 P_m(k) F_\ell(k\chi_1) F^*_\ell(k\chi_2)\,\mathrm{d}k$$



NUMBER COUNTS

- We now have an expression for the observed density per unit inferred volume.
- Now need expression for density per unit redshift per solid angle.
- Two options:
  1. Require that the Jacobian tends to 1 as $\tilde{z} \to 0$.
  2. Use the background FLRW as a reference.
- The two options differ only by influence of long modes.

CHOICE OF BACKGROUND

- Observer terms can be re-absorbed into the background, to get a "locally-measurable", or CFC, background (cf. Dai et al. 2015, Heinesen 2020)
- Or they can be left as perturbations, using the global FLRW background.
- Leads to two formulae:

$$\text{FLRW}: \quad \tilde{n}_g(\tilde{z}, \tilde{\boldsymbol{n}}) = \bar{n}_g(\tilde{z}) \frac{(1+\tilde{z})\left[d_A(\tilde{z})\right]^2}{E(\tilde{z})\,\mathcal{H}_o} \left\{1 - \mathcal{C} - \mathcal{M} + 3\mathcal{T} + \delta_g^{\text{or}}\right\}$$

$$\text{CFC}: \quad \tilde{n}_g(\tilde{z}, \tilde{\boldsymbol{n}}) = \bar{n}_g(\tilde{z}) \frac{(1+\tilde{z})\left[d_A^{\text{CFC}}(\tilde{z})\right]^2}{E(\tilde{z})\, H_0^{\text{CFC}}} \left\{1 - (\mathcal{C} - \mathcal{C}_o) - (\mathcal{M} - \mathcal{M}_o) + 3\mathcal{T} + \delta_g^{\text{or}}\right\}$$

CFC VS. GLOBAL HUBBLE CONSTANT

- The locally-inferred Hubble constant is (to linear order), the divergence of freely-falling time-like geodesics:

$$H_0^{\rm CFC} \equiv \nabla_\mu u_o^\mu(\tau_o)$$
$$= \mathcal{H}_o\left[1 + \left(\frac{\dot{\mathcal{H}}_o}{\mathcal{H}_o} - \mathcal{H}_o\right)\delta\eta_o - \Psi_o - \frac{1}{\mathcal{H}_o}\dot{\Phi}_o + \frac{1}{3\mathcal{H}_o}\left(\partial_i v^i\right)_o\right]$$

- This arises from the redshift-luminosity-distance relation locally around O

$$\mathrm{d}\tilde{z} = \left(H_0^{\rm CFC} + \mathrm{dipole}(\hat{\boldsymbol{n}}) + \mathrm{quadrupole}(\hat{\boldsymbol{n}})\right)|\mathrm{d}\ell|$$

- Supplemented with background

$$E(\tilde{z}) = \sqrt{\Omega_m(1+\tilde{z})^3 + \Omega_\Lambda + \Omega_{\rm rad}(1+\tilde{z})^4 + \Omega_K(1+\tilde{z})^2}.$$

ANGULAR DIAMETER-DISTANCE

- Same procedure for the angular diameter-distance:
- For small $\tilde{z}$

$$\sqrt{\det D_o} \approx \frac{\tilde{z}}{H_0^{CFC}}$$

where

$$\text{FLRW}: \quad \det \mathcal{D}_o(\tilde{z}, \hat{\boldsymbol{n}}) = \left[d_A(\tilde{z})\right]^2 \left\{1 - \mathcal{M} + 2\mathcal{T}\right\}$$

$$\text{CFC}: \quad \det \mathcal{D}_o(\tilde{z}, \hat{\boldsymbol{n}}) = \left[d_A^{\mathrm{CFC}}(\tilde{z})\right]^2 \left\{1 - (\mathcal{M} - \mathcal{M}_o) + 2\mathcal{T}\right\}$$

$$d_A^{\mathrm{CFC}}(\tilde{z}) \equiv \frac{\mathcal{H}_o}{H_0^{\mathrm{CFC}}} \tilde{a}\tilde{\chi} = \frac{1}{(1+\tilde{z})H_0^{\mathrm{CFC}}} \int_0^{\tilde{z}} \mathrm{d}z \, \frac{1+z}{E(z)}$$

SUMMARY

- A null-tetrad can be constructed to be tailored to the task of calculating projection effects.
- Evaluating the volume form of the observer's past light-cone with this tetrad yields a decomposition of the galaxy distribution into gauge-invariant pieces.
- Each piece has a clear physical meaning as an observable cosmic ruler or clock.
- The decomposition is fully non-linear and exact.
- The expressions naturally account for the monopole and dipole (necessary for gauge-invariance).
- A higher-order evaluation of δ_g^{obs} "only" requires a higher order evaluation of the cosmic rulers/clocks.
- Total number count is gauge-invariant, but the way the background-perturbation split is not.