

# Cosmology from weak lensing alone and implications for the Hubble tension

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arXiv: 2104.12880 (AH 2021)



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Why I love Mpc/h units



# The Hubble tension

SH0ES

$$H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$$



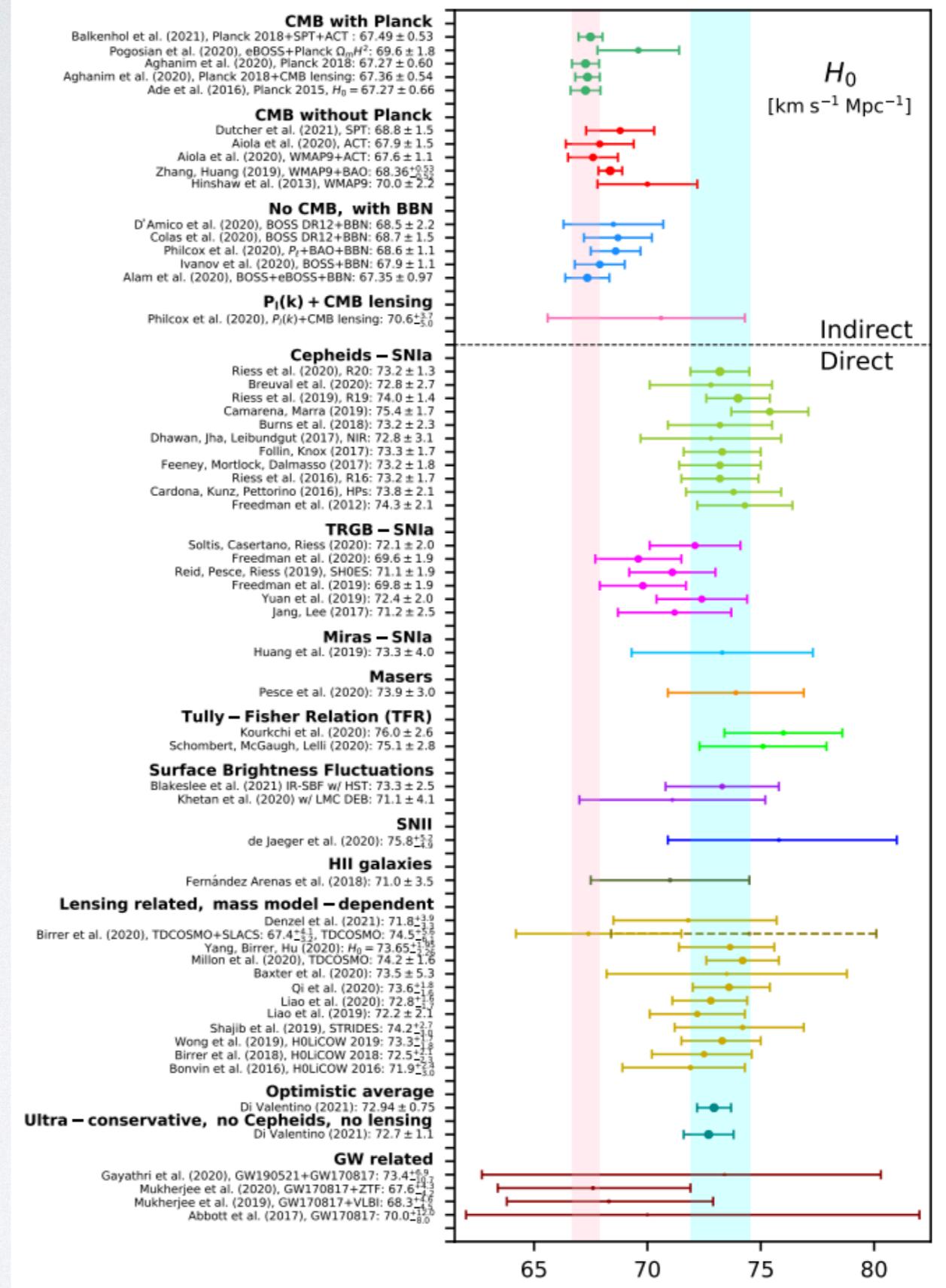
4 - 5 $\sigma$  discrepancy between  
CMB and distance ladder



Planck

$$H_0 = 67.27 \pm 0.60 \text{ km/s/Mpc}$$

- Motivates CMB-independent probes of  $H_0$



Di Valentino et al. 2021

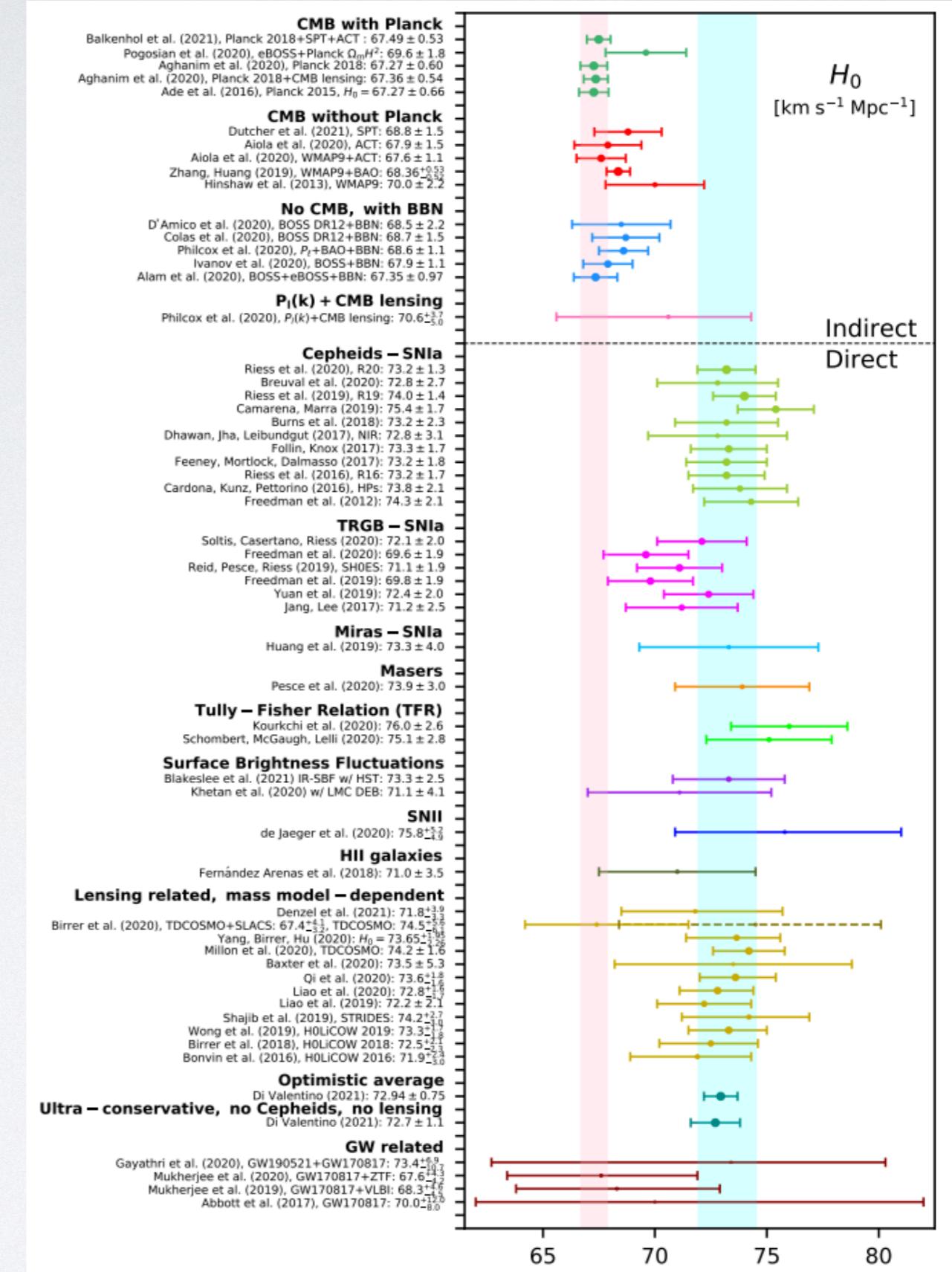
# Late-time cosmology

Weak lensing is maturing into a competitive probe of cosmological parameter space.

Integral part of forthcoming galaxy surveys: Euclid, Rubin-LSST, Roman.

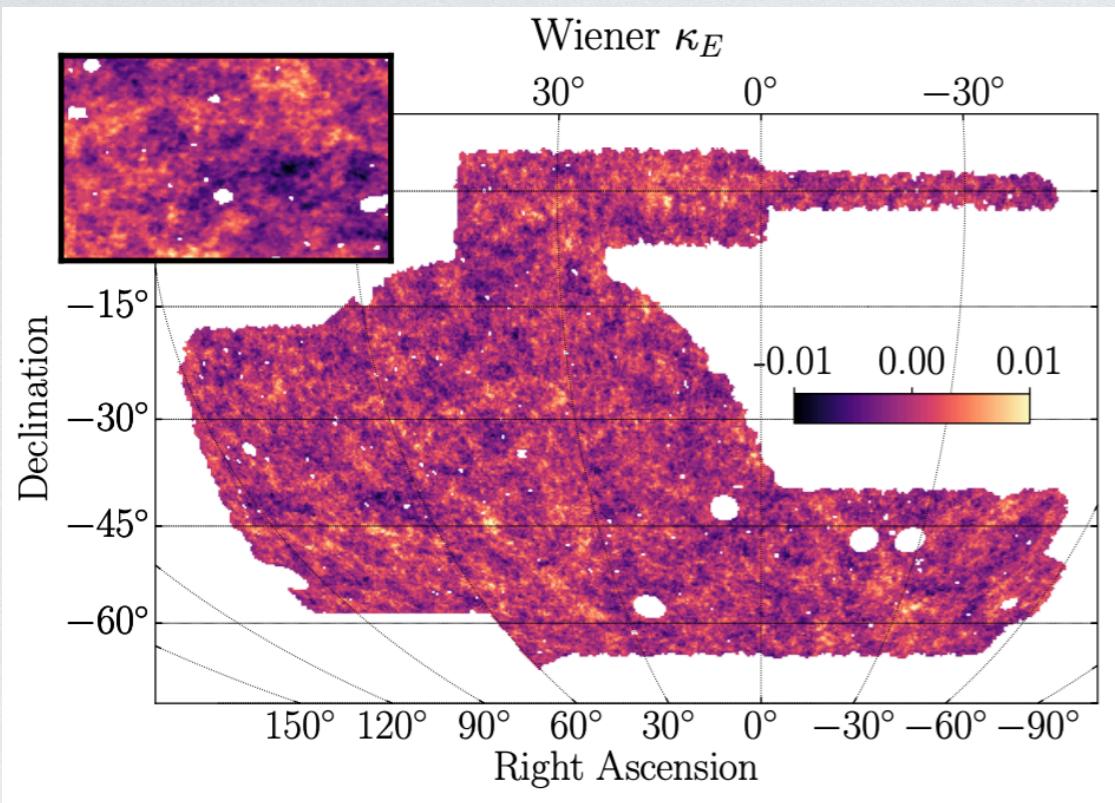
Motivates questions:

- 1) What, if anything, can weak lensing say about the  $H_0$  tension?
- 2) Where does parameter information come from in weak lensing?



Di Valentino et al. 2021

# Weak gravitational lensing



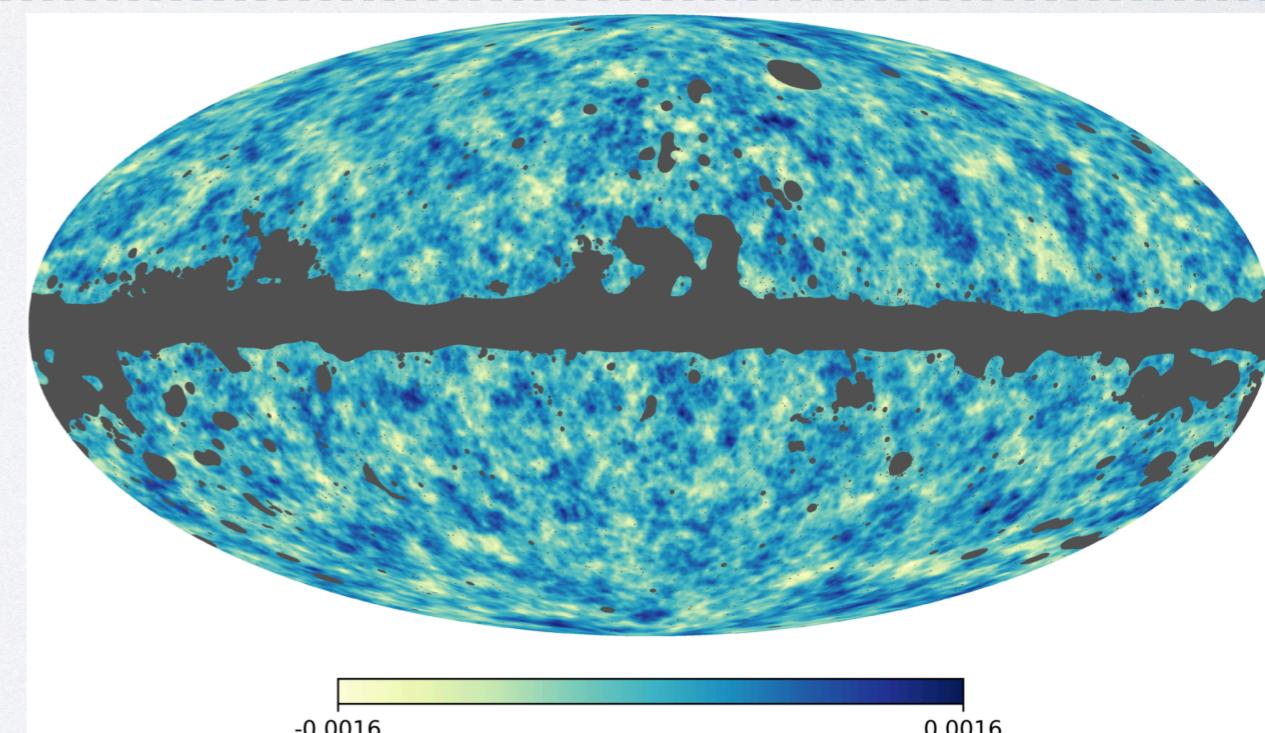
DES Collaboration, Jeffrey et al. 2021

Cosmic shear: coherent distortions in the shapes of galaxies.

- Lensing by **low-redshift** structure spanning **linear, quasi-linear, and non-linear scales**.
- Correlations cleanly measured and modelled over angular separations from a few arcmins to a few degrees.

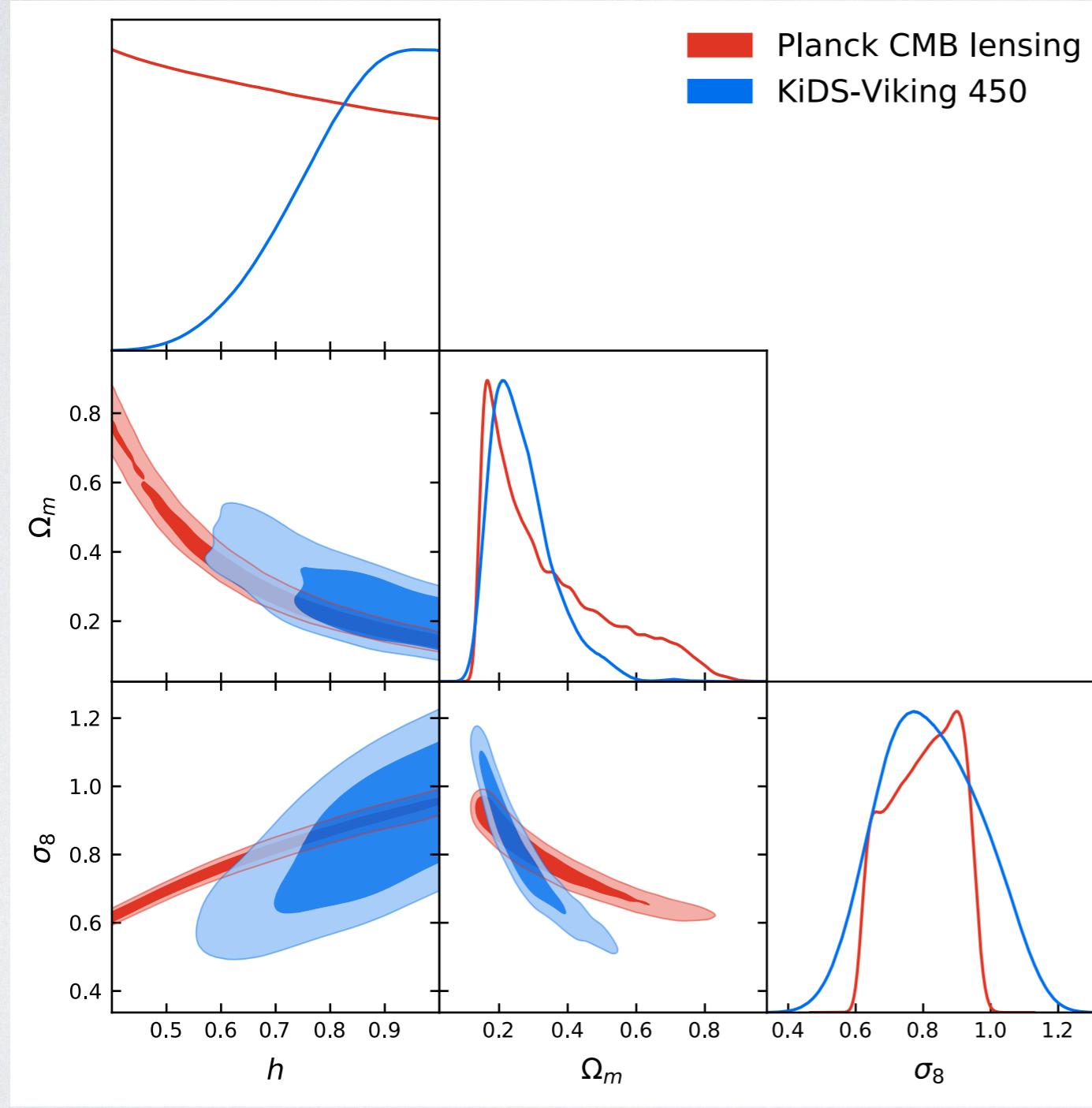
CMB lensing: coherent distortions of CMB anisotropies

- Lensing by **high-redshift** structure on **linear scales**.
- Correlations cleanly measured and modelled over angular separations from a few arcmins to a few degrees.



ESA Planck Collaboration 2018

# $\Lambda$ CDM parameter constraints from weak lensing



## Galaxy weak lensing

Constrain well the combination

$$S_8 \propto \sigma_8 \Omega_m^{0.5}$$

Poor constraints on almost every other parameter combination

## CMB lensing

Constrain well **two** of the combinations

$$\sigma_8 h^{-0.5} \quad \text{Small-scale amplitude}$$

$$\Omega_m^{0.6} h \quad \text{Peak in the matter power spectrum (in projection)}$$

$$\sigma_8 \Omega_m^{0.25} \quad \text{Amplitude and shape}$$

# $\Lambda$ CDM parameter constraints from weak lensing

Do we understand this?

## Galaxy weak lensing

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We understand this

## CMB lensing

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$$\sigma_8 \Omega_m^{0.25} \quad \text{Amplitude and shape}$$

# Why is cosmic shear sensitive to S8 and insensitive to H0?

The building blocks of the cosmic shear data model:

Shear angular power spectrum

$$C_\ell^{\gamma\gamma} = \frac{9}{4}\Omega_m^2 H_0^4 \int_0^{r_{\max}} dr \frac{q(r)^2}{a(r)^2} P_m \left( \frac{\ell + 1/2}{r}; z(r) \right)$$

Lensing kernel + source redshift distribution

$$q(r) = \int_r^{r_{\max}} dr' n_r(r') (1 - r/r')$$

# Why is cosmic shear sensitive to S8 and insensitive to H0?

$$C_\ell^{\gamma\gamma} = \frac{9}{4}\Omega_m^2 H_0^4 \int_0^{r_{\max}} dr \frac{q(r)^2}{a(r)^2} P_m \left( \frac{\ell + 1/2}{r}; z(r) \right)$$

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$$\sim \ell^{-3} \Omega_m^2 \int_0^{z_{\max}} dz \frac{q(z; \Omega_m)^2 [H_0 r(z; \Omega_m)]^3}{E(z; \Omega_m)} \left( \frac{\ell}{r} \right)^3 P_m \left( \frac{\ell}{r}; z \right)$$

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$F(z; \Omega_m) \approx F(z)$  for low-redshift lenses

# Why is cosmic shear sensitive to S8 and insensitive to H0?

$$C_\ell^{\gamma\gamma} \sim \ell^{-3} \Omega_m^2 \int_0^{z_{\max}} dz F(z) \Delta^2 \left( k/H_0 = \frac{\ell}{z}; z \right) \text{ for low-redshift lenses}$$

$$\Delta^2(k) \equiv \frac{k^3}{2\pi^2} P_m(k)$$

The dimensionless matter power spectrum with  $k$  in  $h/\text{Mpc}$  units

**Physically:** lensing introduces no new length scales on top of those already present in the matter distribution.

(Similar arguments for the intrinsic alignment terms in the NLA model)

Aside from overall factors of  $\Omega_m$ , the cosmology dependence of lensing two-point functions is contained entirely within  $P(k)$  when expressed in  $h$ -rescaled units.

# Why is cosmic shear sensitive to S8 and insensitive to H0?

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S8

Usual hand-wavy argument:

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S8

Usual hand-wavy argument:

$$\begin{aligned} \Delta^2 &\sim \sigma_8^2 \\ \implies C_\ell &\sim \sigma_8^2 \Omega_m^2 \end{aligned}$$

i.e. gets the dependence wrong!

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S8

More precise: Jain & Seljak 1997

Using linear theory and the  
Peacock & Dodds 1996 formula  
for the non-linear  $P(k)$

$$\xi_+(\theta) \sim \sigma_8 \Omega_m^\alpha$$

$$\alpha \lesssim 0.5 \quad \theta \lesssim 2'$$

$$\alpha \approx 0.7 \quad \theta > 10'$$

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Non-linearities are important, so it's time we revisited this with a better model: the halo model - also allows the parameter dependence to be understood physically

# Why is cosmic shear sensitive to S8 and insensitive to H0?

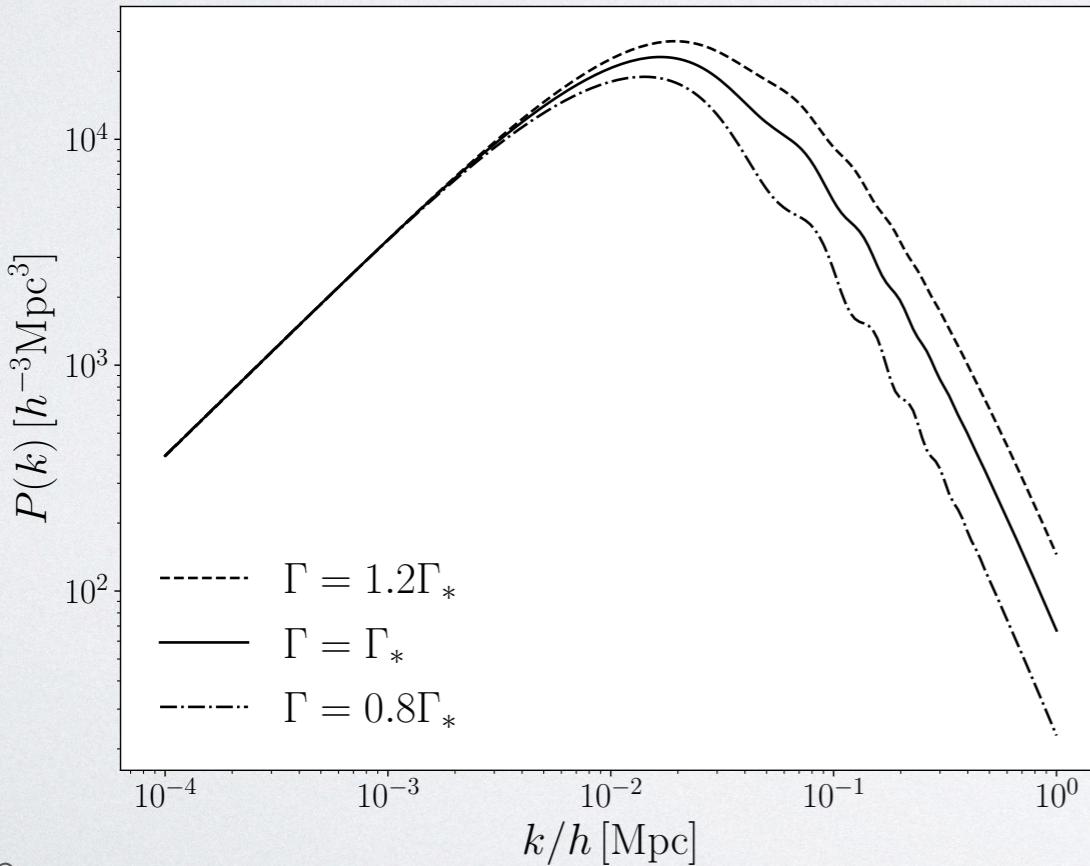
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H0

Fixing  $\Omega_m$  to keep the lensing pre-factors fixed

$\implies \Gamma \equiv \Omega_m h$  changes

(the horizon scale at matter-radiation equality in  $h/\text{Mpc}$  units  
a.k.a. the “shape parameter”)



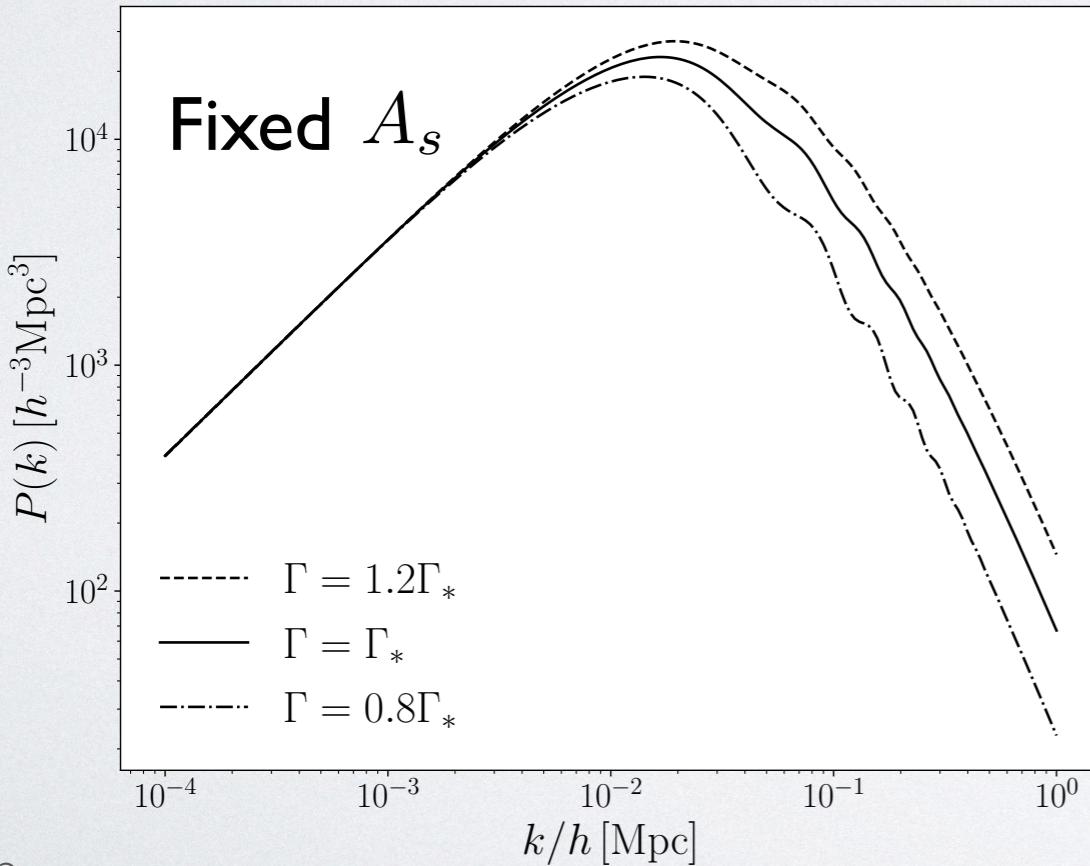
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Fixing  $\Omega_m$  to keep the lensing pre-factors fixed

$\implies \Gamma \equiv \Omega_m h$  changes



$H_0$  changes the small-scale amplitude at fixed  $A_s$

But the amplitude is also controlled by  $\Omega_m$  and  $A_s$  or  $\sigma_8$ .

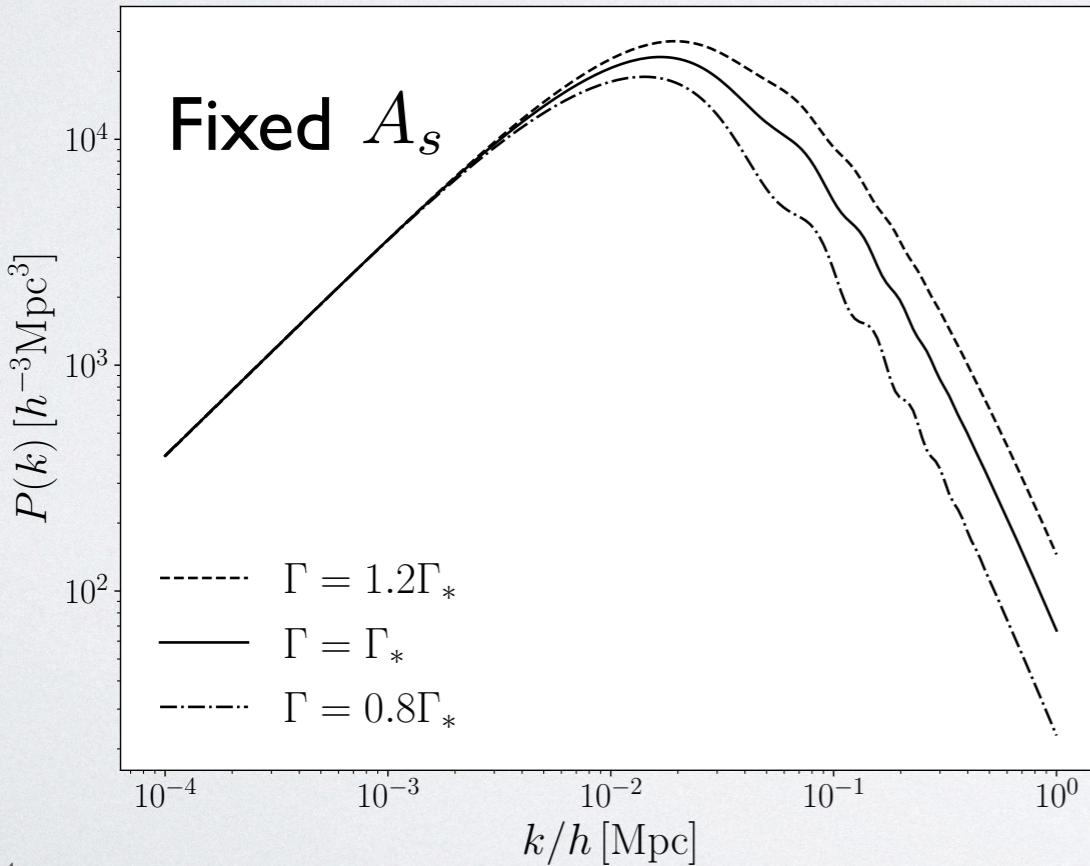
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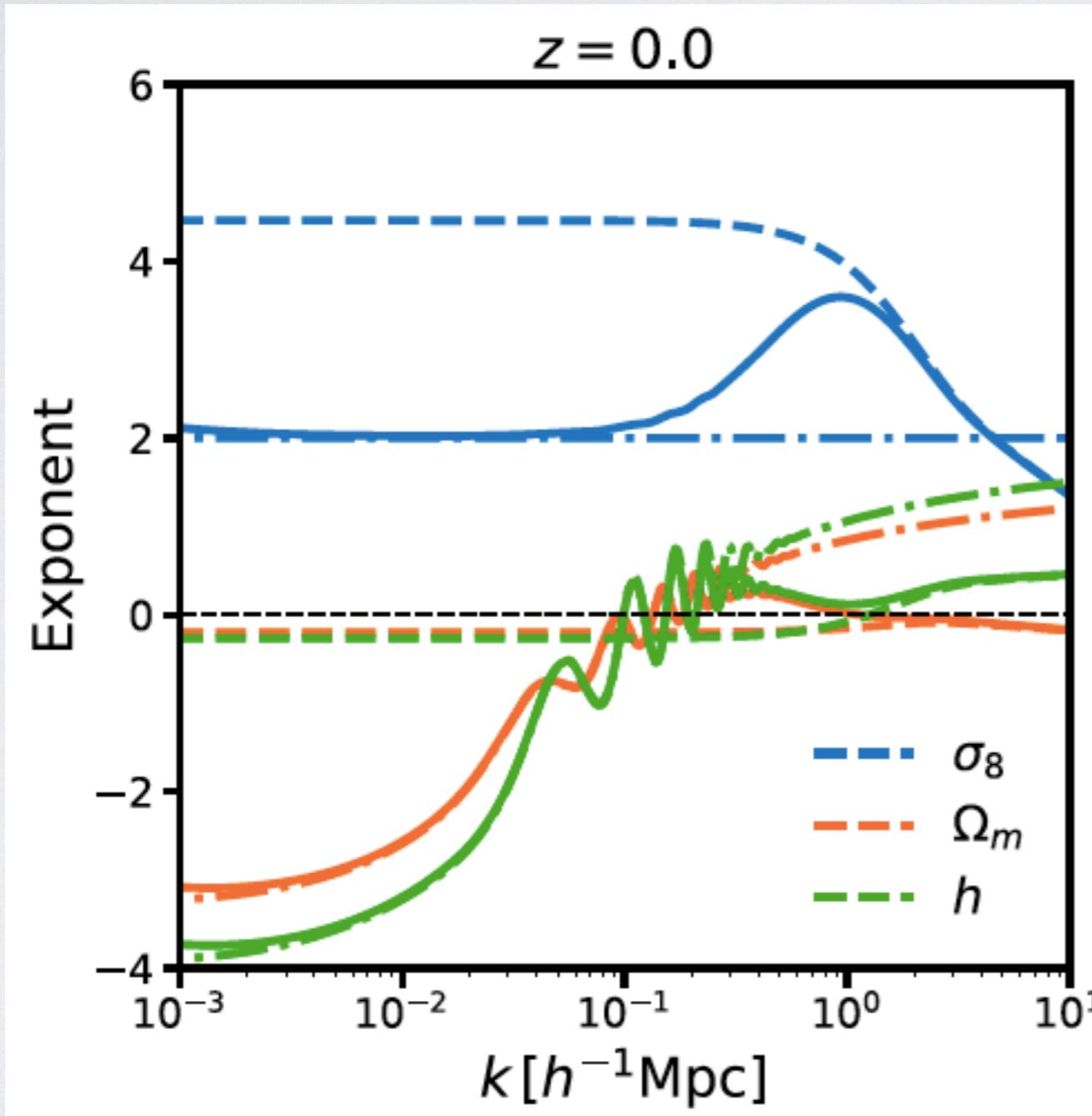
$\implies \Gamma \equiv \Omega_m h$  changes



$H_0$  changes the small-scale amplitude at fixed  $A_s$

Fixing the small-scale amplitude leaves only subtle changes to the shape - not well measured by current surveys!

# Parameter sensitivity of the I-halo term



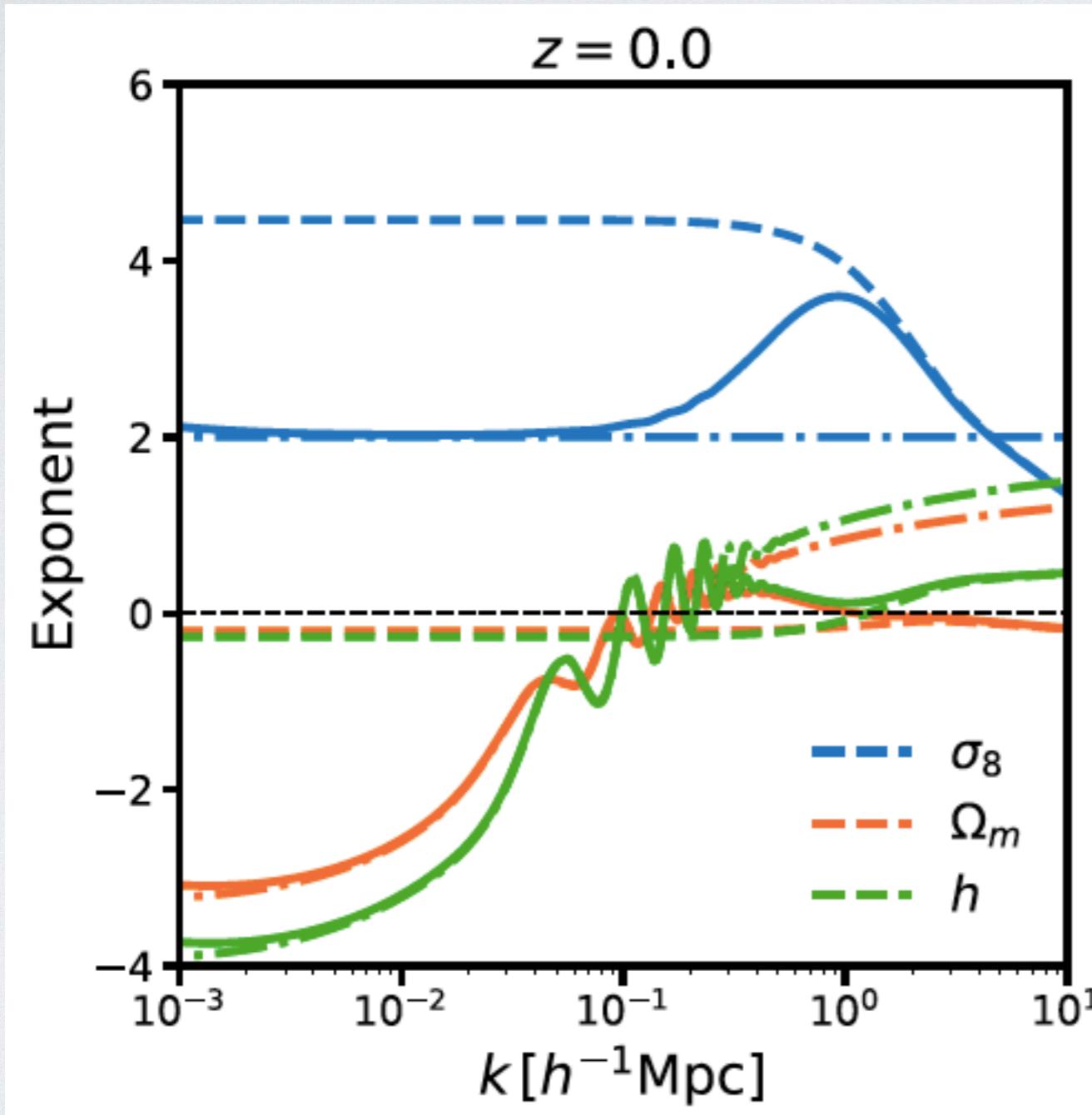
H0 changes the linear shape at fixed amplitude, but further suppression in the I-halo term

$$\lim_{k \rightarrow 0} \Delta_{1H}^2(k) = \frac{(k/h)^3}{2\pi^2} \frac{h^3}{\bar{\rho}^2} \int_0^\infty M^2 n(M) dM$$

$$\Delta^2(k/h) \sim \sigma_8^\alpha \Omega_m^\beta h^\gamma$$

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# Parameter sensitivity of the I-halo term



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$$\lim_{k \rightarrow 0} \Delta_{1H}^2(k) \propto (k/h)^3 \sigma_8^{4.3}$$

Most of the contribution to the I-halo amplitude at  $z=0$  comes from Lagrangian scales around 8 Mpc/h.

# Why is cosmic shear sensitive to S8 and insensitive to H0?

$$C_\ell^{\gamma\gamma} \sim \ell^{-3} \Omega_m^2 \int_0^{z_{\max}} dz F(z) \Delta^2 \left( k/H_0 = \frac{\ell}{z}; z \right)$$

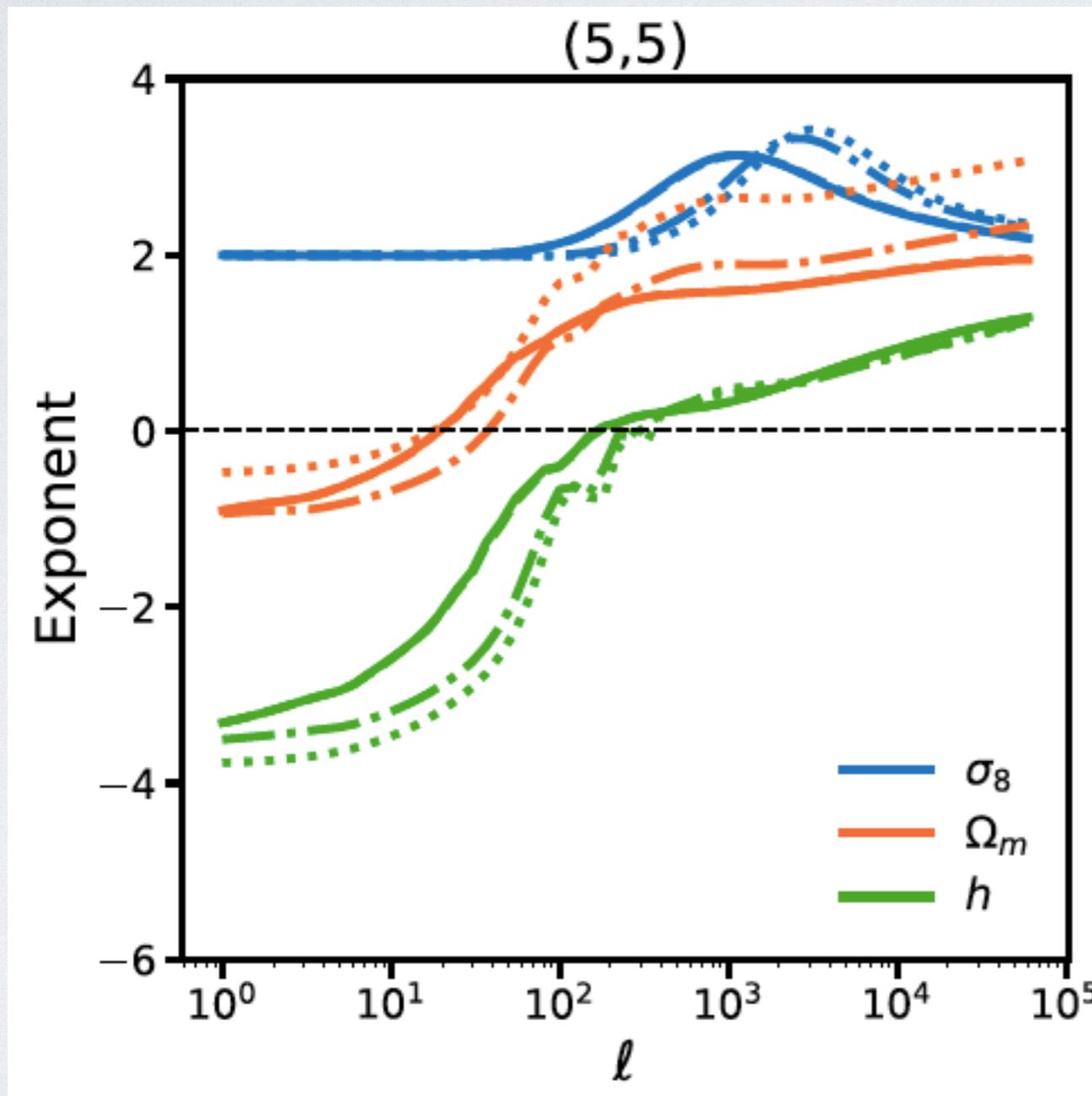
$$\lim_{k \rightarrow 0} \Delta_{1H}^2(k) \propto (k/h)^3 \sigma_8^{4.3}$$

$$\implies C_\ell^{\gamma\gamma} \propto \Omega_m^2 \sigma_8^{4.3} \sim S_8^4$$

On quasi-linear and I-halo scales, h-dependence drops out completely and dependence is entirely on S8

(Not perfect due to baryon smoothing, finite-redshift effects, I-halo shape effects, etc.)

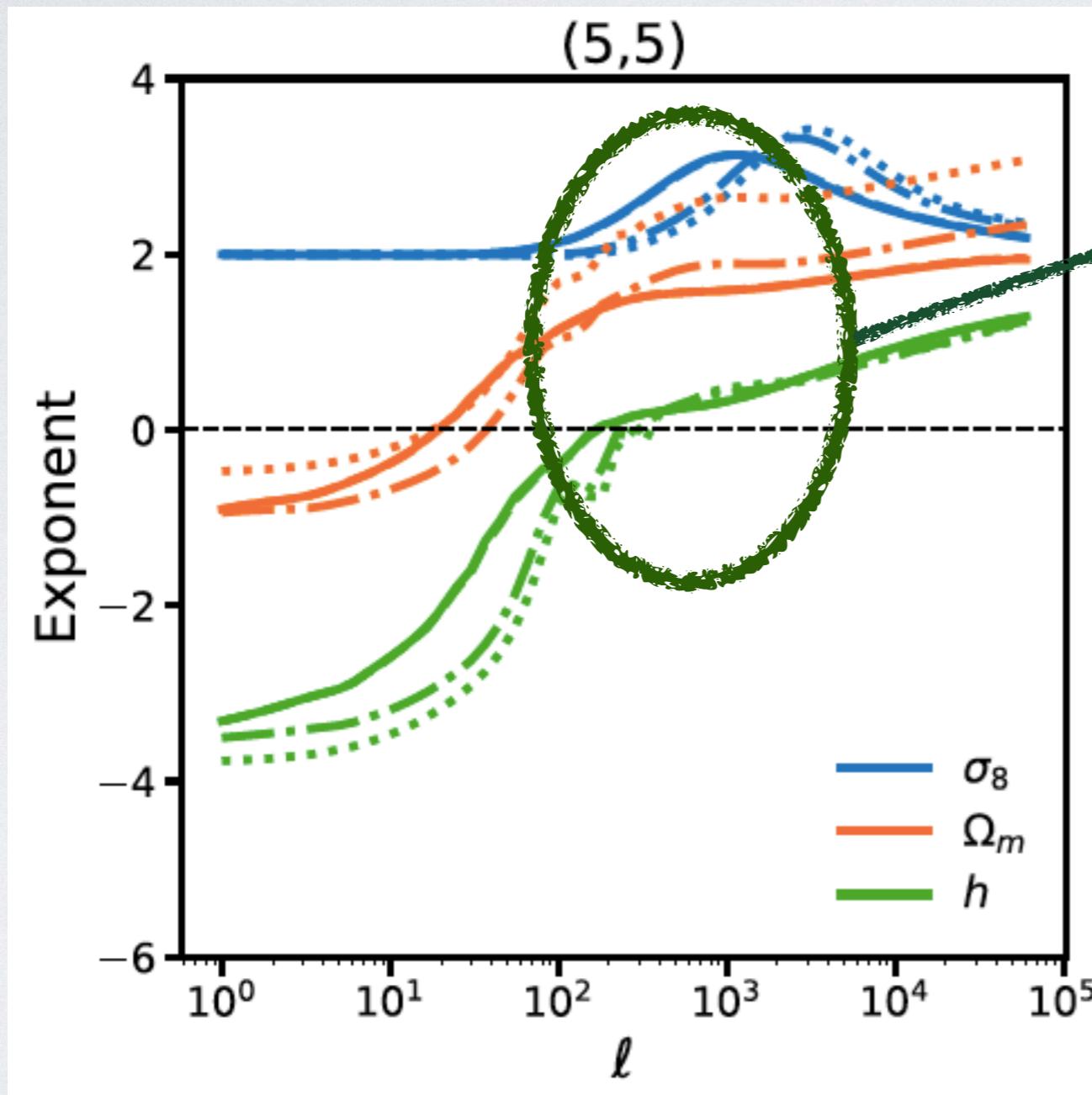
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$$C_\ell \sim \sigma_8^\alpha \Omega_m^\beta h^\gamma$$

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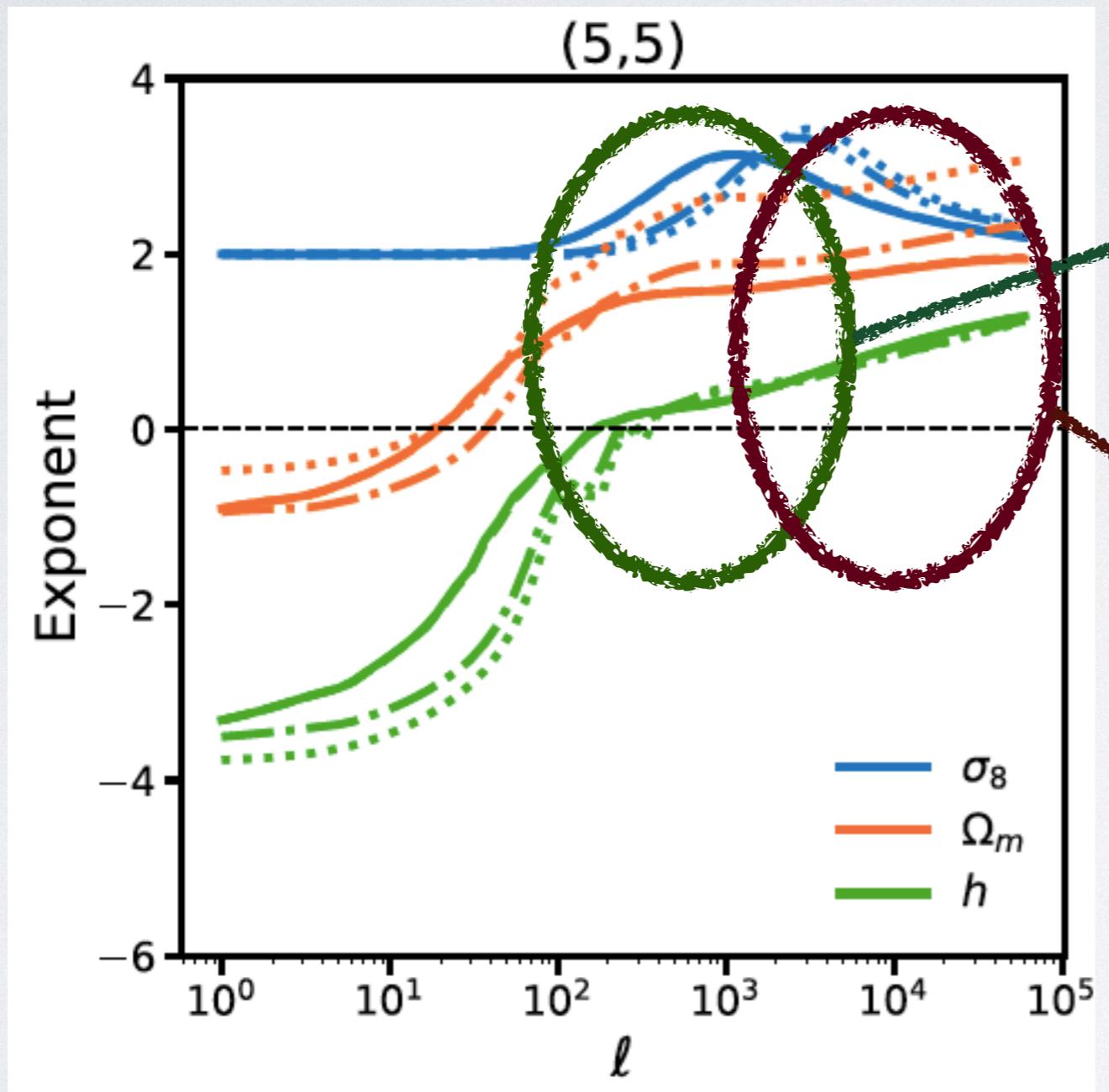


Most of the S/N in current surveys comes from here

$$C_\ell \sim \sigma_8^\alpha \Omega_m^\beta h^\gamma$$

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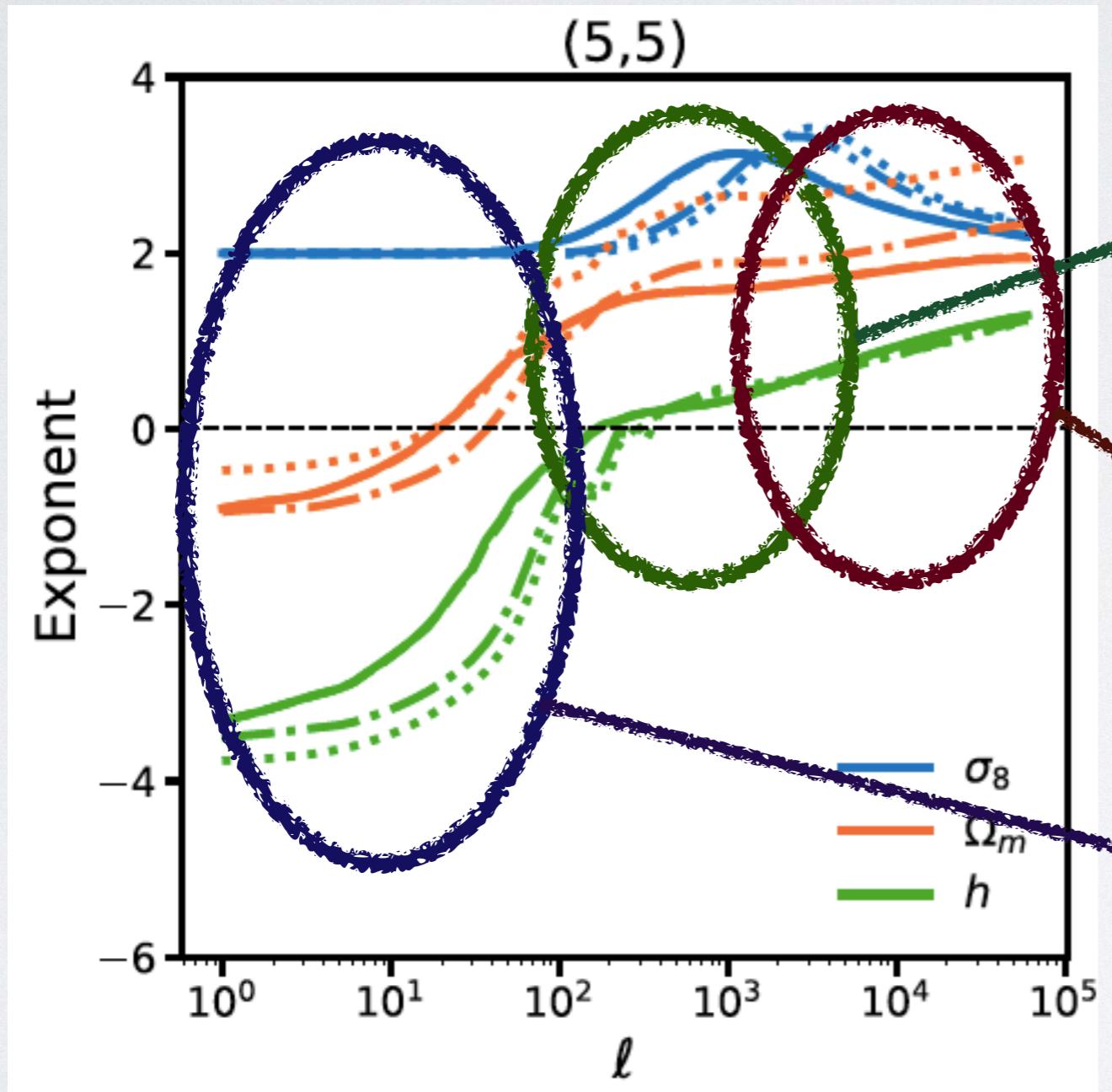
Most of the S/N in current surveys comes from here

Contaminated by baryon feedback!

$$C_\ell \sim \sigma_8^\alpha \Omega_m^\beta h^\gamma$$

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# Why is cosmic shear sensitive to S8 and insensitive to H0?



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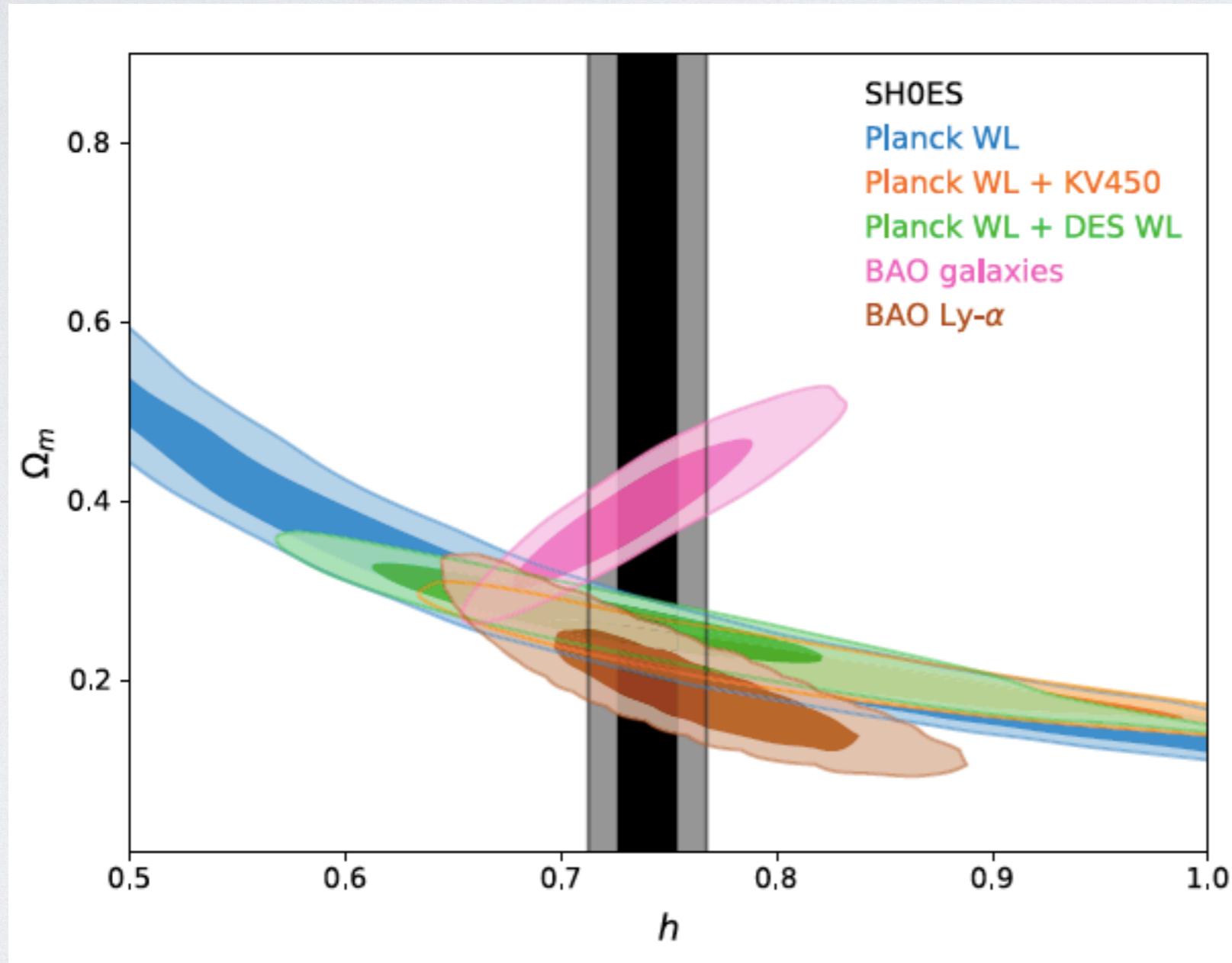
Contaminated by baryon feedback!

Future wide surveys: break degeneracies!

$$C_\ell \sim \sigma_8^\alpha \Omega_m^\beta h^\gamma$$

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# What about combining with other probes?



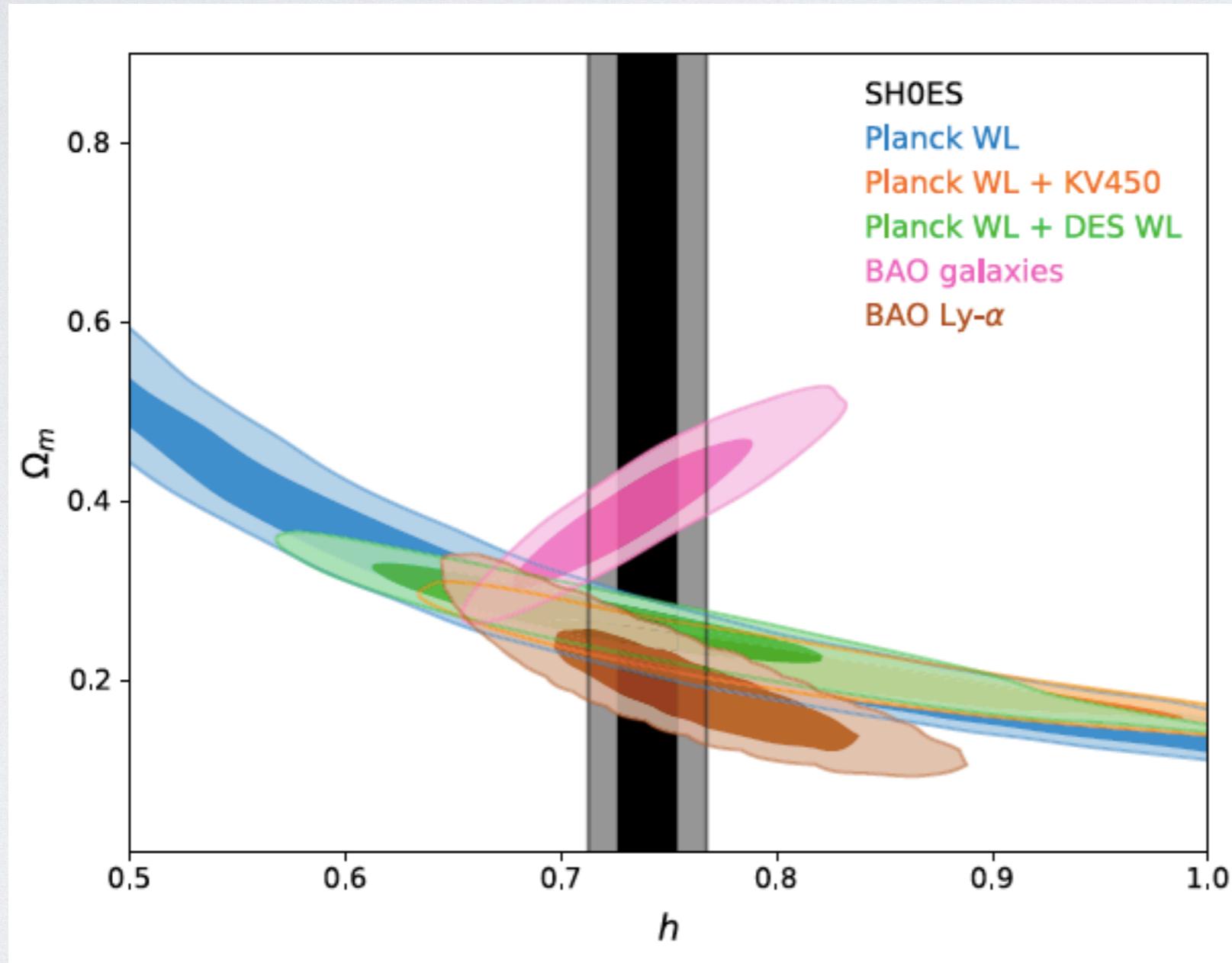
BAO + BBN + WL:

$$H_0 = 67.4 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Galaxy lensing adds basically nothing to  $H_0$  from CMB lensing + BAO.

**Do** get separate  $\Omega_m$  and  $\sigma_8$  constraints.

# What about combining with other probes?



BAO + ~~BBN~~ + WL:

$$H_0 = 70.0 \pm 6.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Galaxy lensing adds basically nothing to  $H_0$  from CMB lensing + BAO.

**Do** get separate  $\Omega_m$  and  $\sigma_8$  constraints.

# Conclusions

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- Current lensing surveys alone **do not provide useful constraints on H<sub>0</sub>.**
- Have shown why current lensing data constrain S<sub>8</sub> well and H<sub>0</sub> poorly, using **analytic arguments** based on the halo model.
- Cleanest probe is the matter-radiation equality scale seen in projection, followed by subtle effects on the shape of the spectrum: partially degenerate with baryon feedback.
- Galaxy lensing adds nothing to the Planck WL + BAO + BBN constraint on H<sub>0</sub>.

# Conclusions

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See the paper for the most complete description of where  $\Lambda$ CDM parameter information comes from in lensing, and why cosmic shear constrains the parameters that it does.

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## Cosmology from weak lensing alone and implications for the Hubble tension

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