Quadratic estimators for the CMB weak lensing

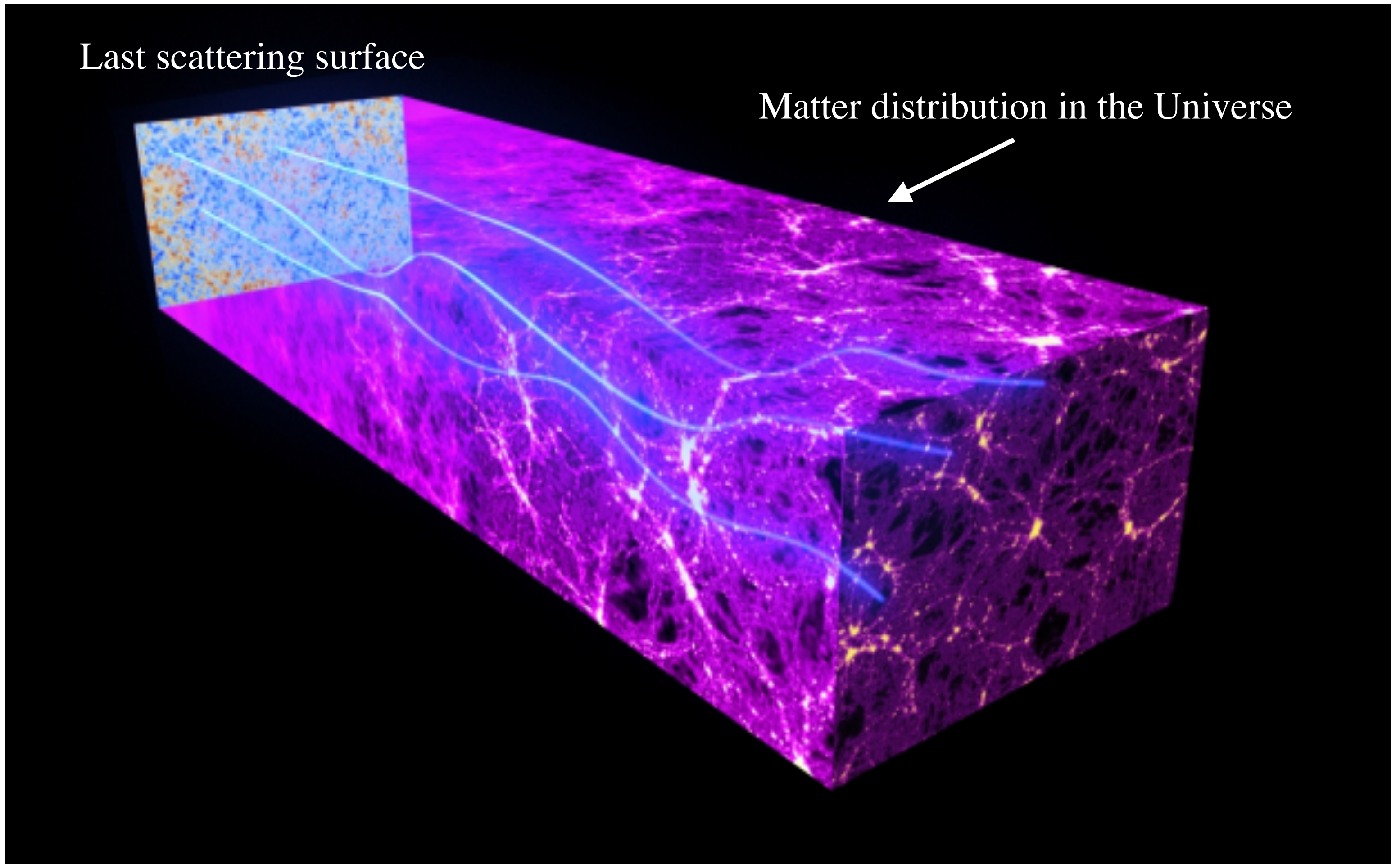
Abhishek S. Maniyar & Yacine Ali-Haïmoud
CCPP, NYU
In collaboration with: Julien Carron, Antony Lewis, Mat Madhavacheril

arXiv:2101.12193

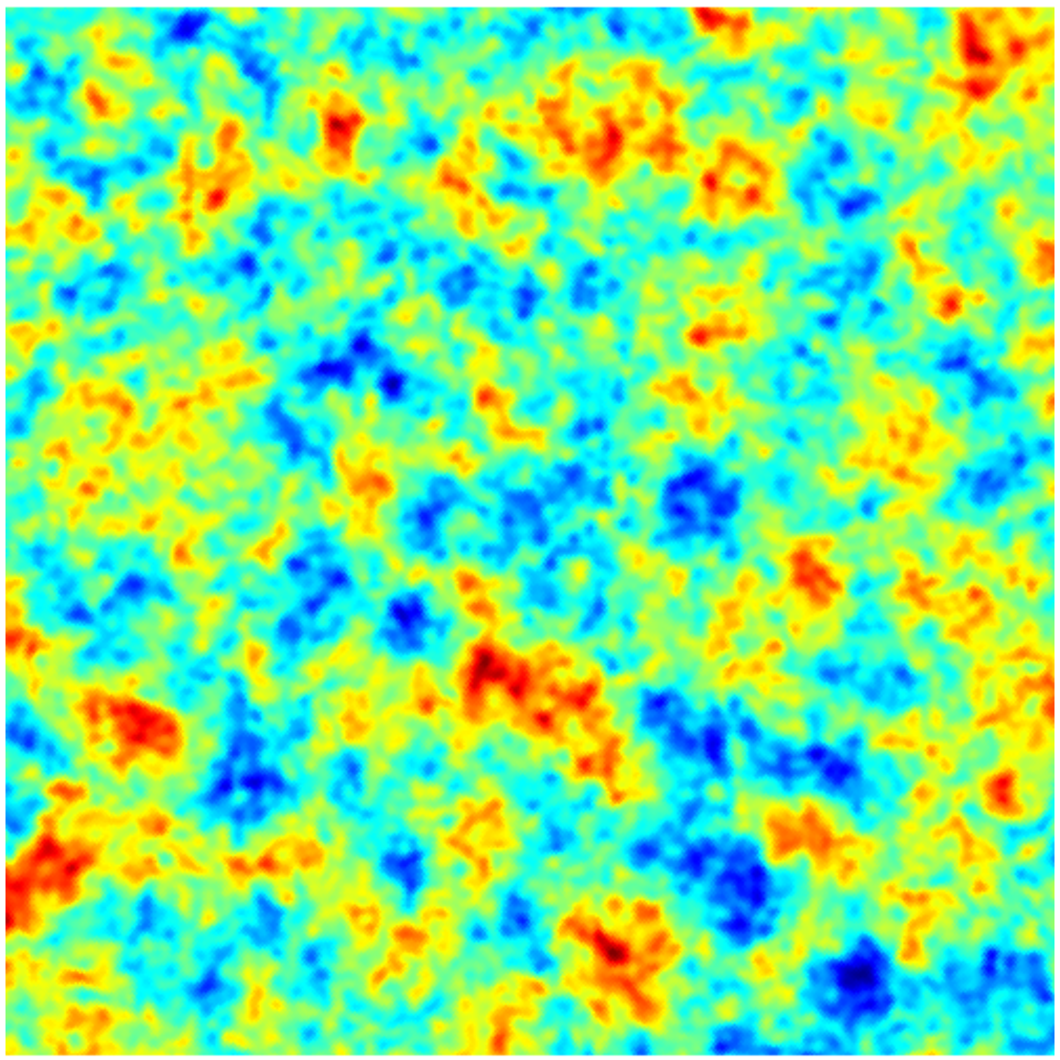
Cosmology from Home 2021
Virtual!
(New York City)

Weak lensing of the CMB

- Distribution of the foreground matter fluctuations deflects CMB photons
- What we see is a distorted CMB map



Weak lensing of the CMB



credit: https://www.earlyuniverse.org/neutrinos/

$$T(\hat{n}) = T^0(\hat{n} + d)$$

lensed map

unlensed map

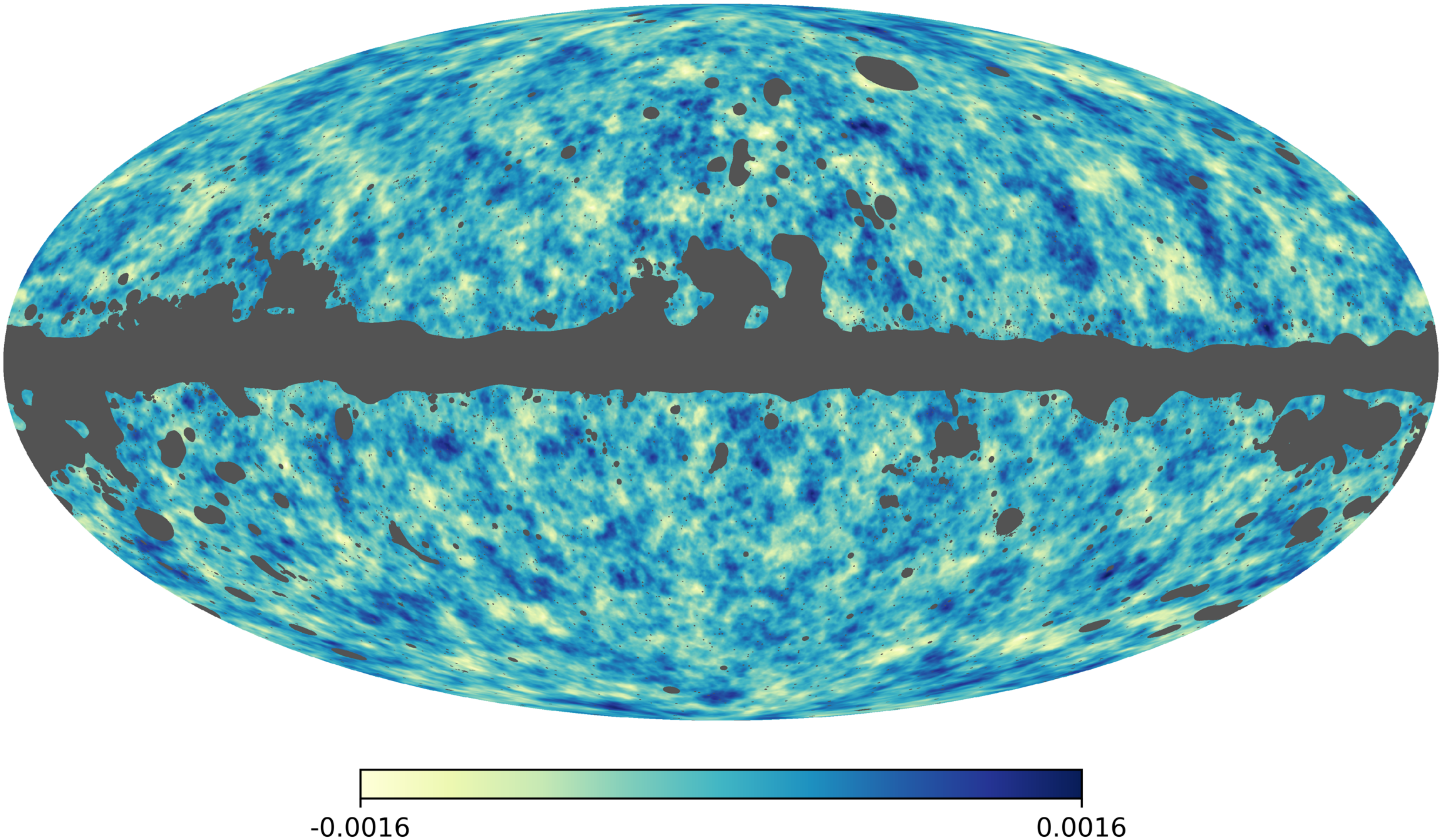
deflection angle

$$d = \nabla\phi \longleftarrow$$ lensing potential

Reconstruction of ϕ

Projected mass distribution along the line of sight
=> projected map of the matter in the Universe!

Planck lensing potential map



Planck 2018

Quadratic estimators

$$\langle x^0(\mathbf{l})x^0(\mathbf{l}')\rangle \equiv (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')C_\ell^0 \longrightarrow$$ Different multipoles uncorrelated

$$\langle x(\mathbf{l})x'(\mathbf{l}')\rangle_{\text{fixed }\phi} = f_\alpha(\mathbf{l},\mathbf{l}')\phi(\mathbf{L}) \longrightarrow$$ Lensing induces correlations between different multipoles!

$$\mathbf{L} = \mathbf{l}+\mathbf{l}' \qquad \mathbf{l} \neq -\mathbf{l}' \qquad x, x' = T, E, B$$

$$\alpha = \{TT, TE, EE, TB, EB, BB\}$$

$$\boxed{\phi(\mathbf{L}) \propto \int_{\mathbf{l}\neq\mathbf{l}'} F(\mathbf{l},\mathbf{l}')x(\mathbf{l})x'(\mathbf{l}')}$$

- Appropriate average of pairs of multipoles can be used to estimate the deflection field!
- Pairs of multipoles => quadratic estimator!

Quadratic Estimators of the CMB weak lensing

- Hu and Okamoto (2002): HO02
- Okamoto and Hu (2003): OH03
- Global minimum variance estimator: GMV
- Suboptimal quadratic estimator: SQE

HO02

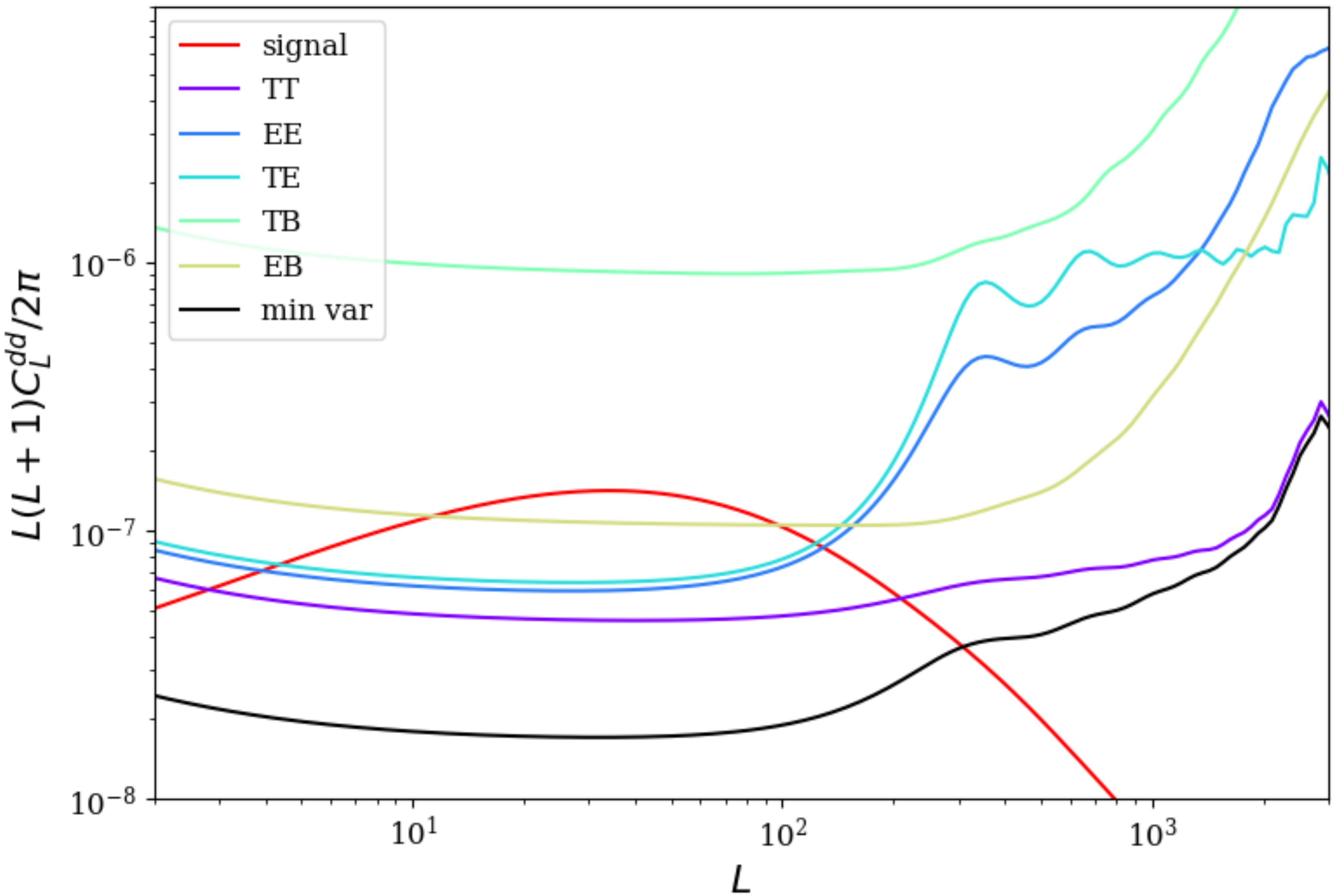
$$\hat{\phi}(\boldsymbol{L}) \propto \int_{\boldsymbol{l}_1 \neq \boldsymbol{l}_2} F_{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2) X(\boldsymbol{l}_1) Y(\boldsymbol{l}_2)$$

- 5 minimum variance estimators: $\hat{\phi}_{TT},\ \hat{\phi}_{EE},\ \hat{\phi}_{TE},\ \hat{\phi}_{TB},\ \hat{\phi}_{EB}$
- Final estimator: minimum variance linear combination of individual estimators

$$\hat{\phi}_{\text{HO02}} = w_{TT}\hat{\phi}_{TT} + w_{EE}\hat{\phi}_{EE} + w_{TE}\hat{\phi}_{TE} + w_{TB}\hat{\phi}_{TB} + w_{EB}\hat{\phi}_{EB}$$
$$w_{TT} + w_{EE} + w_{TE} + w_{TB} + w_{EB} = 1$$

$$\hat{\phi}_{\text{HO02}}(\boldsymbol{L}) = \int_{\boldsymbol{l}_1 \neq \boldsymbol{l}_2} \sum_{XY} F_{XY}^{\text{HO02}}(\boldsymbol{l}_1, \boldsymbol{l}_2) X(\boldsymbol{l}_1) Y(\boldsymbol{l}_2)$$

HO02: SO-like experiment



- Individual TT, EE, TE, TB, and EB estimators
- MV estimator out of combination of individual estimators
- Temperature dominated data

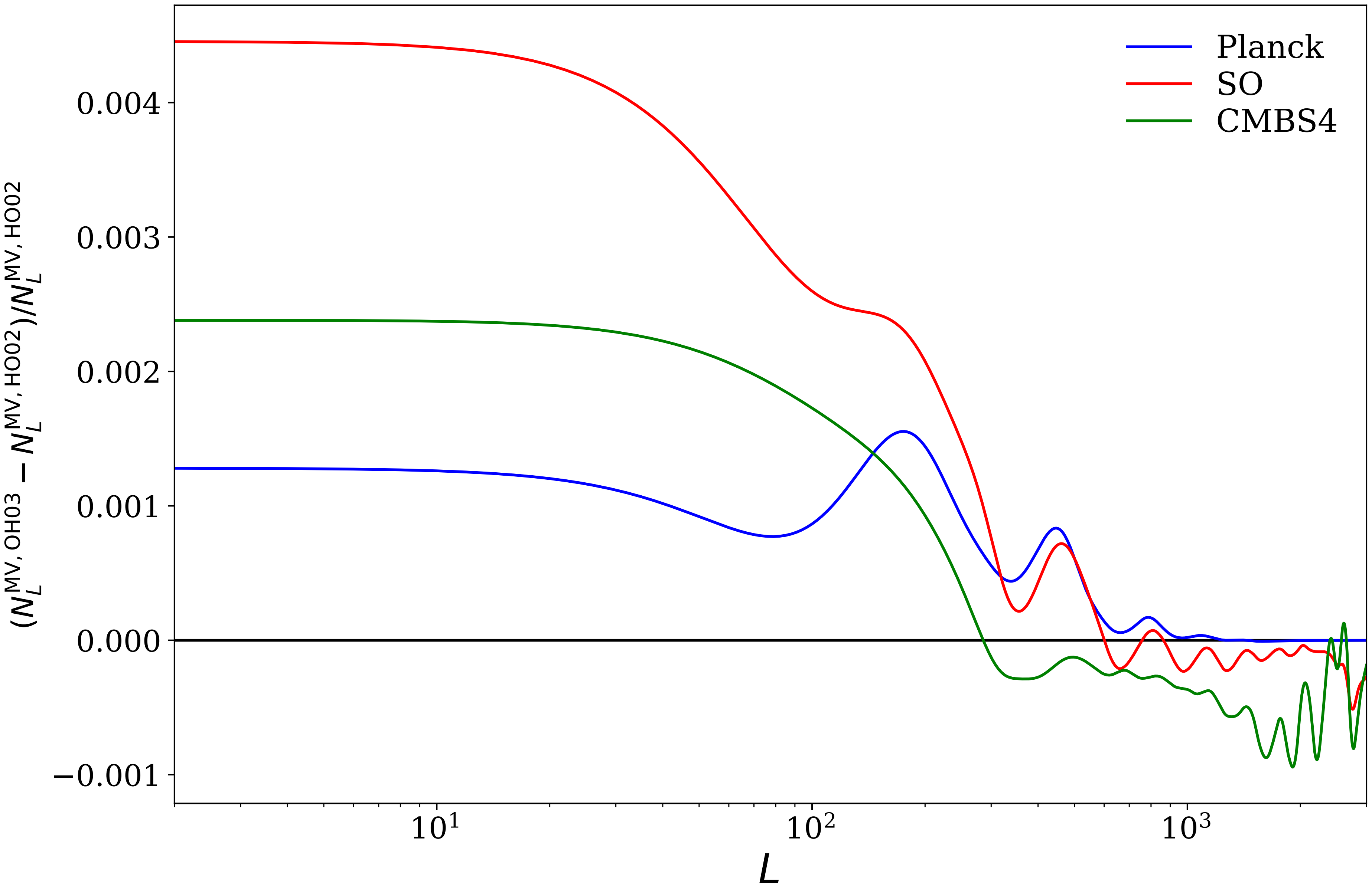
HO02 and OH03

$$F_{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2) = \lambda_{XY}(L) \; \frac{f_{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2)}{(1+\delta_{XY}) C_{l_1}^{XX} C_{l_2}^{YY}}$$

$$F_{TE}(\boldsymbol{l}_1, \boldsymbol{l}_2) = \lambda_{TE}(L) \frac{C_{l_1}^{EE} C_{l_2}^{TT} f_{TE}(\boldsymbol{l}_1, \boldsymbol{l}_2) - C_{l_1}^{TE} C_{l_2}^{TE} f_{TE}(\boldsymbol{l}_2, \boldsymbol{l}_1)}{C_{l_1}^{TT} C_{l_2}^{EE} C_{l_1}^{EE} C_{l_2}^{TT} - \left(C_{l_1}^{TE} C_{l_2}^{TE}\right)^2}$$

- Apart from TE, all estimators separable in l_1 and l_2
  - FFT => speeds up calculations considerably
- Approximation: $C_l^{TE} = 0$ (Okamoto and Hu 2003: **OH03**)
  - TE estimator separable as well, can use FFT
  - Minimal cost: fractional reconstruction noise increases by < 0.5% for a SO-like experiment

HO02 and OH03



- HO02 less noisy than OH03 at low L
- Difference remains below 0.5% for SO-like experiment

GMV

- HO02 consider the correlations between different XY pairs **after** integrating over l_1 and l_2
- GMV: Account for these correlations at each l_1 and l_2
- Less noisy than HO02 and best possible minimum variance quadratic estimator!

$$\phi_{\rm mv} \propto \int \Big(F_{TT} T(\mathbf{l}) T(\mathbf{l}') + F_{EE} E(\mathbf{l}) E(\mathbf{l}') + F_{TE} T(\mathbf{l}) E(\mathbf{l}') + F_{TB} T(\mathbf{l}) B(\mathbf{l}') + F_{EB} E(\mathbf{l}) B(\mathbf{l}') \Big)$$

GMV

$$\hat{\phi}(\boldsymbol{L}) = \int_{\boldsymbol{l}_1 \neq \boldsymbol{l}_2} X^i(\boldsymbol{l}_1)\Xi_{ij}(\boldsymbol{l}_1, \boldsymbol{l}_2)X^j(\boldsymbol{l}_2),$$

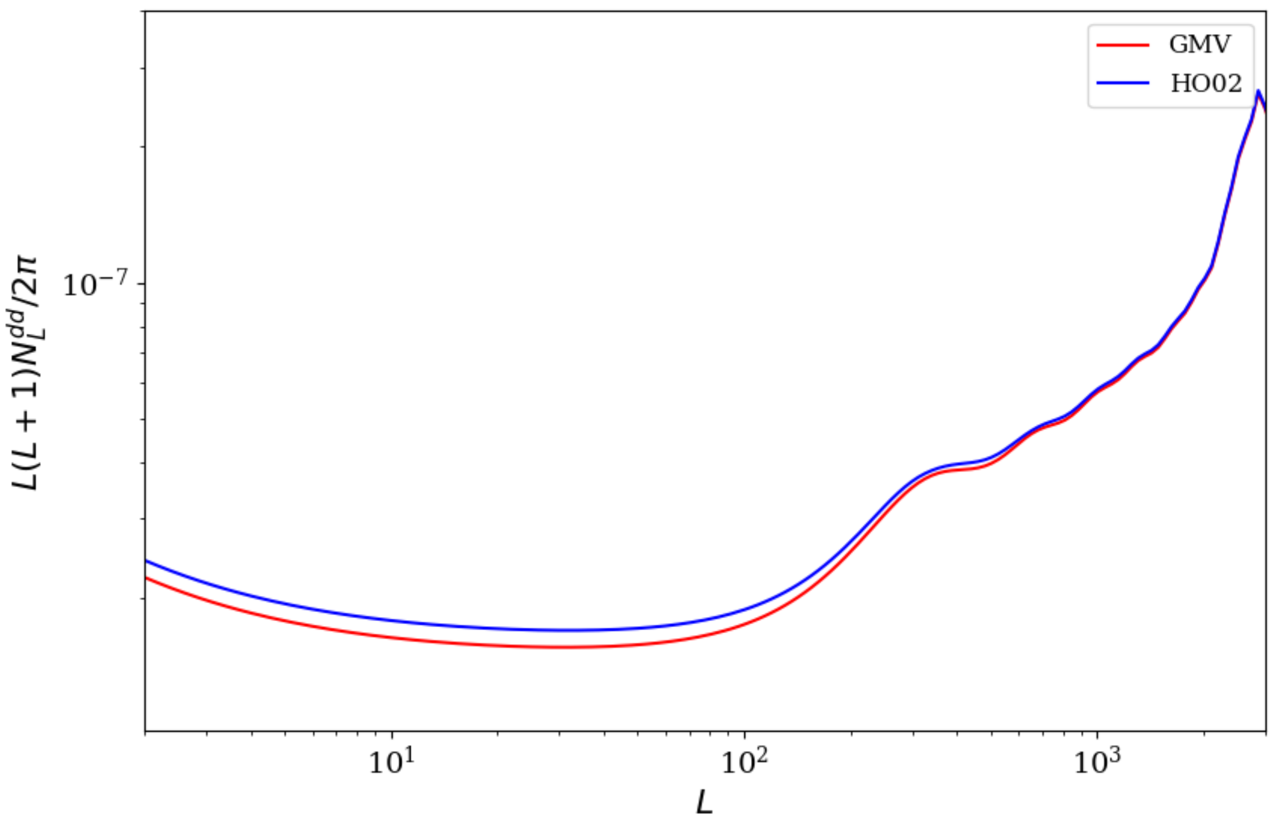
$$[\boldsymbol{\Xi}(\boldsymbol{l}_1, \boldsymbol{l}_2)] = \frac{\lambda(L)}{2}[\boldsymbol{C}_{l_1}]^{-1}[\boldsymbol{f}(\boldsymbol{l}_1, \boldsymbol{l}_2)][\boldsymbol{C}_{l_2}]^{-1}$$

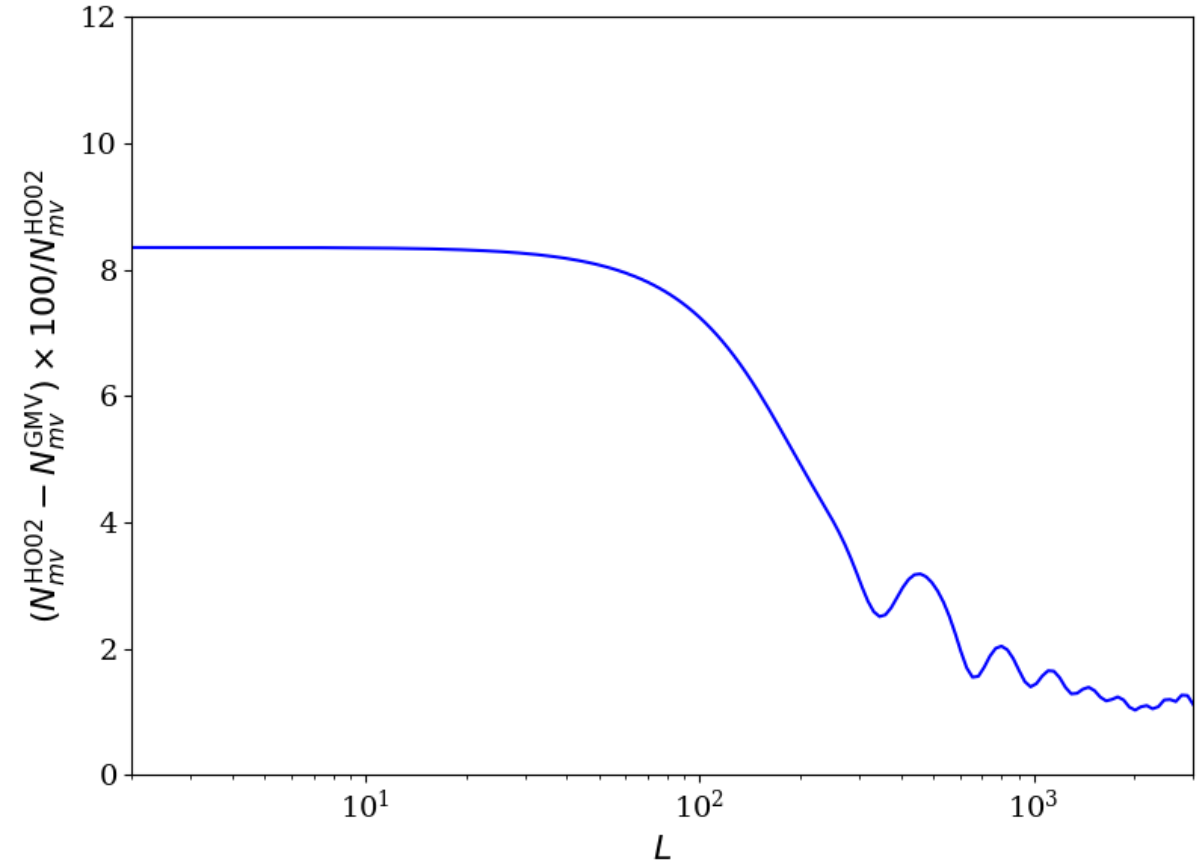
HO02

$$F_{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2) = \lambda_{XY}(L)\,\frac{f_{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2)}{(1+\delta_{XY})C^{XX}_{l_1}C^{YY}_{l_2}}$$

- $\boldsymbol{C}_l$ and $\boldsymbol{f}(\boldsymbol{l}_1, \boldsymbol{l}_2)$: 3 x 3 symmetric matrices
- Separable in l_1 and l_2 without any approximations! => FFT
- Previously derived, but erroneously described as equivalent to HO02 estimator!!

GMV: SO-like experiment





- 8-10% smaller noise than HO02 on small L
- More information out of the same maps!

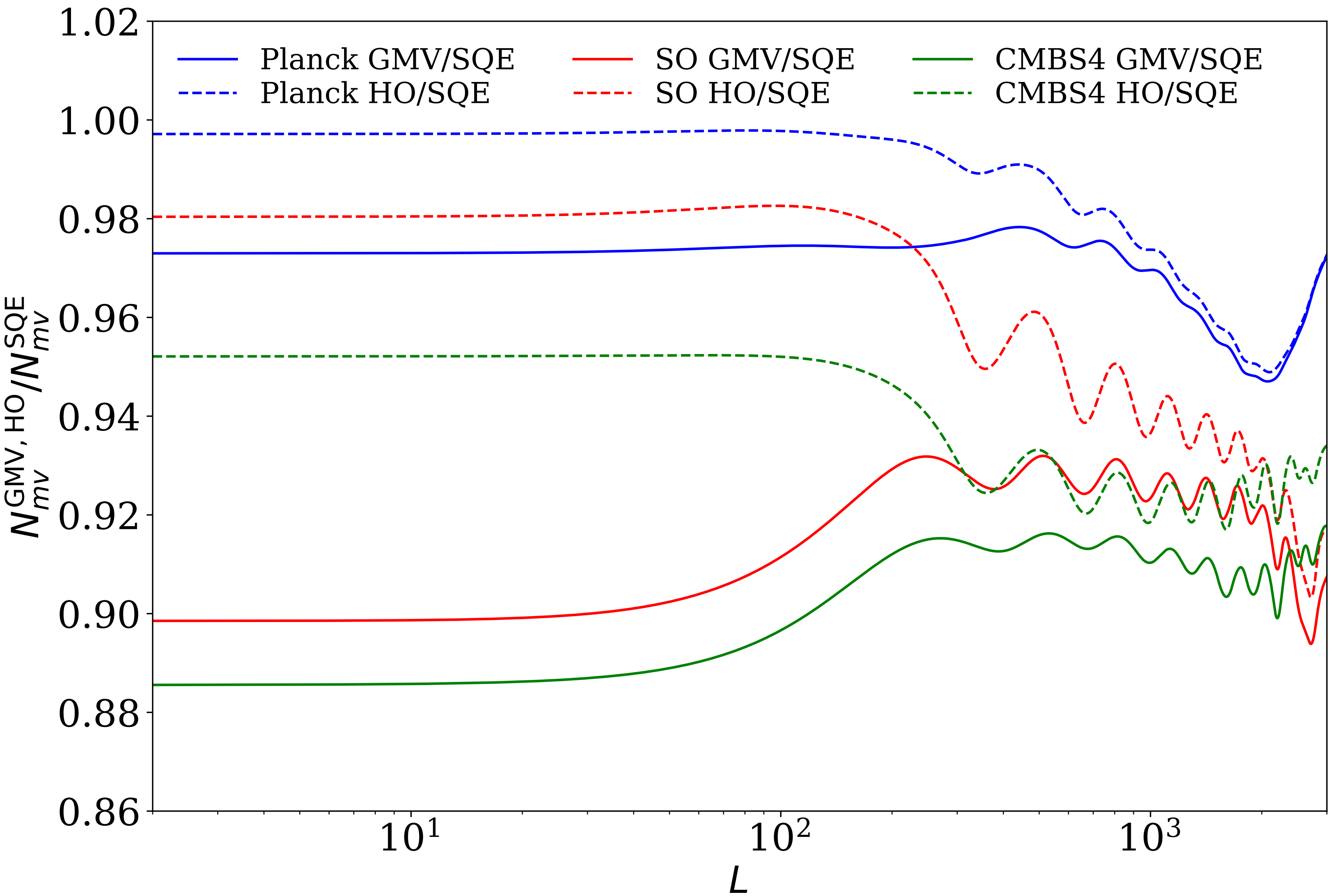
SQE

$$\hat{\phi}(\boldsymbol{L}) = \int_{\boldsymbol{l}_1 \neq \boldsymbol{l}_2} X^i(\boldsymbol{l}_1)\Xi_{ij}(\boldsymbol{l}_1, \boldsymbol{l}_2)X^j(\boldsymbol{l}_2), \qquad [\boldsymbol{\Xi}(\boldsymbol{l}_1, \boldsymbol{l}_2)] = \frac{\lambda(L)}{2}[\boldsymbol{C}_{l_1}]^{-1}[\boldsymbol{f}(\boldsymbol{l}_1, \boldsymbol{l}_2)][\boldsymbol{C}_{l_2}]^{-1}$$

- Planck (2016, 2020) and SPT (2019) use an approximated version: SQE
- $C_l^{TE} = 0$ in $\boldsymbol{C}_l$
- Allows to deal with cut-sky setup with lower computational cost
- Preserves separability in l_1 and l_2
- 3% noise penalty for Planck
- Suboptimal to HO02 as well!

$$F_{XY}^{\mathrm{SQE}}(\boldsymbol{l}_1, \boldsymbol{l}_2) = \lambda_{\mathrm{SQE}}(L)\frac{f_{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2)}{(1+\delta_{XY})C_{l_1}^{XX}C_{l_2}^{YY}}$$

Comparison of all estimators



- SQE to GMV difference:
    - 3-6% for Planck-like experiments
    - 11-12% for SO-like experiments
- Should motivate use of full covariance matrix rather than setting $C_l^{TE} = 0$

Conclusions

- HO02 optimisation procedure does not lead to absolute minimum-variance QE
- GMV is the global minimum-variance QE
- HO02 is not equivalent to GMV as previously thought
- SQE used in data analysis: suboptimal to all: HO02, OH03 and GMV
- Arguments applicable to full-sky as well
- Cross-correlation studies of lensing will benefit by smaller noise on reconstruction: GMV
- Lower reconstruction noise also beneficial for delensing
- CMB-S4 will use likelihood based iterative methods for reconstruction; QE still relevant for forecasting and cross-checking

Thank you!

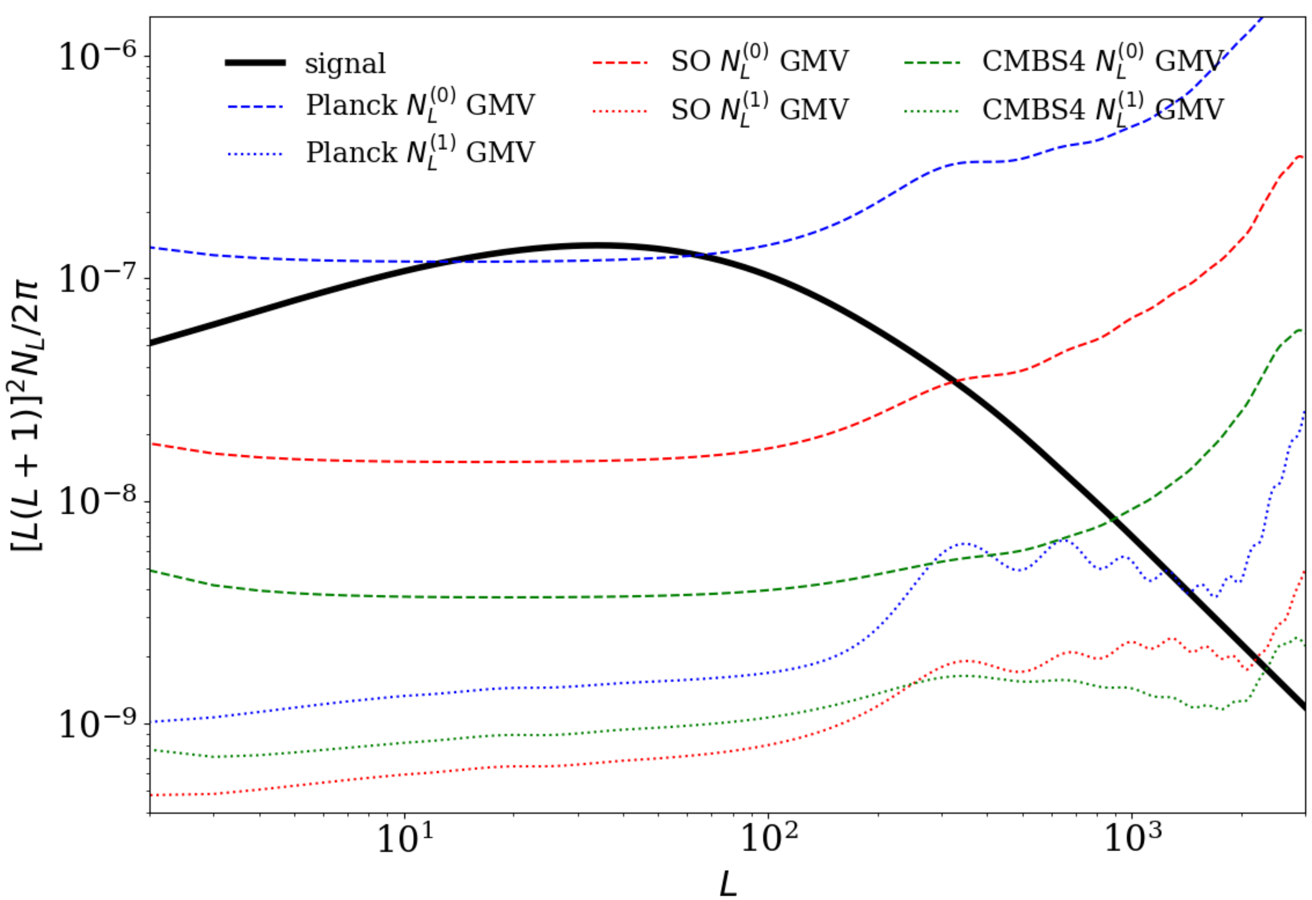
Experimental specs

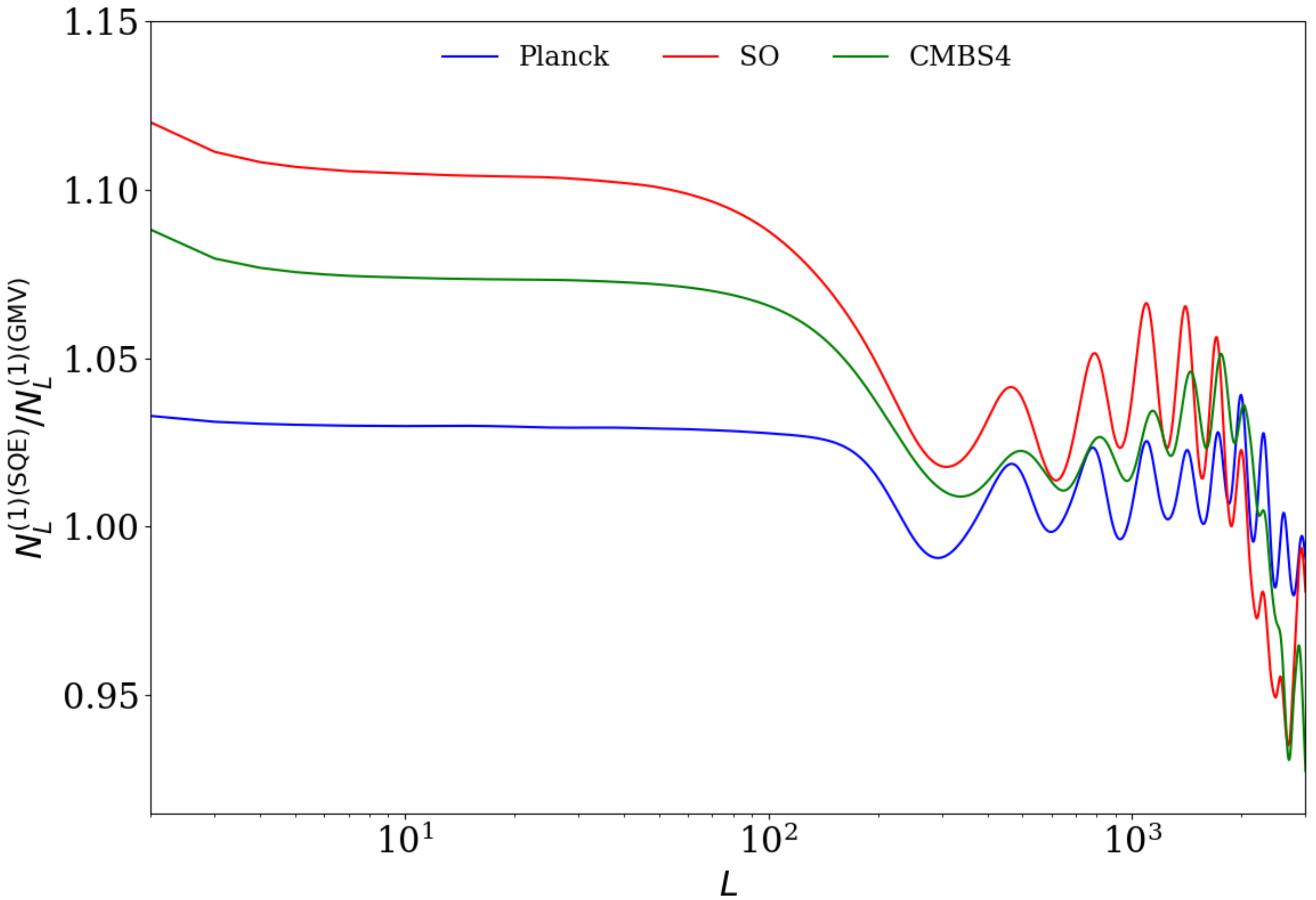
Experiment	$\ell_{\max}$	Δ_T	Δ_P	σ
		μK-arcmin	μK-arcmin	arcmin
Planck	3000	35.0	60.0	5.0
SO	3000	8.0	$8.0\sqrt{2}$	1.4
CMBS4	3000	1.0	$1.0\sqrt{2}$	1.0

TABLE II: Experimental specifications used in this work.

$$\ell_{\max}^{T} = \ell_{\max}^{P}$$

N(0) and N(1) bias





- N(1) factor of few to ~2 orders of magnitude smaller than N(0)
- N(1) important to model in likelihood analysis for more sensitive experiments on smaller scales
- N(1) smaller for GMV than SQE especially at large angular scales