

# ***Measurement of the B-band Galaxy Luminosity function with Approximate Bayesian Computation***

**arXiv: 2001.07727**

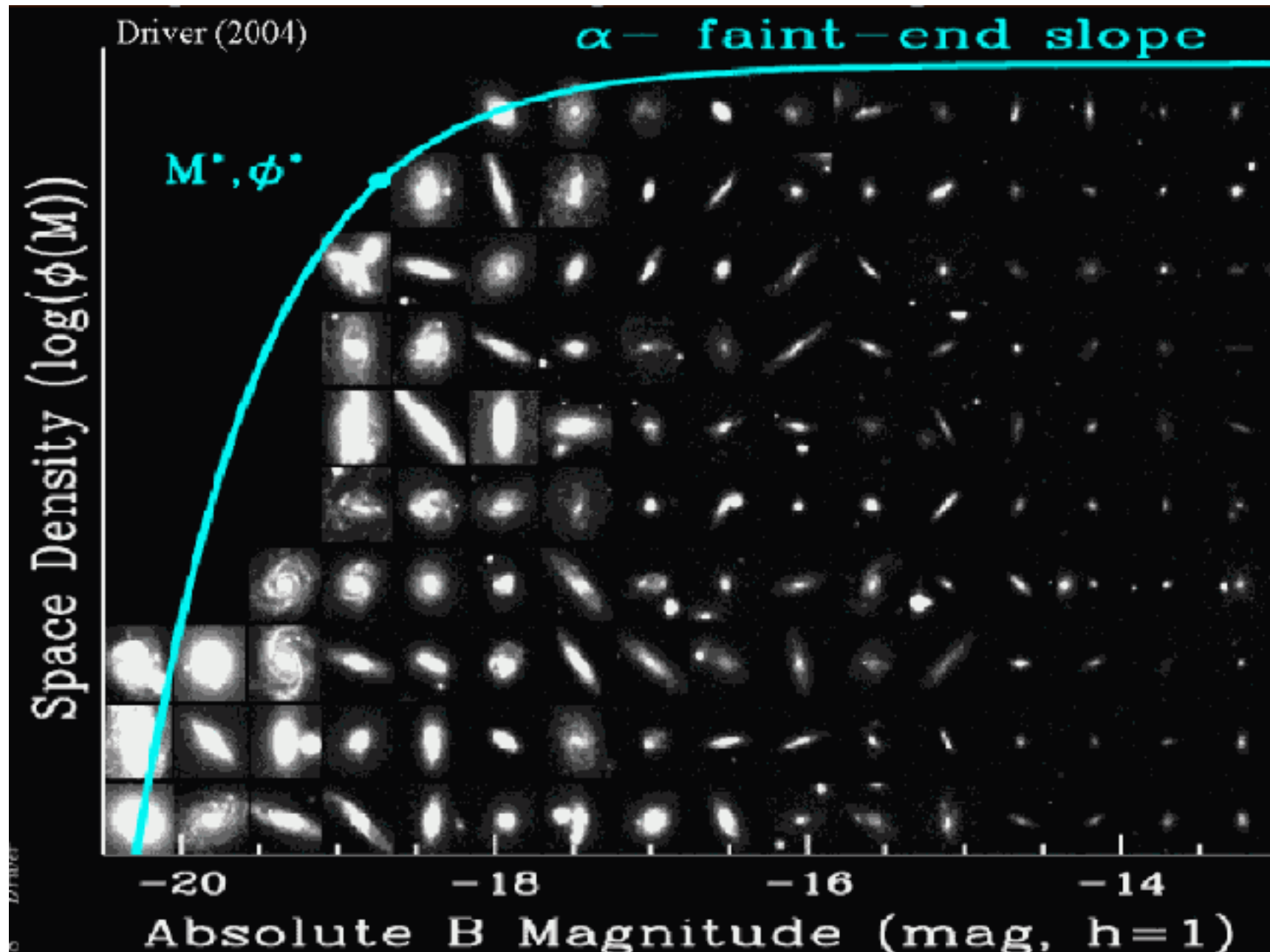
**Luca Tortorelli**

**&**

**Martina Fagioli, Jörg Herbel, Adam Amara,  
Tomasz Kacprzak, Alexandre Refregier**

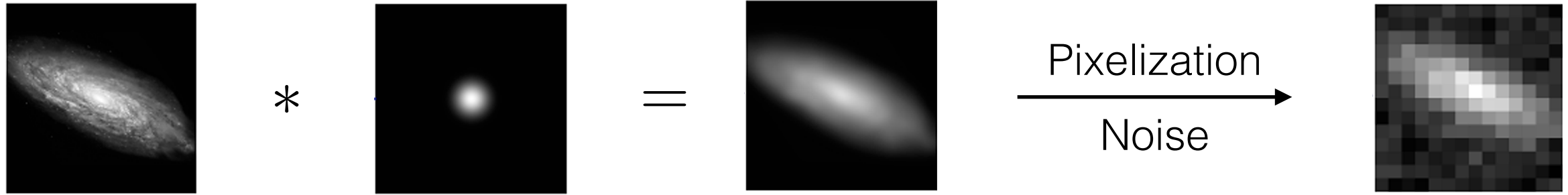


# The Galaxy Luminosity function

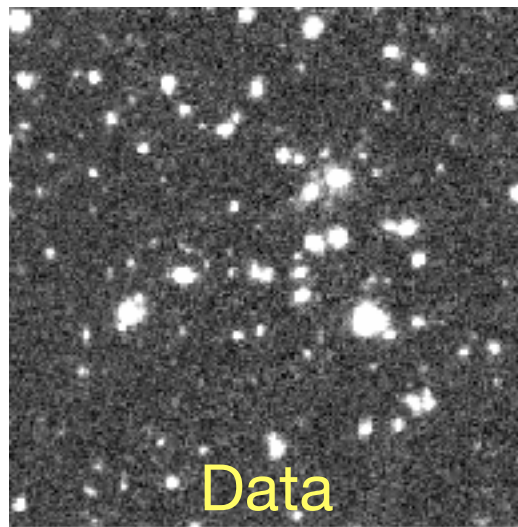


$\Phi(M, z)$  = Number of galaxies per unit volume per unit magnitude at redshift  $z$

# What is the Forward Modeling approach?



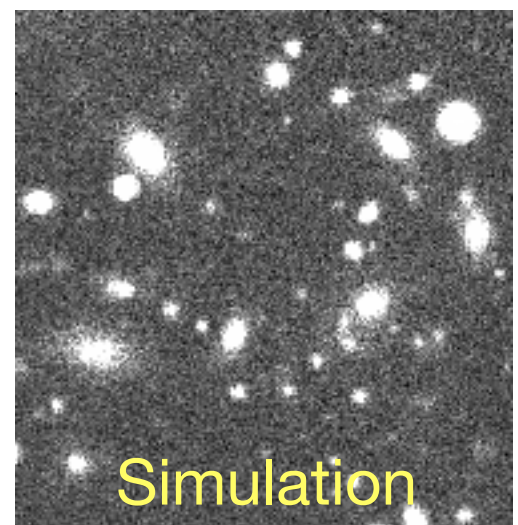
Bridle et al. 2009



Analysis  
→  
Selection cuts

Target sample

	Mag	Size	...
Obj 1	...	...	...
Obj 2	...	...	...
Obj 3	...	...	...
...	...	...	...



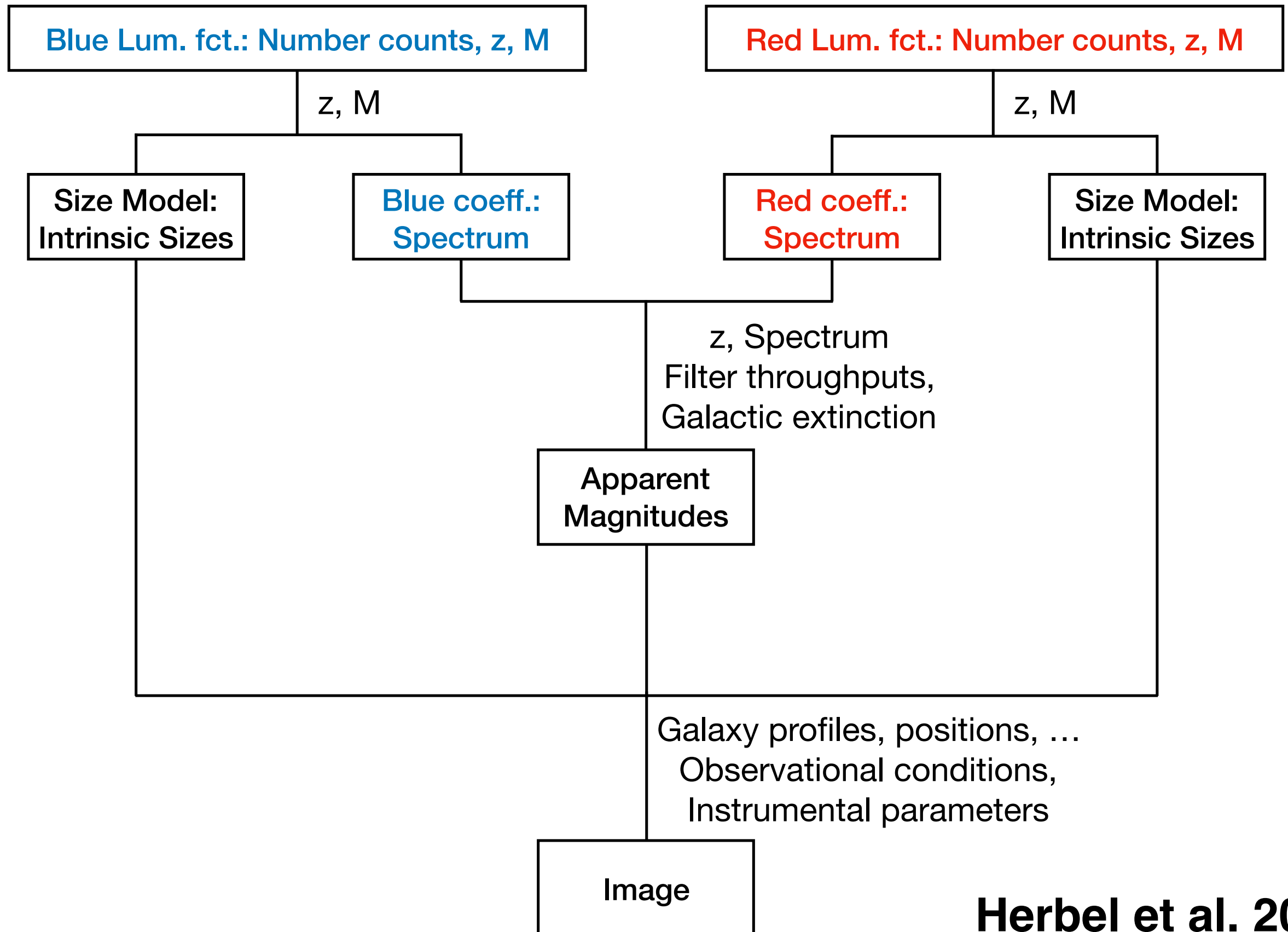
Analysis  
→  
Selection cuts

Simulated target sample

	Mag	Size	...
Obj 1	...	...	...
Obj 2	...	...	...
Obj 3	...	...	...
...	...	...	...

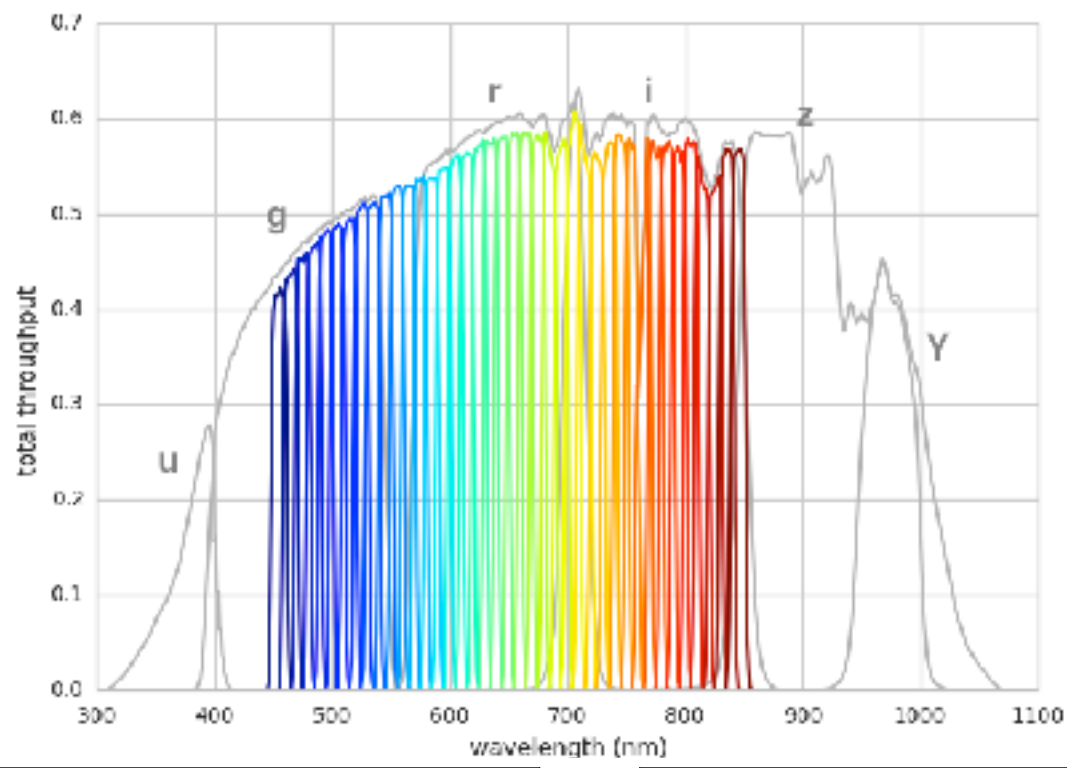
	z	...
Obj 1	...	...
Obj 2	...	...
Obj 3	...	...
...	...	...

# Galaxy population model in UFig

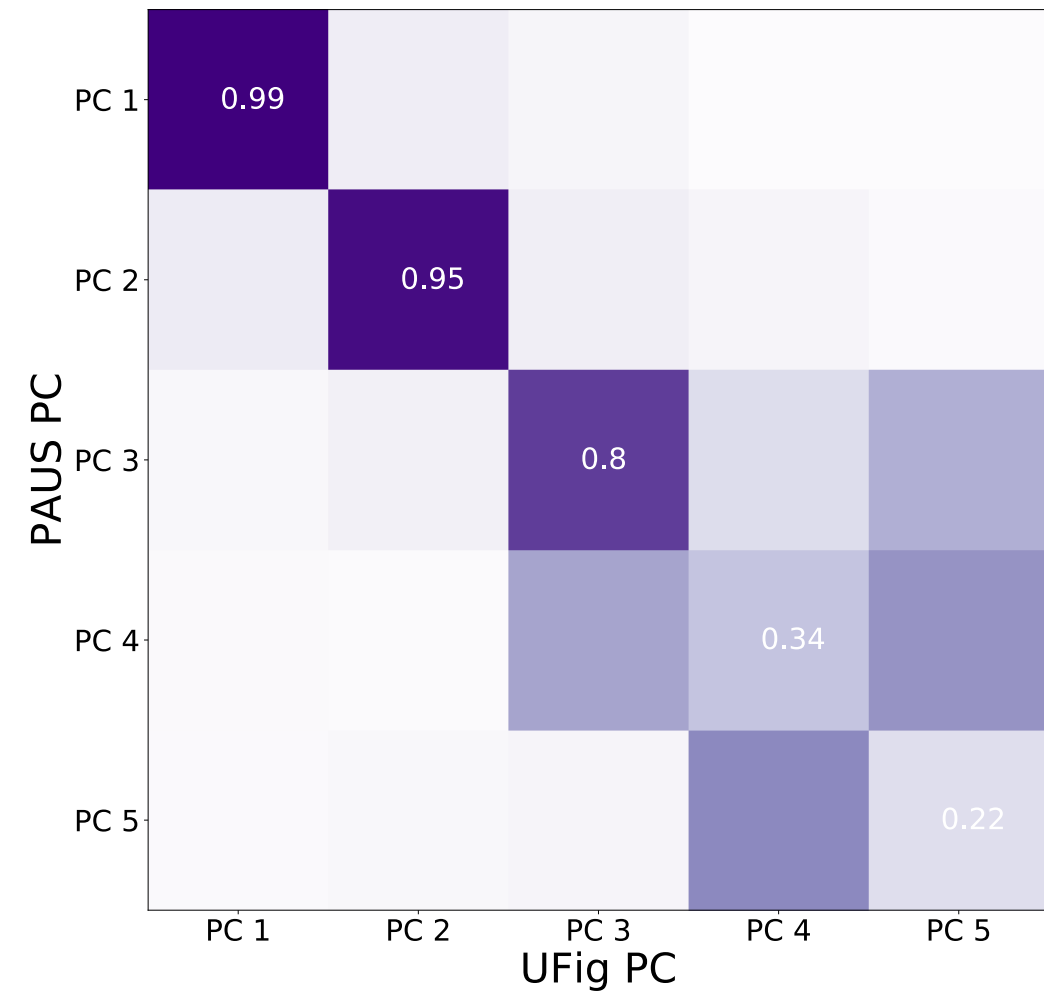
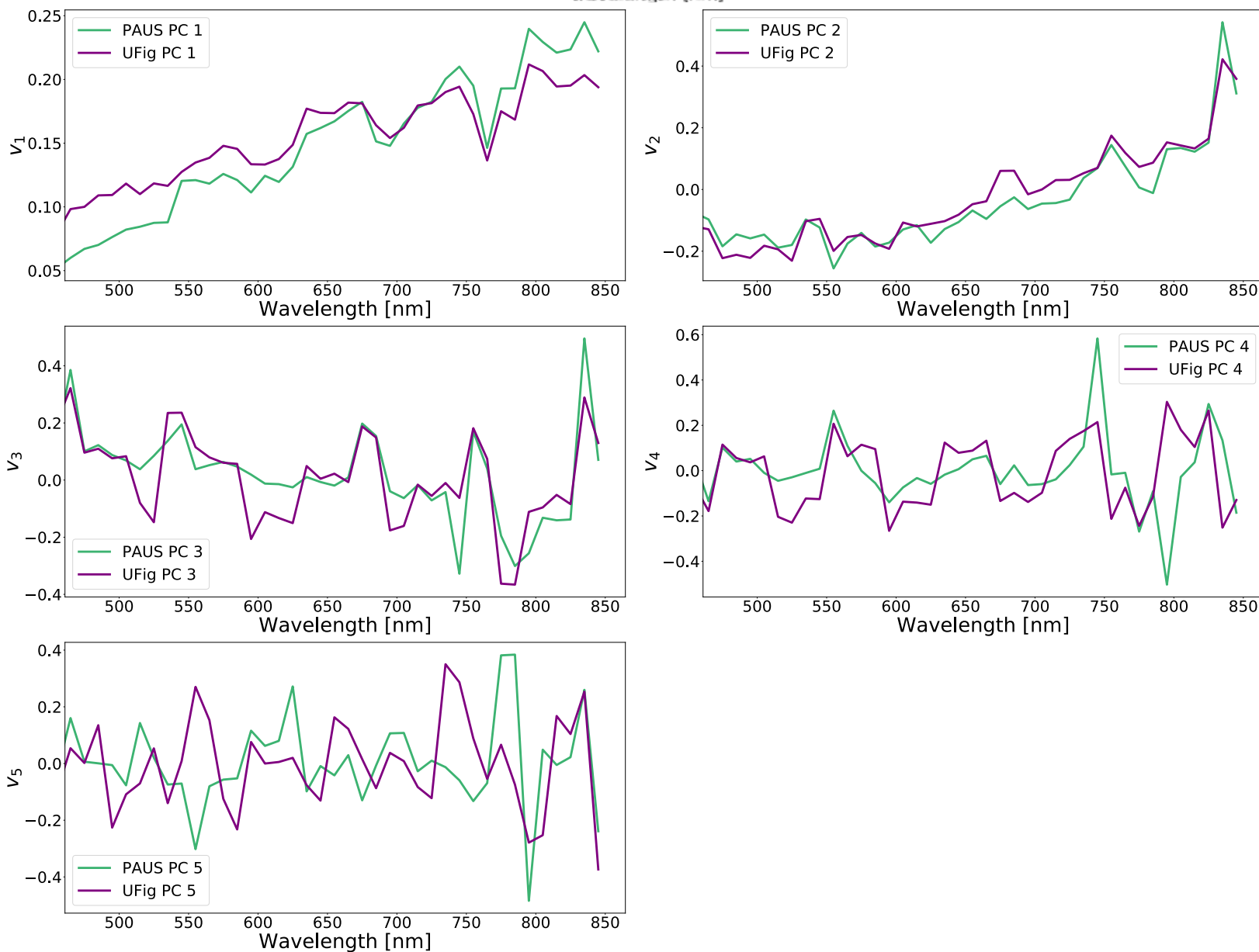


**Herbel et al. 2017**





# Tortorelli et al. 2018



# The ABC inference scheme

Calibration, but no likelihood for Bayesian Analysis



*Approximate Bayesian Computation (ABC).*

$$p(\theta|y) \simeq p(\theta|\rho(x, y) \leq \epsilon)$$

---

## ABC algorithm

---

**while** ( $p_{\text{acc}} > p_{\text{acc, min}}$ ) **do**:

**if**  $T = 1$  **do**:

**for**  $i = 1$  **to**  $N$  **do**:

      Sample  $\theta_{i, T=1}^*$  from pre-defined prior:  $\theta_{i, T=1}^* \sim p(\theta)$

      Create dataset  $x$  from  $\theta_{i, T=1}^*$ :  $x \sim \text{Model}(\theta_{i, T=1}^*)$

      Set  $\theta_{i, T=1} = \theta_{i, T=1}^*$

      Set  $\rho_{i, T=1} = \rho_{i, T=1}(x, y)$

**end for**

**else do**:

**for**  $i = 1$  **to**  $N'$  **do**:

      Sample  $\theta_{i, T}^*$  from  $\text{GMM}(\theta_{T-1})$

      Create dataset  $x$  from  $\theta_{i, T}^*$ :  $x \sim \text{Model}(\theta_{i, T}^*)$

      Set  $\theta_{i, T} = \theta_{i, T}^*$

      Set  $\rho_{i, T} = \rho_{i, T}(x, y)$

**end for**

    Set  $p_{\text{acc}} = \frac{1}{N'} \sum_{k=1}^{N'} \mathbb{I}_{\rho_{i, T} < \epsilon_{T-1}}$

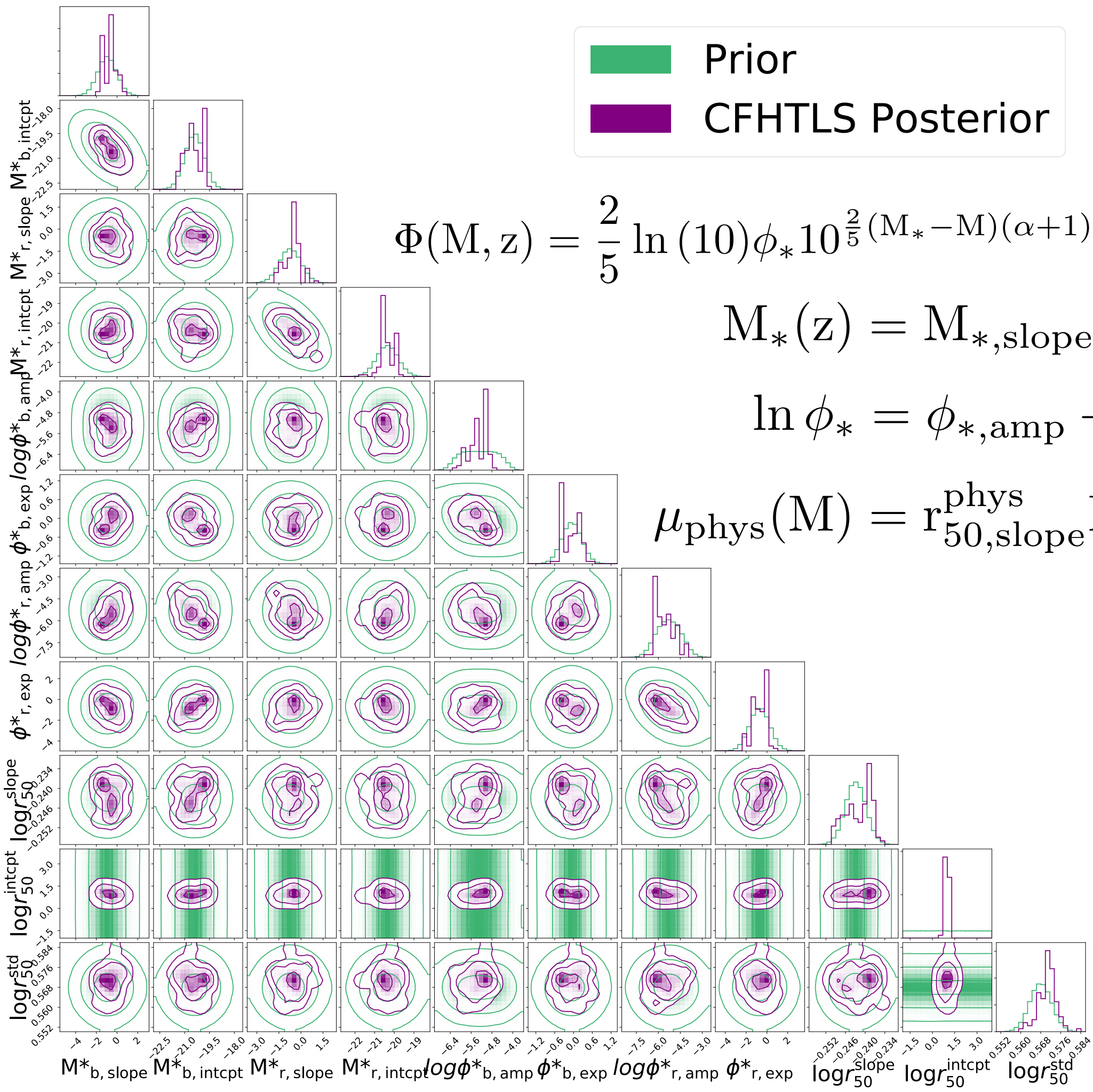
  Let  $\epsilon_T = Q_{\rho^{(T)}}(q)$  the  $q$ -th percentile value of  $\rho^{(T)}$ , where  $\rho^{(T)} = \{\rho_{i, T}\}_{i=1, \dots, N'}$

  Let  $\{(\theta_i^{(T)}, \rho_i^{(T)})\} = \{(\theta_{i, T}, \rho_{i, T}) | \rho_{i, T} \leq \epsilon_T, i = 1, \dots, N_{q, T}\}$

# Prior and Distance metrics

Parameter	Distribution	Prior
$\alpha$ (blue)	Fixed value	-1.3
$\alpha$ (red)	Fixed value	-0.5
$M_{B,\text{slope}}^*$ (blue)	Multivariate Normal	$\mu = -9.44 \times 10^{-1}, \sigma^2 = 8.29 \times 10^{-1}$
$M_{B,\text{slope}}^*$ (red)	Multivariate Normal	$\mu = -7.33 \times 10^{-1}, \sigma^2 = 5.30 \times 10^{-1}$
$M_{B,\text{intcpt}}^* - 5 \log h_{70}$ (blue)	Multivariate Normal	$\mu = -2.041 \times 10^1, \sigma^2 = 3.312 \times 10^{-1}$
$M_{B,\text{intcpt}}^* - 5 \log h_{70}$ (red)	Multivariate Normal	$\mu = -2.035 \times 10^{-1}, \sigma^2 = 2.968 \times 10^{-1}$
$\phi_{\text{exp}}^*$ (blue)	Multivariate Normal	$\mu = -5.66 \times 10^{-2}, \sigma^2 = 9.96 \times 10^{-2}$
$\phi_{\text{exp}}^*$ (red)	Multivariate Normal	$\mu = -6.97 \times 10^{-1}, \sigma^2 = 9.21 \times 10^{-1}$
$\ln \phi_{\text{amp}}^* / 10^{-3} h_{70}^3 \text{ Mpc}^{-3} \text{ mag}^{-1}$ (blue)	Multivariate Normal	$\mu = -5.28 \times 10^0, \sigma^2 = 4.1 \times 10^{-1}$
$\ln \phi_{\text{amp}}^* / 10^{-3} h_{70}^3 \text{ Mpc}^{-3} \text{ mag}^{-1}$ (red)	Multivariate Normal	$\mu = -5.28 \times 10^0, \sigma^2 = 6.5 \times 10^{-1}$
$r_{50,\text{slope}}^{\text{phys}}$	Multivariate Normal	$\mu = -2.4 \times 10^{-1}, \sigma^2 = 9.8 \times 10^{-6}$
$r_{50,\text{intcpt}}^{\text{phys}}$	Uniform	[-2, 4]
$\sigma_{\text{phys}}$	Multivariate Normal	$\mu = 5.7 \times 10^{-1}, \sigma^2 = 1.9 \times 10^{-5}$
$a_{i,0}$	Dirichlet $\times$ Uniform	[1., 1., 1., 1., 1.] $\times$ [5, 15]
$a_{i,1}$	Dirichlet $\times$ Uniform	[1., 1., 1., 1., 1.] $\times$ [5, 15]

Distance Metric	Label
Absolute difference in the number of detected galaxies	$d_1$
Random Forest distance with 21 summary statistics	$d_2$
Random Forest distance with 31 summary statistics	$d_3$
Maximum Mean Discrepancy distance on $u^*, g', r', i', z'$ band properties	$d_{4,\dots,7}$
Maximum Mean Discrepancy distance on $u^*, g', i'$ band properties	$d_{8,\dots,11}$
Maximum Mean Discrepancy distance on $i'$ band magnitudes and redshift distributions	$d_{12}$
Magnitude histogram distance on $u^*, g', r', i', z'$ bands separately	$d_{\{13,\dots,17\}}$
Size histogram distance on $u^*, g', r', i', z'$ bands separately	$d_{\{18,\dots,22\}}$
Maximum value among all previously defined rescaled distances	$d_{23} = \max(\underline{d}_{\{1,\dots,22\}})$
Maximum value between the rescaled MMD distance on 5 bands and the rescaled absolute difference	$d_{24,\dots,27} = \max(\underline{d}_1, \underline{d}_{4,\dots,7})$
Maximum value among the rescaled MMD distance and the rescaled magnitude histogram distance on 5 bands	$d_{28,\dots,31} = \max(\underline{d}_{4,\dots,7}, \underline{d}_{\{13,\dots,17\}})$



■ Prior  
■ CFHTLS Posterior

$$\Phi(M, z) = \frac{2}{5} \ln(10) \phi_* 10^{\frac{2}{5}(M_* - M)(\alpha + 1)} \exp \left[ -10^{\frac{2}{5}(M_* - M)} \right]$$

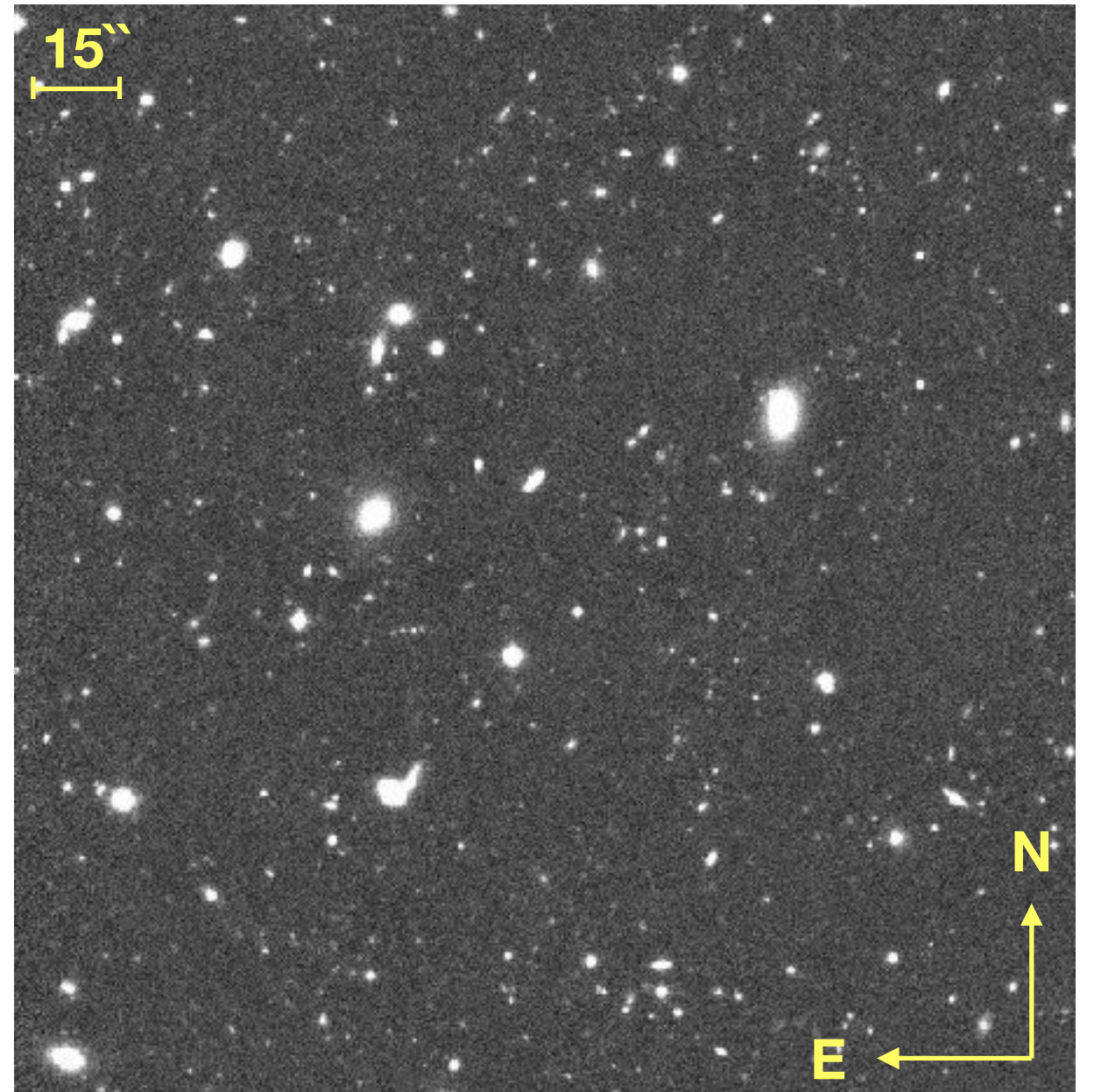
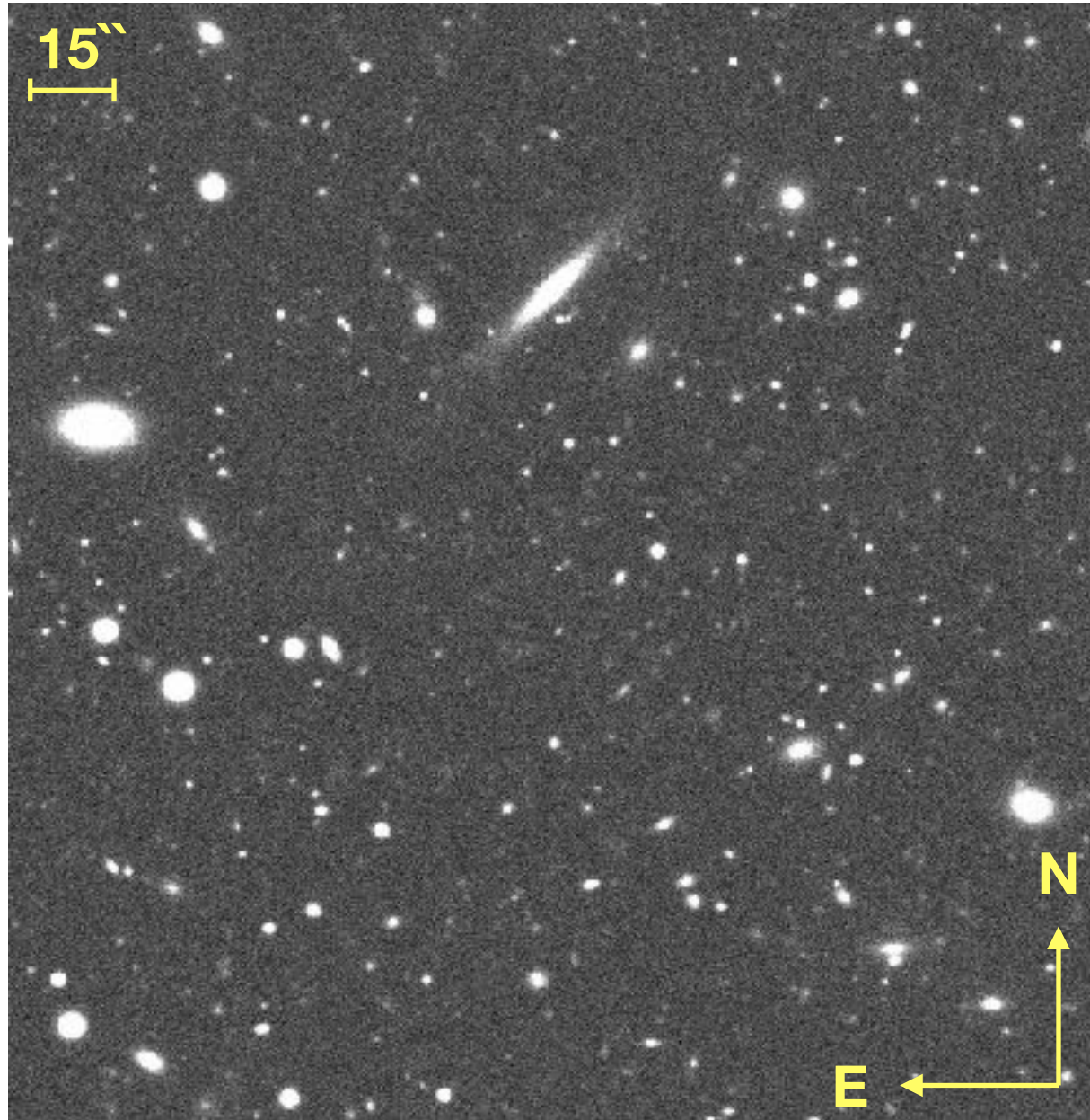
$$M_*(z) = M_{*,\text{slope}} z + M_{*,\text{intcpt}}$$

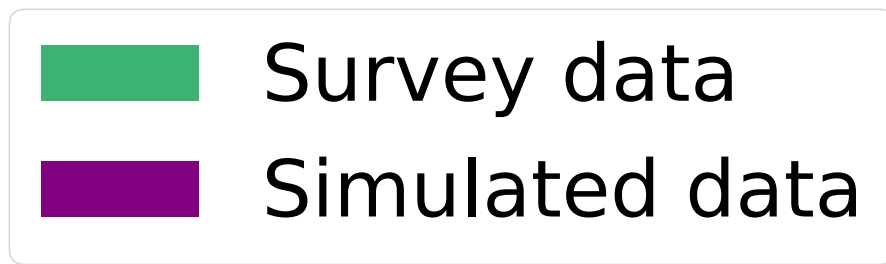
$$\ln \phi_* = \phi_{*,\text{amp}} + \phi_{*,\text{exp}} z$$

$$\mu_{\text{phys}}(M) = r_{50,\text{slope}}^{\text{phys}} M + r_{50,\text{intcpt}}^{\text{phys}}$$

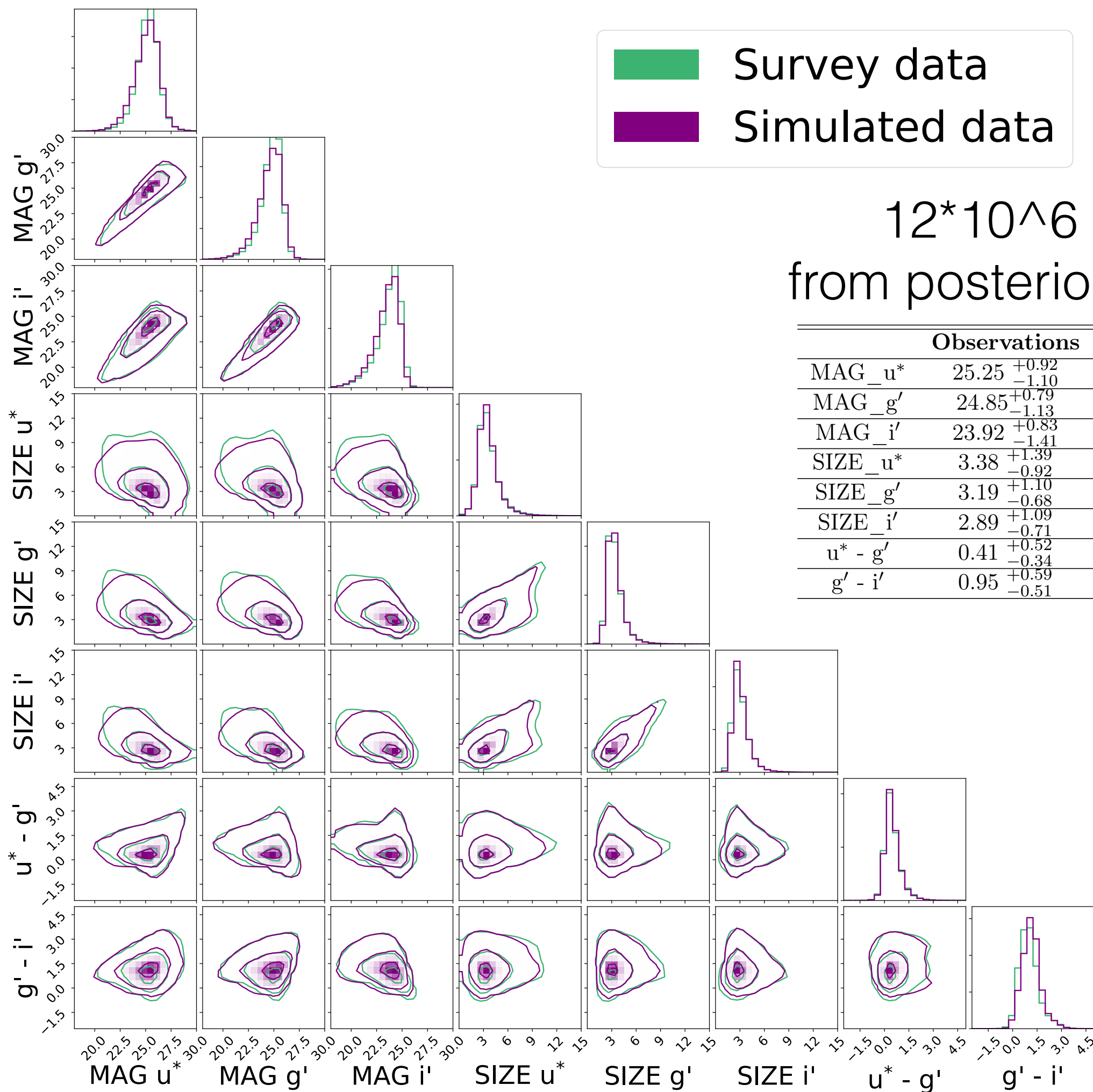


# Example images



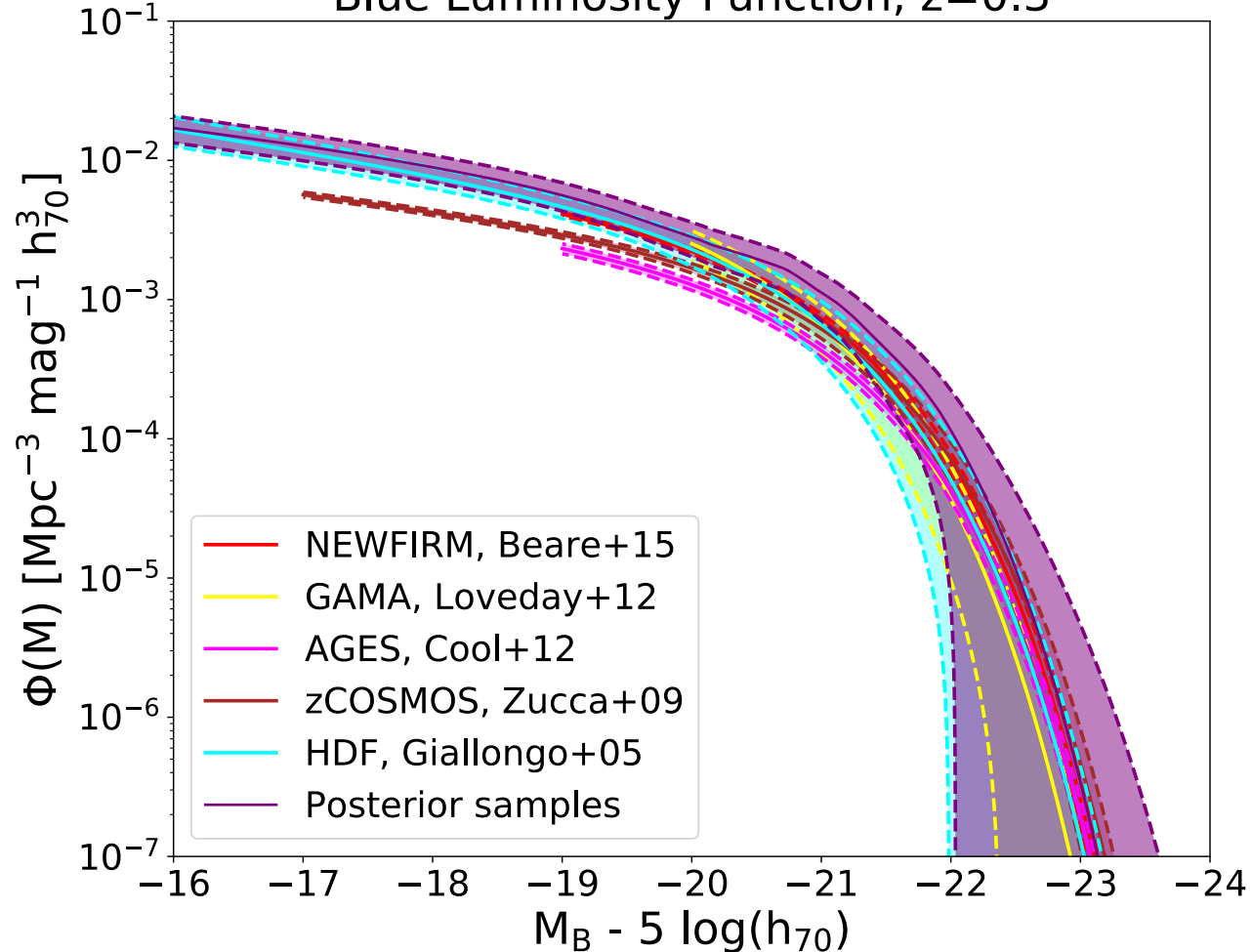
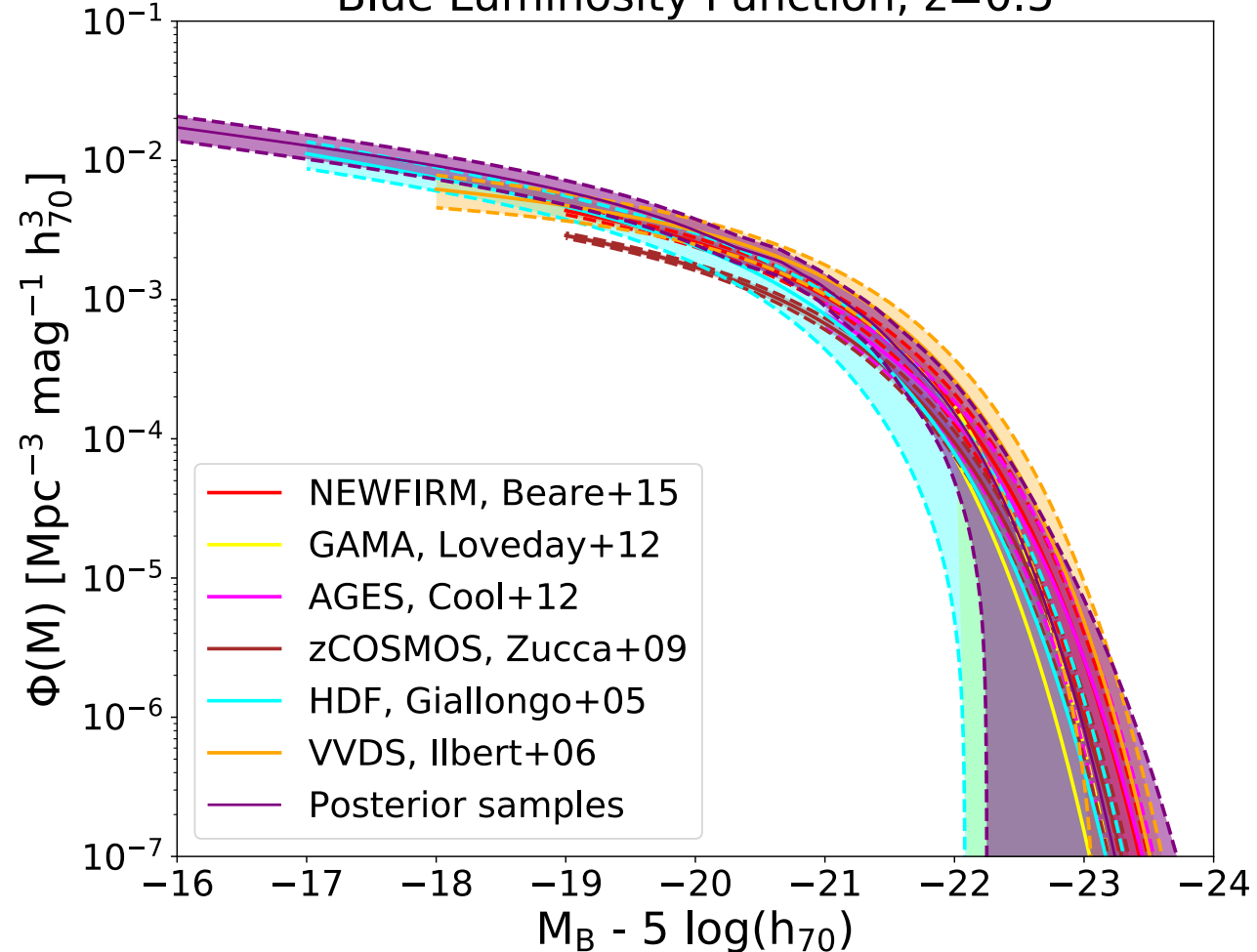
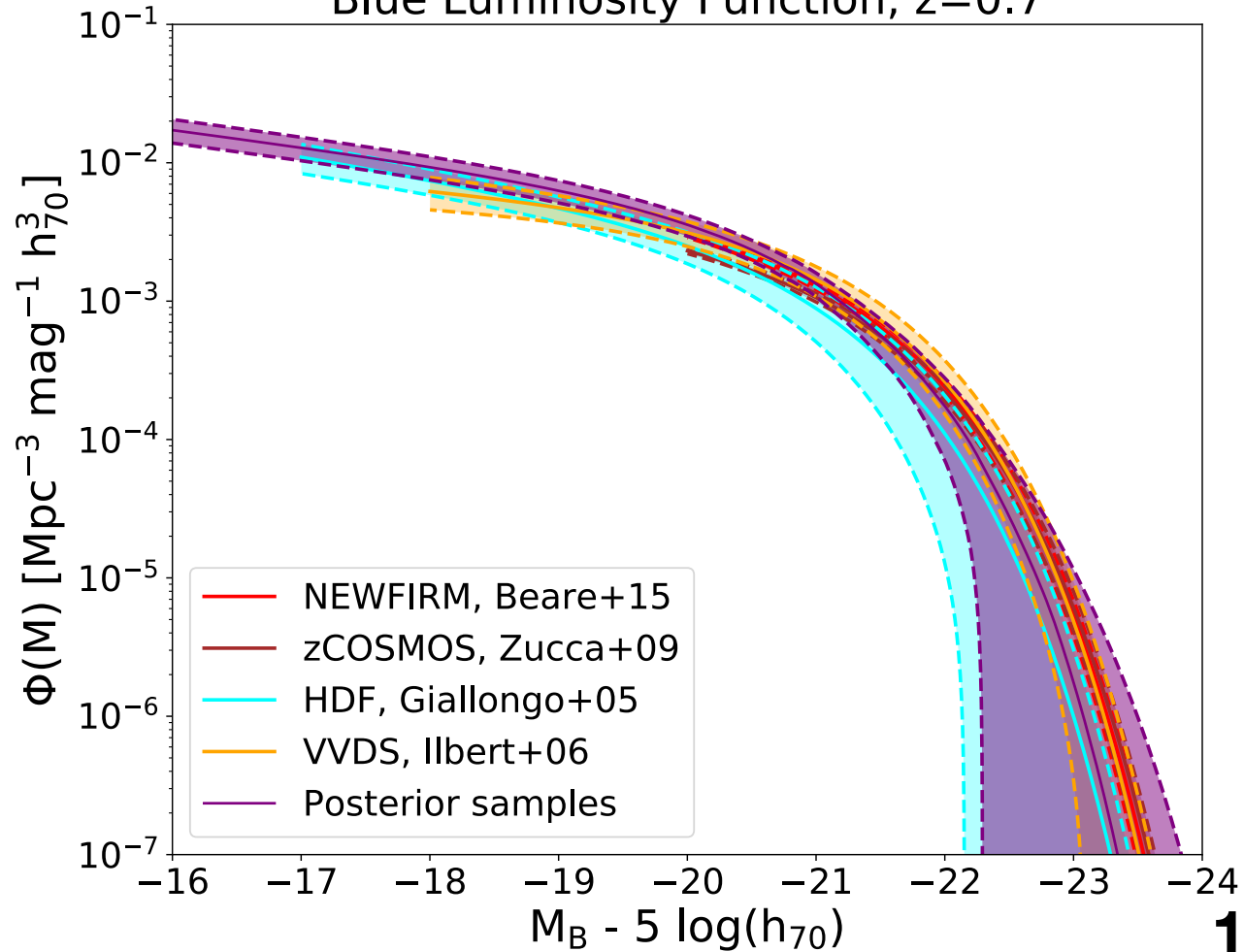
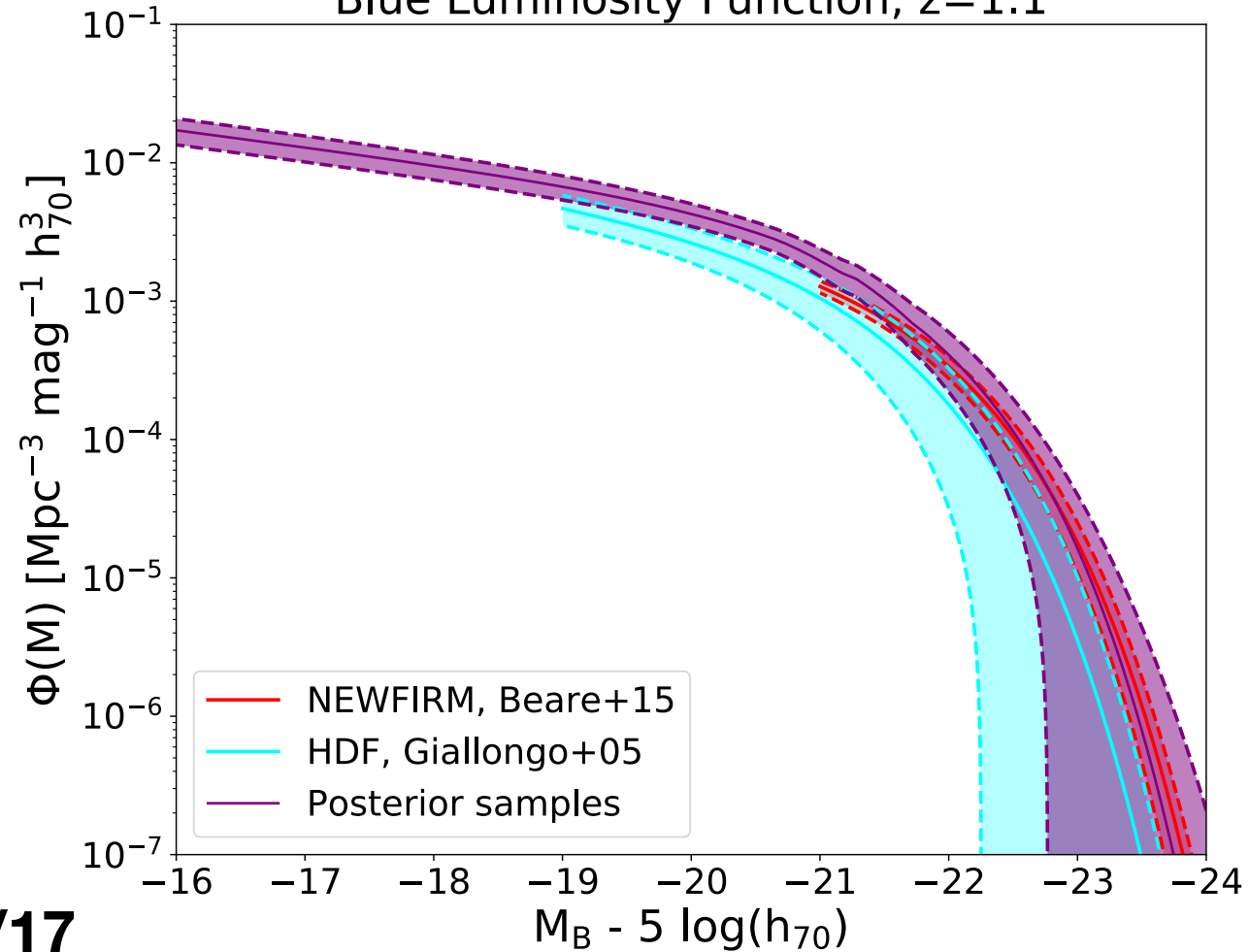


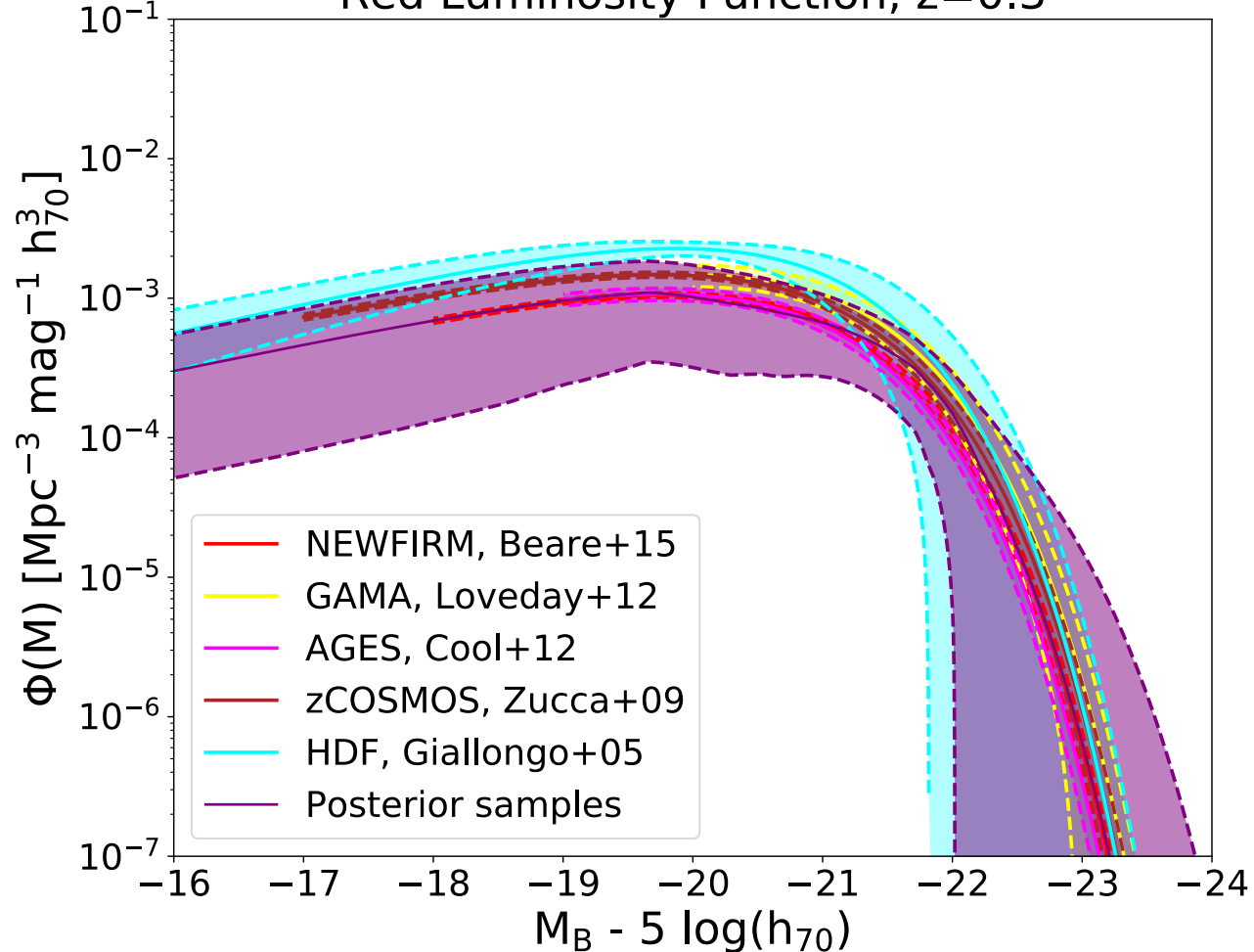
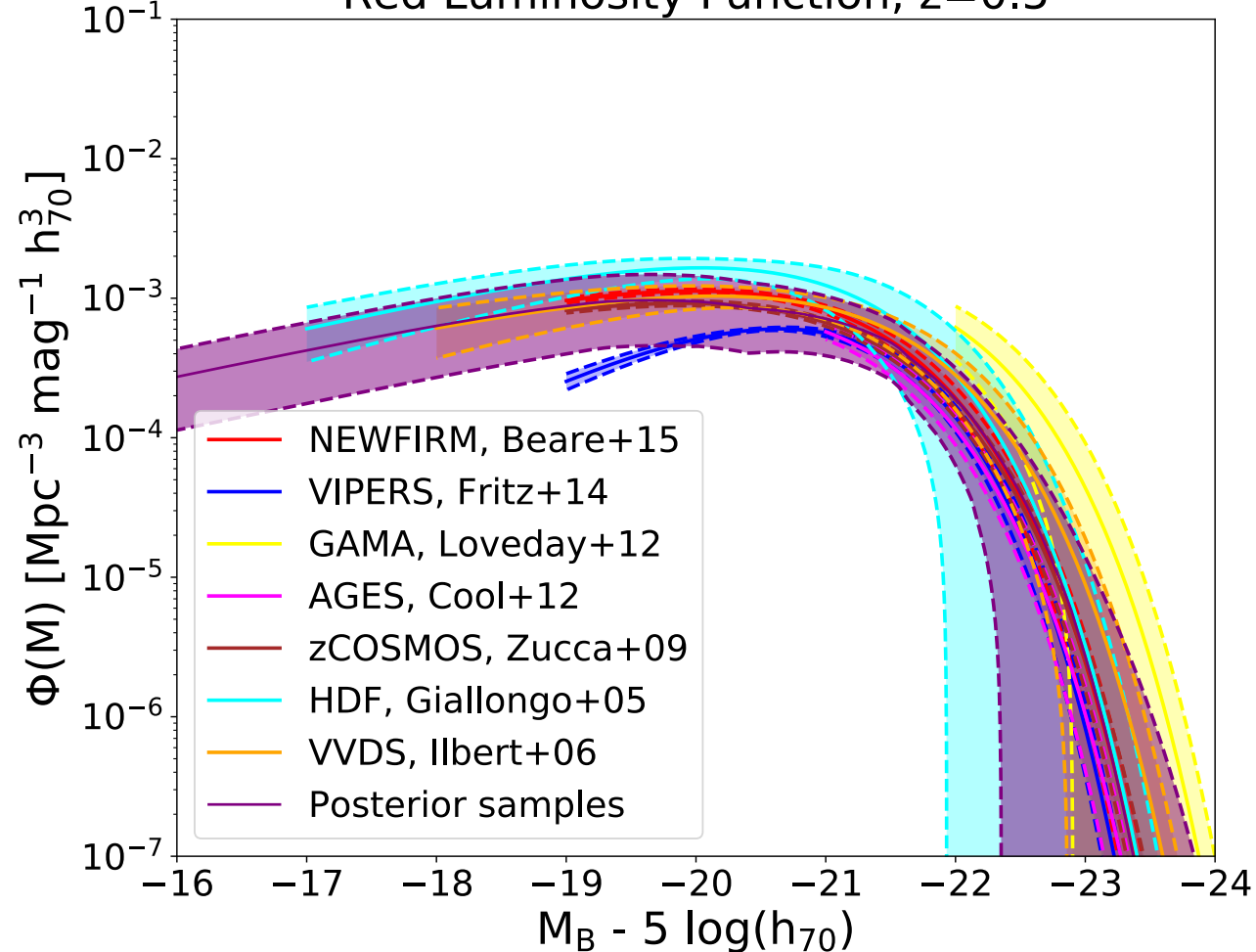
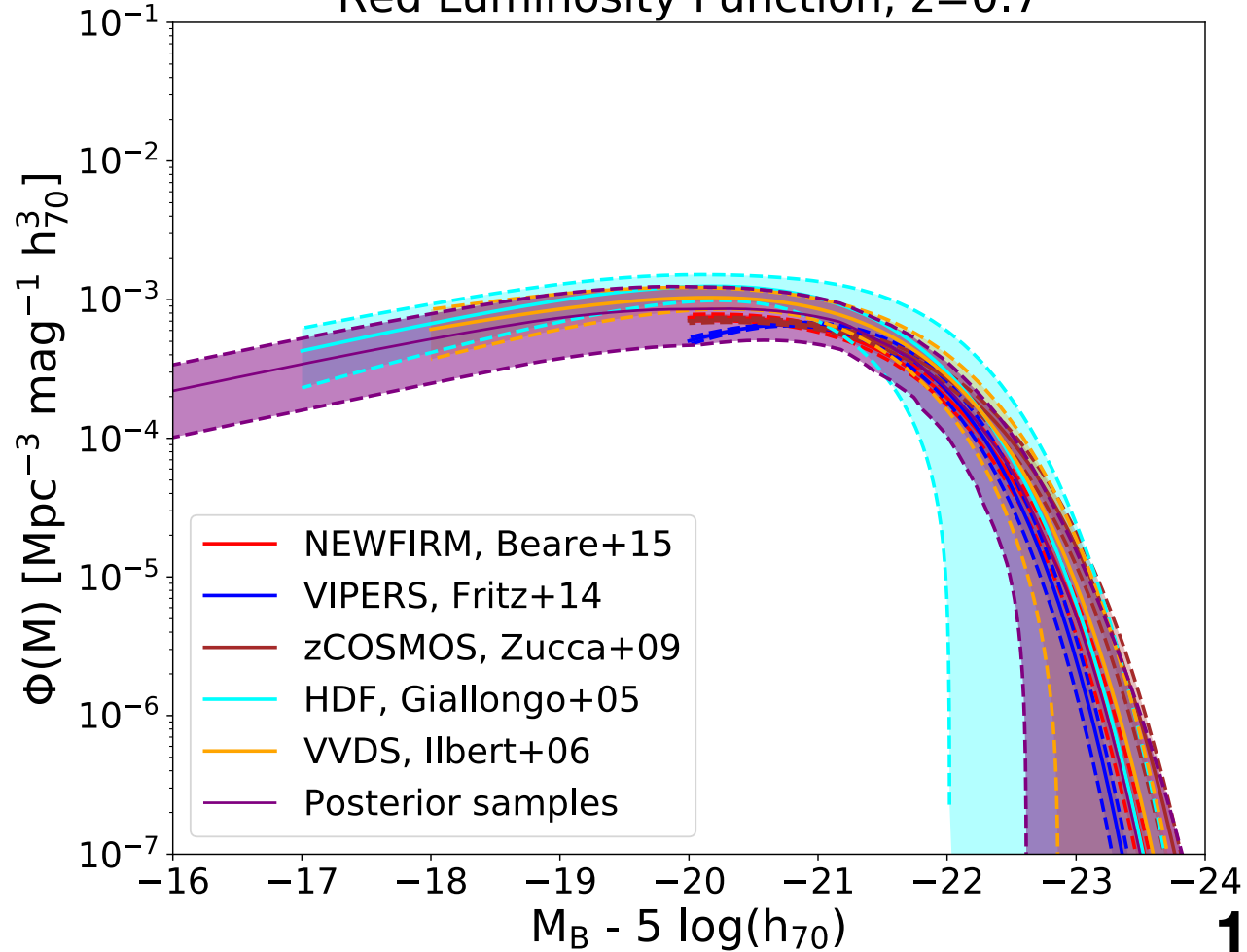
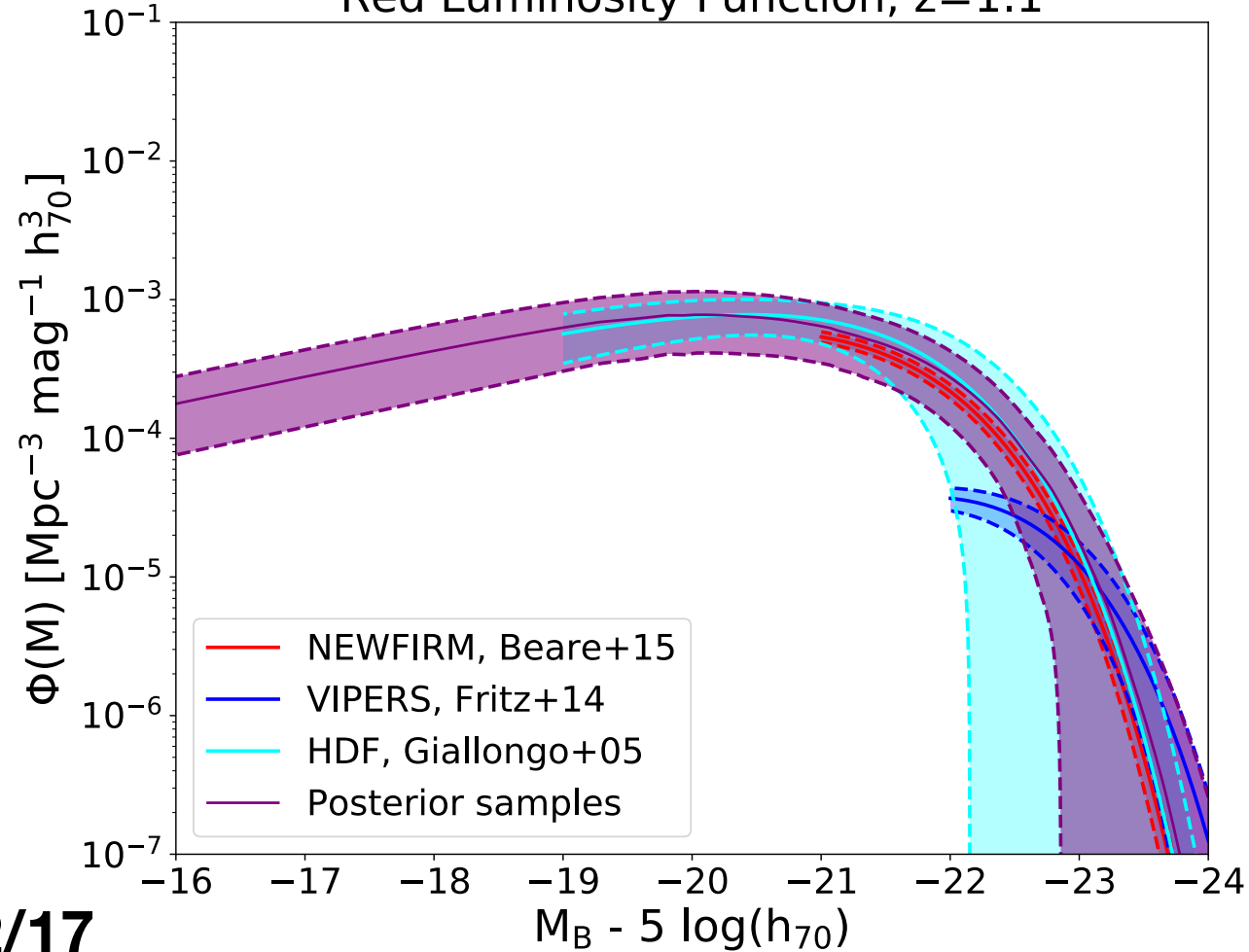
12\*10<sup>6</sup> galaxies  
from posterior distribution

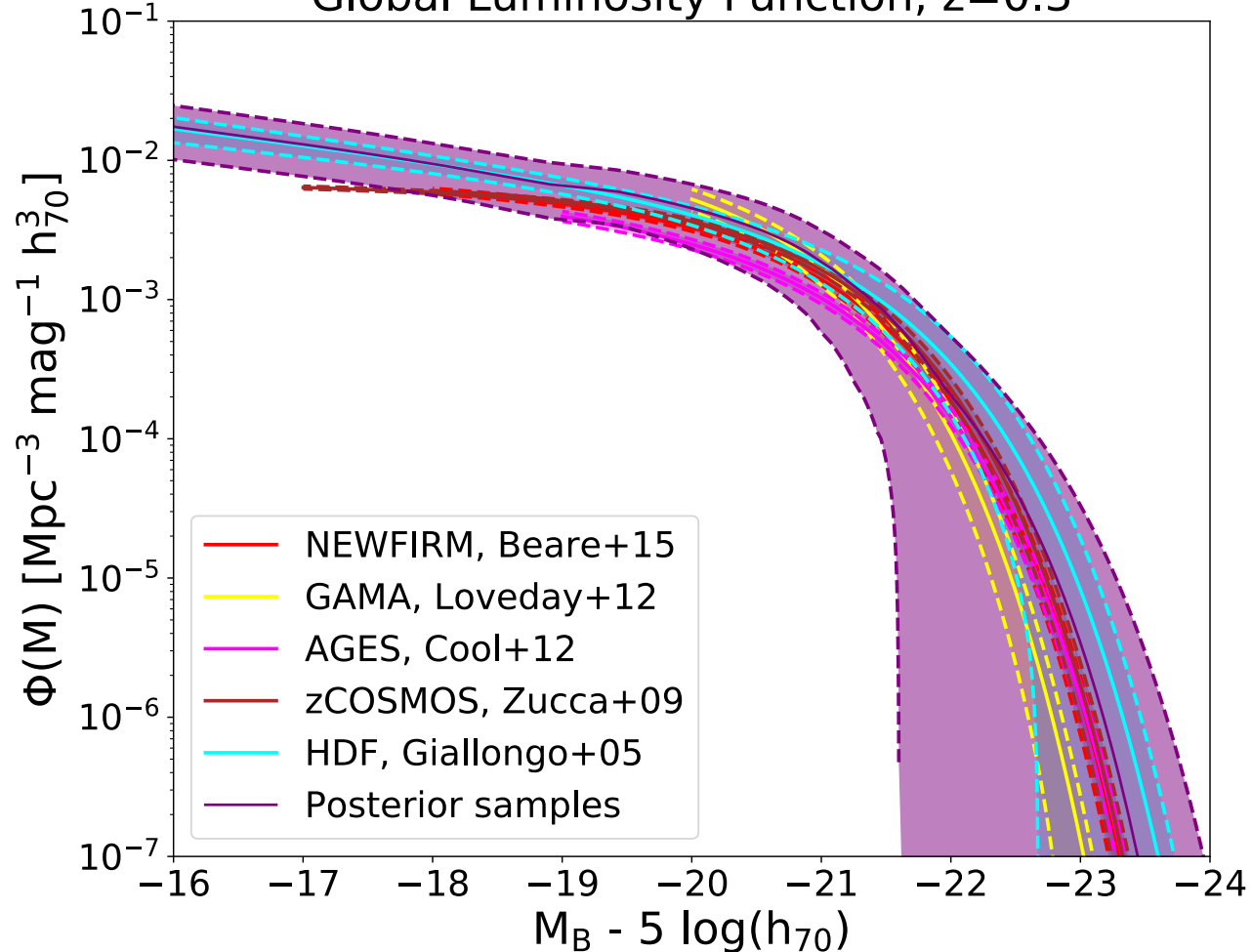
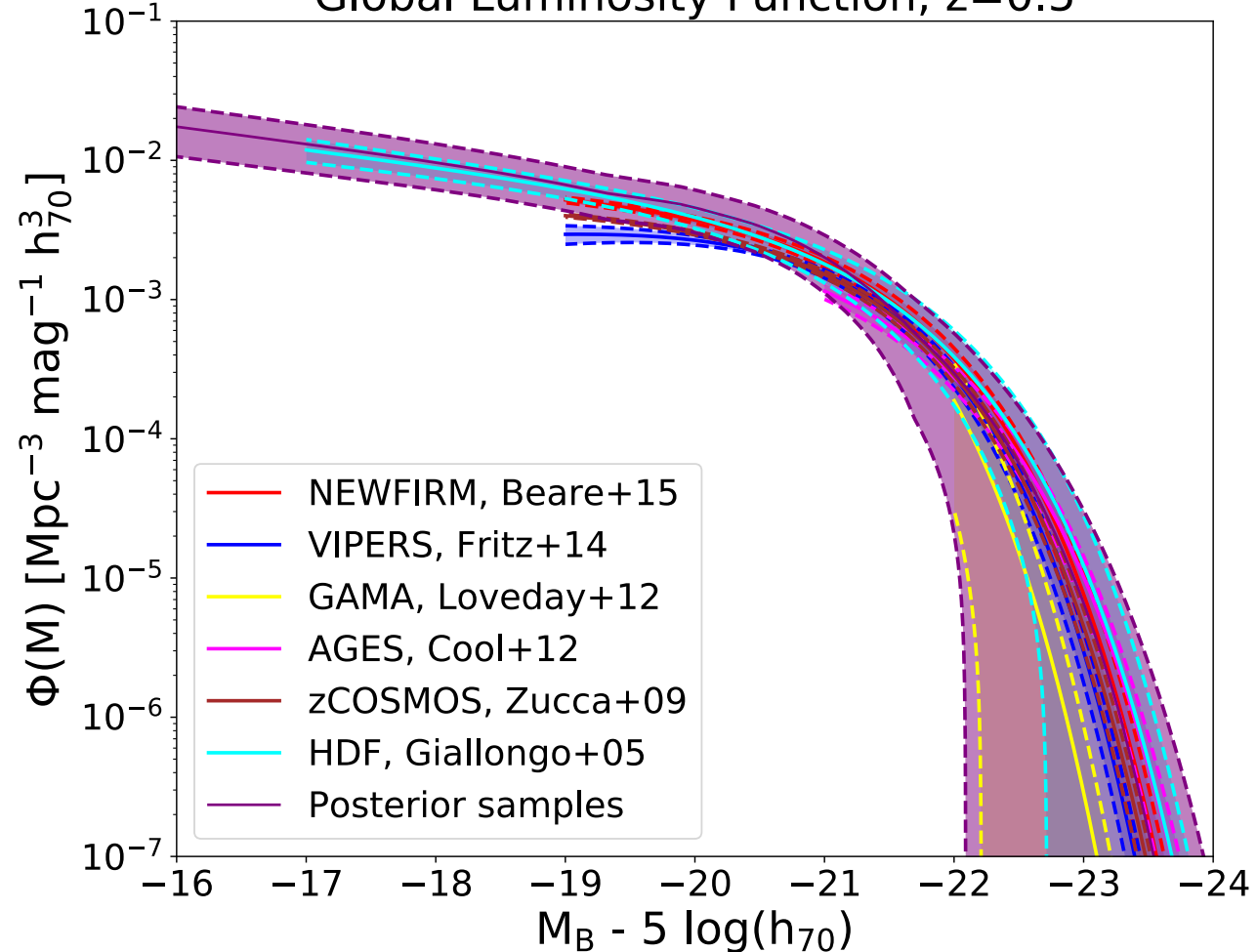
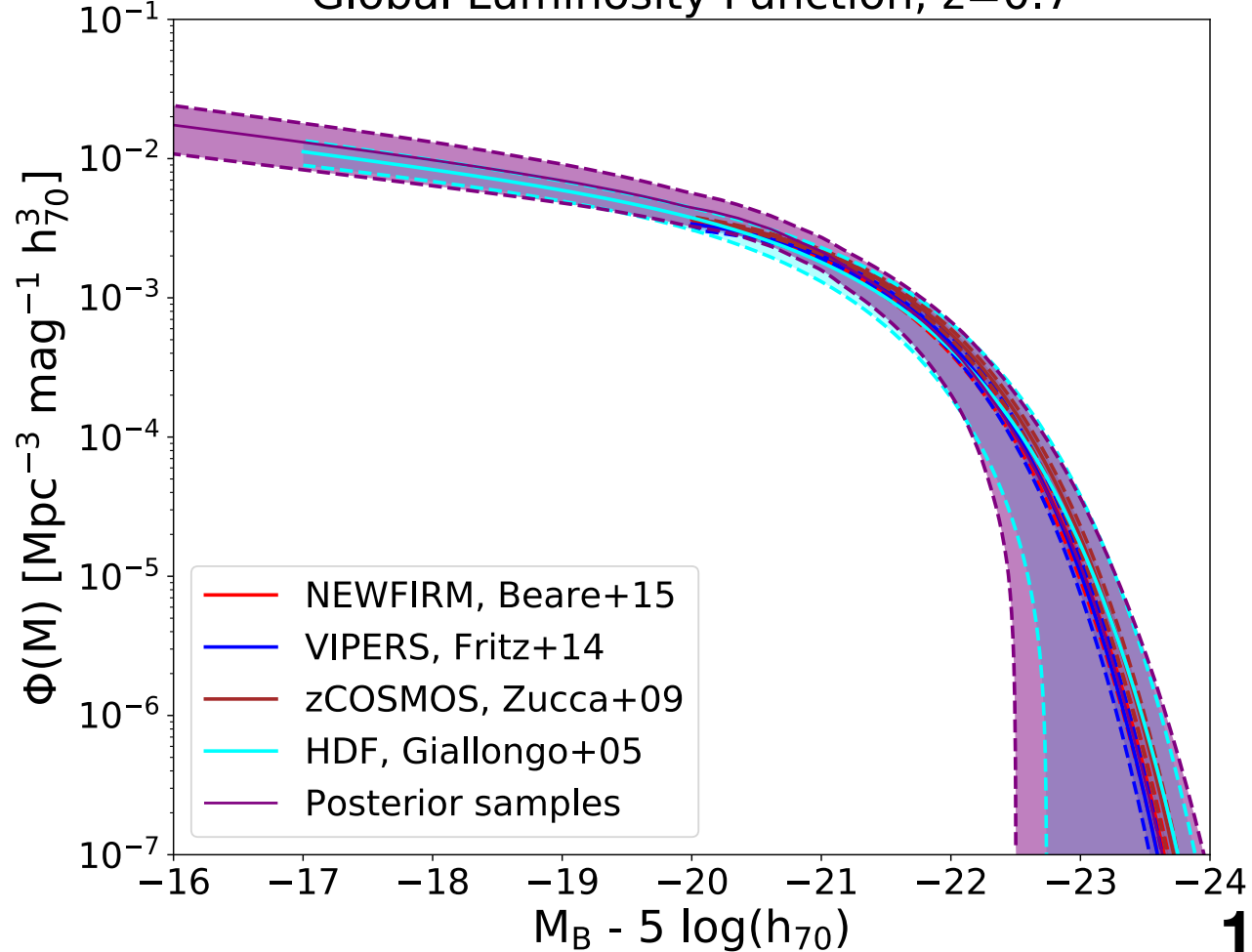
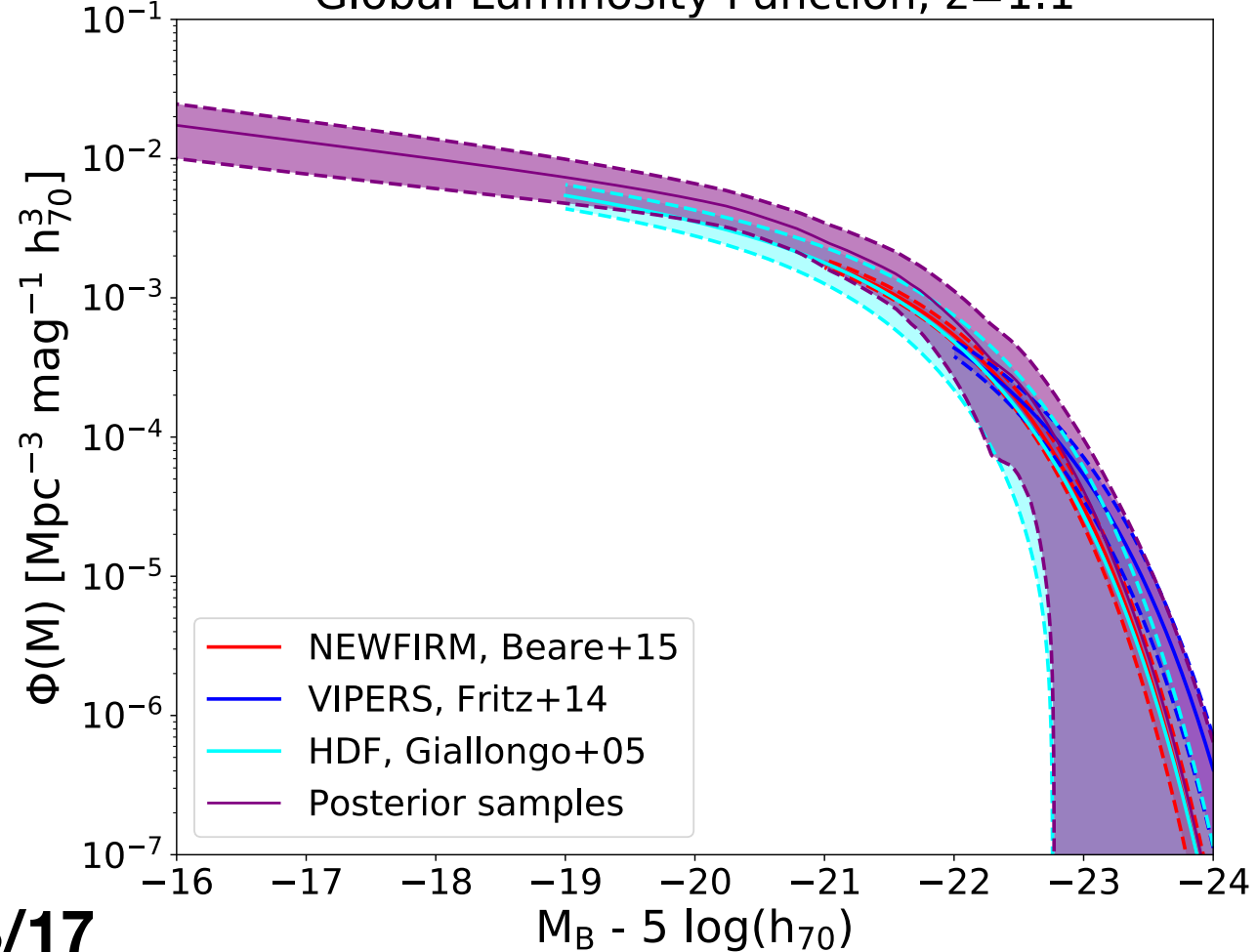


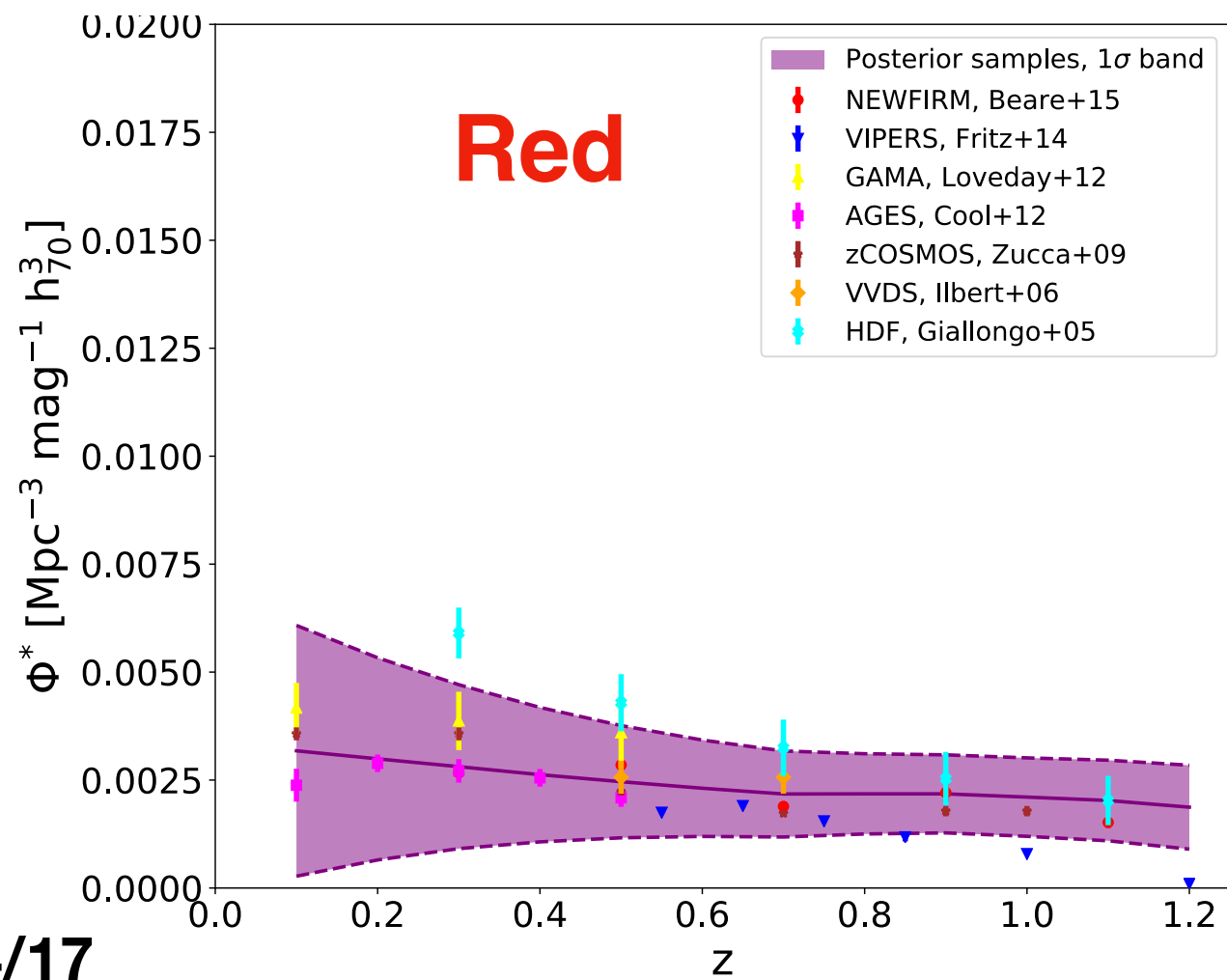
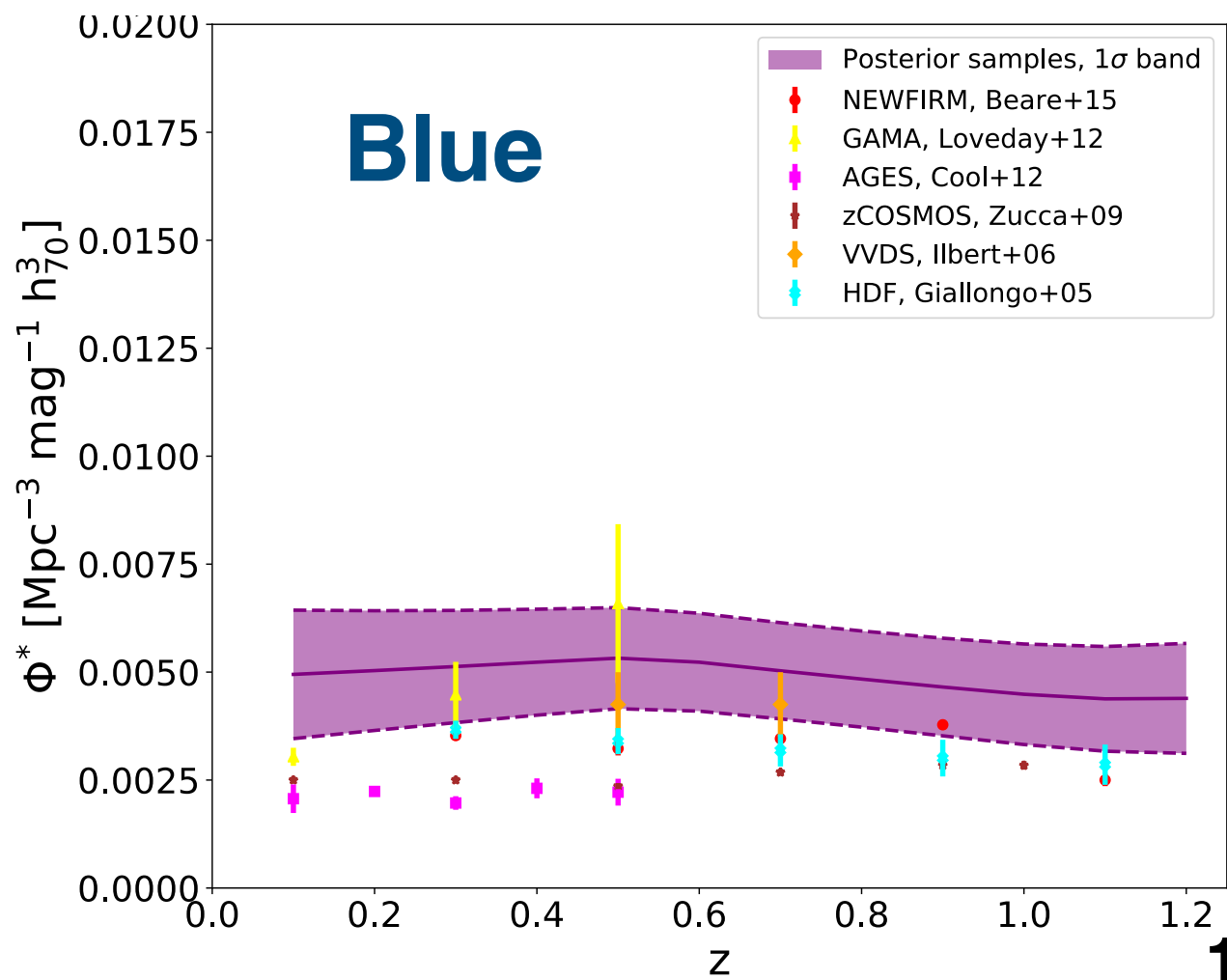
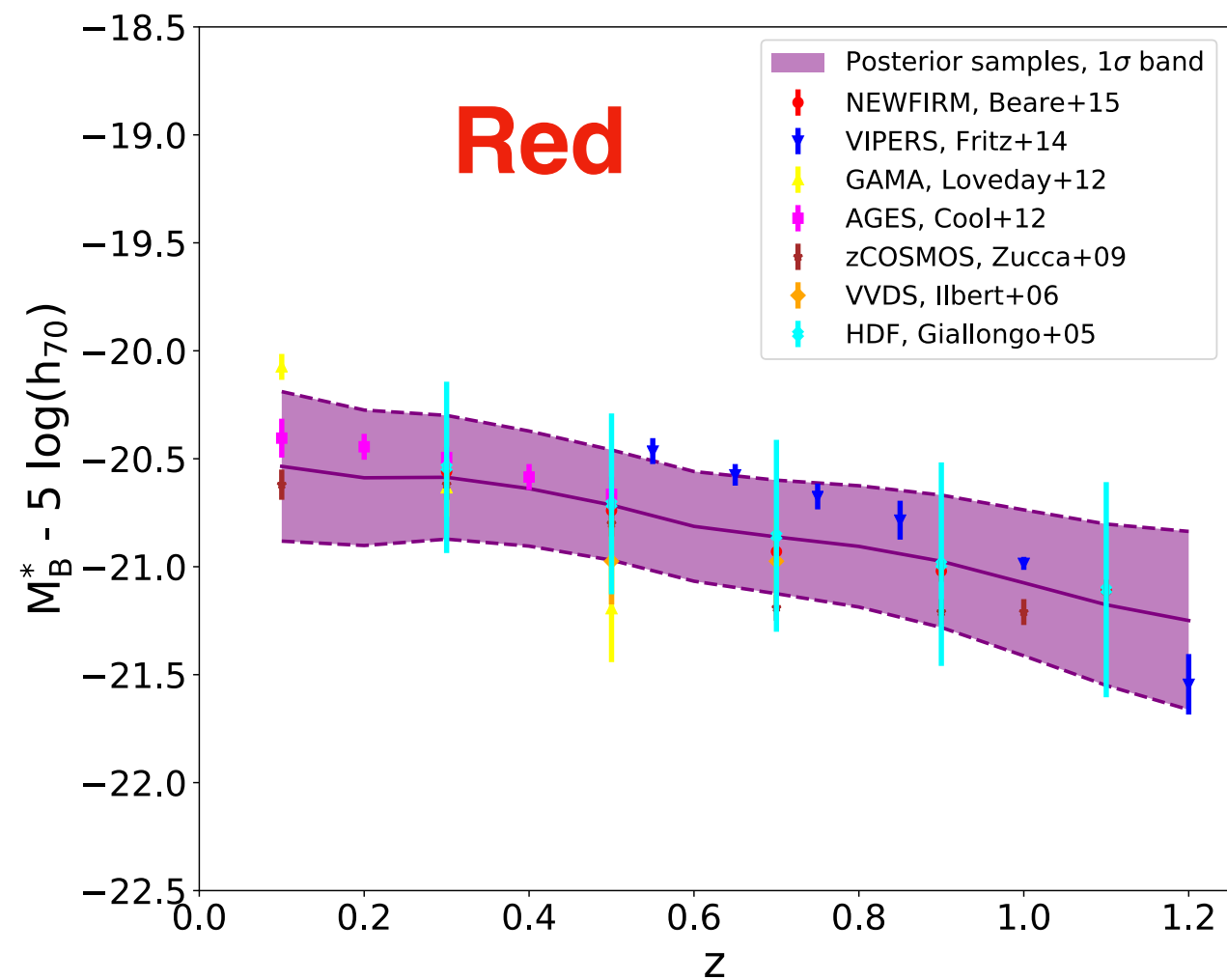
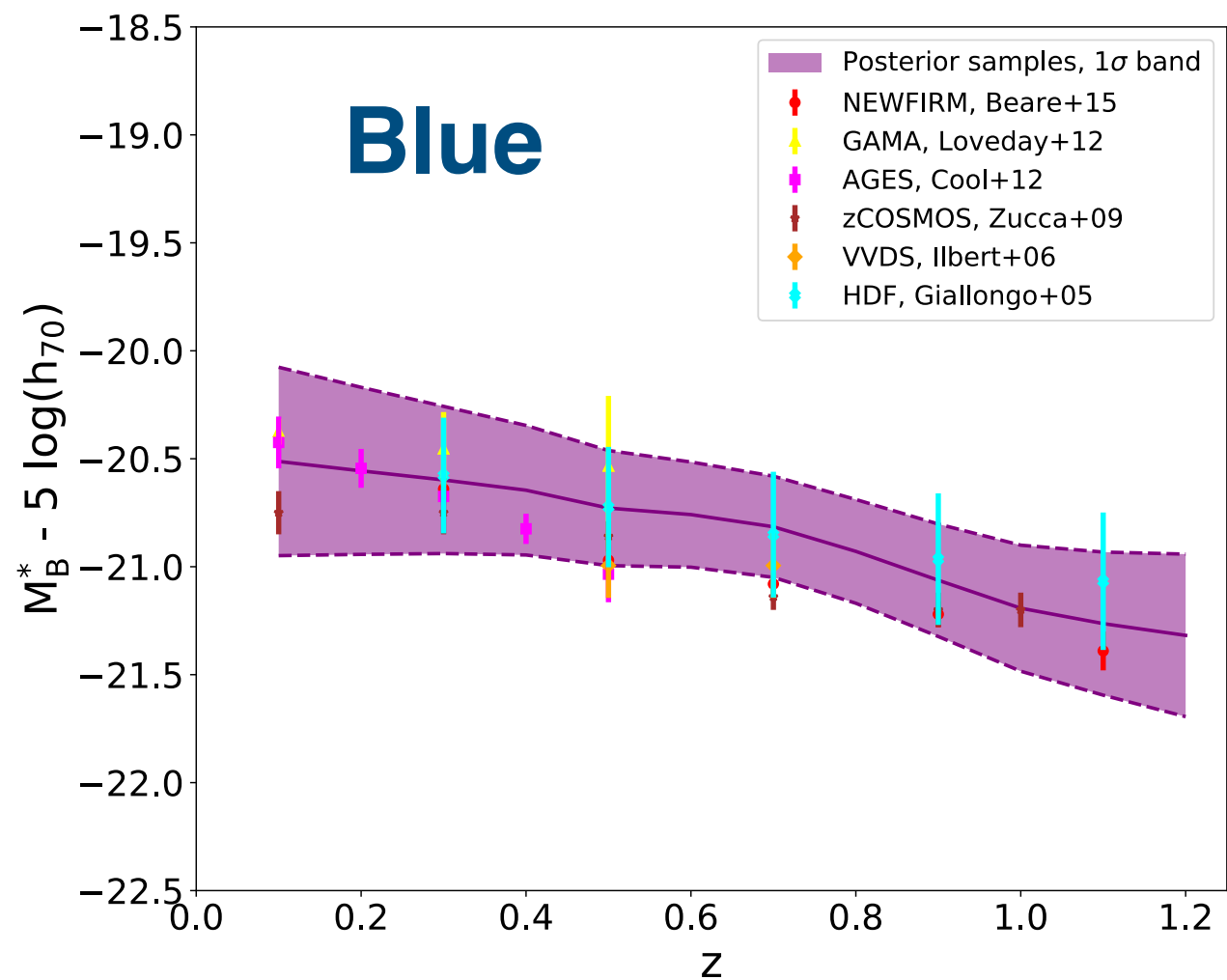
	Observations	Posterior simulations
MAG_u*	25.25 <sup>+0.92</sup> <sub>-1.10</sub>	25.21 <sup>+1.02</sup> <sub>-1.34</sub>
MAG_g'	24.85 <sup>+0.79</sup> <sub>-1.13</sub>	24.75 <sup>+0.95</sup> <sub>-1.34</sub>
MAG_i'	23.92 <sup>+0.83</sup> <sub>-1.41</sub>	23.66 <sup>+0.91</sup> <sub>-1.44</sub>
SIZE_u*	3.38 <sup>+1.39</sup> <sub>-0.92</sub>	3.40 <sup>+1.23</sup> <sub>-0.89</sub>
SIZE_g'	3.19 <sup>+1.10</sup> <sub>-0.68</sub>	3.23 <sup>+1.01</sup> <sub>-0.67</sub>
SIZE_i'	2.89 <sup>+1.09</sup> <sub>-0.71</sub>	2.94 <sup>+1.02</sup> <sub>-0.64</sub>
u* - g'	0.41 <sup>+0.52</sup> <sub>-0.34</sub>	0.42 <sup>+0.49</sup> <sub>-0.32</sub>
g' - i'	0.95 <sup>+0.59</sup> <sub>-0.51</sub>	1.12 <sup>+0.53</sup> <sub>-0.53</sub>



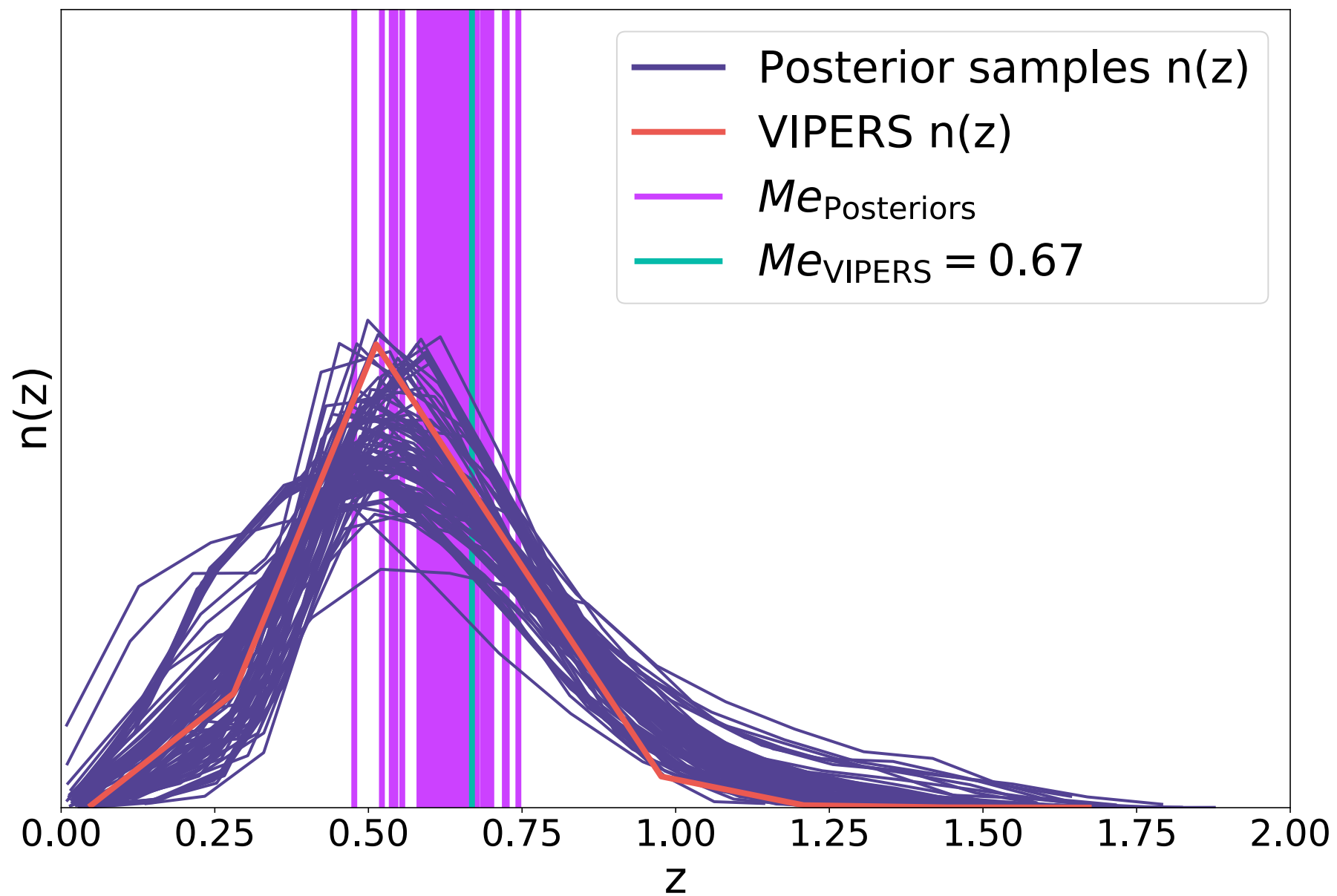
Blue Luminosity Function,  $z=0.3$ Blue Luminosity Function,  $z=0.5$ Blue Luminosity Function,  $z=0.7$ Blue Luminosity Function,  $z=1.1$ 

Red Luminosity Function,  $z=0.3$ Red Luminosity Function,  $z=0.5$ Red Luminosity Function,  $z=0.7$ Red Luminosity Function,  $z=1.1$ 

Global Luminosity Function,  $z=0.3$ Global Luminosity Function,  $z=0.5$ Global Luminosity Function,  $z=0.7$ Global Luminosity Function,  $z=1.1$ 

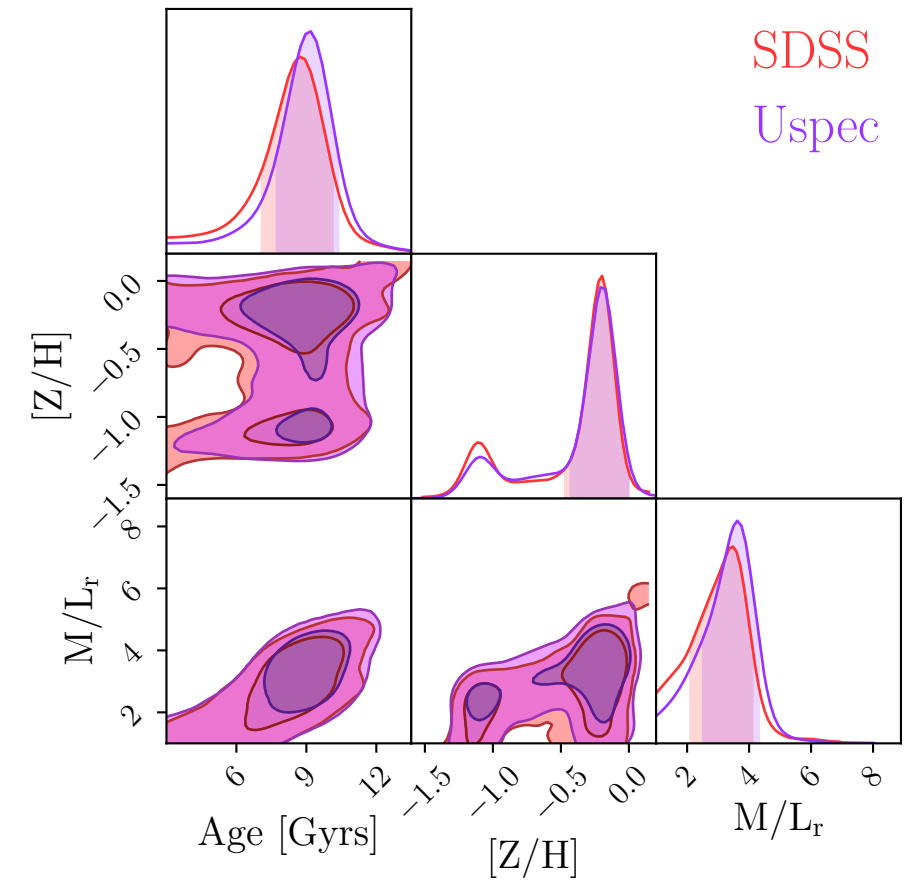
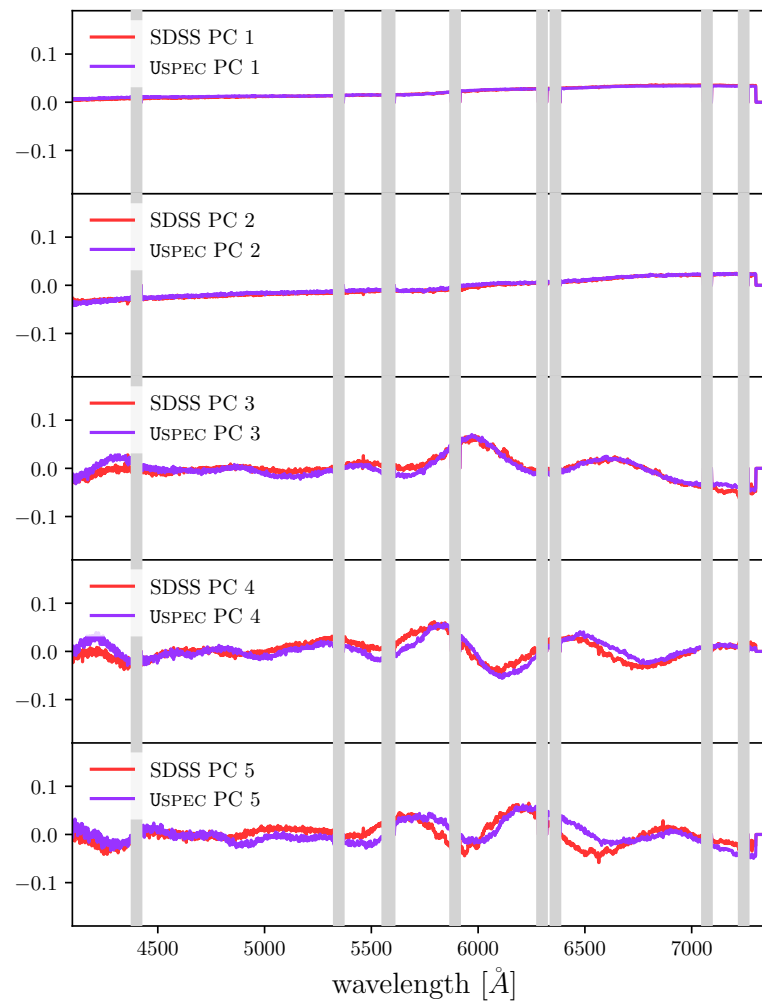
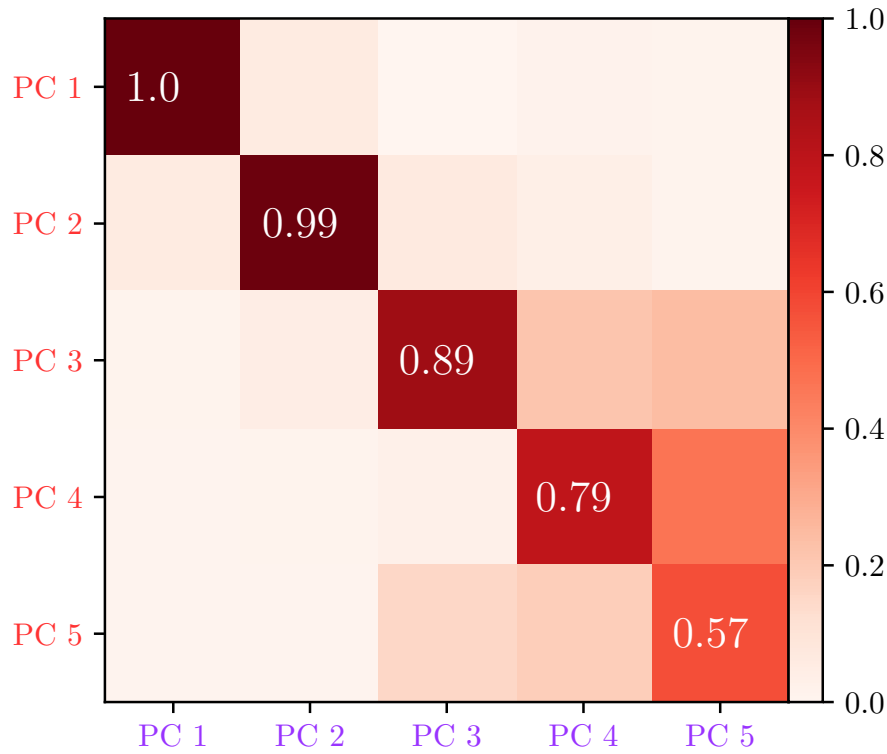


# Redshift distribution $n(z)$ for VIPERS



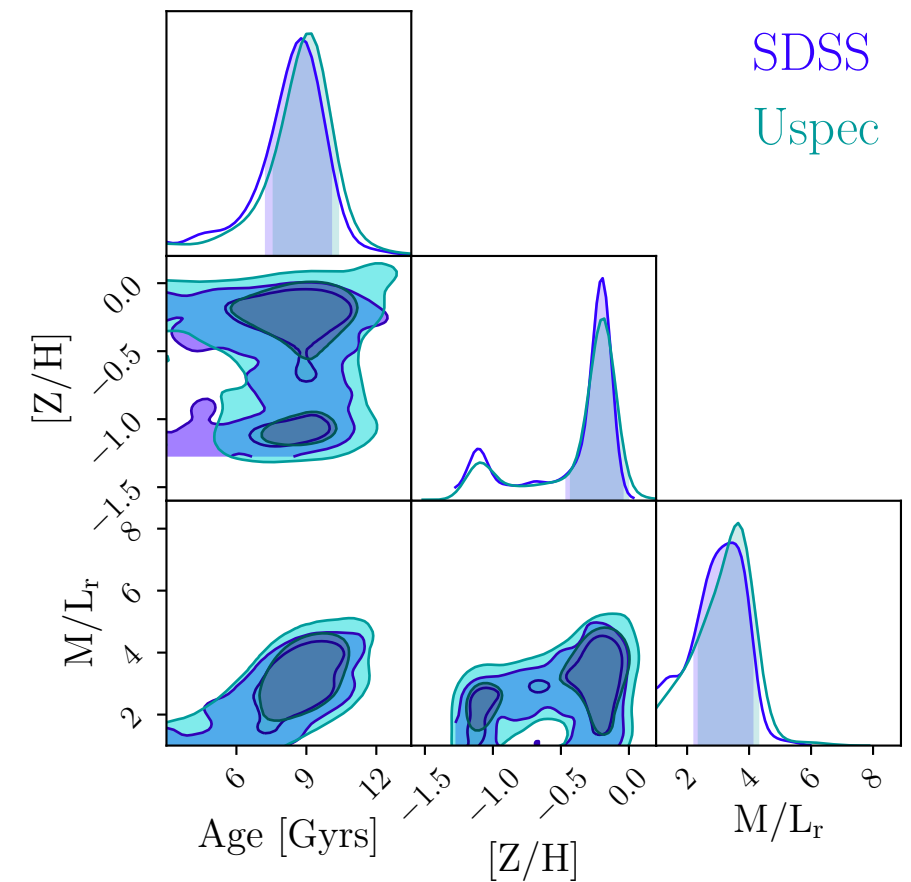
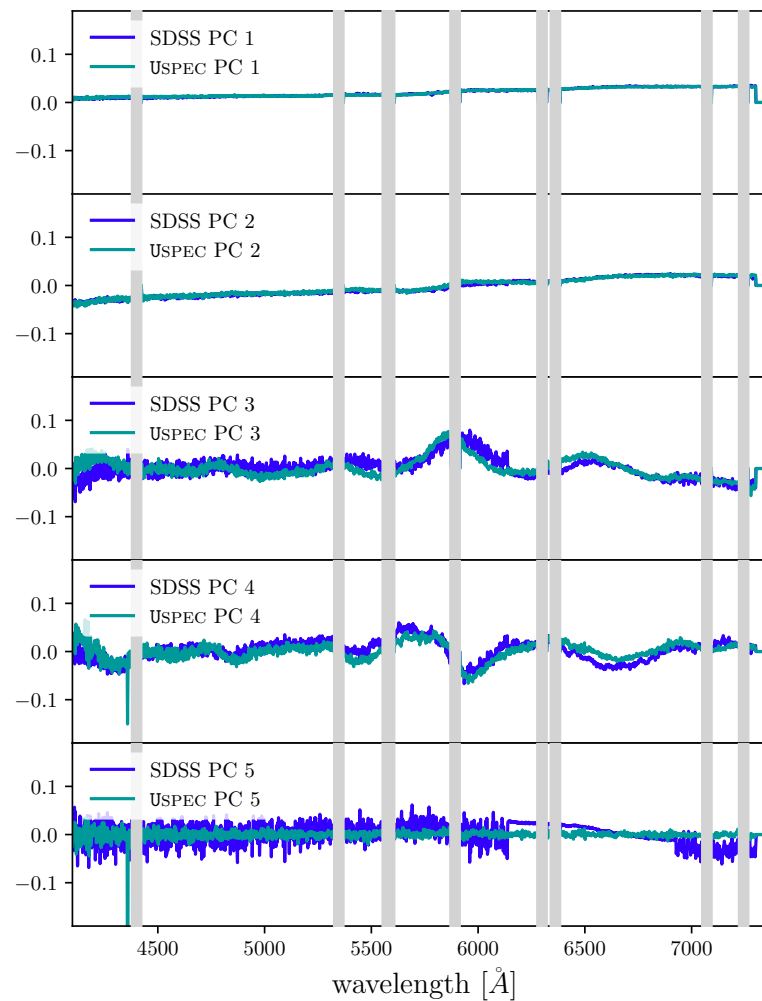
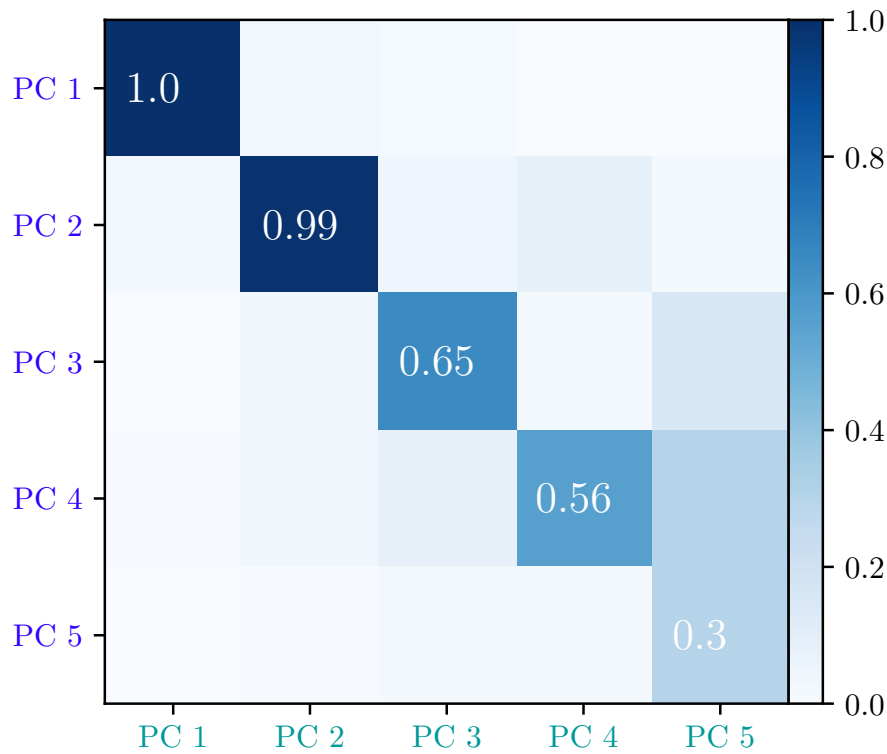


# SDSS CMASS



SDSS  
Uspec

# SDSS CMASS SPARSE



SDSS  
Uspec

Fagioli, Tortorelli et al. 2020,  
arXiv:2002.04039



# Conclusions

- For  $M_B - 5 \log h_{70} > -21$ , at all redshifts, the number density at fixed absolute magnitude of blue galaxies is greater than that of red galaxies.
- $M^*$  for blue galaxies fades more than that for red galaxies from  $z = 1$  to  $z = 0.1$ .
- $\Phi^*$  for blue galaxies stay roughly constant between  $z = 0.1$  and  $z = 1$ .
- $\Phi^*$  for red galaxies decreases by 35% in the same redshift range.
- The number density of blue galaxies at  $M^*$  is always higher than the red one.